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TESTS OF REINFORCED CONCRETE FLAT SLAB STRUCTURES

BY
ARTHUR N. TALBOT
And
WILLIS A. SLATER

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TESTS OF REINFORCED CONCRETE FLAT SLAB STRUCTURES

BY

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AND

WILLIS A. SLATER,
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TESTS OF REINFORCED CONCRETE FLAT SLAB STRUCTURES

I. Introduction.

1. *Purpose and Scope.*—It is the purpose of this bulletin to present the results of certain tests made on four reinforced concrete buildings and one reinforced concrete test structure. These tests were made with a view of getting experimental information on the action of the concrete and the reinforcing bars in floor slabs of the flat slab type of building construction. Data were obtained also on the bending action of the supporting columns. Efforts were made to find the distribution of stress in the bands of reinforcement both laterally and longitudinally, and that of the compressive stresses in the concrete on the opposite face of the slab; these in the regions of both the negative bending moment and of the positive bending moment.

The description of the methods used in making the test is limited to those features which are different from the methods used in the tests described in Bulletin No. 64 of the University of Illinois Engineering Experiment Station, "Tests of Reinforced Concrete Buildings Under Load", and from the methods described in a paper on "The Use of the Strain Gage in the Testing of Materials."*

It will be appreciated that the circumstances surrounding the floor test of a building are unfavorable to securing definite and uniform quantitative results. The structure is not homogeneous. There is a distribution of the resistance afforded by the structure to parts beyond the portion which is loaded. Effects of changes in temperature are troublesome. The stresses developed in the steel and in the concrete are small and there is considerable variation between parts which are supposedly similar in action. The conditions under which the measurement of deformation must be made are unfavorable to securing exactness. The location and presence of the loading material also add to the difficulties of the work.

It will be seen that it is impracticable to obtain complete information or to formulate conclusions which are entirely definite. Only general results and conclusions of a qualitative character may be expected. However, it is believed that the tests herein recorded bring out information of value on the action of reinforced concrete flat slabs and of the supporting columns. Since among engineers there is such a marked variation of opinion concerning the action of the flat slab and since there is such uncertainty in the analysis of the flat slab, it is believed that the information given will be regarded as adding to the general knowledge of this subject, and that it will be useful in considering many features of the design of buildings of the type tested.

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*Proceedings of the American Society for Testing Materials, vol. 13, p. 1019.

2. *General Statement of Tests.*—The tests were largely cooperative work. The interests of engineers and contractors in learning the properties of the flat slab and their willingness to bear expenses of the tests made the test work possible. In two cases the tests were made by engineering firms, Mr. Slater being connected with the work, and the data of the tests were placed at the disposal of the Engineering Experiment Station. Mr. Slater was an observer in all of the tests. In every case the data have been carefully worked over in the office of the Engineering Experiment Station and the results discussed and put into form. In the long work of studying the data, many inconsistencies and uncertainties were found, and the presentation of many matters on which it was hoped that the tests would give information had to be abandoned because the results of the tests were indefinite or inconclusive. Results from which at least qualitative conclusions may be drawn have been recorded in the bulletin.

The structures tested and the arrangements for the tests were as follows:

(a) Shredded Wheat Factory, Niagara Falls, N. Y. Flat slab floor with two-way reinforcement. Designed by Corrugated Bar Company, Buffalo, N. Y. Building built by Braas Bros., contractors, Niagara Falls, N. Y. Tested by Corrugated Bar Company.

(b) Soo Line Freight Terminal, Chicago, Ill. Flat slab floor with four-way reinforcement. Designed and built by the Leonard Construction Company, engineers and contractors, Chicago. Tested by cooperation between Leonard Construction Company, Central Terminal Railway Company, and the Engineering Experiment Station of the University of Illinois.

(c) Schulze Baking Company Building, Chicago, Ill. Flat slab floor with four-way reinforcement. Designed by Lieberman and Klein, engineers, Chicago. Built by McLennan Construction Company, contractors, Chicago. Tested by Mr. Slater for American System of Reinforcing. The contractors placed and removed the loading material.

(d) Worcester Slab Test, Worcester, Mass. A sixteen-panel slab having four different designs of reinforcement. Constructed especially for the test. Built according to plans prepared by B. S. Brown, consulting engineer, Boston. Tested by cooperation between Mr. Brown, Worcester Polytechnic Institute, and the Engineering Experiment Station of the University of Illinois.

(e) Curtis-Leger Company Building, Chicago, Ill. Flat slab floor having four-way reinforcement at interior of panel, and two-way reinforcement in region of columns.

Designed by Barton Spider Web System, Chicago, and built by the Simpson Construction Company, contractors, Chicago. Tested by the Engineering Experiment Station of the University of Illinois with the assistance of the engineers and contractors.

Table 1 gives general data concerning the tests.

3. *Acknowledgment.*—The test of the Shredded Wheat Factory Building was conducted by Mr. F. J. Trelease, Chief of the Research Department of the Corrugated Bar Company, and Mr. Slater. The Corrugated Bar Company bore the expense of the test. Acknowledgment is made to this company for the courtesy in permitting the use of the test data.

The technical part of making the test of the Soo Terminal Structure was done as the work of the Engineering Experiment Station. The Leonard Construction Company bore the expense of the preparation for the test and of matters connected with the application of the load. Mr. A. R. Lord, consulting engineer, represented the Leonard Construction Company in planning and carrying out the test. The Central Terminal Railway Company placed the track for the test, and applied and removed the test load. Mr. Slater was in immediate charge of the preparations for the test and of its conduct. The following observers and recorders from the University assisted in the test: Messrs. H. F. Moore, D. A. Abrams, N. E. Ensign, H. F. Gonnerman,H. R. Thomas, G. A. Maney, and M. Abe. Messrs. Meyer, O. R. Erickson, C. J. Nelson, and O. R. Kellner of the force of the Leonard Construction Company also assisted in the test.

The floor test of the Schulze Baking Company Building was conducted by Mr. Slater for the American System of Reinforcing, Chicago. This company, and the contractors, McLennan Construction Company, bore the expense of the test. Acknowledgment is made to the American System of Reinforcing for permission to use the data for publication.

The Worcester slab test structure was conceived and planned by Mr. B. S. Brown, consulting engineer, of Boston, and the expense of carrying out the test was borne principally by Mr. Brown. The materials, labor, and supervision of the construction were furnished by the Allentown Portland Cement Company, Boston, Carnegie Steel Company, Boston, Varnum P. Curtis Gravel Company, Worcester, Aberthaw Construction Company, Boston, and Mr. Brown. The plans for the test were laid out by Mr. Brown, Professor French, of Worcester Polytechnic Institute, and the writers. The conduct of the test was directed by Mr. Slater. Professor H. F. Moore of the Engineering Experiment Station cooperated in the work.

The expense of the Curtis-Leger Building test was borne by F. M. Barton, engineer and architect, Chicago, and the Simpson Construction Company, contractors, Chicago. Mr. Slater conducted the test.

II. The Shredded Wheat Factory Building Test.

4. *The Building.*—The building of the Shredded Wheat Company is of reinforced concrete construction of the flat slab type, three stories in height. The main portion of the building is 265 ft. 4 in. long and 81 ft. wide. The panels are 20 ft. by 22 ft. The floor is the type of flat slab designated by the trade name, Corr-Plate Floor. The floor on which the test load was applied is the first floor above the basement. This floor is nominally 7 in. thick in the central portion of the panel and 9 in. thick throughout an area 8 ft. 6 in. square (the depressed head) surrounding each column. It was designed for 125 lb. per sq. ft. live load. The interior columns have a pyramidal capital of octagonal form 42 in. in diameter at the top and sloping 45° with the horizontal. The slab has two-way reinforcement, designed to resist negative moment at all points across the edges of the panels and positive moment across the center lines of the panels. Corrugated bars were used for reinforcement. Fig. 1 and 2 show the distribution of the reinforcing bars in the test floor of this building. Fig. 2 contains information on the bending and supporting of bars and on other details of the slab. Readings with an engineer's level at numerous positions on the floor gave an average floor thickness of 9.13 in. at the columns and 7.29 in. midway between columns. The average measured depths from the compression surface of the concrete to the center of gravity of the reinforcement of the central panel of the test area were 6.82 in. and 4.95 in. for positions of negative moment in the depressed head and in the thinner portions of the slab

TABLE 1.

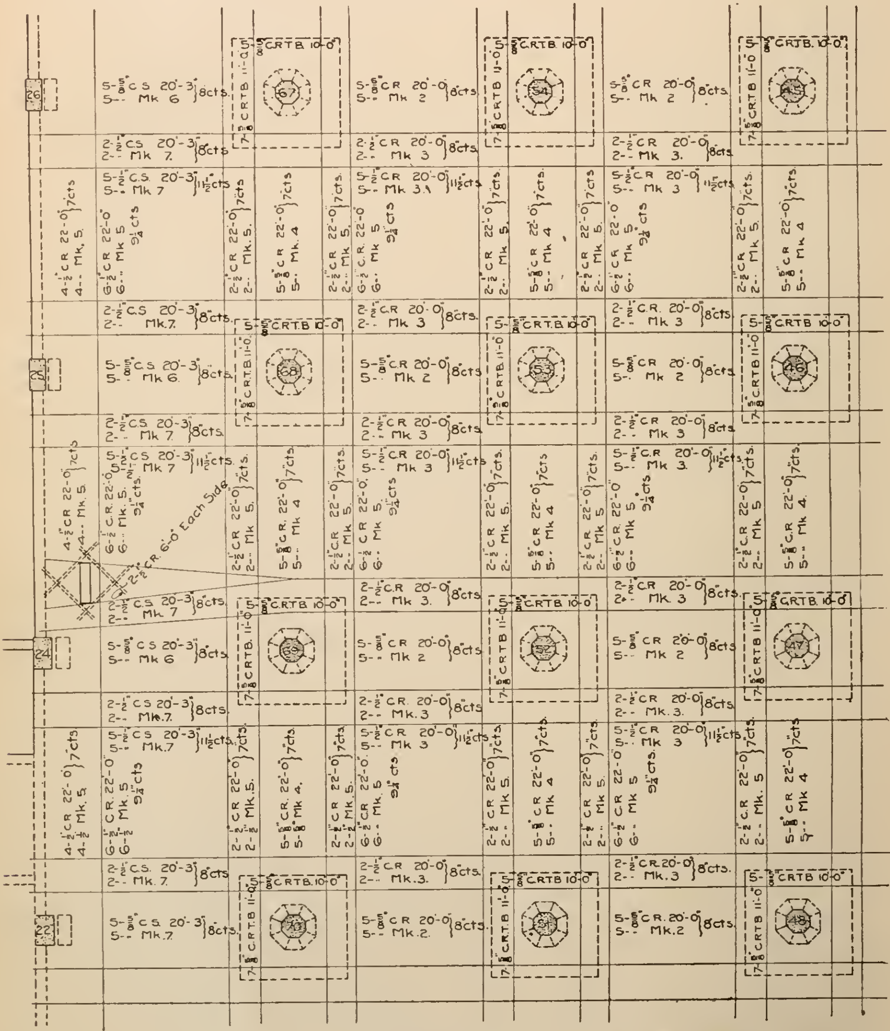
GENERAL DATA OF TESTS.

Structure	Type	Test Area		Loading Material	Design Load lb. per sq. ft.	Test Load lb. per sq. ft.	Load Handled tons	No. of Gage Lines	No. of Observers	Days required for	
		In square feet	Per cent of Total Area							Preparation	Test
Shredded Wheat Factory	Two-way reinforcement; depressed head	3960	18.4	Gravel	125	(c) 243	(g) 390	288	2		7
Soo Line Terminal	Four-way reinforcement; depressed head	2304	0.38	Ore cars loaded with broken stone	(b)	700	1006	816	6	13	8
Schulze Baking Company Building	Four-way reinforcement; depressed head	1400	2.95	Brick stacked in piers	300	(d) 722	437	263	1	7	6
Worcester Slab	Four-way reinforcement; no depressed head	3280	100.0	Gravel		(e) 328	459	320	2	11	6
Curtis-Leger Company Building	Four-way reinforcement; no depressed head	1355	11.1	Bags of Cement	200	(f) 500	(h) 237	142	1	7	8

(a) 4-way in central portions of panel, 2-way at column.
(b) Designed for E-50 Wheel-loading.
(c) 191 lb. per sq. ft. over entire 9-panel area.
243 lb. per sq. ft. over only one panel.
(d) 722 lb. per sq. ft. over south two panels, and part of north two panels; 435 lb. per sq. ft. over remainder of 4-panel test area.
(e) 328 lb. per sq. ft. over about 11 panels.
215 lb. per sq. ft. over about 5 panels.
(f) 500 lb. per sq. ft. over 678 sq. ft.
200 lb. per sq. ft. over 1355 sq. ft.
(g) Includes the rehandling of 3 per cent of the material.
(h) Includes the rehandling of 28 per cent of the material.

respectively, and 6.16 in. for positions of positive moment. The columns are octagonal, 25 in. in short diameter for the basement columns, and 22 in. in short diameter for the first story.

The concrete in the building was of excellent quality. Gravel was used as aggregate. At the time of making the test the concrete in the floor was about 80 days old. Four test cylinders, made at the time of pouring the test floor, gave an average strength of about 3500 lb. per sq. in. at an age of 115 days in tests made at the University of Illinois, none falling below 3200 lb. per sq. in.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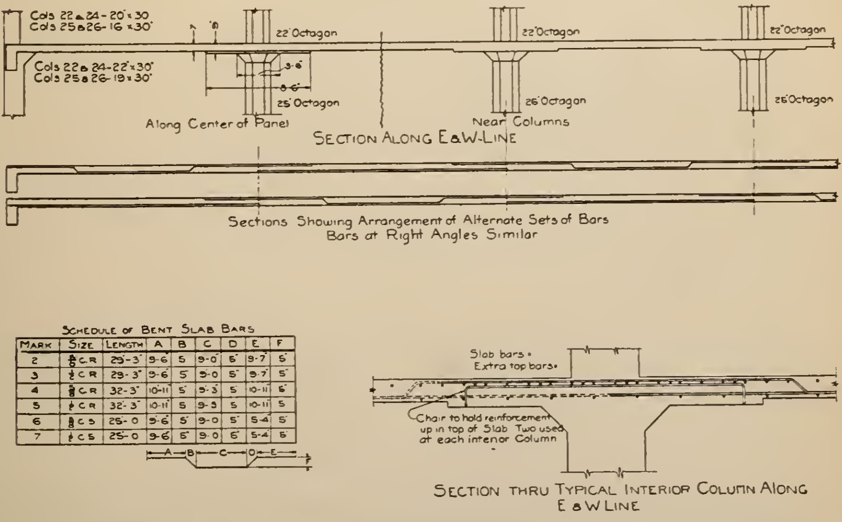
5-⅝ C.R.T.B. 10-0
5-⅝ C.R.T.B. 11-0
5-⅝ C.S. 20-3 8 cts. 5-- Mk 6
5-⅝ C.R. 20-0 8 cts. 5-- Mk 2
2-½ C.S. 20-3 8 cts. 2-- Mk 7
2-½ C.R. 20-0 8 cts. 2-- Mk 3
5-½ C.S. 20-3 11½ cts. 5-- Mk 7
5-½ C.R. 20-0 11½ cts. 5-- Mk 3
4-½ C.R. 22-0 7 cts. 4-- Mk 5
6-½ C.R. 22-0 9¼ cts. 6-- Mk 5
2-½ C.R. 22-0 7 cts. 2-- Mk 5
5-⅝ C.R. 22-0 7 cts. 5-- Mk 4
2-½ C.R. 6-0 Each Side

FIG. 1. DISTRIBUTION OF REINFORCEMENT FOR TEST FLOOR OF SHREDDED WHEAT FACTORY.

5. *Testing.*—The deformation measurements were taken on 137 gage lines on the reinforcement and 151 gage lines on the concrete. Fig. 3 and 4 show the location of the gage lines, and Fig. 5 shows the key to the grouping of the gage lines for purposes of comparison. Deflection readings were taken at 33 points (see Fig. 6).

The loaded area is shown in Fig. 6. Gravel was used as the loading material. It was raised by means of a concrete hoist to the second floor and there deposited into the hopper of a concrete chute. By moving the chute the gravel was distributed as desired. Fig. 7 shows the load in position. The load covered the entire panel areas except for aisles 2 ft. wide extending from column to column and boxes about 3½ ft. square placed at the center of each panel. These areas were left uncovered to afford access to the gage lines. Each increment of load was leveled carefully. At the load of 191 lb. per sq. ft., in order to ascertain the total weight per cubic foot of gravel as compacted, a bottomless box was sunk through the gravel after the fashion of an open caisson. The gravel was shoveled out from the inside of the box, measured and weighed. The weight per cubic foot of loose gravel was found to be 113 lb. per cubic foot, and of the compacted gravel, 134½ lb. per cubic foot. As only 79 per cent of the floor area was covered, the corresponding average load per square foot for the total test area was 106½ lb. per ft. of height of compacted gravel. The exact degree of compactness of the gravel at all times was not known, but it is thought that the values used are representative of the test load.



SCHEDULE OF BENT SLAB BARS

MARK	SIZE	LENGTH	A	B	C	D	E	F
2	5/8 C R	29'-3"	9-6	5	9-0"	6"	9-7"	5"
3	1/2 C R	29-3"	9-6"	5"	9-0	5"	9-7"	5"
4	5/8 C R	32-3"	10-11"	5"	9-3"	5	10-11"	6"
5	1/2 C R	32'-3"	10-11"	5	9-3	5	10-11	5
6	5/8 C S	25'-0	9'-6"	5"	9-0	5"	5-4	5"
7	1/2 C S	25'-0	9-6	6"	9-0	6"	5-4"	6"

FIG. 2. BENDING OF BARS FOR TEST FLOOR OF SHREDDED WHEAT FACTORY.

Nine panels were first loaded (see Fig. 6). Strain gage observations were made at loads of 56, 120, and 191 lb. per sq. ft. The 191-lb. test load was allowed to remain in position two and one-half days. The load was then removed from the three wall panels and from the three panels farthest from the wall, leaving three panels loaded as shown in the figure. Strain gage observations were taken with this load in place. Next, the two outer panels of the three were unloaded and the load on the center panel was increased to 243 lb. per sq. ft.

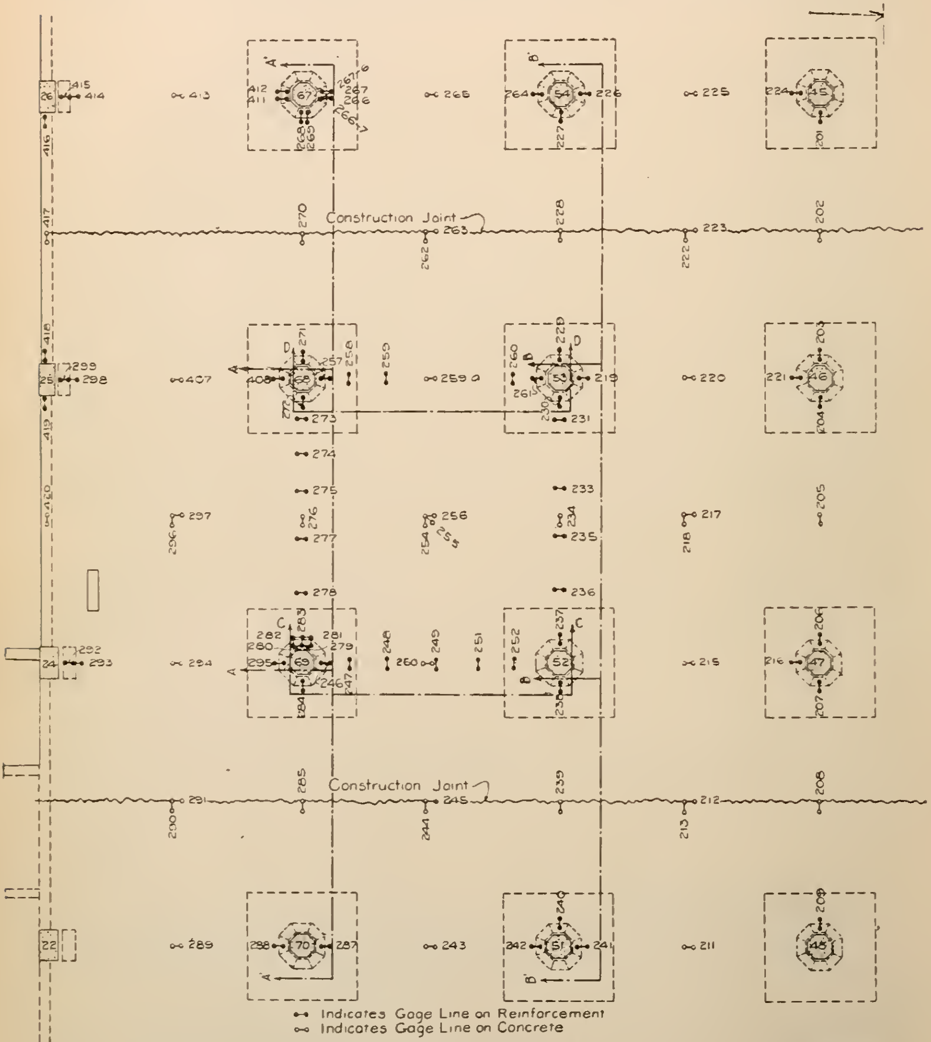
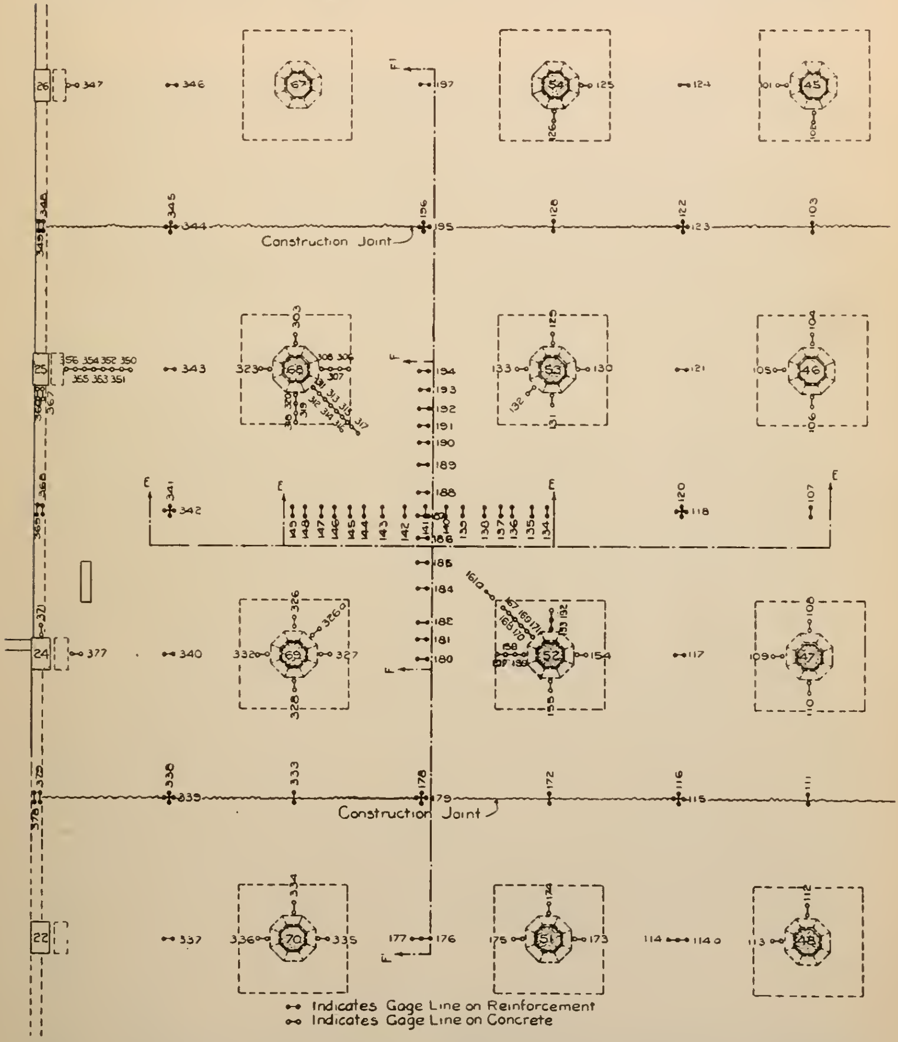


FIG. 3. LOCATION OF GAGE LINES ON UPPER SURFACE OF TEST FLOOR OF SHREDDED WHEAT FACTORY.

At each stage of the test the load was allowed to remain in position at least 12 hours before the final strain gage readings were taken. In order to obtain information on the effect of time on deformation under load, readings were taken at the 191-lb. load immediately after completing the loading operation and again after the load had been in place about 60 hours.

6. *Load-deformation Diagrams.*—The load-deformation diagrams have been plotted in Fig. 8 and 9, the grouping being such as to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Construction Joint
25
68
53
46
24
69
52
47
Construction Joint
22
70
51
48
⊷ Indicates Gage Line on Reinforcement
⊶ Indicates Gage Line on Concrete

FIG. 4. LOCATION OF GAGE LINES ON UNDER SURFACE OF TEST FLOOR OF SHREDDED WHEAT FACTORY.

place close together the diagrams for gage lines located in similar positions on the floor. In these diagrams the values from the zero load to the first point plotted for the load of 191 lb. per sq. ft. are for nine panels loaded (test area No. 1, Fig. 6). The second point at load of 191 lb. per sq. ft. in each diagram is for three panels loaded (test area No. 2). The point corresponding to 243 lb. per sq. ft. is for the final one-panel load (test area No. 3). The number of panels loaded at various stages of the test is indicated for the diagram marked (1) a, Fig. 8. This is typical of all the diagrams in Fig. 8

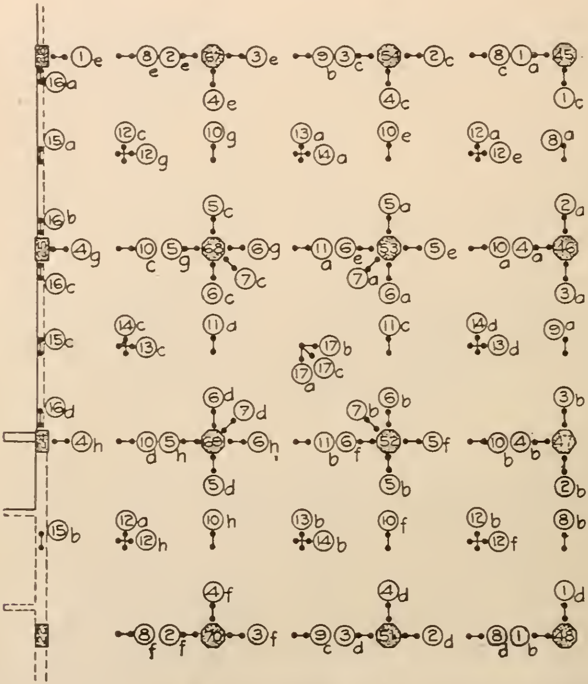

Fig. 5. Key to Grouping of Gage Lines; Test Floor of Shredded Wheat Factory.

and 9. The numbers on the diagrams correspond to the gage line numbers given in Fig. 3 and 4. The grouping of the diagrams is indicated by the numbers which correspond to those given in the "Key to Grouping of Gage Lines," (Fig. 5).

In some cases the load-deformation diagrams give peculiar results; however, the similarity of the deformations found in positions remote from each other but similarly situated is marked and tends to give confidence in the results. An example of this is found in Groups 1, 2, and 3 in which, in several instances, slight tension was found where compression would be expected. It is quite possible that these apparently erratic results may be due to general changes in temperature in the concrete.

Examination of these diagrams shows that in all but a small number of cases the removal of load from six panels caused the kind of change in stress which would be expected from the nature of the change in loading.

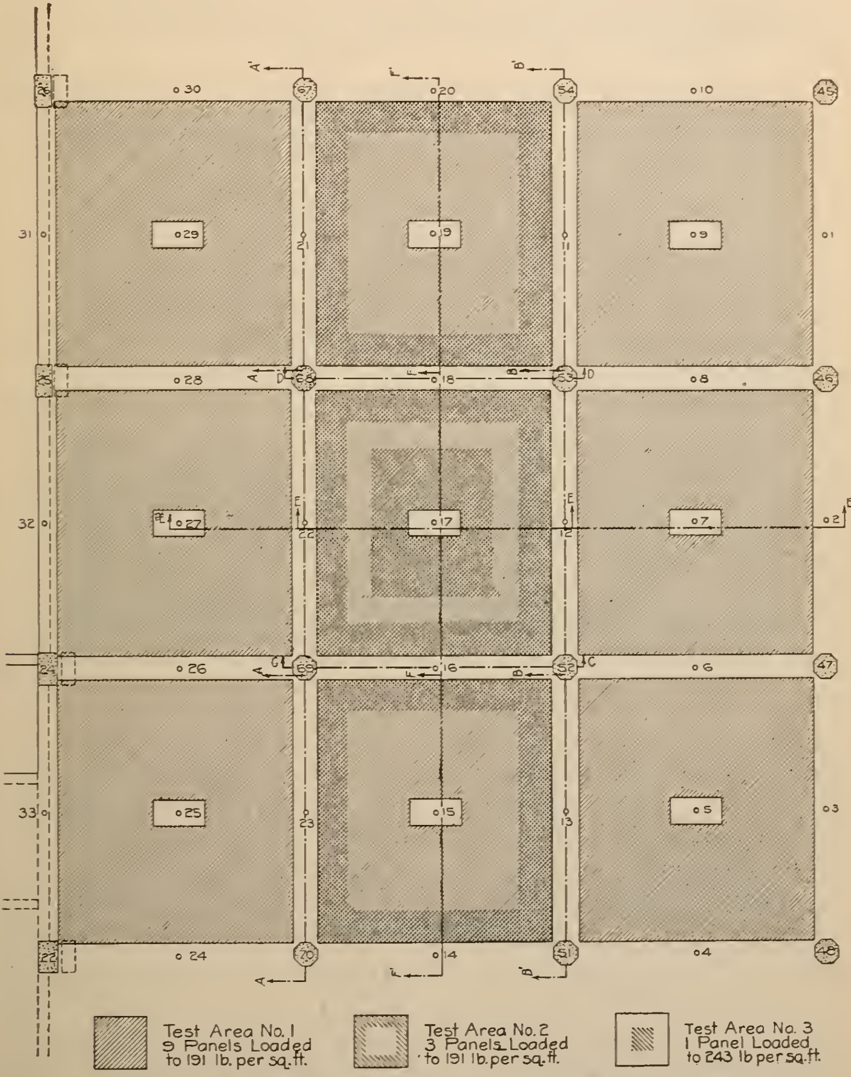

Fig. 6. Location of Deflection Points and Plan of Load Distribution at Three Successive Stages of Test of Shredded Wheat Factory Floor.

7. *Effect on Deformations of Standing Under Load.*—With 191 lb. per sq. ft. on the nine-panel area, strain gage readings were taken immediately after placing the load in position and again after it had been in place for about 56 hours. With few exceptions there was a material increase in the deformation during this time, and the increase for the gage lines on the concrete nearly always was greater than for the gage lines on the steel. An examination of 66 gage lines indicates an average increase in unit-deformation approximating 30 per cent for the concrete and 20 per cent for the steel over that which existed just after the load was applied. There was no apparent systematic difference in the amount of increase between positions at the columns and positions near centers of panels and none between gage lines at the edge of the loaded area and those in the interior portions.

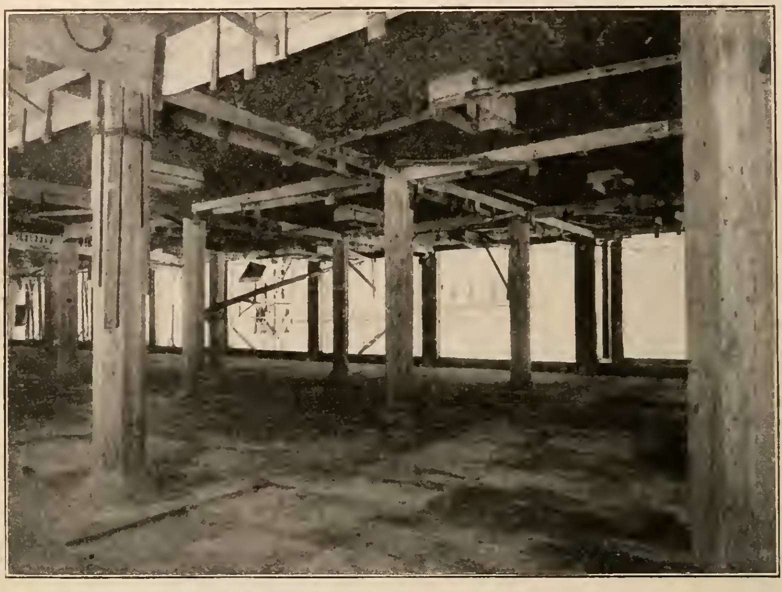

FIG. 7. VIEW OF TEST LOAD IN SHREDDED WHEAT FACTORY.

With the load of 243 lb. per sq. ft. on one panel only, the measurements indicate little or no general increase in deformation, but most of the gage lines on which these observations were taken were so situated that they were affected by the removal of load from the outer panels, and it seems probable that the recovery may have been progressing at the same time that the deformations were increasing due to standing under load.

TABLE 2.

EFFECT ON UNIT-DEFORMATION OF CHANGES IN LOAD DISTRIBUTION.

Plus indicates increase and minus indicates decrease in deformation

Position of Gage lines	Direction of Measurement	Average Change in Unit-deformation in per cent of Unit-deformation before Moving Load	
		Change from 9 panels to 3 panels both at 191 lb. per sq. ft.	Change from 3 panels at 191 lb. per sq. ft. to 1 panel at 243 lb. per sq. ft.
Across center line of panel, Section F′–F′; on steel	E–W	+66	
Across center line of panel, Section F–F; on steel	E–W	+56	0
Across center line of panel, Section F′–F′; on concrete	E–W	+116	
Across center line of panel, Section F–F; on concrete	E–W		—33
Across edges of panel, Sections A′–A′ and B′–B′; on steel	E–W	—19	
Across edges of panel, Sections A–A and B–B; on steel	E–W		+32
Across center line of panel, Section E–E; on steel	N–S	—5	+23
Across center line of panel, Section E–E; on concrete	N–S	—31	
Across edges of panel, Sections C–C and D–D; on steel	N–S	—22	—5

8. *Effect of Change in Number of Panels Loaded.*—Table 2 gives a summary of the results of an examination of the change of unit-deformation caused by the change of loading from nine panels to three panels, and from three panels to one panel (see also Fig. 10 and 11). The areas occupied by these loads and their intensities are indicated as test areas No. 1, No. 2, and No. 3, Fig. 6. In this table increases in the deformation are given a positive sign and decreases are given a negative sign. The most striking features of this table are the great increases in deformation in gage lines which cross the north and south center line of the panel (section F′-F′) in both the concrete and the steel. It is of note also that, in the three instances in this table in which changes in deformation in steel and concrete on the same section can be compared, the changes in concrete are much greater than those in the steel.

The change from nine panels to three panels caused an average increase in deformation of 66 per cent in gage lines which cross section F′-F′. Taking only the gage lines of the middle panel of the three, the average increase was only 56 per cent. (See Fig. 11). The greatest changes of all were in gage lines 179 and 195 both of which lay close to and parallel with construction joints. It is not apparent that the

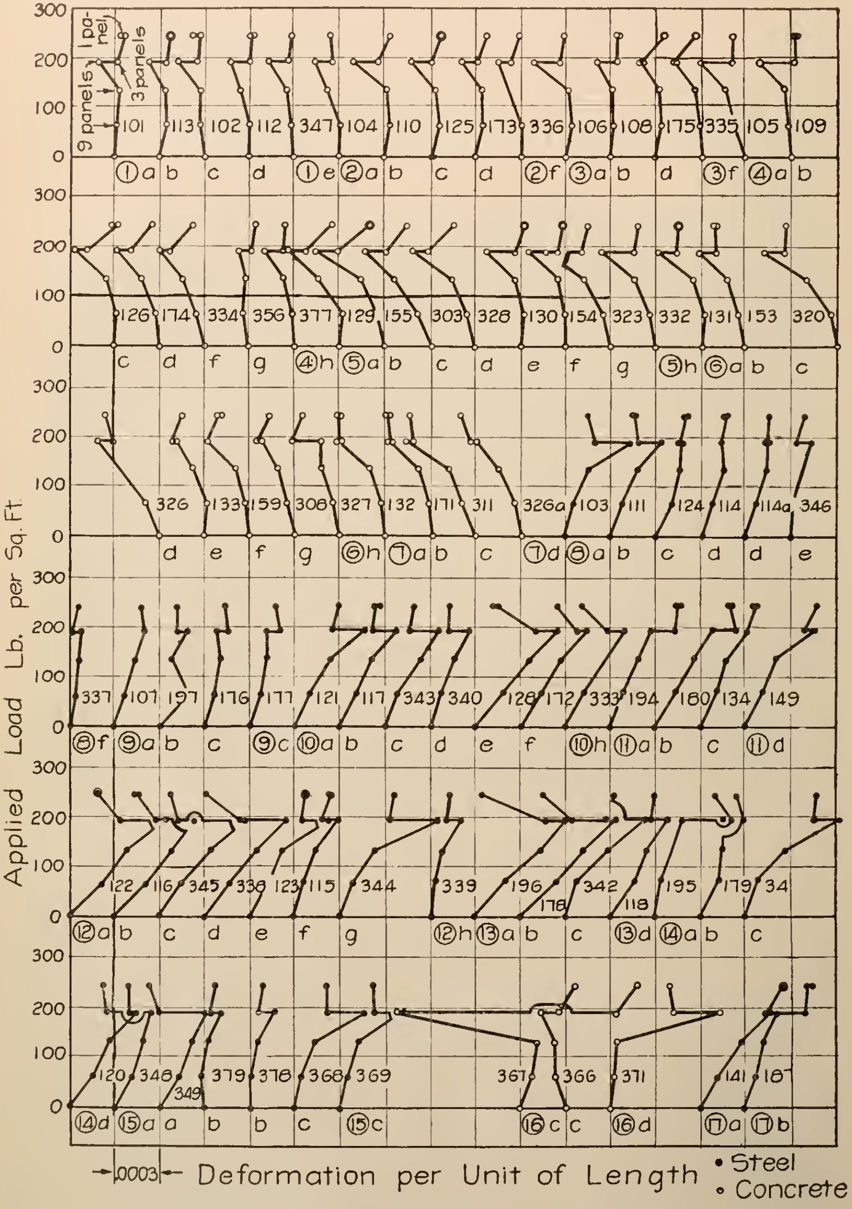

FIG. 8. LOAD-DEFORMATION DIAGRAMS FOR GAGE LINES ON UPPER SURFACE OF TEST FLOOR OF SHREDDED WHEAT FACTORY.

proximity of the construction joint should influence the change since the gage lines do not cross the joint. Gage lines 176 and 177 (at the edge of the test area) showed very large increases also, but that in 197 was below the average.

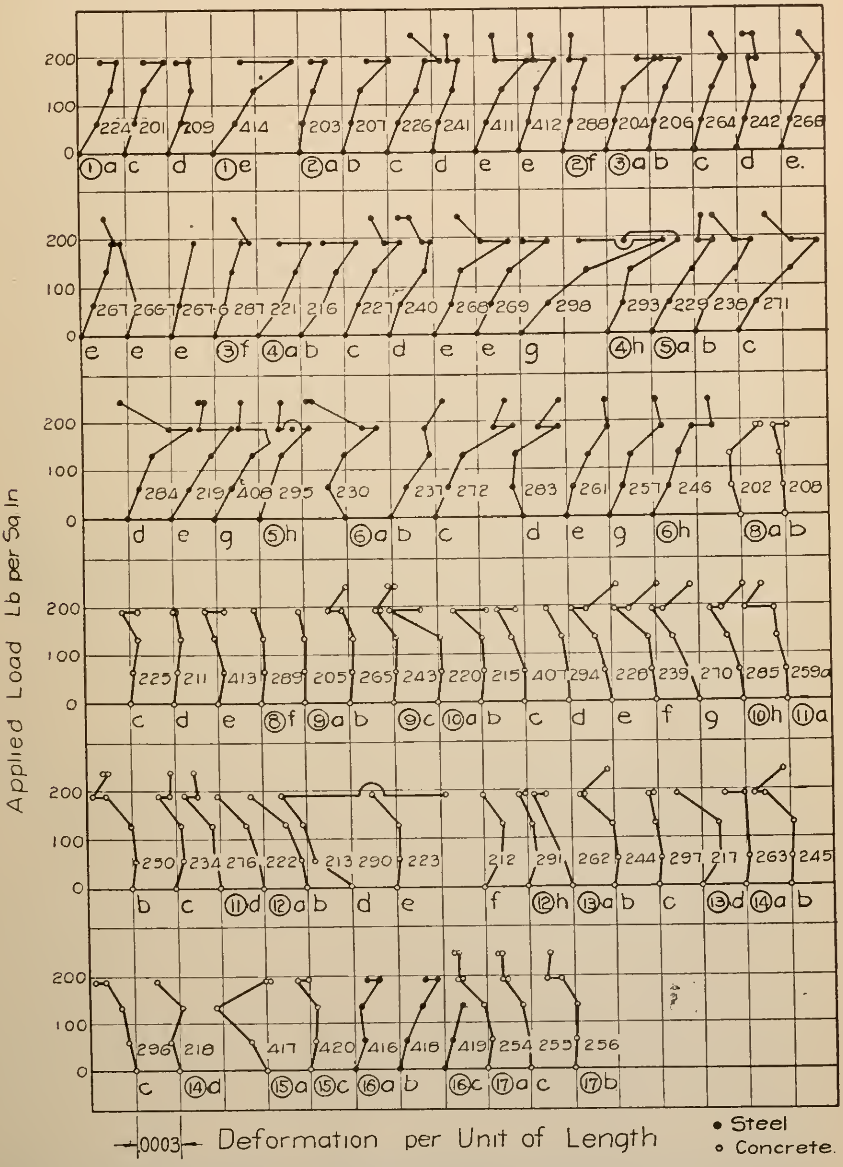

FIG. 9.—LOAD-DEFORMATION DIAGRAMS FOR GAGE LINES ON UNDER SURFACE OF TEST FLOOR OF SHREDDED WHEAT FACTORY.

With few exceptions the deformations in the reinforcement across the edges of the panel (that is, across sections A-A and B-B) decreased with the change of load from nine panels to three panels. These exceptions all are in the case of bars which pass through a column or over a column capital. In all cases the bars which cross the panel edge at points intermediate between the columns lost part of their stress on changing the load from nine panels to three panels. These phenomena would indicate that if, for purposes of comparison, the slab were to be conceived of as a double system of strip beams, the strips passing through the columns could be considered as fixed or nearly fixed at their supports while those passing between columns must be considered to have appreciably less end restraint. The average for all gage lines crossing sections A-A and B-B shows a decrease in deformation of 19 per cent due to the change from nine panels to one panel loaded.

Gage lines were located on the under side of the slab close to the columns in the center panel and in an outer panel. On the removal of the load from the outer panels, the load of 191 lb. per sq. ft. being left on the row of three panels, the deformation in the concrete on the

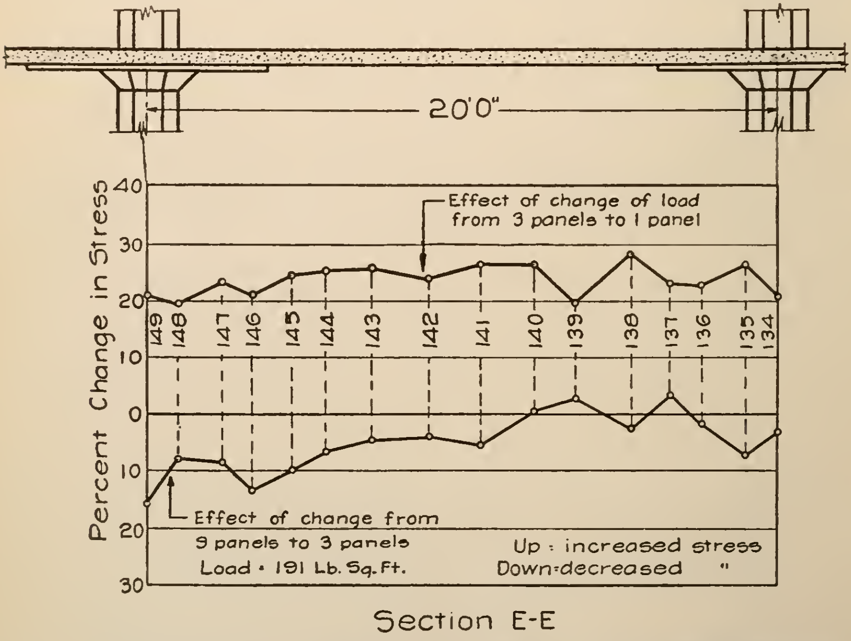

Fig. 10. Change in Stress Accompanying Changes in Loading for Gage Lines across East-west Center Line of Central Panel, Shredded Wheat Factory.

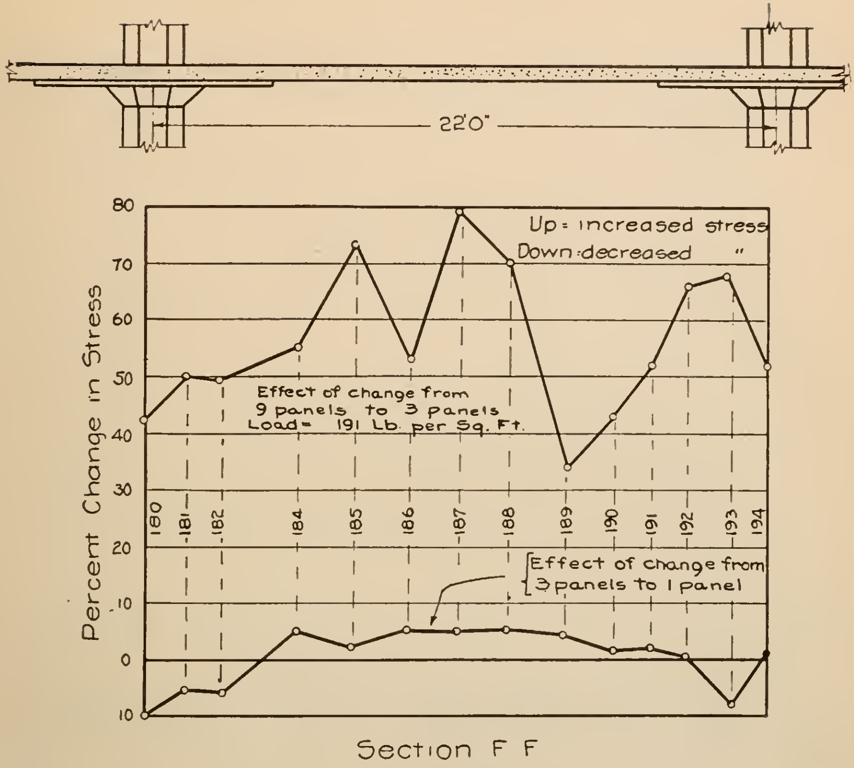

FIG. 11. CHANGE IN STRESS ACCOMPANYING CHANGES IN LOADING FOR GAGE LINES ACROSS NORTH-SOUTH CENTER LINE OF CENTRAL PANEL, SHREDDED WHEAT FACTORY.

under side of the slab at the unloaded side of the column (gage lines 323, 332, 154, and 130) decreased markedly, and the deformation in the concrete on the loaded side of the column (gage lines 327, 159, 133, and 308) increased somewhat. Assuming a modulus of elasticity of 3,000,000 lb. per sq. in. for the concrete, the amount of the compressive stress in the concrete on the loaded side amounted to about 800 lb. per. sq. in., and on the unloaded side to about 300 lb. per. sq. in.

Fig. 13 and 15 show the deformations in the gage lines which cross the center lines of the central panel (sections F-F and E-E) for the three panels loaded and for the one panel loaded. Fig. 10 and 11 show the change in stress in these sections caused by the changes in loading in per cent of the stress present before moving the load.

The decrease in the deformation in bars lying under the load but near the panel edge was more marked than that in bars parallel to them under the central part of the load. Taking into account the

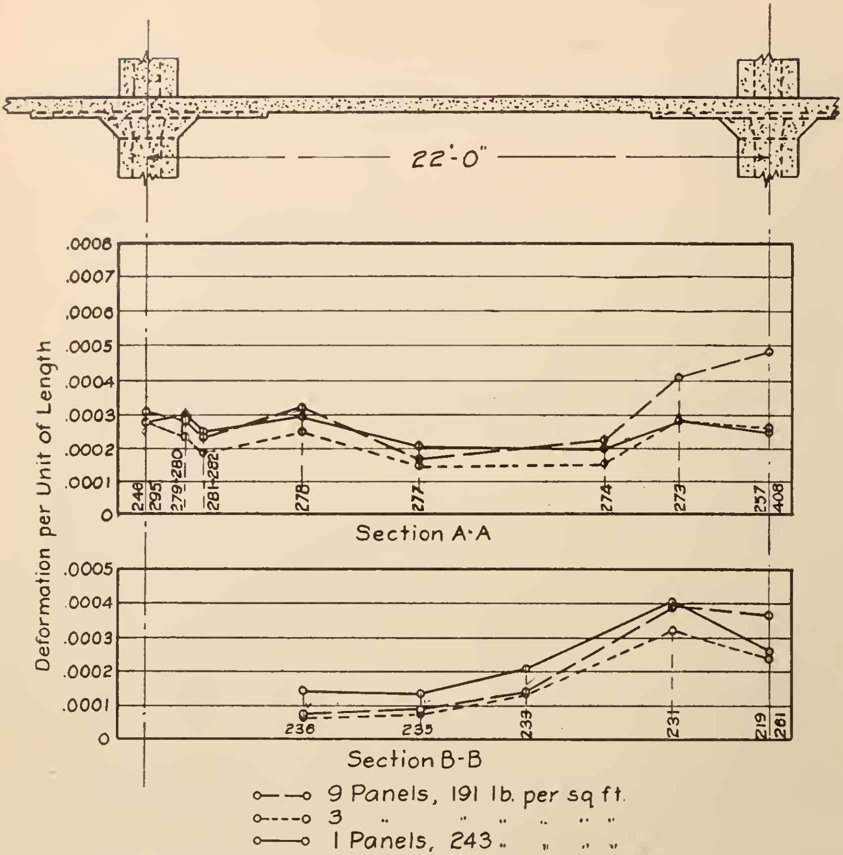

FIG. 12. NORTH-SOUTH DISTRIBUTION OF DEFORMATION IN EAST-WEST BARS ACROSS EDGE OF CENTRAL TEST PANEL IN SHREDDED WHEAT FACTORY.

increase in the intensity of the load simultaneously with the removal of load from the two outer panels, leaving only the central panel loaded, it seems that there must have been a distribution of stress laterally to bars outside the loaded area (or assistance given by the development of stress in the reinforcement in the unloaded panels adjacent to the loaded panel). The effect seems to have disappeared at a distance of about one-quarter panel width from the edge of the loaded area.

For some reasons it might be expected that the change from the three-panel load to the one-panel load would cause an increase in deformations on gage lines which cross section E-E (see Fig. 10). However, the increase in deformation on section E-E (23 per cent)

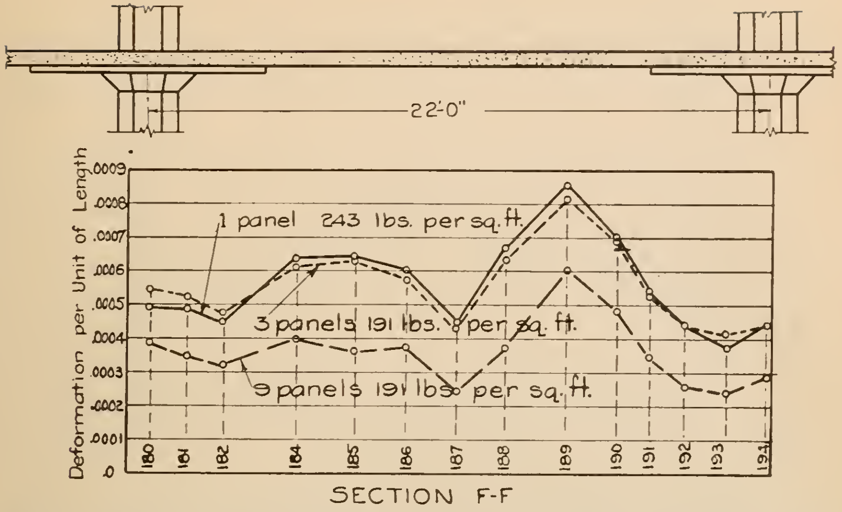

FIG. 13. NORTH-SOUTH DISTRIBUTION OF DEFORMATION IN EAST-WEST BARS ACROSS CENTER LINE OF CENTRAL TEST PANEL IN SHREDDED WHEAT FACTORY.

following the change from three panels at 191 lb. per sq. ft. to one panel at 243 lb. per sq. ft. was less than the increase in intensity of the load (27 per cent).

9. *Distribution of Stress and of Moments.*—In the design of the floor slab the reinforcing bars were distributed across the panel width in accordance with a method which the Corrugated Bar Company derived from the test made by Mr. Trelease on a small rubber slab.* To show the distribution of stress over the width of the slab, as measured in the test, the deformations developed in the reinforcing bars at the load of 191 lb. per sq. ft. over nine panels have been plotted in Fig. 12, 13, 14, and 15, using deformations as ordinates and distances at right angles to the direction of the length of bar as abscissas (width of slab). It will be seen that rather large differences were found in the deformations in bars which were close together. That these differences are not generally errors of observation is indicated by the fact that a set of check readings on a large number of gage lines gave stresses practically identical with those of previous observations at the same load.

Fig. 16 gives moment factors showing the distribution of the positive resisting moment across the width of the slab based upon the measured deformations in the reinforcing bars, and Fig. 17 gives similar

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*See Proceedings National Association of Cement Users, vol. VIII., p. 218.

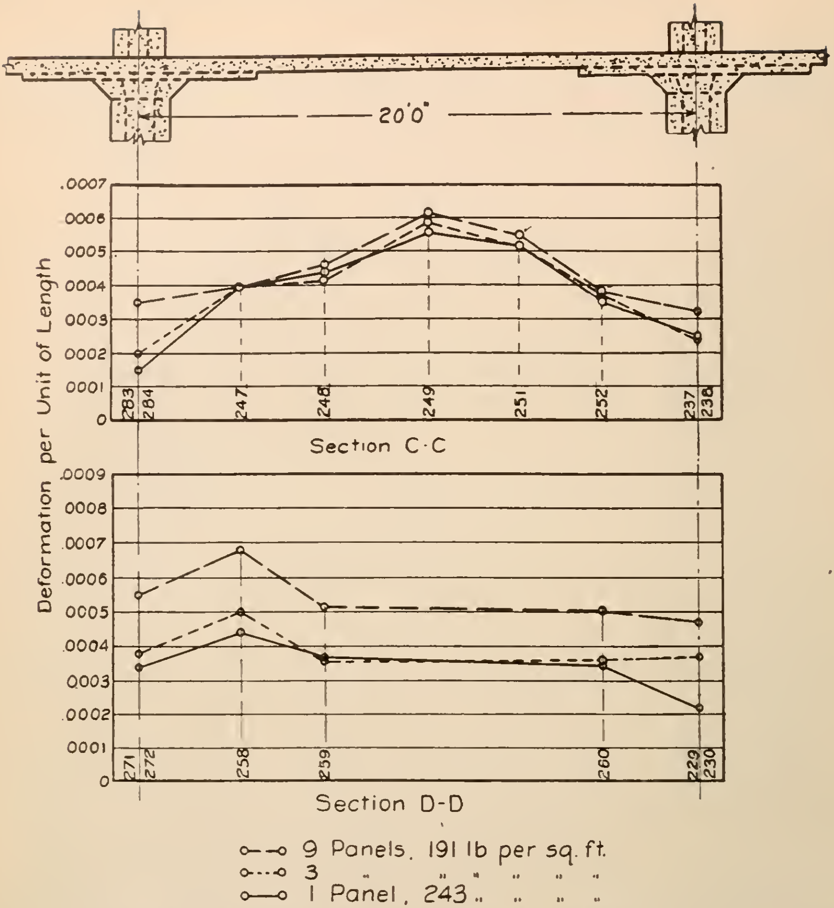


FIG. 14. EAST-WEST DISTRIBUTION OF DEFORMATION IN NORTH-SOUTH BARS ACROSS EDGE OF CENTRAL TEST PANEL IN SHREDDED WHEAT FACTORY.

moment factors for the negative resisting moment based upon the measured deformations in the steel. These moment factors represent the coefficients by which wl^2 must be multiplied to obtain the positive (or negative) resisting moment per unit of width of section developed by the stress found in the steel, w being the load per unit of area and l the panel length center to center of columns in the direction of the stress considered.

In making up these two diagrams the width of the slab was divided for convenience into several portions, and for each portion the average stress in the reinforcement was multiplied by the effective moment arm times the cross-sectional area of the reinforcement. The moments for the several portions were divided by the

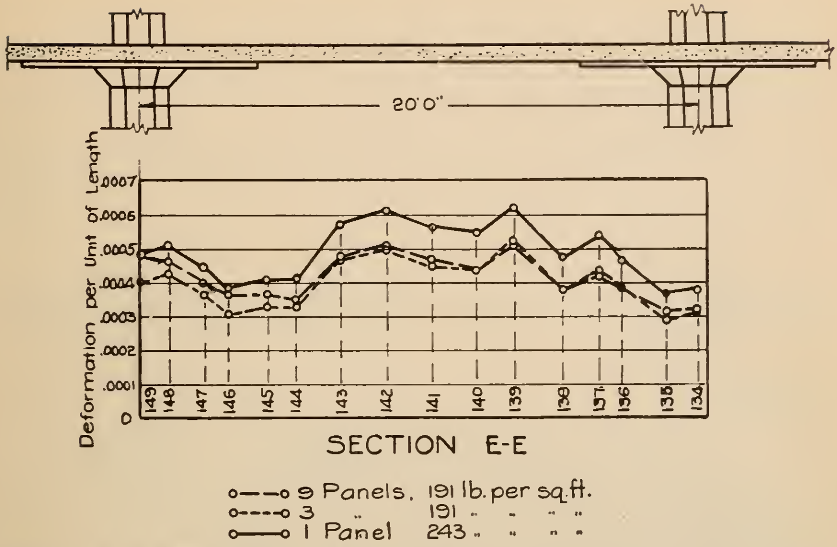


FIG. 15. EAST-WEST DISTRIBUTION OF DEFORMATION IN NORTH-SOUTH BARS ACROSS CENTER LINE OF CENTRAL TEST PANEL IN SHREDDED WHEAT FACTORY.

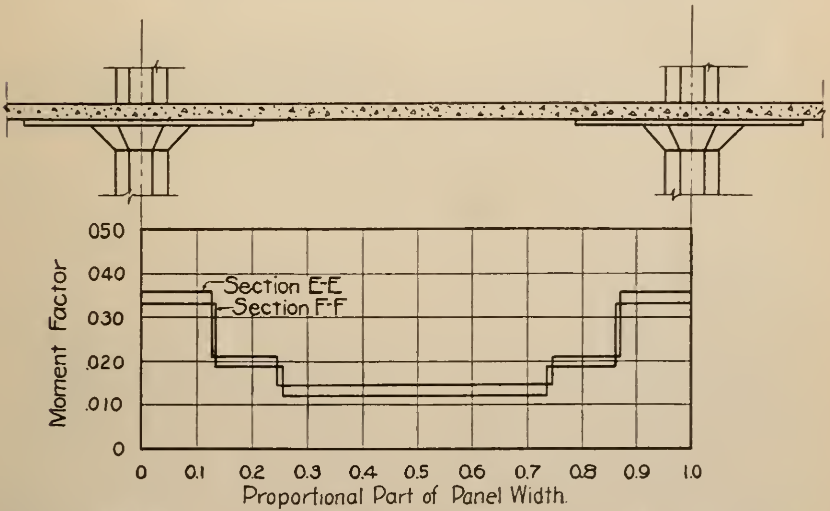


FIG. 16. MOMENT FACTOR DIAGRAM FOR SECTIONS OF POSITIVE MOMENT (SECTIONS E-E AND F-F) OF CENTRAL TEST PANEL, SHREDDED WHEAT FACTORY.

widths and by the quantity wl^2. Calculations were thus made for two sections (E-E and F-F) at positions of maximum positive moment and for four sections (A-A to D-D) at positions of negative moment. The stresses used were taken from Fig. 12, 13, 14, and 15 for the load of 191 lb. per sq. ft. over nine panels in such a way as seemed best to represent the stresses over the entire section. In some cases a uniform stress represented the distribution as accurately as any symmetrical curve which could be used.

It will be noted that the distribution of the values of the moment factors in Fig. 16 and 17 is largely dependent upon the distribution of the reinforcing bars over the width of the section. If the bars had been grouped less closely over the column head there probably would have been less variation in the values of the moment factors.

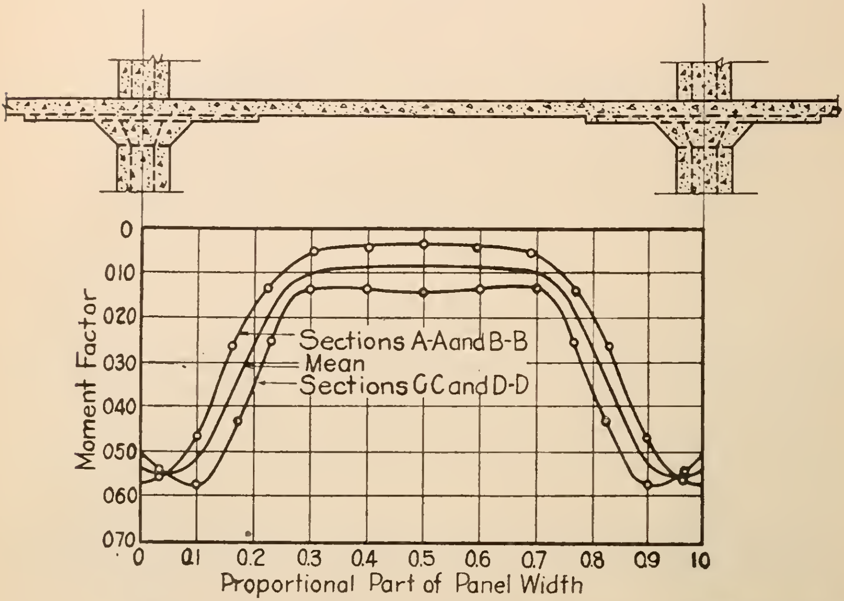


Fig. 17. Moment Factor Diagram for Sections of Negative Moment (Sections A-A to D-D), Shredded Wheat Factory.

By integration of the areas between the moment factor curve and the axis, the value for the positive resisting moment due to the measured stresses in the steel for a width equal to the width of the panel is found to be 0.021 Wl and that for the negative resisting moment 0.026 Wl, where W is the total load on the panel and l is panel length. These moments are the averages of the values obtained from the two curves for the positive moment and from the two curves for negative moment. These moments represent the value of the resisting moment developed by the steel for one direction only, as determined by average unit-deformations over 8-in. gage lengths. With the increased deformations at certain places when only three panels were loaded the average positive moment would be found to be larger than the value given here. As the tensile resistance of the concrete may be expected to be considerable for beams or slabs having as low percentages of reinforcement as these and as low deformations as were found in the steel, the moments given above can not be taken to be the actual resisting moments developed in the slab.

10. *Stresses and Moments in Columns.*—In case not all of the panels are loaded, the bending moment developed in the slab at the

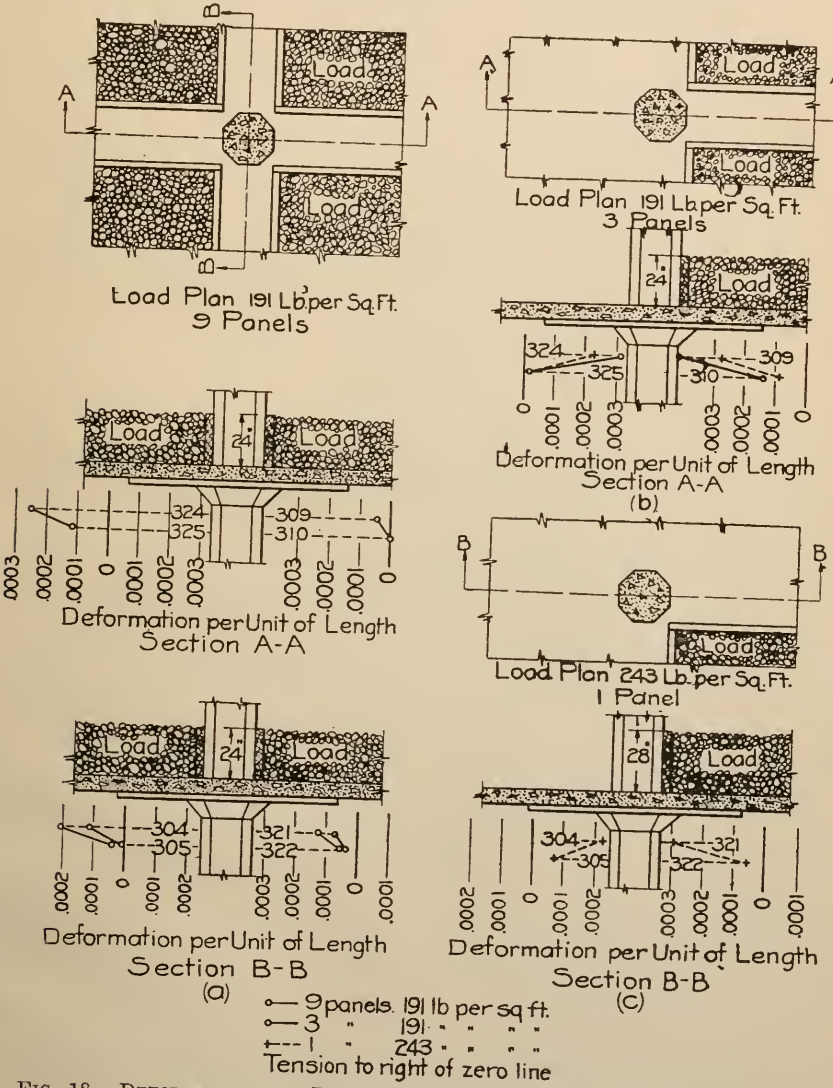

Load
A
A
B
B
Load Plan 191 Lb. per Sq. Ft.
9 Panels
Load Plan 191 Lb. per Sq. Ft.
3 Panels
Load Plan 243 Lb. per Sq. Ft.
1 Panel
24
28
324
325
309
310
304
305
321
322
.0003
.0002
.0001
0
Deformation per Unit of Length
Section A-A
Section B-B
(a)
(b)
(c)
9 panels. 191 lb per sq ft.
3 " 191 " " " "
1 " 243 " " " "
Tension to right of zero line

Fig. 18. Deformations in Interior Column No. 68 of Shredded Wheat Factory.

edge of the loaded panels is resisted by a restraining moment taken by the columns at the edge of the loaded area and by the slab of the adjacent unloaded area. The division of this restraining moment into the three restraining moments, that taken by the column above the floor, that taken by the column below the floor, and that taken by the adjacent unloaded slab, is dependent upon the relative stiffness of these members (represented by the moments of inertia of the members and by the relative length of the members). As the modulus of elasticity of the concrete in the structure is not known and as the effect of the tensile strength and stiffness of the concrete is quite uncertain, it is not practicable to make a definite statement concerning the amount of bending stresses in the columns or the exact relative bending moments taken by the columns and by the unloaded floor. It may be interesting, however, to note the phenomena as they were observed (see Fig. 18).

Gage lines 309, 324, 304, and 321 were located on the four sides of a basement column just below the column capital. The deformations

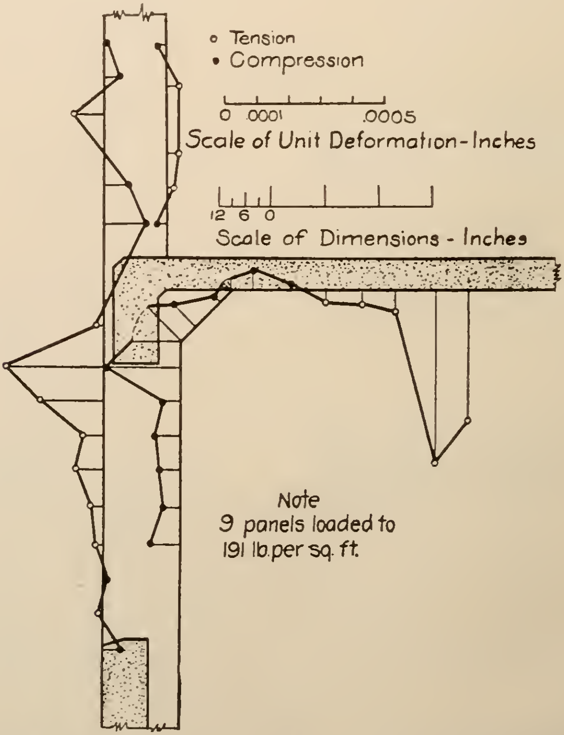


Fig. 19. Deformations in Wall Column No. 25 and Point of Zero Unit-deformation on Under Surface of Test Floor near Wall Column, Shredded Wheat Factory.

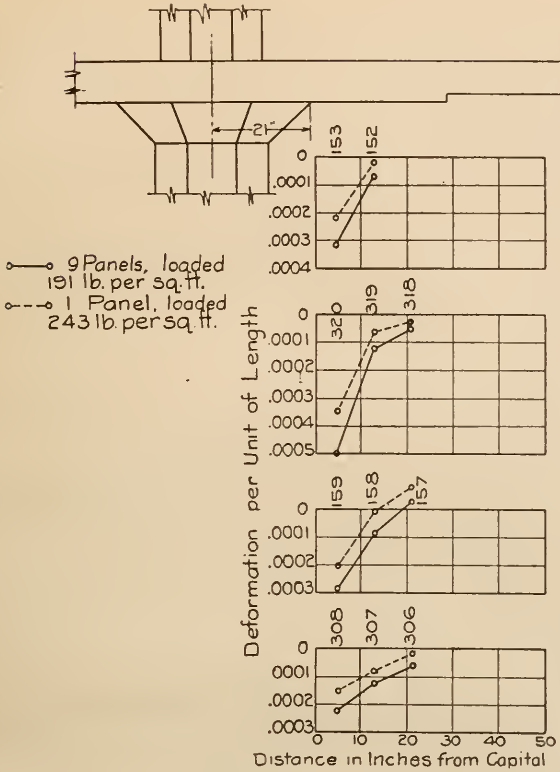


FIG. 20. POINTS OF ZERO UNIT-DEFORMATION ON UNDER SURFACE OF TEST FLOOR NEAR INTERIOR COLUMNS OF SHREDDED WHEAT FACTORY.

in this column show (see Fig. 18) that the partial loading of the slab developed a severe bending in the columns of the basement story. No measurements were taken on the columns of the first story.

11. *Point of Inflection.*—With a view of determining the position of the point of inflection, several series of measurements of deformations in the concrete on the under side of the slab were taken. The results are shown in Fig. 19, 20, and 21. Reference to Fig. 4 will show the location of gage lines. In Fig. 20 only one series of measurements is carried far enough to cover a change from compression to tension. The tendency of these curves to change their slope abruptly makes it impracticable to find the point of zero stress by producing the line till it crosses the axis. Some floor tests have shown the position of zero-deformation of the under surface of the slab to be closer to the column than that for the upper surface, while other tests are quite the opposite. No reason is known for such variation in

results. It seems unwise to base conclusions as to the positions of the point of inflection upon data obtained on the under surface only The curves in Fig. 20 and 21 show a tendency for the point of zero-stress to move toward the column when the load is changed from test area No. 1 to test area No. 2.

12. *Lintel Beams.*—During the progress of the tests, minute cracks were found in the concrete in various places in the structure,

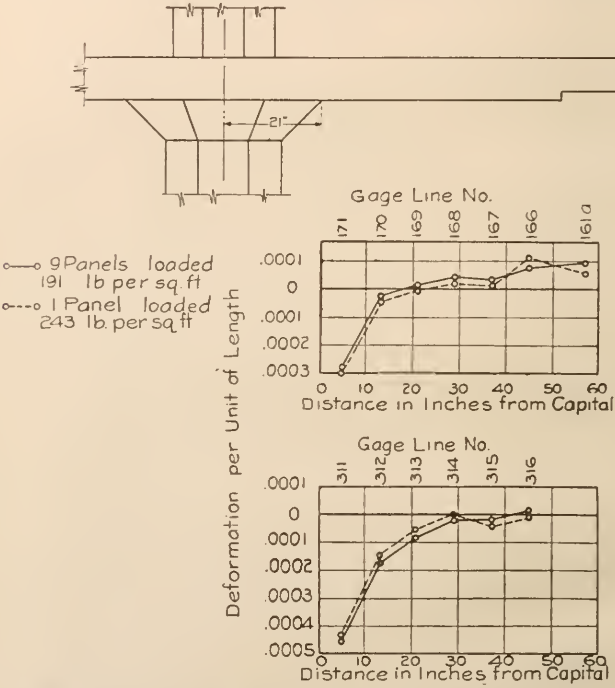


FIG. 21. POINTS OF ZERO UNIT-DEFORMATION ON UNDER SURFACE OF TEST FLOO NEAR INTERIOR COLUMNS OF SHREDDED WHEAT FACTORY.

and the presence of these cracks confirmed the results of the strai gage measurements. The most important indications of this kind wer cracks which were found on the interior side of the lintel beams nea their ends. The cracks did not appear on the exterior of these beam and except for this, the cracks, by their position and direction resembled the diagonal tension cracks ordinarily found in beam subjected to high shearing stress. The web reinforcement in the linte beams consisted of both inclined members and vertical members whic were wrapped about the horizontal bars. The presence of crack on the interior surface and the absence of cracks on the exterio surface may indicate that this phenomenon was due to torsion. Th slab and the lintel beams were built monolithically, and the bending moment developed in the slab at its end by the load on the wall panel would produce torsion in the beam. The formation of the cracks shows that stresses exist in the lintel beam which are not ordinarily considered. The effect of the bending of the slab on the lintel beam to which it is attached should be considered.

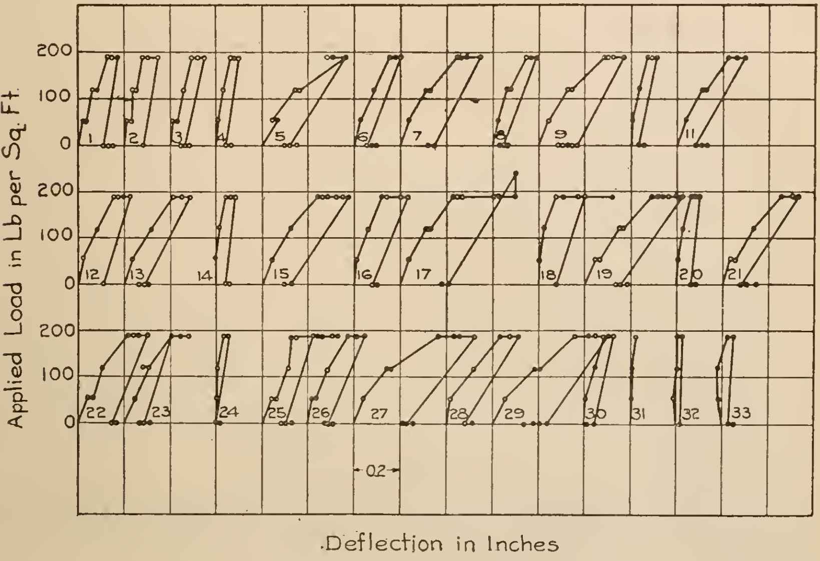


FIG. 22. LOAD-DEFLECTION DIAGRAMS FOR TEST FLOOR OF SHREDDED WHEAT FACTORY.

13. *Deflections.*—Little use has been made in this report of the deflections which were measured in the tests, but since in many cities the building regulations make certain requirements for deflection under load, it is believed that the presentation of deflection data may serve a useful purpose. In Fig. 22 are given diagrams in which the ordinates represent the load in pounds per square foot, and the abscissas represent the deflection in inches. Accompanying these diagrams are the numbers of the deflection points, the locations of which are shown in Fig. 6.

14. *Summary of Results.*—The principal results brought out in the foregoing discussion are as follows:

1 There was a considerable increase in the deformations in both steel and concrete during the fifty-six hours of retention of the load.

2 Upon the removal of the load from the six panels, there was an

increase in the deformations across the center line of the three panels which remained loaded (section of positive bending moment). There was a decrease in the deformations across the side edges of the area remaining loaded (sections of negative bending moment). There was also a decrease in the deformations in the direction of the side edge of the loaded area in those bars under the load which lay near this edge.

3 The positive bending moment for a panel width corresponding to the deformations measured in the reinforcing bars in one direction was found to average 0.021 Wl for a panel width; the negative bending moment found in the same way was 0.026 Wl. These values may be of interest in comparing the results of this test with the results of other tests. It must be understood, however, that these do not represent values of the bending moment coefficients which should be used in design.

4 The distribution of stresses in the reinforcement across panel edges and across panel center lines was substantially uniform, taking averages of the several sections. The variation from uniform distribution of the moment factors for these sections corresponds closely to variation in the slab thickness and in the distribution of the reinforcement.

5 The measurements show that a large bending moment was developed in the basement columns under a partial loading of the slab.

6 In the lintel beams cracks were found on the interior side near the ends which probably were caused by the twisting action produced by bending moment developed in the slab at its edge by the load on the wall panel.

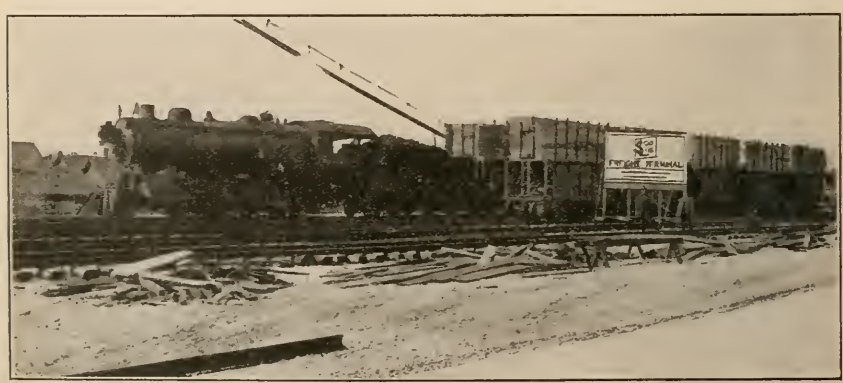


Fig. 23. View of Test Load, Soo Terminal Structure.

III. THE TEST OF THE FLAT SLAB OF THE SOO LINE FREIGHT TERMINAL.

15. *The Structure.*—The Soo Line Freight Terminal of Chicago, comprises a freight yard and a central receiving and distributing building for the Central Terminal Railway Company of the Soo Line. The freight yard is elevated above the ground so that the intersecting streets cross underneath the terminal without a depression of their grades. Both across these streets and between them for a distance of more than half a mile the yard is on a reinforced concrete slab structure which is supported on columns. The space under the slab not occupied by streets is arranged to be utilized for storage purposes.

A more complete general description of the structure is given in Engineering Record, August 16, 1913 (Vol. 68, No. 7), in

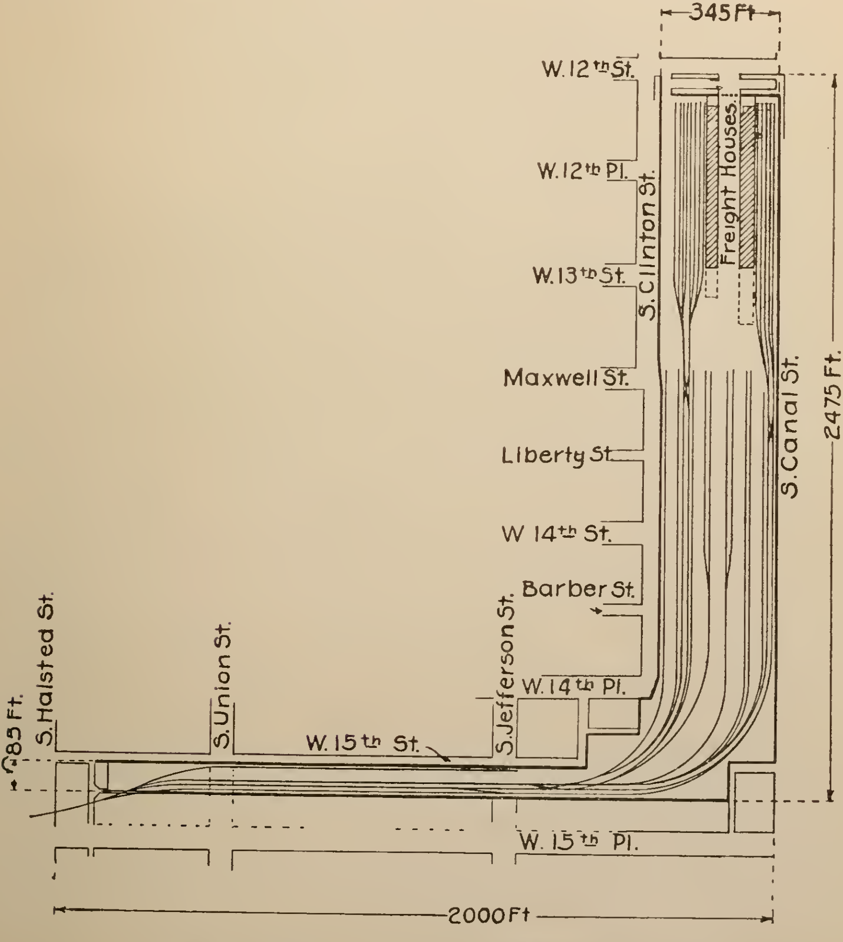


FIG. 24. GENERAL PLAN OF SOO TERMINAL STRUCTURE.

Engineering News, August 21, 1913 (Vol. 70, No. 8), and in Railway Age Gazette, August 22, 1913.

The general plan of the structure is shown in Fig. 24 and the design of a typical panel is shown in Fig. 25. For the test a four panel area was chosen which represents the typical flat slab of this structure, which is an unusually heavy one. The test area was located between West Fourteenth Street and Barber Street,

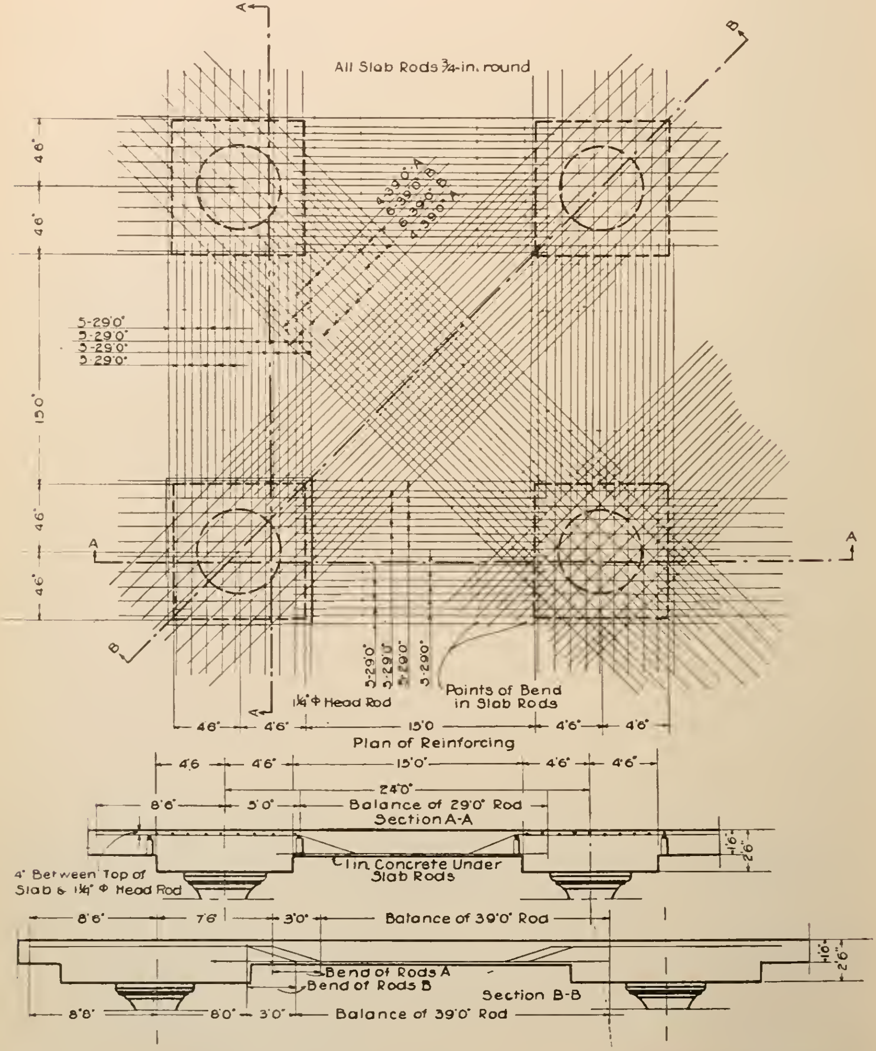


FIG. 25. DIMENSIONS AND REINFORCEMENT OF TYPICAL PANEL OF SOO TERMINAL TEST FLOOR.

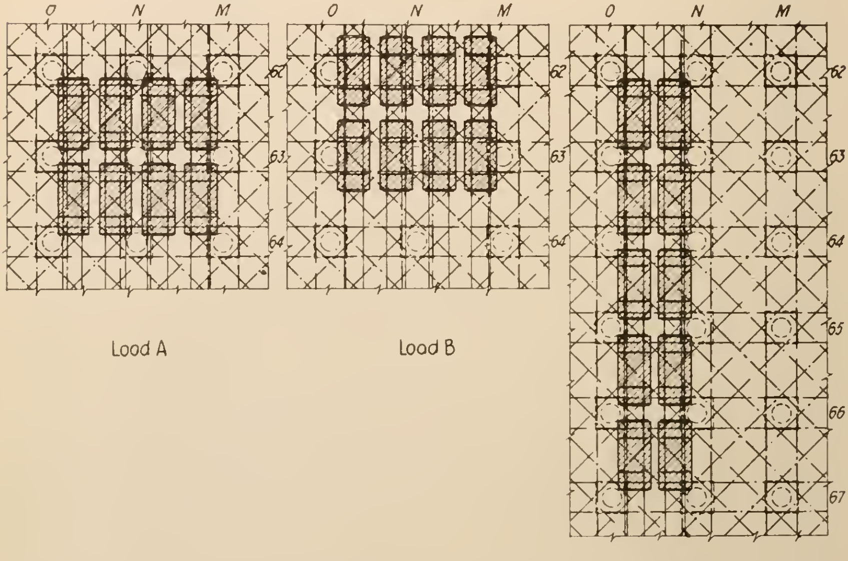
Chicago. The test panels are 24 ft. square. The floor slab in the central portion of the panel is nominally 18 in. thick. Its nominal thickness is 30 in. throughout the area of a 9-ft. square surrounding each column, this portion of the slab being referred to as the depressed head. The necessary slope for drainage purposes was obtained by increasing the thickness of the slab to form alternate ridges and valleys. The thickness of the slab along the ridges was in some places as much as 2 in. greater than the nominal thickness. The additional thickness did not affect the position of the reinforcing bars. In regions of negative moment it is possible that an additional thickness of concrete affected the amount of the contribution of the tensile strength of the concrete to the resisting moment. In regions of positive moment an additional depth would increase the moment arm of the reinforcement stress.

The slab contains four-way reinforcement, as shown in Fig. 25. In slabs of ordinary thickness and span it has been a common practice to depend upon the weight of the reinforcing bars to pull the bars down into place in the center of the span, bars of one-half inch diameter or less being used. In a floor built for very heavy loading as in the case of the terminal slab under consideration, it is necessary to bend the bars to a definite shape. This bending was done after the steel had been placed in position. The average distance from the compression surface of the slab to the center of gravity of the reinforcement at the edge of the capital of column N63 was 26¼ in. The distance to the center of gravity of the reinforcement midway between columns varied somewhat due to the presence of the ridges previously referred to. The greatest and least depths found were 20¾ in. and 16¾ in., the least being fully as great as the depth which would be expected from the nominal thickness given above.

The columns are cylindrical, 32 in. in diameter, and end above in bell-shaped heads or capitals 5 ft. 6 in. in diameter at the top. The columns are reinforced with a spiral of ½-in. round steel, having a mean diameter of 28½ in. and a pitch of 2 in. In addition, 20 1⅛-in. diameter longitudinal reinforcing bars are placed just within the spiral and wired fast to it. The spiral is carried from a point 16 in. below the level of the basement floor to a point well into the bottom of the depressed head. Bearing and anchorage of the longitudinal rods are afforded by right angle hooks 8 in. long at the bottom and at the top. For the portion of the structure tested, the length of the columns from the top of the basement floor to the bottom of the depressed head averages about 12 ft. 10 in.

The footings are of reinforced concrete 14 ft. square with a concrete pier 4 ft. square extending from the top of the footing upward to a point 18 in. below the top of the basement floor.

16. *The Test.*—The concreting of the panels on which the test was made was done on June 12, 1913, the building being 112 days old at the time of the test, and the four-panel test load was applied on October 2, 1913. Ore cars having bins built on top to an average height of 6 ft. 2¼ in. above the regular height of the car were filled with broken stone and run upon the test area to furnish the load. Fig. 23 shows the loaded cars in position for the five-panel test (Load C).



Load A

Load B

Load C

Fig. 26. Plan Showing Positions of Cars for Loads A, B, and C on Soo Terminal Test Floor.

Portions of the ballasted and of the uncovered slab are seen in the foreground. The cars held on an average 1740 cu. ft. each., and the average weight including the car was 200,800 lb. The intensity of this load is considerably greater than the Terminal Railway Company expects the slab to be subjected to in service. Table 3 gives data of the loaded cars. It may be said in anticipation of results reported hereinafter that this load caused stresses in the slab so small that their interpretation is difficult.

The three different positions of load, designated as A, B, and C, are indicated in Fig. 26. Load A was designed to give the representative stresses around a central column approaching the condition of all panels loaded. The movement into position B was designed to determine the effect of a change of a half panel length in the position of the load. Load C was designed to find the effect of loading adjacent tracks lying within a row of panels.

The preparation for the test covered the time from Sept. 17 to Sept. 29, inclusive. As showing the extent of the preparations, it may be stated that during this time from eight to twelve laborers were employed in cutting or drilling into the concrete to expose the steel and to place gage lines for measurements on the concrete. The test occupied the time from Sept. 30, 1913 to Oct. 8, 1913. Observations were taken on 816 gage lines, of which 338 were on the upper surface and 478 on the lower surface of the slab. Locations of slab gage lines are shown in Fig. 27, and 28 and locations of deflection points in Fig. 30. Locations of gage lines on columns may be determined by a study of Fig. 37 to 41 inclusive. In the carrying out of the test considerable delay was experienced because of rains which flooded the gage holes and necessitated a large amount of tedious work in draining and cleaning.

TABLE 3.

WEIGHTS OF LOADING MATERIAL AND DIMENSIONS OF CARS.

Average weight of cars alone	32,000 lb.
Average weight of stone in cars normally	74,600 lb.
Average additional height of bin	6 ft. 2¼ in.
Average length of bin inside	18 ft. 7½ in.
Average width of bin inside	8 ft. 5¼ in.
Average additional capacity	970 cu. ft.
Average unit weight of stone used	97 lb. per cu. ft.
Average additional weight of stone per car	94,200 lb.
Average total weight of car and stone	200,800 lb.
Number of cars loaded	10
Maximum variation of total weight of any car from average	400 lb.
Length of car center to center of coupling	24 ft.
Center of coupling to center of first wheel	2 ft.
Center of first wheel to center of second wheel	5 ft.

In order to lessen the chances of error in readings, two complete sets of strain gage readings were taken before applying a load, and also two complete sets under Load A (see Fig. 26). The load was then moved to the position indicated as load B, Fig. 26, and two series of readings were taken in only the significant positions. The cars were then removed from the test area and a complete series of no-load readings was taken. With Load C in position readings were taken at the most important positions.

17. *Settlement and Deflections.*—Levels were taken on the slab at several of the columns and at deflection points before loading,

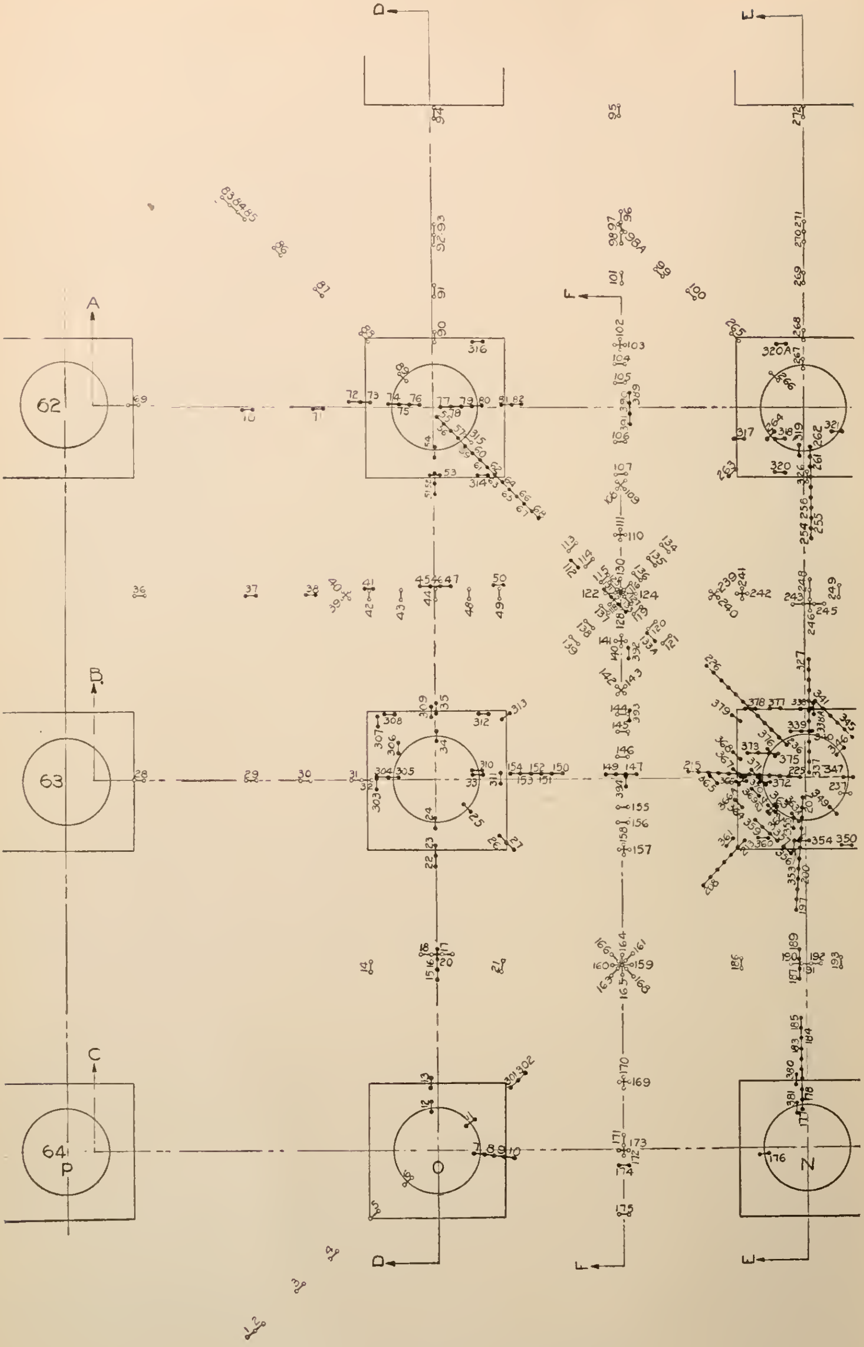
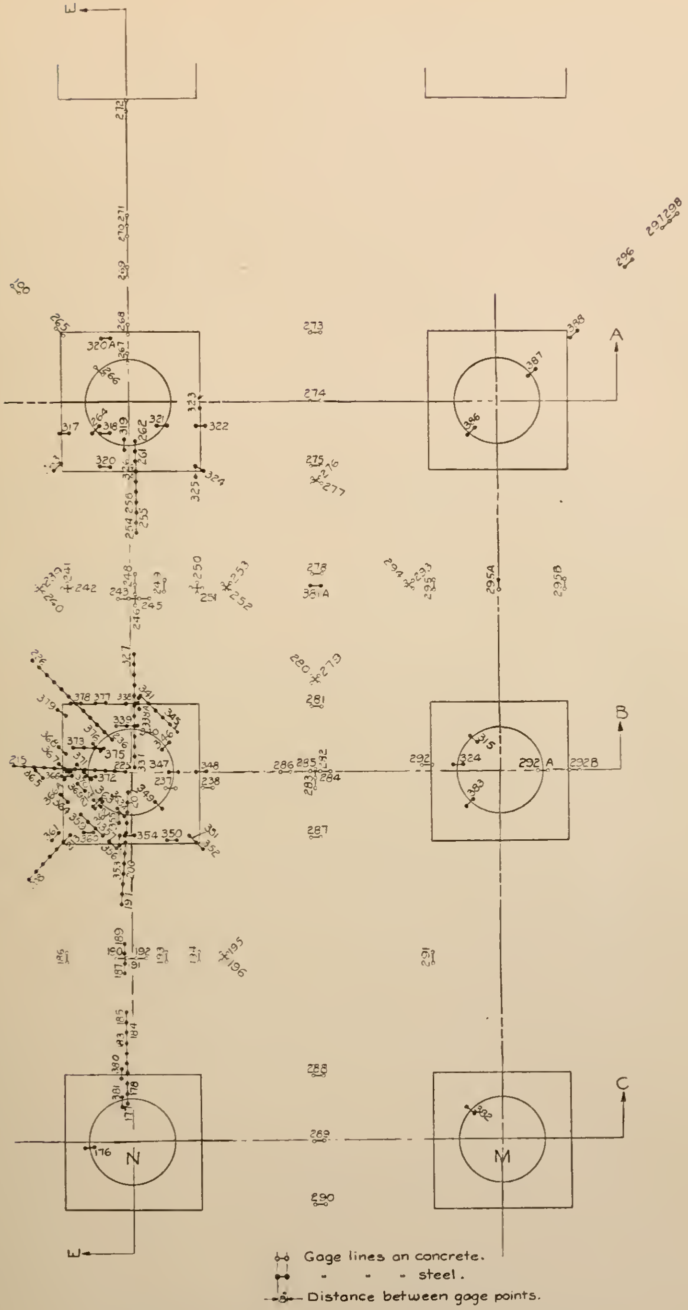


FIG. 27. LOCATION OF GAGE LINES



ON UPPER SURFACE OF SOO TERMINAL TEST FLOOR.

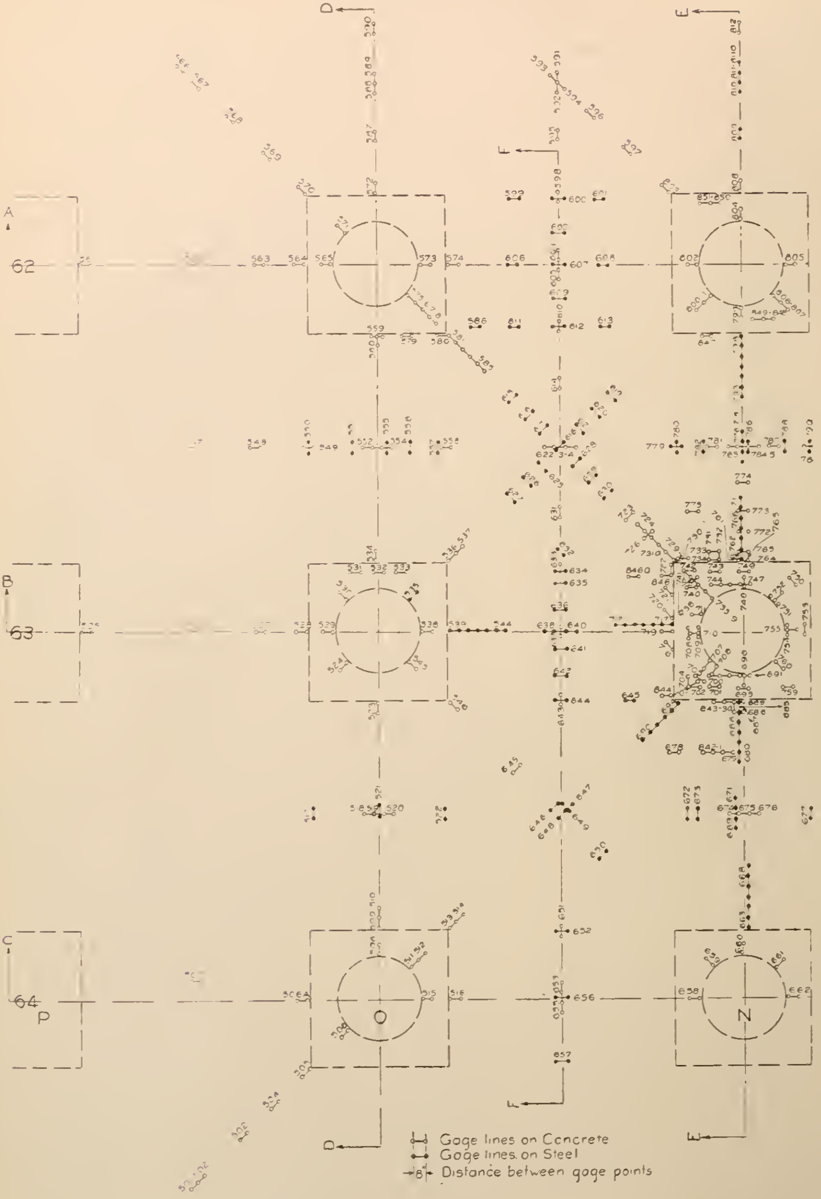
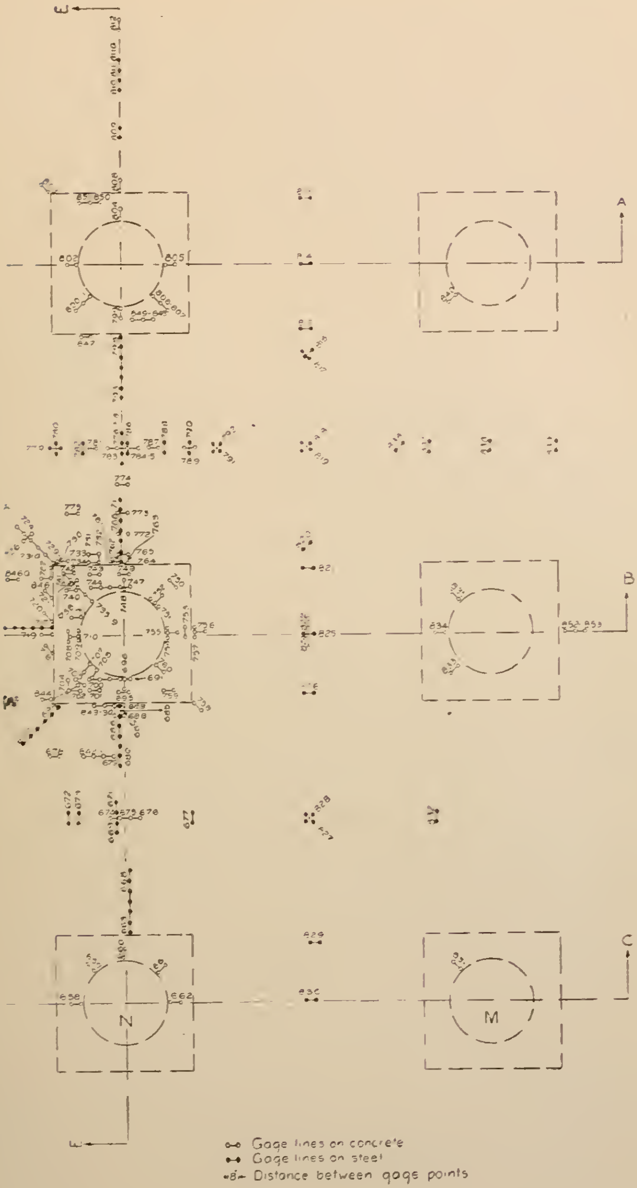


FIG. 28. LOCATION OF GAGE LINES



ON UNDER SURFACE OF SOO TERMINAL TEST FLOOR.

with load on, and with load removed. The levels were taken by engineers connected with the construction and the work is known to have been carefully done, but the results in some respects seem conflicting and it is quite possible that they do not show the actual changes which took place. It is possible that the bench mark was affected by the movement of the structure under load. As the measurement of deflections of slabs was taken from the floor below, uncertainties about the settlement of the structure will also affect the observations on deflections.

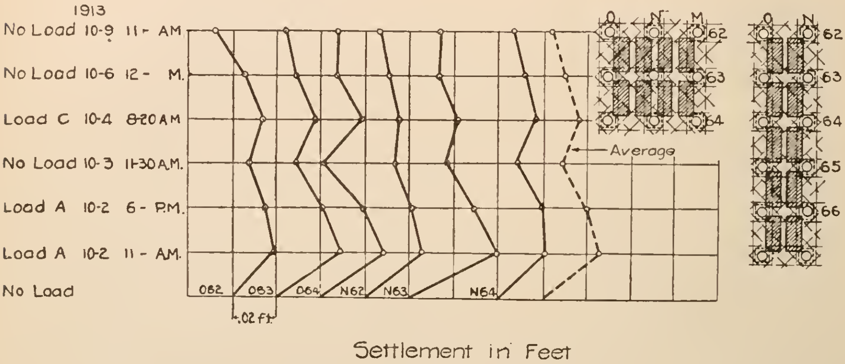


FIG. 29. SETTLEMENT OF COLUMNS SUPPORTING TEST FLOOR OF SOO TERMINAL STRUCTURE.

The data of settlement and deflections are given in Fig. 29 and 30. The maximum settlement indicated by the level notes was 0.04 feet which appears to have occurred at Column N63, the central column of the four-panel test area. The almost complete recovery from this settlement on removal of the load and the large amount of recovery during the time from 11:00 a. m. to 6:00 p. m., October 2, while the full load was in place, seem improbable and hence raise serious question as to whether as much settlement as that indicated above was present at any time. Besides, the maximum measured deflection of 0.14 in. is so small as to make this value also seem not to represent the actual deflection. However the settlement is not large for buildings on Chicago soil. There are many inconsistencies between the deflections and settlements observed at different places and under different loads, and the uncertainty of the conditions makes it futile to attempt to account for the recovery from settlement or otherwise to interpret the data. However, Fig. 29 and 30 are given for the purpose of record. It is evident that even slight settlement under load will modify the distribution and

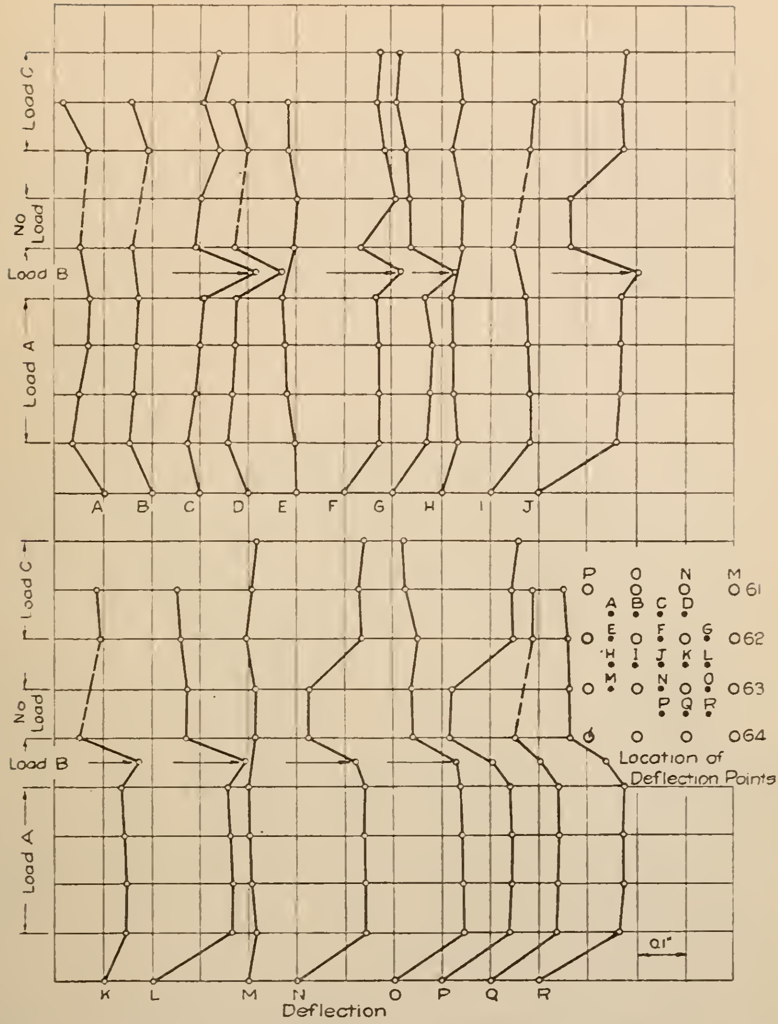


FIG. 30. DEFLECTION OF TEST FLOOR AT POINTS MIDWAY BETWEEN COLUMNS OF SOO TERMINAL STRUCTURE.

the amount of the stresses in the structure. The reference to settlement is given in the discussion as an aid in interpreting the observed phenomena and not in any way as having a bearing on the stability of the structure.

18. *Deformation in the Slab.*—The diagrams in Fig. 31 to 34 give the results of measurements taken to determine the distribution of deformations in the slab along the edges of panels, the measurements being taken in the direction of these edges, that is, along or parallel

to the sections shown in these figures. The diagrams in Fig. 35 and 36 give the results of measurements taken to determine the distribution of deformations across the center line of panels (section F-F) on the top and on the bottom of the slab. In this case the measurements were taken normal to the sections shown.

The deformations measured in the slab for the several loads were very small, in general, much smaller than were anticipated. As a result the data are not such as to throw much light on several of the questions on which information was sought, as for example, the effect of shifting the load a short distance. In many cases the changes in length were smaller than the possible errors of observation, and such results are therefore meaningless. Besides, with the low stresses found, the tensile strength of the concrete must have played an important but uncertain part in the resistances developed. To complicate the matter further, unequal settlement of the footings evidently modified the action of the structure. Altogether, it may be said that on account of the smallness of the deformations and complications with the tensile resistance of concrete and the settlement of the foot-

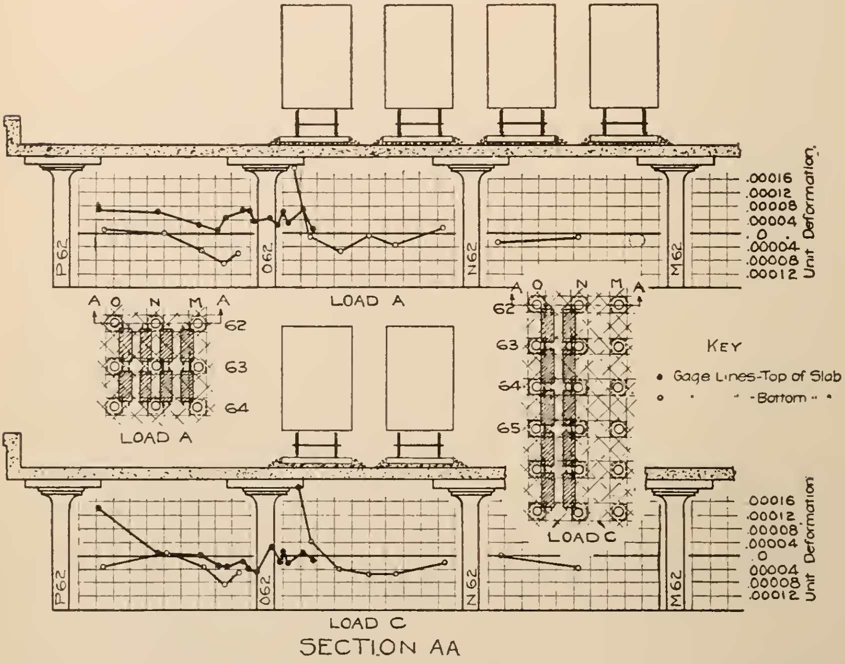


Fig. 31. Deformation along Section A-A (Normal to Tracks) of Soo Terminal Test Floor.

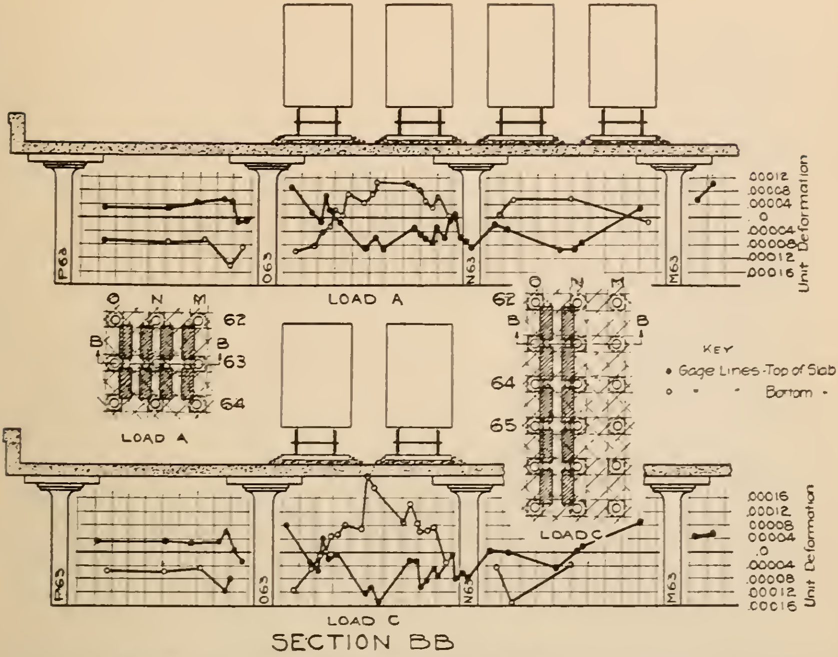


FIG. 32. DEFORMATION ALONG SECTION B-B (NORMAL TO TRACKS) OF SOO TERMINAL TEST FLOOR.

ings little conclusive information was obtained on the main questions connected with the action of the slab, except that the smallness of the stresses developed may be considered as important in judging of the action of such flat slab structures. In some respects the observations give indications of the way in which the stresses are distributed, but frequently they are so masked by uncertainties that comparison cannot be made with any degree of confidence. However, comment will be made as best it can.

Some of the difficulties may be seen from the following examples. For the negative bending moment around the central column (column N63, Fig. 32), with four panels loaded, the compressive stresses in the concrete due to both dead and live loads would not be expected, from calculations made according to current practice, to exceed 500 lb. per sq. in. If the concrete on the tension side remained intact, the tensile stresses in the concrete in this region would be less than this amount and the stresses in the reinforcement would be correspondingly low. As no cracks were observed in these positions and as the observed tensile deformations here were very small, it seems probable that the tensile strength of the concrete was sufficient

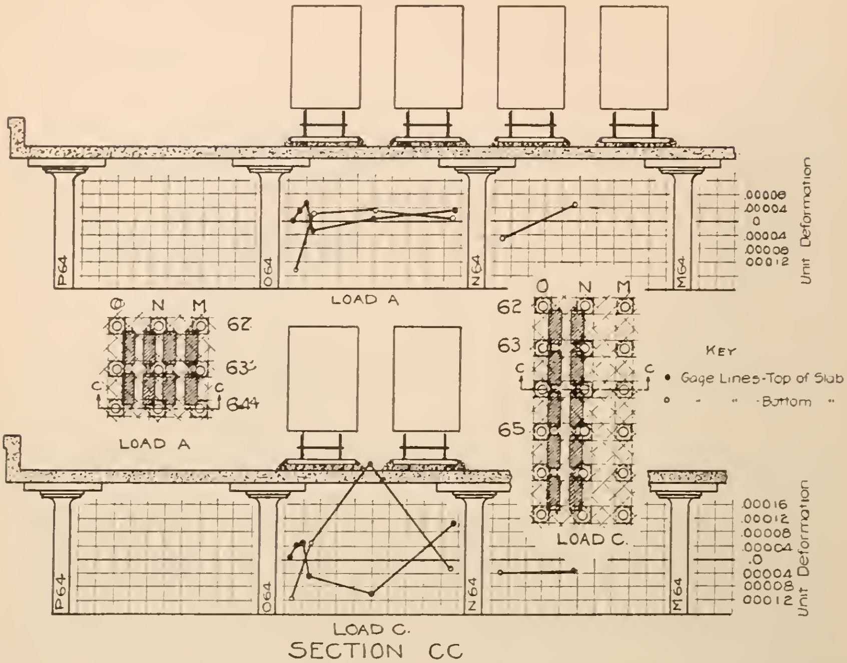


FIG. 33. DEFORMATION ALONG SECTION C-C (NORMAL TO TRACKS) OF SOO TERMINAL TEST FLOOR.

to prevent cracking in regions of negative moment even without relief of the stresses here by settlement of the column. The differences in depth of embedment of the reinforcing bars in the different bands also added to the uncertainties. In view of these difficulties, no quantitative values of the negative bending moment developed in the slab around the central column can be given. Again, for the four-panel loading, the highest deformation measured in any of the reinforcing bars of the bands at the central column corresponded to 2700 lbs. per sq. in. tension in the bar and to, say, 270 lb. per sq. in. tension in the concrete on the upper surface of the slab and the range was from this value to a small compressive deformation. It is evident that with such stresses as these it would be of little value to try to make comparison of stresses in the different bands or in different bars of the same band.

The deformations in the reinforcing bars of the rectangular bands at the bottom of the slab half way from the central column to the adjoining column were larger than those in the top of the slab near the central column (see Fig. 32). The settlement of columns may have had some influence on this. Even here the highest deformation

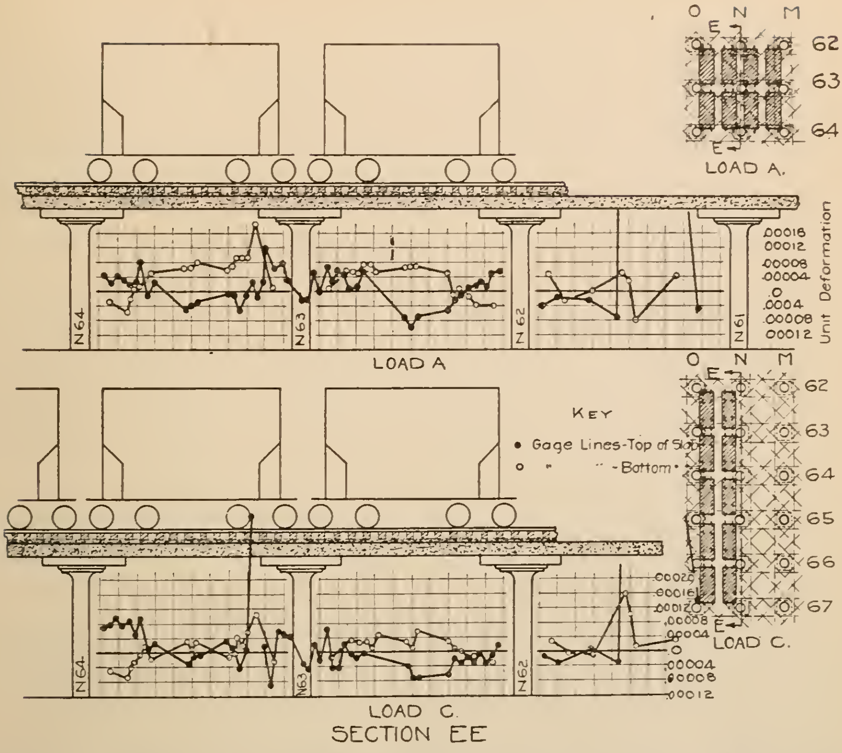


FIG. 34. DEFORMATION ALONG SECTION E-E (PARALLEL TO TRACKS) OF SOO TERMINAL TEST FLOOR.

measured in any bar under load A corresponds to a stress of less than 4000 lb. per sq. in., and most of the bars show values much smaller than this.

In the four-panel loading (see plan of load A, Fig. 26) the bars in a band of reinforcement which has load on both sides of it (as the rectangular band from Column N62 to N63 and on to N64, and also the rectangular band from Column M63 to N63 and on to O63) may be expected to take their full share of the stresses due to the load. In bands at the edge of the loaded area (as in the band extending from Column O62 to O63 to O64) the bars at the edge of the band which lie within the loaded area will be stressed higher than the bars at the other edge of the same band which lie outside the loaded area, but the latter bars will carry a considerable proportion of stress, and the stresses developed in the former bars will be materially less than if the latter bars were not present. This condition will need to be taken

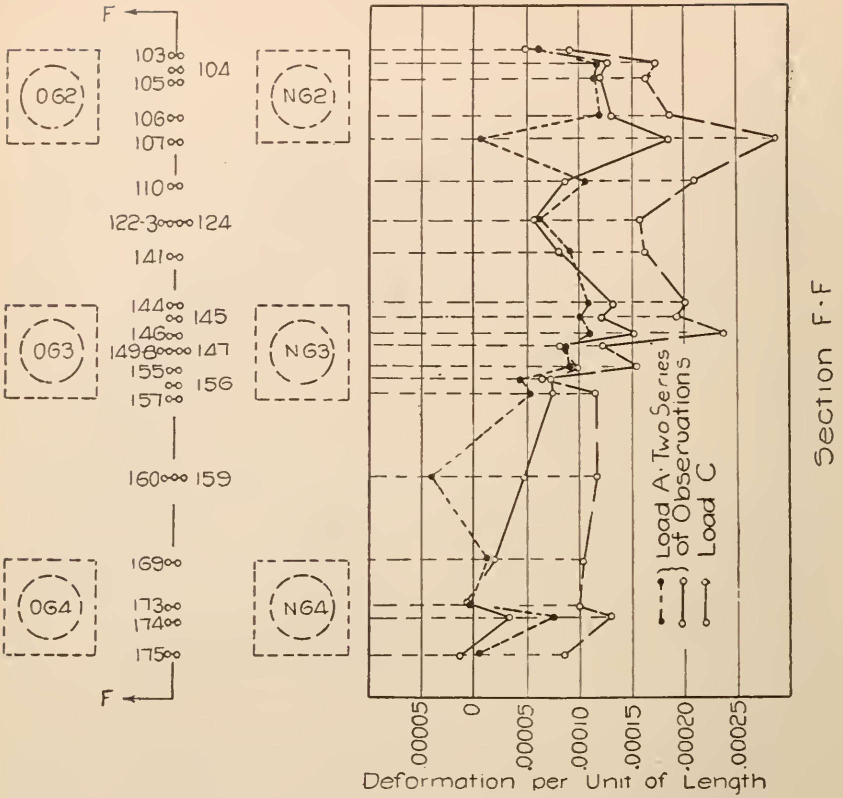


Fig. 35. Effect of Change from Load A to Load C on Deformation in Gage Lines Crossing Section F-F on Top of Soo Terminal Test Floor.

into consideration in comparing the deformations found when the load is removed from a panel but is left on the adjoining panel.

In the change from a four-panel loading (load A) to a load on a row of panels (load C), besides the condition just referred to, a change may be expected in the stresses in the bands which run across the row of loaded panels. Due to the bending of any column under the new arrangement of load (like Column N63, Fig. 32, which in the four-panel loading was surrounded by a symmetrical load) and with the resulting change in slope at the top of the column, and due also to whatever reverse bending occurs in the slab outside the loaded area, it may be expected that for a band such as the one running from Column N63 to O63 the stresses at Column N63 (due to negative bending moment) would be less for the row of panels loaded than it

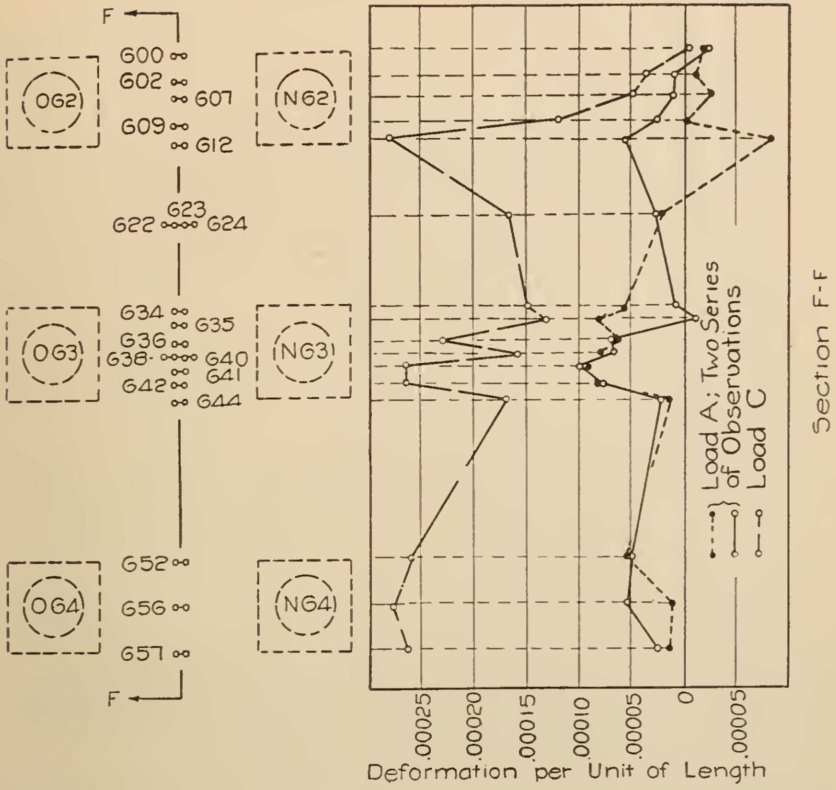


Fig. 36. Effect of Change from Load A to Load C on Deformation in Gage Lines Crossing Section F-F on Bottom of Soo Terminal Test Floor.

had been for the four-panel load, and that at points half way between Columns N63 and O63 the stresses (due to positive bending moment) would show an increase over those for the four-panel load. Section C-C, Fig. 33, shows the effect of loading a row of panels and of extending the load entirely across a section instead of loading on only one side of the section. This effect is seen in the considerable increase in deformation midway between Columns N64 and O64 (see also Fig. 35 and 36.)

When the loading was changed from load A to load C the measurements at Column N63 showed very little change in the stresses in the bars of the band which extends from Column N63 to O63 (see Fig. 32). At a point half way between these columns the stresses in the bars increased, averaging about 1500 lb. per sq. in. with load A and 6000 lb. per sq. in. with load C (see Fig. 32 and 36.) Taking into account the stresses produced by the dead load, it seems that the tensile resistance of the concrete must have been exceeded with load C, and this makes impracticable the use of the resulting stresses for comparing quantitatively the bending moments developed by the two loads.

For the band running from Column O62 to O63 the average stress observed in the reinforcement at mid-span under the four-panel loading (load A) was only 500 lb. per sq. in. and under the loading of a row of panels (load C) 900 lb. per sq. in. For the band running from Column N62 to N63 (see Fig. 34), the average stress observed at mid-span under load A was 2000 lb. per sq. in. and under load C 500 lb. per sq. in.

It will be seen that for load A, there is considerable similarity in the distribution of stresses along section B-B (Fig. 32) and section E-E (Fig. 34).

19. *Columns.*—Columns located at the edges of the loaded area showed considerable bending stress. The stresses in columns were

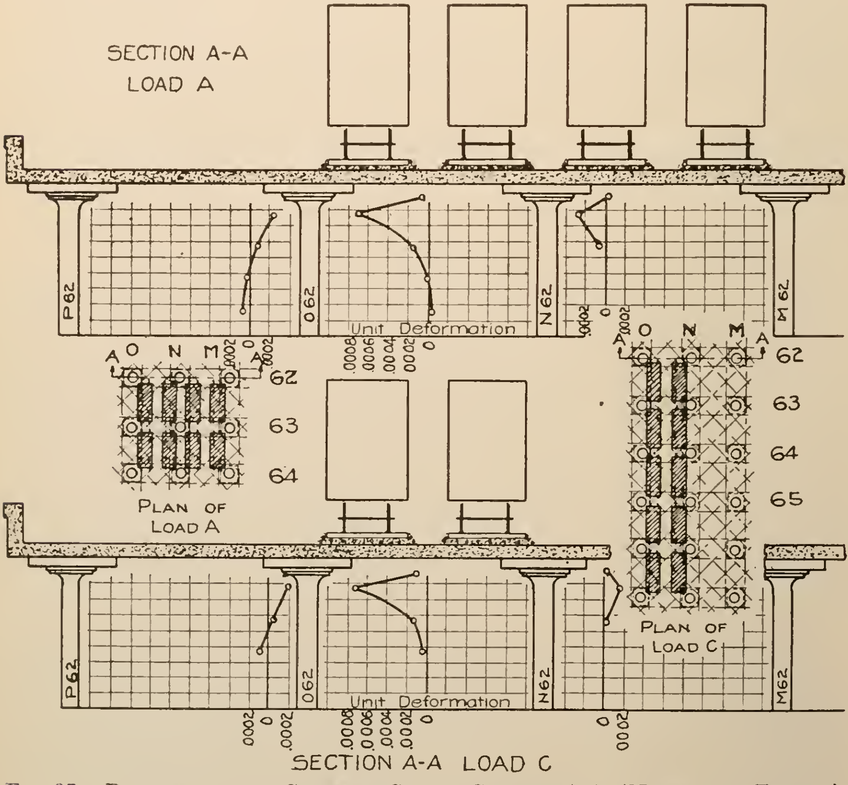


FIG. 37. DEFORMATION IN COLUMNS CUT BY SECTION A-A (NORMAL TO TRACKS) OF SOO TERMINAL STRUCTURE.

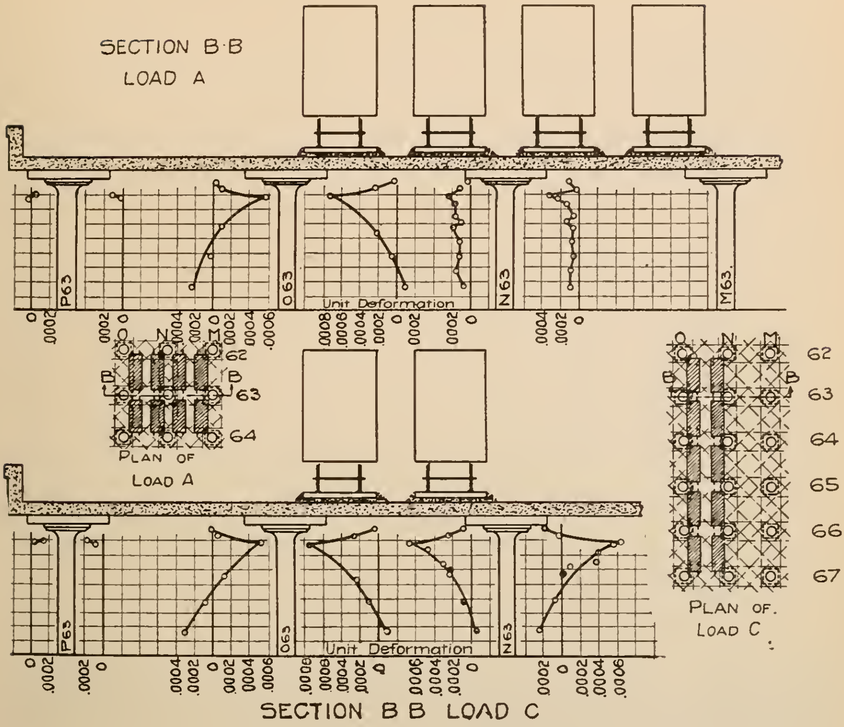


FIG. 38. DEFORMATION IN COLUMNS CUT BY SECTION B-B (NORMAL TO TRACKS) OF SOO TERMINAL STRUCTURE.

higher and more definite than the stresses in the slab. The amount of the deformations in the concrete on opposite sides of the column is shown graphically in Fig. 37 to 41. The large tensile deformations in the columns show that the reinforcing bars were subjected to considerable tensile stress, and even considering the compression due to dead load the tensile strength of the concrete must have been exceeded. This action is also shown by the formation of cracks on the tension side of the columns. Measurements made on the reinforcing bars (not plotted in the figures) show tensile deformations smaller than those observed in the concrete, but still considerable. The distribution of the flexural deformations along the column length is also shown in Fig. 37 to 41. In general the point of zero deformation is lower on the compression side than on the tension side of the column, a difference which may be due in part to the direct compression caused by the test load. Table 4 gives the distances from the bottom of the depressed head to the point of zero deformation on the two sides of their columns and their averages. The distances are

TABLE 4.

POSITIONS OF POINTS OF ZERO DEFORMATION ON COLUMNS.

Column No.	Load	Distance in Inches from Depressed Head to Point of Zero Deformation			Ratio of Average to Column Length
		Loaded side	Unloaded side	Average	
063	A	East 115	West 81	98	.64
063	C	East 105	West 85	95	.62
N63	C	West 106	East 78	92	.60
062	A	East 103	West 103	103	.67
062	C	East 98	West 78	88	.57
064	A	East 108	West 106	107	.70
064	C	East 108	West 100	104	.68
064	A	North 106			
062	A	South 103			
062	C	South 105			

expressed also as proportional parts of the total distance between the bottom of the depressed head and the upper surface of the basement floor. The average of the ratios is 0.64. If the case of a column

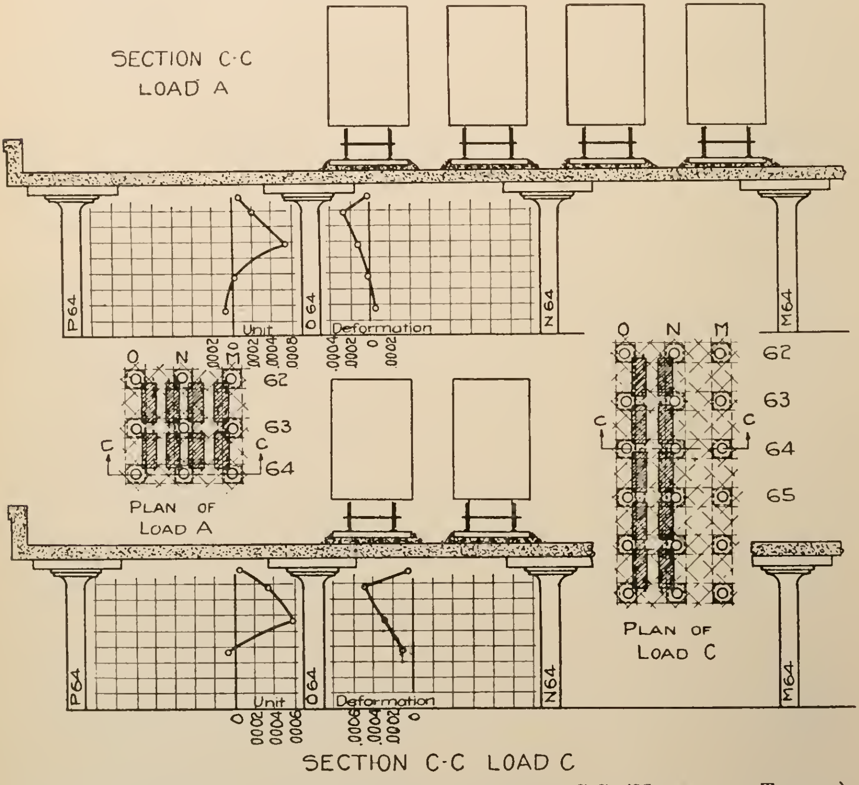


FIG. 39. DEFORMATION IN COLUMNS CUT BY SECTION C-C (NORMAL TO TRACKS) OF SOO TERMINAL STRUCTURE.

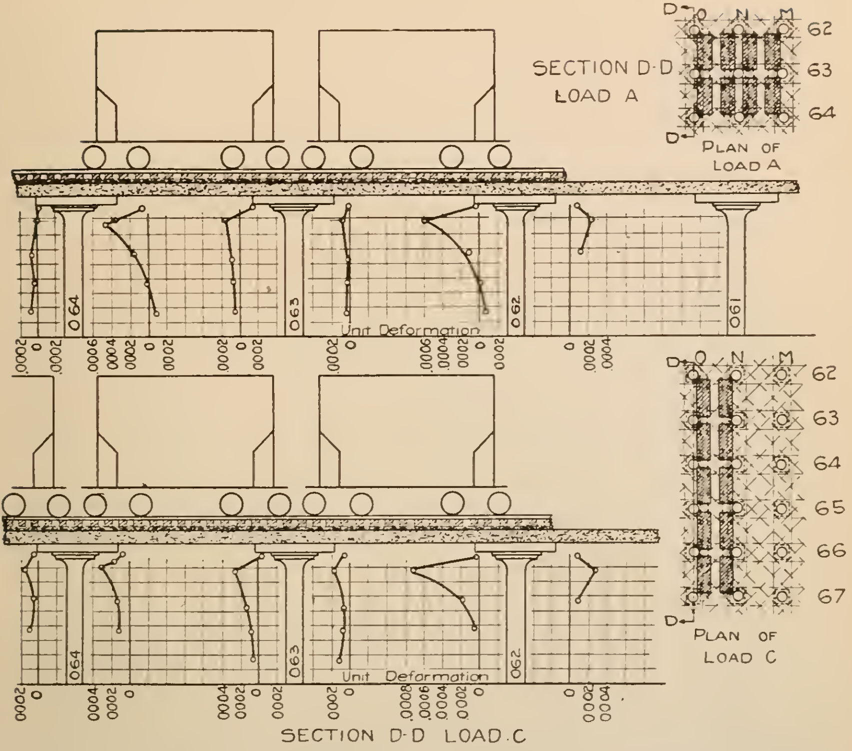


FIG. 40. DEFORMATION IN COLUMNS CUT BY SECTION D-D (PARALLEL TO TRACKS) OF SOO TERMINAL STRUCTURE.

fixed at one end and having a rigid connection with the slab or beam at the other end be subjected to analysis, the point of inflection will be found to be two-thirds of the distance from the point of rigid connection to the point of fixity. The results are in fair agreement with the analysis if the column be considered to be fixed at the basement floor.

Because of the uncertainty of the effect of tensile stresses in the concrete and because the value of the modulus of elasticity of the concrete of the columns is unknown, it is not feasible to calculate the bending moment produced in the columns. It can easily be seen, however, that the bending moment developed in this case is very large, as may be expected from an analysis of a structure made up of thick slabs, and not having the columns continued upward to other stories. The fact that such bending is developed in columns and is shown by measurements may be worth recording.

20. *Cracks.*—The location of small cracks which were observed

in the slab when the four-panel load (load A) was in position is shown in Fig. 42. All these cracks were found on the bottom of the slab. No cracks were found on the top. It is seen that in general they extend in a direction parallel with the rows of columns, displaying also a slight tendency to extend along the diagonal of the panel. Portions of the upper side of the slab were covered with ballast and no examination for cracks could be made in these places, but as the ballasted regions occupied the central portions of the panels where compression should be found cracks would not be expected at these places. The top of the slab was available for examination over all columns, and no cracks could be found in such positions although a careful search for them was made. It is seen that the cracks referred to are in regions where compression would be expected. Their appearance may be explained by even a slight settlement of columns; those on the under side of the slab close to the depressed heads of some of the columns at the edge of

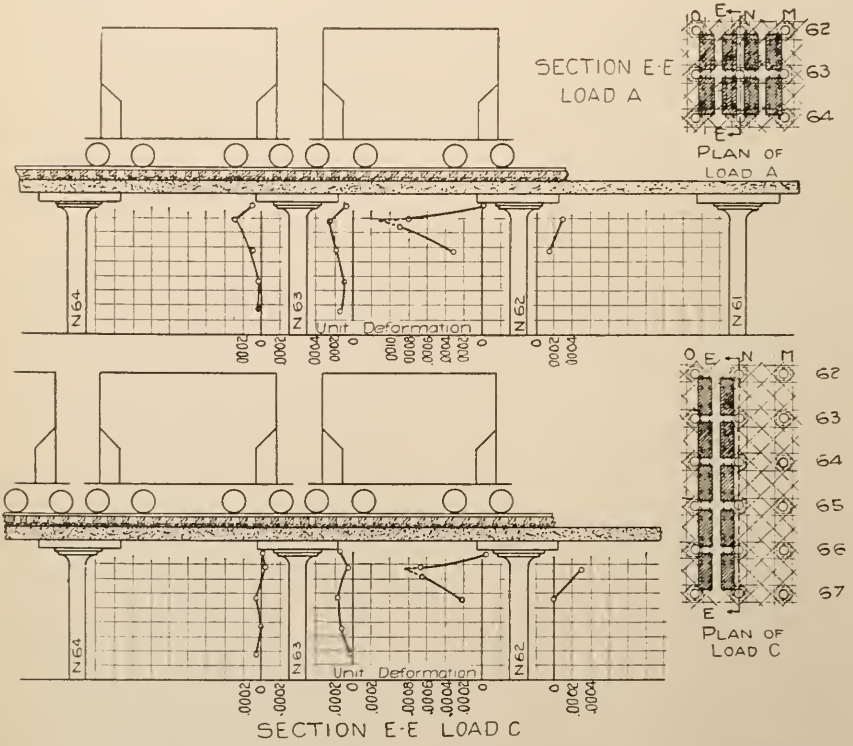


FIG. 41. DEFORMATION IN COLUMNS CUT BY SECTION E-E (PARALLEL TO TRACKS) OF SOO TERMINAL STRUCTURE.

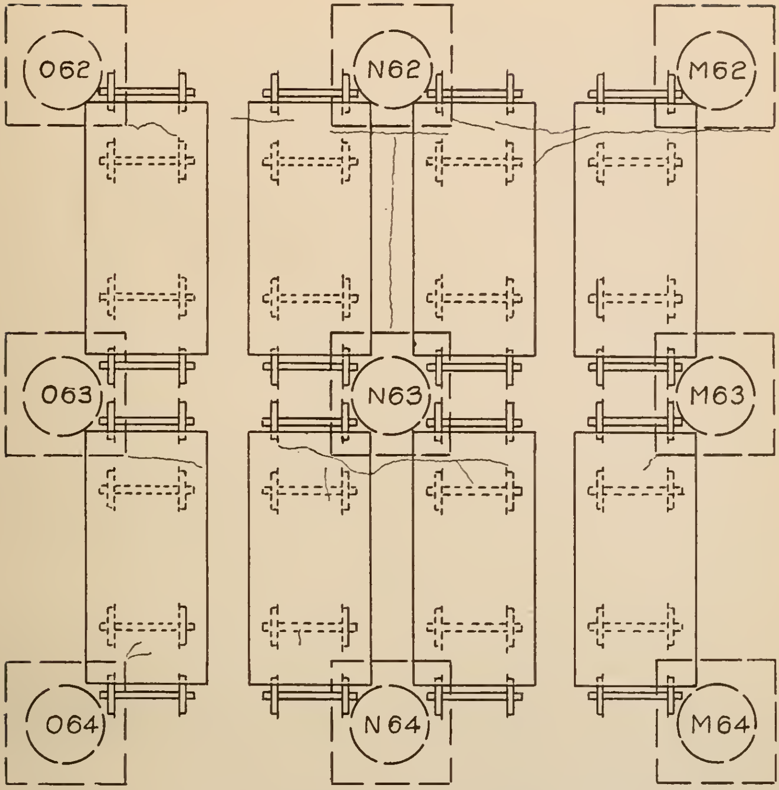


FIG. 42. LOCATION OF CRACKS IN SOO TERMINAL TEST FLOOR.

the loaded area may be explained if there were settlement of these columns, and those which were found on the under side of the slab close to the depressed head of the central column if there were a settlement of this column larger than that of any other.

Under Load A, cracks were observed in several columns located on the edges of the loaded area. These were found on the column capital a little above its bottom edge and on the side away from the loaded area. It was not easy in every case to distinguish between these cracks and other cracks found on the columns which evidently were due to expansion and contraction of the floor caused by changes in temperature, but a survey of the temperature cracks disclosed that they were systematically arranged and could be distinguished from the cracks which were due to eccentricity of loading and which were indicative of bending moment developed in the columns as already discussed under 19. *Columns.*

21. *Measurement of Dead Load Stress.*—An effort was made to obtain information on the amount of stress developed in the slab by the weight of the floor upon the removal of the forms. The test was made upon a 24 by 24-ft. panel. As the floor slab is very thick it cannot be expected that the stresses developed by the dead load would be very high, but the test indicates that it is practicable to measure the effect produced when the forms are removed.

The forms were left standing until the time of the test. Gage lines were placed where readings were desired and initial or zero readings were taken. The forms were then removed and other sets of readings were taken when the slab was supporting its own weight. In order to place gage lines on the bottom of the slab before the forms were removed, it was necessary to cut holes in the forms. As the deformations were expected to be small, special care was taken to insure correctness of the observations. Both initial readings and final readings on all gage lines were taken twice by each of two observers.

TABLE 5.

AVERAGE UNIT-DEFORMATION UNDER DEAD LOAD.

Plus indicates extension and minus indicates shortening.

Location	Average Observed Unit-deformation
Bottom; outside of capital; rectangular direction	—.000042
Top; outside of capital; rectangular direction	+.000013
Bottom; outside of capital; diagonal direction	—.000024
Top; outside of capital; diagonal direction	+.000005
Bottom; outside of capital; Depressed head, rectangular direction	—.000013
Top; outside of capital; Depressed head, rectangular direction	+.000005
Bottom; midway between columns; rectangular direction	+.000013
Top; midway between columns; rectangular direction	—.000048
Bottom; midway between columns; diagonal direction	+.000014
Top; midway between columns; diagonal direction	—.000032

The averages of unit deformations found are given in Table 5. It is seen that for measurements at gage lines located near the column capital the deformations were greater on the bottom of the slab than on the top, while for the locations midway between columns, the deformation was greater on the top of the slab than on the bottom; in other words, the deformation was greater in the regions of compression than in the regions of tension. From the amounts of the deformations measured it may not be expected that the concrete had failed in tension. In judging of the amount of these deformations it must be considered that there is a possibility that some of the dead weight is assumed by the slab in advance of the removal of the forms because of the shrinkage of the forms or of very slight settlement. The fact that the compressive deformations are higher than the tensile deformations may indicate that arch action played a part in the support of the dead load.

22. *Summary of Results.*—The following comments may be made on the test of the Soo Terminal Building:

1 The deformations measured in the steel and in the concrete of the slab were very small, in many cases smaller than the possible errors of observation. The tensile stresses developed in the reinforcement being so small, the tensile strength of the concrete must have played a very large part in the bending resistance of the slab. It appears also that uneven settlement of the footings under the applied load modified the action of the structure.

2 With the development of such low stresses and the uncertain action due to uneven settlement of the footings, the results of the test may not be used to throw light on the mechanics of the slab and on the distribution of stresses over the slab in the way it was hoped they could be used. As would be expected, an increase in the stress in a cross band under the loaded area was found when the load was changed from four panels in the form of a square to five panels in a row.

3 Marked bending was found in the columns at the edge of the loaded area. The point of inflection of the elastic curve of flexure of the columns was about two-thirds of the distance from the bottom of the depressed head to the upper surface of the basement floor, which is the location to be expected for a column fixed at the bottom and having a rigid connection with the slab at the top.

4 The location of the cracks found on the under side of the slab indicates that stresses in a structure subject even to slightly uneven settlement of footings may be of different character from those found by the ordinary assumptions of design.

5 The measurements made to determine the stresses produced in the slab by dead load upon striking centers indicate that while the deformations were small in the test made, it is practicable to measure the deformations in a reinforced concrete structure due to dead load.

IV. THE SCHULZE BAKING COMPANY BUILDING TEST.

23. *The Building.*—The building of the Schulze Baking Company is located at 55th St. and Wabash Ave., Chicago. It is five stories

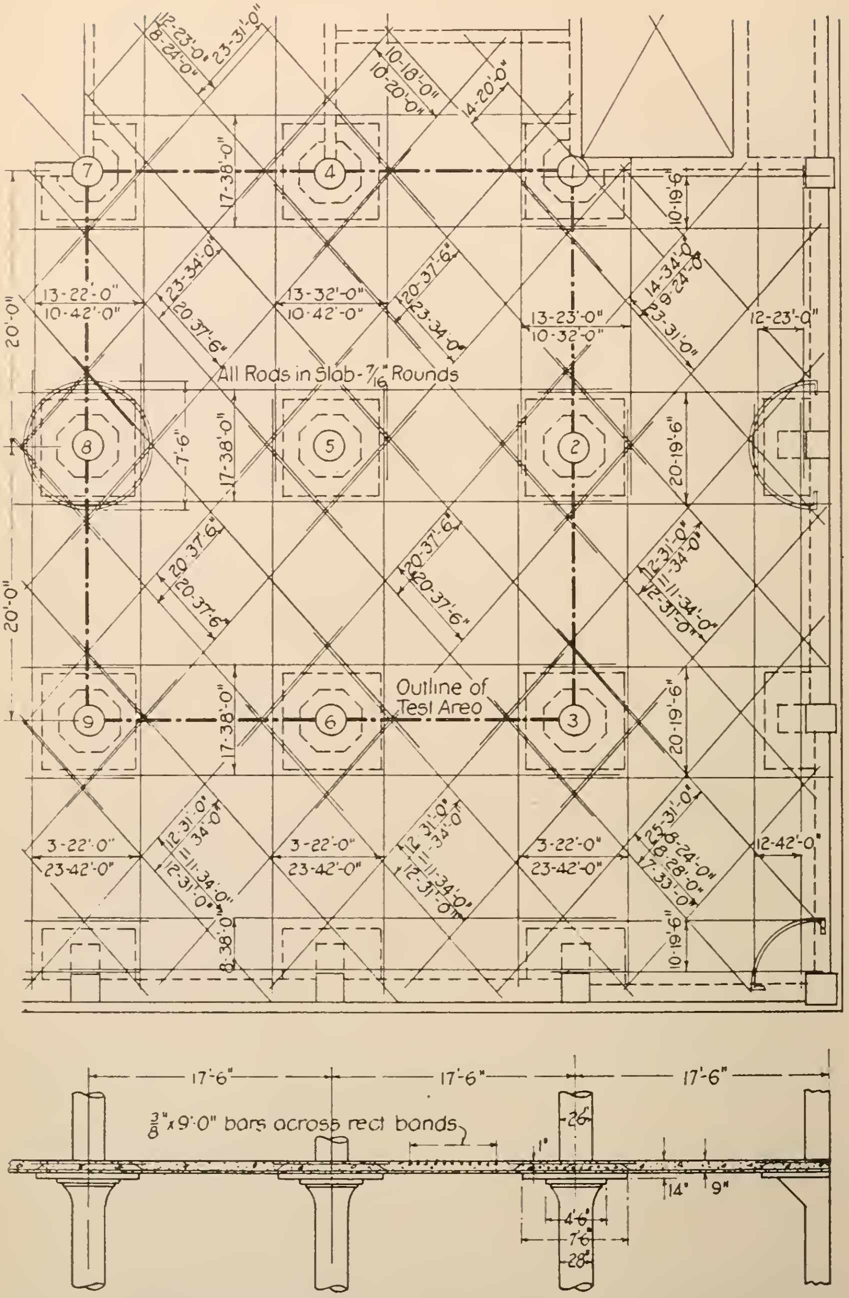


Fig. 43. Dimensions and Reinforcement Plans of Test Floor of Schulze Baking Company Building.

in height and covers an area 298 ft. 4 in. by 160 ft. The floor space is divided into panels 17 ft. 6 in. by 20 ft. The test load was applied to panels of the second floor (one story above sidewalk level). The floor construction in this part of the building consists of a four-way reinforced concrete slab nominally 9 in. thick in the central portion of the panel and 14 in. thick throughout the area of a 7-ft. 6-in. square surrounding each column. It was designed for a live load of 300 lb. per sq. ft.

The columns below the second floor are circular, 28 in. in diameter, and terminate at the top in octagonal bell-shaped heads

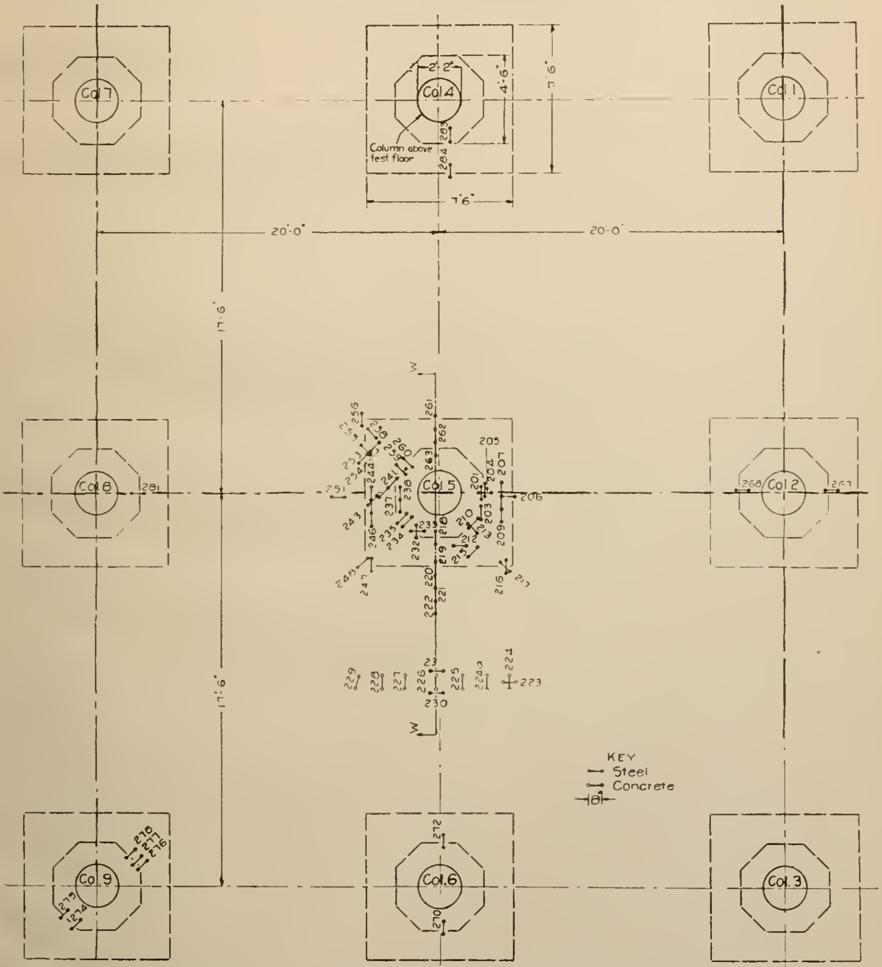


Fig. 44. Gage Lines on Upper Surface of Test Floor of Schulze Baking Company Building.

54 in. in diameter at their upper ends. The columns between the second and third floors are circular, and are 26 in. in diameter.

The distribution of the reinforcement in the portion of the floor to which the test load was applied is shown in Fig. 43.

Two-thirds of the slab rods of the rectangular bands are located in the top of the slab where they pass over the columns. These rods drop down to the bottom of the slab at a point about two-tenths of the panel length from the center line of the columns. At the corresponding point relative to the next column the rods are bent up again and pass over the column in the top of the slab, ending 9 in.

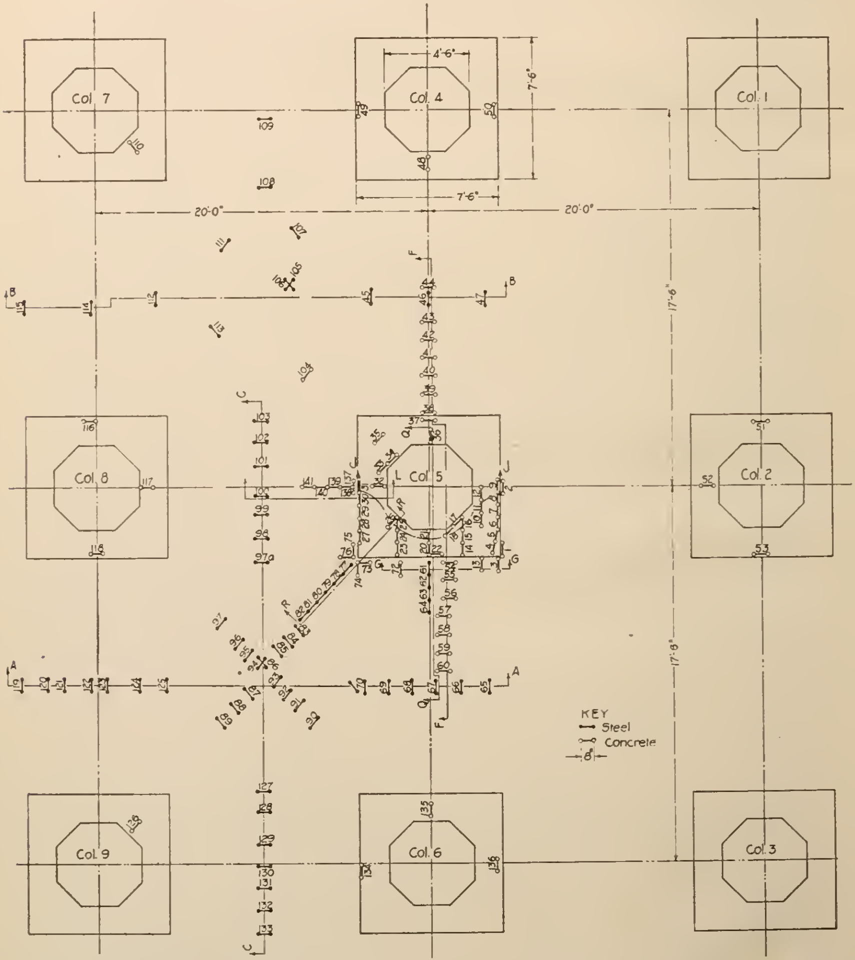


FIG. 45. GAGE LINES ON UNDER SURFACE OF TEST FLOOR OF SCHULZE BAKING COMPANY BUILDING.

to 18 in. beyond the center line of columns. The remaining one-third of the bars of the rectangular bands extend through the bottom of the slab throughout the length of the bars. All diagonal rods bend up to pass over the column in the top of the slab, and extend 5 ft. 3 in. beyond the center lines of columns. Measurements taken at the time of the test indicate an average depth from the compression surface to the center of gravity of the reinforcement of 10.15 in. for positions of negative moment and 7.45 in. for positions of positive moment.

To prevent concentration of cracks along the center line of the band of reinforcement from column to column, $\frac{3}{8}$-in. bars 9 ft. long spaced about 12 in. apart were placed across the boundary line between panels.

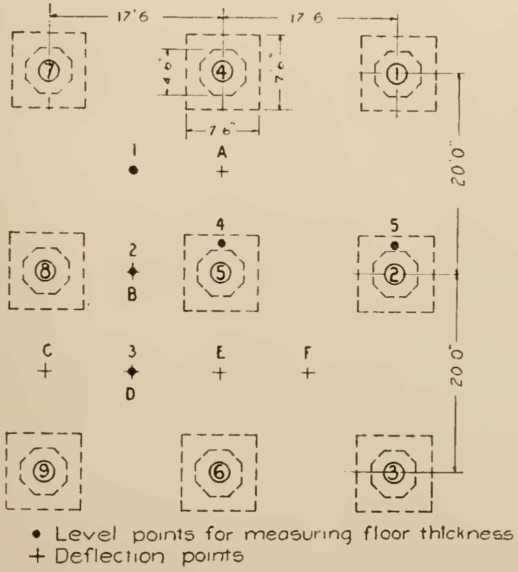


FIG. 46. LOCATION OF POINTS FOR MEASUREMENT OF THICKNESS AND OF DEFLECTION OF TEST FLOOR IN SCHULZE BAKING COMPANY BUILDING.

The concrete was placed late in October, 1913, and there was considerable cold weather during the time of hardening. For this reason it was expected that the concrete would not show up as well as concrete poured and set under the more favorable conditions of summer, and because of the unfavorable conditions a little more time than usual was allowed before making the test. At the time of the test it appeared that there still was considerable moisture in the concrete, but no unfavorable indications were found in the concrete.

24. *The Test.*—The period of preparation for the test covered the time from January 13 to January 19, 1914. Deformations were measured in 123 gage lines on the reinforcement of the slab, 82 gage lines on the concrete of the slab, and 58 gage lines on the concrete of the columns. The gage lines on the upper and under surfaces of the slab are shown in Fig. 44 and 45. All the strain gage readings were taken by Mr. Slater. Deflections and floor thicknesses were measured in five places, the locations of which are shown in Fig. 46. The floor thicknesses for the various points in order were as follows: (1), 9⅛ in.; (2), 8 3/16 in.; (3), 8 11/16 in.; (4), 14⅛ in.; (5), 14 7/16 in. This gives an average thickness for the thin portion of the floor of 8.87 in. and for the thick portion of 14.28 in.

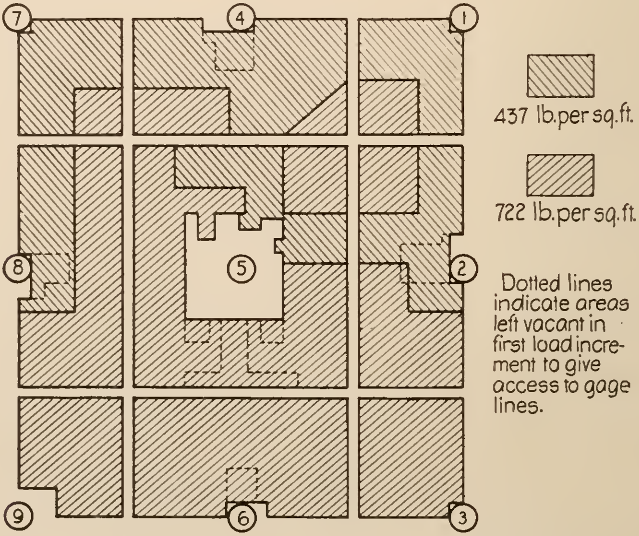

⑦
④
①
⑧
⑤
②
⑨
⑥
③
437 lb. per sq. ft.
722 lb. per sq. ft.
Dotted lines indicate areas left vacant in first load increment to give access to gage lines.

FIG. 47. PLAN SHOWING DISTRIBUTION OF LOAD ON TEST FLOOR OF SCHULZE BAKING COMPANY BUILDING.

Brick was used as the loading material. Previous to the test its weight was determined by weighing a number of wheelbarrow loads, noting both the number of bricks and measuring the cubic contents when stacked. The weight per cubic foot was found to be 96 lb. The load was applied to four panels as shown in Fig. 47, the brick being stacked in piers. Fig. 48 is a view of the load in place. On account of shortages of brick the load was not always uniformly distributed over the floor at the time of taking the readings. Table 6 shows the loads at which strain gage readings were taken and indi-

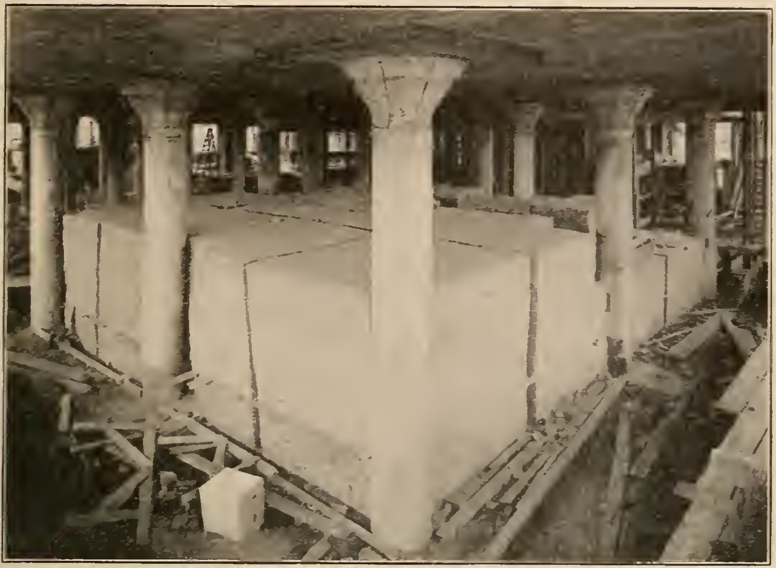


FIG. 48. VIEW OF TEST LOAD ON FLOOR OF SCHULZE BAKING COMPANY BUILDING.

cates the nominal loads used in plotting the load-deformation curves.

TABLE 6.

LOADING OF FLOOR IN SCHULZE BAKING COMPANY BUILDING.

Stage of Loading	Height in inches	Courses of Brick	Load lb. per sq. ft.
1	29	12	189
2	48¼	20	319
3	65	27	437
4	106	44	722

The maximum load was 722 lb. per sq. ft. (twice the design live load plus the dead load), except for a portion of the two north panels (Fig. 47) which had 437 lb. per sq. ft. The deficiency was due to a shortage of material, but the load on these panels was distributed with a view to making the material at hand as effective as possible in producing bending moment. It is believed that the moment in the fully loaded panels, both at the central column and midway between columns was not less than 90 per cent of the moment which would have been developed if the load had been 722 lb. per sq. ft. over the entire area.

Expansion and contraction of the steel and concrete with change in temperature made complications which partially obscure the results of the test. In an effort to eliminate possible errors from this source, corrections were determined by taking readings on an unstressed gage line in the floor. In spite of the indications that the expansion and contraction of the concrete go through the same cycles of changes as the temperature of the air, corrections made on the basis of these observations do not remove all the inconsistencies from the load-deformation curves. However, it is believed that such errors have been greatly reduced by the corrections used.

25. *Tension in Slab.*—The measured unit-deformations observed in this test were unusually small, both in the steel and in the concrete. Very few unit-deformations were more than 0.0002 and the majority were less than this at the maximum load of 722 lb. per sq. ft. Assuming a modulus of elasticity of 30,000,000 lb. per sq. in. for steel, the stress at a unit deformation of 0.0002 is 6,000 lb. per sq. in.

TABLE 7.

AVERAGE STRESSES IN TENSION REINFORCEMENT AT MAXIMUM LOAD.

Gage Lines	Location and Direction of Gage Lines	Average Stress lb. per sq. in.
203, 208, 216, 232, 237	Long rectangular direction at central column	7200
205, 212, 233	Short rectangular direction at central column	3900
215, 210, 234, 235, 242, 253	Diagonal across loaded area at central column	5400
272, 283	Long rectangular direction at columns at edge of loaded area	4300
268, 281	Short rectangular direction at columns at edge of loaded area	3300
276, 277, 278	Diagonal at column at corner of loaded area	7100
65 to 70, inclusive	Long rectangular direction midway between columns	8700
97a to 103, inclusive	Short rectangular direction midway between columns	6300
90 to 97, inclusive	Diagonal across loaded area midway between columns	3900
83 to 89, inclusive	Diagonal across corner of loaded area midway between columns	3000
127 to 133, inclusive	Long rectangular direction midway between columns on edge of loaded area	4900
119 to 125, inclusive	Short rectangular direction midway between columns on edge of loaded area	3600
267	Short rectangular direction just outside of column at edge of loaded area	1200
274, 275	Diagonal just outside of column at corner of loaded area	2800

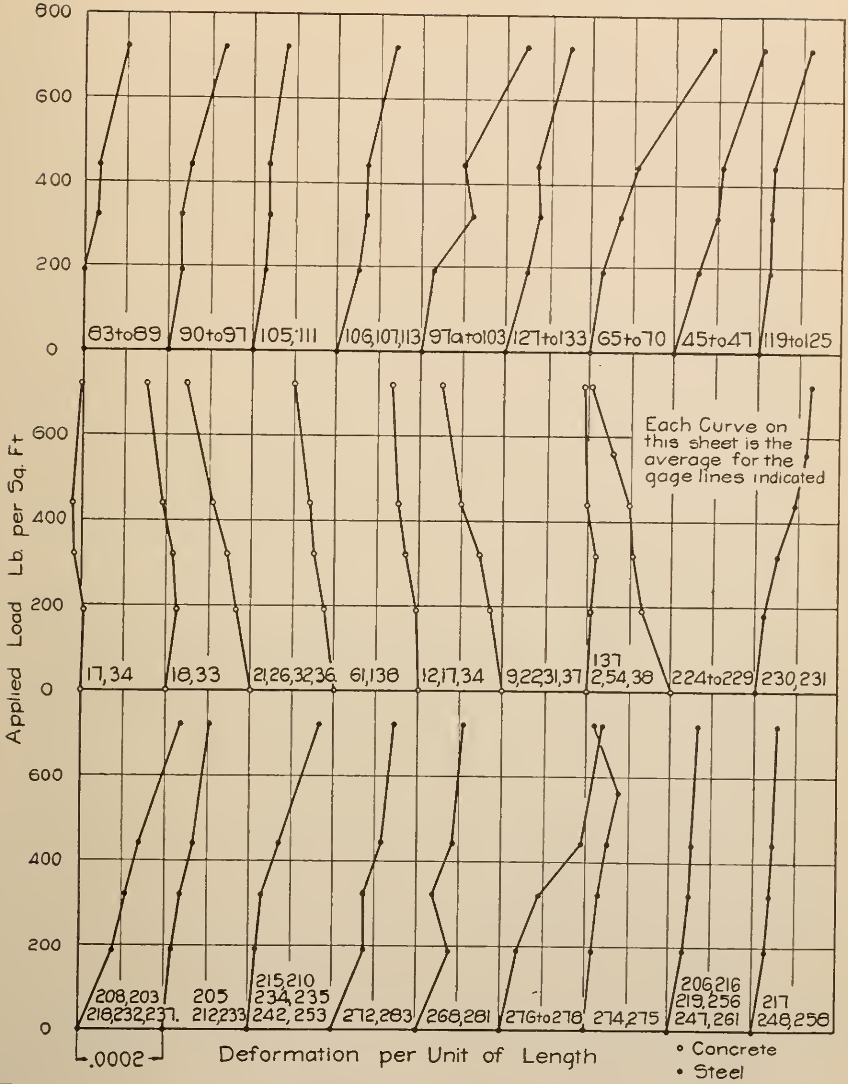


Fig. 49. Average Load-deformation Diagram for Test in Schulze Baking Company Building.

There were only two cases in which a value as high as 10,000 lb per sq. in. was reached.

In Table 7 the average stresses in steel for groups of gage lines at various locations are shown. While these figures may indicate correctly the relation of the stresses of the several groups, it is recognized that in a test where stresses generally were so small, unknown variations in conditions in different parts of the slab, such as the formation of cracks at certain parts, may influence the distribution of stresses more than some of the known elements of the design.

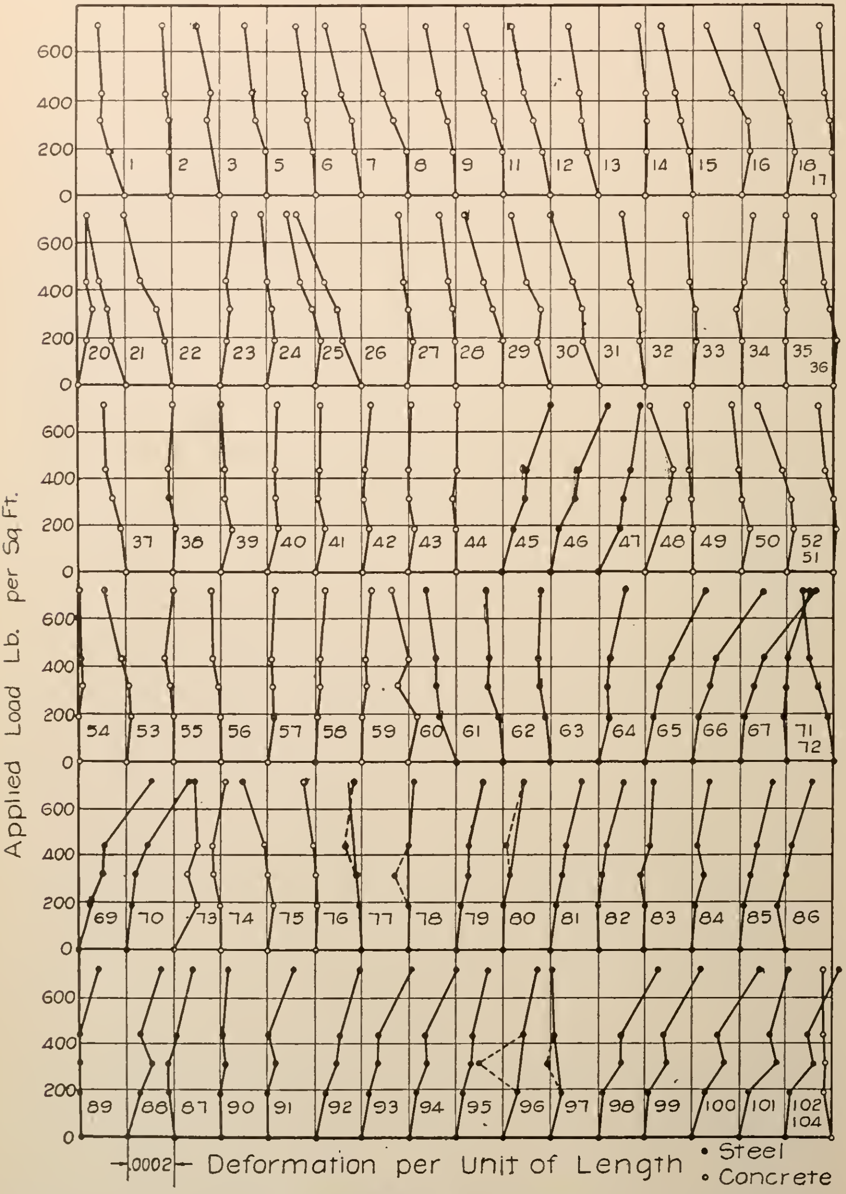


FIG. 50. LOAD-DEFORMATION DIAGRAMS FOR GAGE LINES 1 TO 102 OF TEST IN SCHULZE BAKING COMPANY BUILDING.

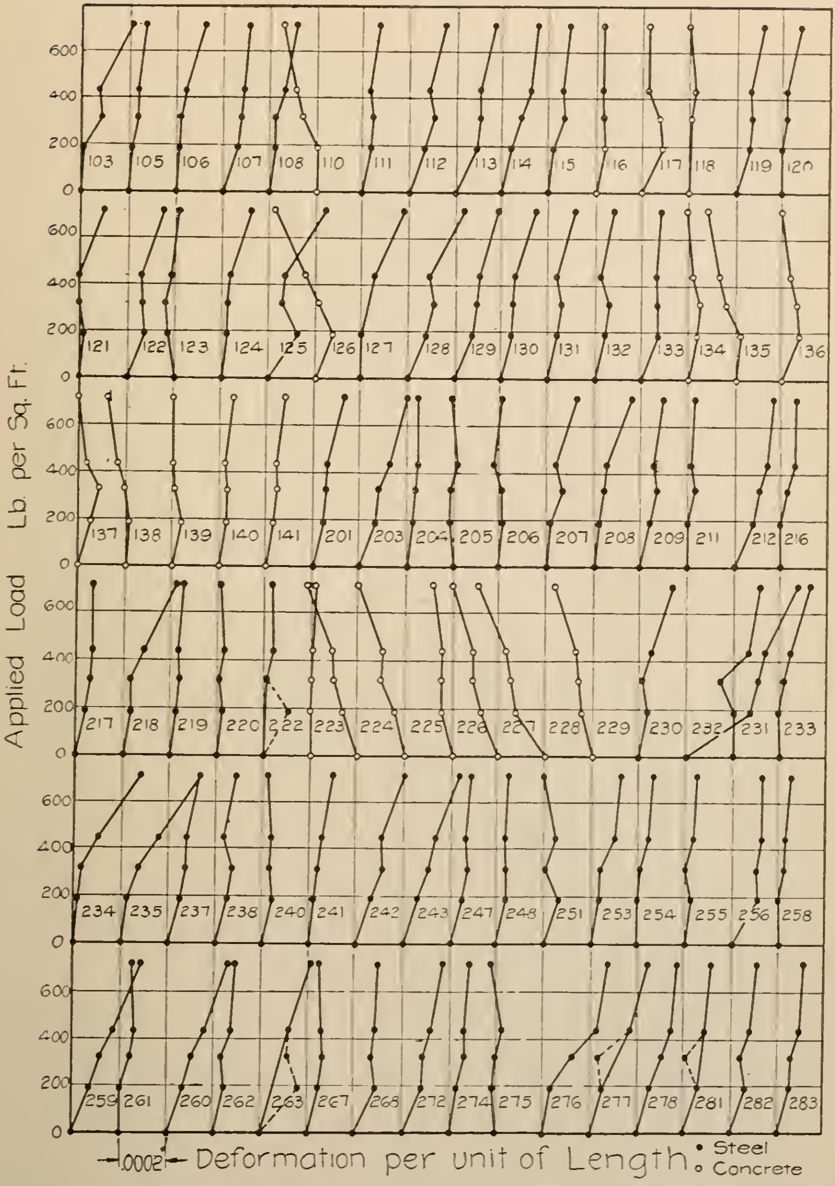


FIG. 51. LOAD-DEFORMATION DIAGRAMS FOR GAGE LINES 100 TO 283 OF TEST IN SCHULZE BAKING COMPANY BUILDING.

For instance, the formation of cracks at certain positions may be a more important factor in stress distribution than the difference in panel length in the two directions. In the use of this table such limitations must be kept in mind. Load-deformation diagrams, averages for groups of gage lines, are given in Fig. 49 and the diagrams for the individual gage lines are given in Fig. 50 and 51. Unit-deformations across or along various sections through the floor are shown in Fig. 52 to 55. The locations of the sections are shown in Fig. 44 and 45.

Table 7 indicates that the stress at the central column and the stress at the column on the edge of the loaded area were greater in the direction of the longer side of the panel than in the direction of

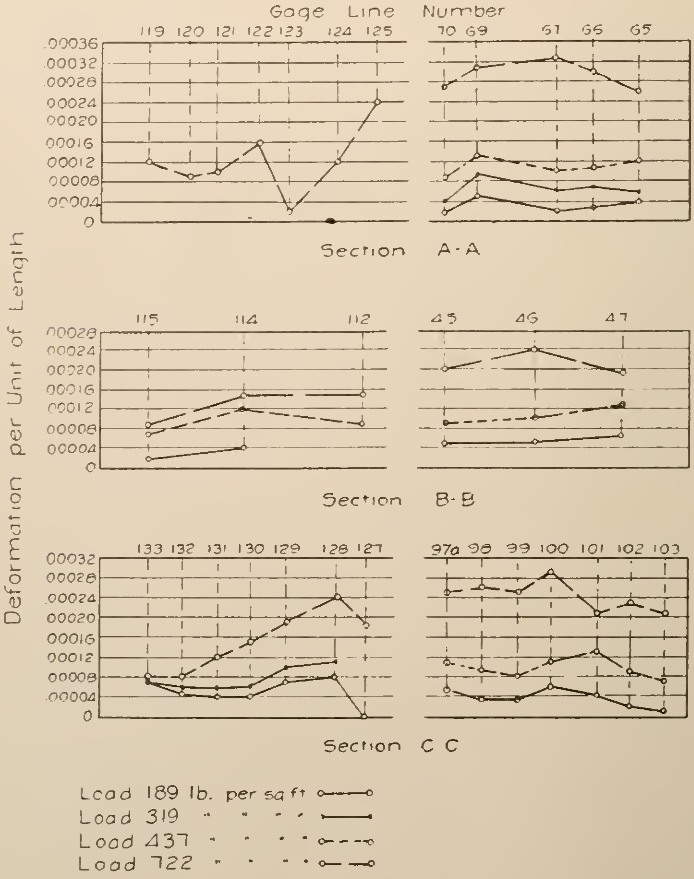


FIG. 52. DEFORMATION IN REINFORCEMENT IN RECTANGULAR BANDS MIDWAY BETWEEN COLUMNS IN TEST FLOOR OF SCHULZE BAKING COMPANY BUILDING.

the shorter side. For the center of the span a similar relation is shown.

A comparison of the deformations at the left hand portions of the sections shown in Fig. 52 with those at the right hand portions shows that the average stress in the rectangular band at the edge of the loaded area was considerably less than that in the corresponding band at the central portion of the loaded area. There is also an indication that the stress increased (somewhat irregularly) from the outer portion of the outer band at the edge of the loaded area toward the inner edge of this band.

In the diagonal bands of reinforcement the stress was about the mean of the average stresses in the two rectangular bands for positions at the central column and at the center of the panel (see Table 7). However, the stress in the diagonal band at the corner of the loaded area was greater than that in the rectangular bands at the columns on the edges of the loaded area. Only a few gage lines were read at the positions near the edge of the loaded area and the stresses found there may not represent the normal conditions for such positions.

The measurements on gage lines 230 and 231 (taken on the bars placed across the boundary line between two panels) indicate an average stress of about 3000 lb. per sq. in. at the maximum load (see Fig. 51). This would indicate that such rods may be effective in accomplishing the purpose for which they were designed, namely, the distribution or prevention of cracks which are likely to occur along the line from column to column. It indicates also that reinforcing bars in such positions may be of considerable value as reinforcement for negative moment. Of course, the reinforcing bars should be carried well beyond the usual position of the point of inflection to provide sufficient anchorage for cases of partial loading.

26. *Compressive Stresses and Unit-Deformations.*—Table 8 gives average values of the compressive unit-deformation in the concrete for representative positions. In tests of girderless slabs high unit-deformations are expected on the bottom of the depressed head close to the capital, and those on the bottom of the thin portion of the slab close to the edge of the depressed head have been found to be nearly as high. Examination of the data of this test shows that in the rectangular directions the deformations in these two places are not far different from each other.

The largest compressive unit-deformation, 0.00028, was found in the diagonal direction at the central column on gage line 26. Using a value of 3,000,000 lb. per sq. in. for the modulus of elasticity of

TABLE 8.

AVERAGE COMPRESSIVE UNIT-DEFORMATION AT MAXIMUM LOAD.

Gage Lines	Location of Gage Lines	Unit-deformation
8, 11, 16, 21, 25, 31	Long rectangular direction on bottom of depressed head	.00016
22, 32, 37	Short rectangular direction on bottom of depressed head	.00013
13, 61, 72	Long rectangular direction on slab near depressed head	.00012
21, 32, 36	Average of long and short rectangular directions near column capital	.00011
61, 138	Average of long and short rectangular directions near depressed head	.00013
9, 22, 31, 37	Across panel lines, bottom of depressed head near edge	.00015
*224 to 229 inclusive	Long rectangular direction midway between columns	.00015

*On upper surface of slab; all others on under surface.

concrete (frequently assumed in such cases), the corresponding compressive stress is 840 lb. per sq. in. A relatively large unit-deformation is found at the other end of the same diagonal in gage line 126. All other compressions in the diagonal direction were small.

In the long rectangular direction it is found that the maximum deformations would be cut by a curved section through gage lines 8, 11, 16, 21, 25, and 31 (see section J-J, Fig. 45). The average unit-deformation across this section was 0.00016 (see Table 8). In the short rectangular direction the average unit-deformation on gage lines 22, 32, and 37 was 0.00013.

It is seen that in general the deformations in the concrete were low. With the concrete intact in tension the neutral axis would be farther from the compression face than it would be otherwise. For the same resisting moment this would result in a smaller compressive

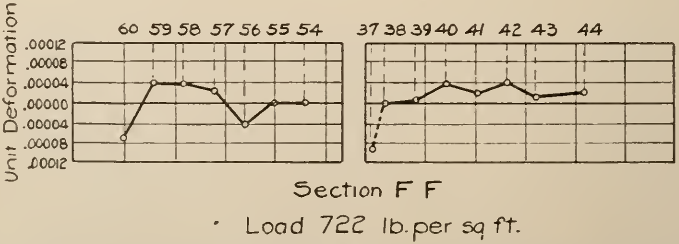


FIG. 53. LATERAL DISTRIBUTION OF COMPRESSIVE DEFORMATION ON BOTTOM OF TEST FLOOR NEAR DEPRESSED HEAD IN SCHULZE BAKING COMPANY BUILDING.

stress in the concrete than if the concrete had cracked generally on the tension side.

27. *Lateral Distribution of Compression.*—Little compression was detected in the two groups of gage lines 54 to 60 and 38 to 44 (see section F-F, Fig. 45 and 53). In fact, some of the measurements indicate a slight tension, but this is so small that the evidence should not be taken as conclusive. This indicates that the moment arm of the tensile stresses in the concrete in the thin portion of the slab may be determined as much by the thickness of the slab within the depressed head as by the thickness of the thinner portion of the slab. The tensile resistance of the concrete in the thinner portion of such a slab may thus give an increased resisting moment over that for a slab without the depressed head. This result suggests also that with a depressed head of the thickness here used the neutral axis is likely to be so close to the level of the bottom of the thin portion of the slab that the section of the slab outside of the depressed head may not be considered to contribute very much to the compressive stresses of the negative resisting moment. Tests to be used for comparing slabs having depressed heads with slabs which do not have them should employ loads large enough to eliminate effects of the kind here discussed.

Fig. 54, section G-G, shows the distribution of the compression which is developed across a section close to the capital, and J-J shows the distribution of compression which is developed across a section in the thin portion of the slab close to the edge of the depressed head.

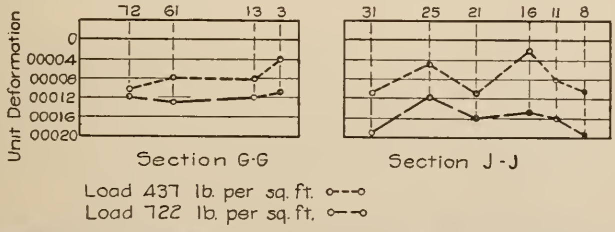


FIG. 54. COMPRESSIVE DEFORMATION ON BOTTOM OF TEST FLOOR NEAR DEPRESSED HEAD, AND NEAR COLUMN CAPITAL IN SCHULZE BAKING COMPANY BUILDING.

28. *Moment Coefficients.*—Assuming that all the load is carried by flexure, it is apparent that moment coefficients calculated on the basis of the stress in the steel only, will not represent the full resisting moment if the concrete assists in carrying the tensile stresses. In this test the tensile stresses were so low that it is certain that the concrete must have assisted greatly. However, while values of the

moment coefficient obtained by using the observed stress in the steel cannot be used for design, a comparison of the coefficients obtained for different parts of the panel may be of interest.

The resisting moments of the stresses in the reinforcement as given by calculations for the load of 722 lb. per sq. ft. are 0.0050 Wl for the positive and 0.0097 Wl for the negative moment, in which W is the total applied load on the panel and l is the average of the long and the short panel length center to center of columns. These are the components, in a direction parallel with a panel side, of the moment resisted by the stresses in all the reinforcement cut by a section across the entire panel at a position of maximum negative moment and at one of maximum positive moment. The values given are averages of the moment found for the long direction and the short direction of the panel.

Since no cracks could be found in the concrete it is certain that the tensile stresses in the concrete must have assisted very greatly in resisting the bending moment. The bending moment coefficients given, therefore, are not values which may be used for design purposes, and as the concrete may have given more assistance relatively at one of these positions than at the other, the values may not even give the correct ratio between the negative bending moment and the positive bending moment.

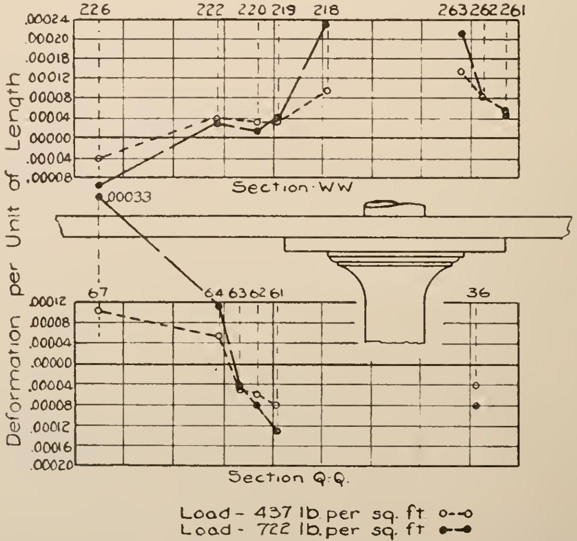


FIG. 55. LOCATION OF POINTS OF ZERO UNIT-DEFORMATION ON UPPER AND UNDER SURFACES OF TEST FLOOR OF SCHULZE BAKING COMPANY BUILDING.

29. *Point of Inflection.*—Points of zero unit-deformation on the upper and under surfaces of the slab in the direct lines between columns may be taken from the diagrams showing unit-deformation along sections W-W, Q-Q, L-L, and R-R, Fig. 55 and 56. These data are inconclusive, but the indications are that as in other tests reported in this bulletin the point of zero unit-deformation for the under side of the slab was closer to the column than that for the upper side; or, in other words, the total tensile stress on any section seems to be greater than the total compressive stress. The presence of arch action would imply an excess of compression. This phenomenon has been observed in several tests and its occurrence need not be doubted. No explanation of this matter is attempted.

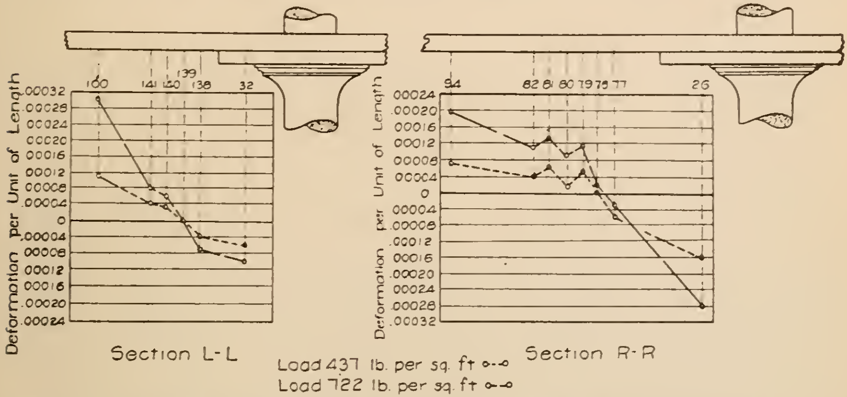


FIG. 56. LOCATION OF POINTS OF ZERO UNIT-DEFORMATION ON UNDER SURFACE OF TEST FLOOR OF SCHULTZE BAKING COMPANY BUILDING.

30. *Columns.*—The deformations in the columns were rather erratic and allow interpretations of only the most general sort. The differences in amount and character of deformations on opposite sides of the same column were sufficient to make it clear that there was considerable bending of the columns. The largest compressive deformation was found in column 2 which is on the edge of the loaded area, and the largest difference between deformations on opposite sides of any column is found for column 9 at the corner of the loaded area.

The significance of these observations seems to lie in their indication that the most important element of the deformation developed in the columns by load tests is that caused by the moment of unbalanced loads and that there is a larger amount of flexure in the corner columns than in the columns at the side of the loaded area. The latter is indicated also by the observation that the stress was greater in the diagonal band of slab reinforcement at the corner of the loaded area than in the rectangular bands at the edge of the loaded area.

31. *Examination for Cracks.*—A careful examination of the surfaces of the slabs and columns disclosed no cracks. This was unexpected and it is the only case known personally to the writers in which a floor loaded to twice the design live load plus the dead load did not develop cracks which were large enough to be found by a reasonably careful examination. The absence of visible cracks on the shaft of the columns may be due to the influence of the dead weight of the floors above in overcoming a portion of the flexural tension. However, at certain portions of the slab it seems probable that there were very minute cracks because in several instances steel stresses in the neighborhood of 10,000 lb. per sq. in. were found. Even though there may have been very minute cracks a considerable portion of the concrete must have been intact, and this undoubtedly had a great deal to do with keeping down the steel stresses.

32. *Deflections.*—Load-deflection diagrams are shown in Fig. 57.

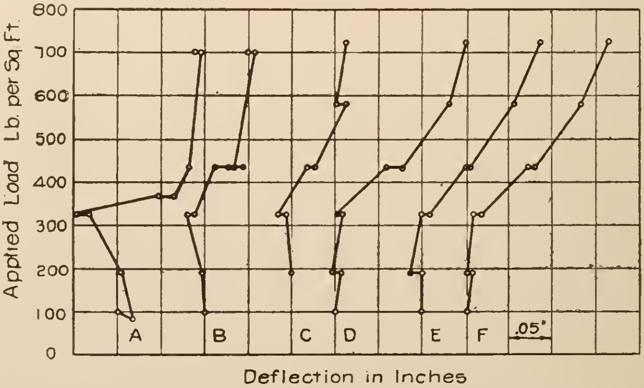


Fig. 57. Load-deflection Diagrams for Floor Test in Schulze Baking Company Building.

Distance to the right of the zero line represents downward deflection. The first set of readings was not obtained until a load of about 100 lb. per sq. ft. was on the floor and at deflection point C the apparatus was disturbed after the first set of readings had been taken; hence the deflection cannot be shown below 190 lb. per sq. ft. for C nor below 100 lb. per sq. ft. for all other points. The indications are that the failure to get readings for zero load makes little difference in the appearance of the curves. The small deflections and the tendencies toward upward deflection for the loads below 319 lb. per sq. ft. are ascribed to the rather extreme temperature changes. The air temperature rose from 20° F. at the time the load of 319 lb. per sq. ft. was placed to 50° F. at the time the load of 437 lb. per sq. ft. was placed, and then fell again to 25° F.

33. *Summary of Results.*—The results of the test here reported are conditioned upon a correct interpretation of the effect of the rather extreme temperature variation during the time of the test and of the assistance given by the strength of the concrete in tension. A continuation of the test to a point at which the concrete in tension had cracked generally, probably would modify many of the conclusions.

The main results pointed out in the foregoing paragraphs or shown in the diagrams are:

1 Very few steel stresses higher than 6000 lb. per sq. in. were found. On only two gage lines did the observed deformations indicate steel stresses as high as 10,000 lb. per sq. in. The highest of these was 14,400 lb. per sq. in., but the form of the curve indicates that the initial reading may have been in error and that 10,000 lb. per sq. in. is a more probable value.

2 The averages of the stresses in the bands of reinforcement passing under the central portion of the loaded area were higher than averages in the bands under the edges of the loaded area. In the latter also the stresses in the inner bars of the band (the bars on the side toward the center of the loaded area) were larger than the stresses in the outer bars (lying outside the loaded area).

3 The stresses in the reinforcement of the diagonal bands fell between the averages for the two rectangular bands for positions around the central column and midway between columns. The stresses were larger in a diagonal band at the corner of the loaded area than in a rectangular band where it crosses the edge of the loaded area.

4 The stresses in the short bars placed across the panel boundary lines were low but large enough to indicate that the bars may be effective in distributing or preventing cracks along the edge of the panel.

5 The compressive unit-deformations were low.

6 The compressive deformations across a section of the slab for gage lines as near as possible to the edge of the depressed head were nearly as high as those across the section of the depressed head near the edge of the capital.

7 The largest compressive unit-deformation was found in the diagonal direction at the central column.

8 The portions of the slab beyond the edge of the depressed head did not develop compressive stresses on the under side in a direction parallel to that edge.

9 Moment coefficients calculated on the basis of the steel stresses developed are exceedingly low. That for a position of maximum negative moment is about twice as large as that for a position of maximum positive moment.

Especial emphasis should be placed on the fact that these coefficients cannot be taken as indicating the total resisting moment developed.

10 The indications are that the bending of the columns was an important feature of the action of the structure. The largest bending apparently occurred in a column at the corner of the loaded area.

V. THE WORCESTER SLAB TEST.

34. *The Test Structure.*—The structure on which the test was made was built especially for the test and was located near Worcester, Mass. The slab was designed with the object of obtaining information on the effect of (1) different methods of arranging and distributing the reinforcement, and (2) variation in size of column capital. Fig. 58 gives the general design of the structure.

In order to avoid as far as possible lack of uniformity in conditions of building and testing the different parts and in order to reduce the proportion of the number of wall panels to interior panels, the four types of design used were placed in the four quadrants of a single slab four panel lengths (56 ft.) square. This gave a group of four panels to each of the four designs and a column in the center of each group. The details of the slab are shown in Fig. 58. The arrangements of slab reinforcement at the column capitals for the various groups were as follows:

Group I All tension reinforcement was placed in the diagonal bands. The rectangular bands lay in the bottom of the slab and afforded compression reinforcement at the column capitals.

Group II Both rectangular and diagonal reinforcement were in the top of the slab. There was no reinforcement in the bottom of the slab at the columns.

Group III All tension reinforcement was placed in the diagonal bands. The bars of rectangular bands did not pass over the column capitals. There was no reinforcement in the bottom of the slab at these positions.

Group IV Reinforcement was the same as in Group II but the column capital was smaller.

The amount and distribution of the tension reinforcement at the column capitals in each of the four groups were such that if planes were passed cutting each band at right angles outside the column capital the total area of steel in the top of the slab so cut in an angular distance of 180° around the column was 4.86 sq. in., the same for all groups. It should be noted, however, that the effect of the reinforcement in producing resisting moment across a panel edge will not be exactly the same for all groups. In the group having only diagonal bands a calculation of the rectangular component of the resisting moment (the component in a direction at right angles to the edge of the panel) will be about one-sixth greater than the rectangular component of the resisting moment of the reinforcement in the groups having both diagonal bands and cross bands. This area includes the section of tension reinforcement which in Groups II and IV consists of two rectangular bands and two diagonal bands; in Groups I and III this area is the section of two diagonal bands.

At points midway between columns, both in the rectangular and in the diagonal directions, the amount and distribution of the reinforcement were the same for all groups. In Group III the bars in the rectangular bands ended near the probable points of inflection and were not bent up in any way. The total area of cross-section of two rectangular bands and two diagonal bands was 4.2 sq. in.

Midway between columns two bars $\frac{3}{8}$ in. in diameter, 6 ft. long, were placed in the top of the slab across the panel boundary, that is, normal to the direction of the rectangular bands of reinforcement.

The panel length was 14 ft. center to center of columns in all panels. The diameter of the top of column capital was 4 ft. 6 in. for Groups I, II, and III, and 2 ft. 9 in. for Group IV. The ratios of these capital diameters to the panel length were 0.321 and 0.196 respectively. The average of all the measured thicknesses of the slab was 4.93 in. The average measured depth to the center of gravity of

TABLE 9.

CALCULATED SOIL PRESSURE AND MEASURED SETTLEMENT FOR UNIFORM LOAD OF 215 LB. PER SQ. FT.

Column Numbers	Soil Pressure lb. per sq. ft.	Average Settlement in.
E1 A1 E5 A5	1600	.11
E2 D1 B1 A2 E4 D5 B5 A4	2300	.235
C1 A3 C5 E3	2000	.27
D2 B2 D4 B4	3000	.39
C2 B3 C4 D3	2700	.27+
C3	2400	.37

the bands of reinforcement was 1.04 in. midway between columns and 1.63 in. near the columns for the four groups.

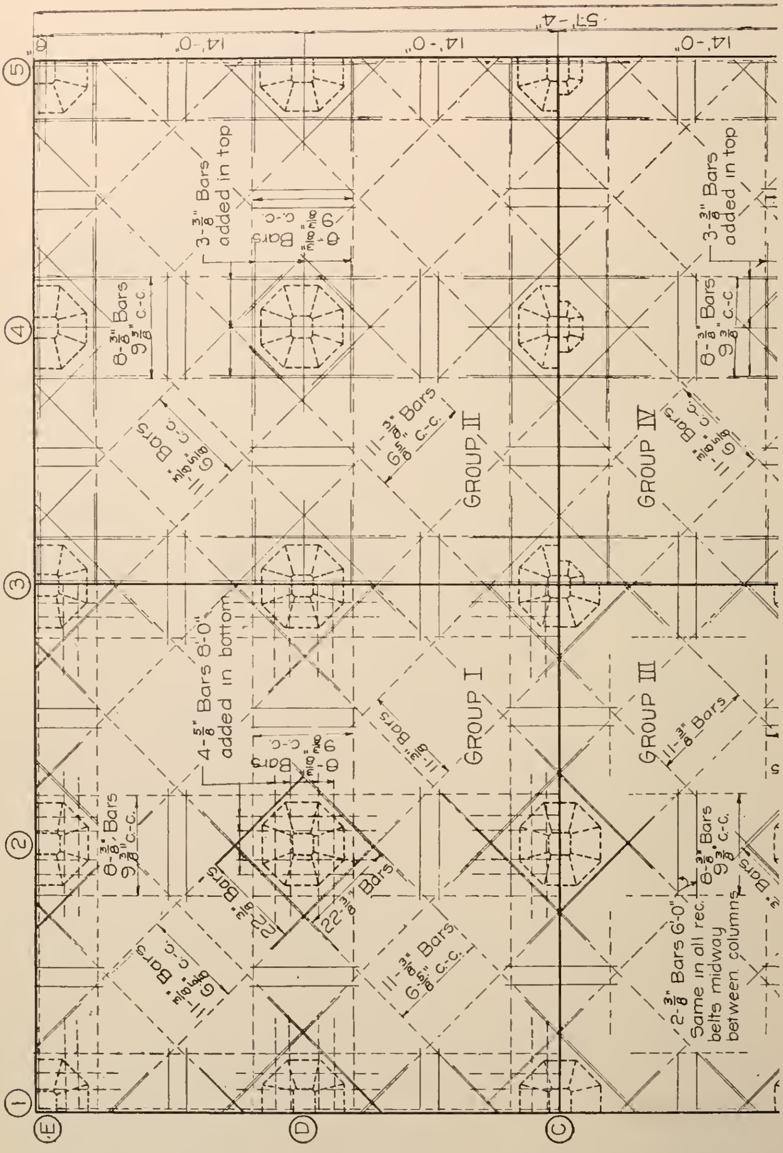
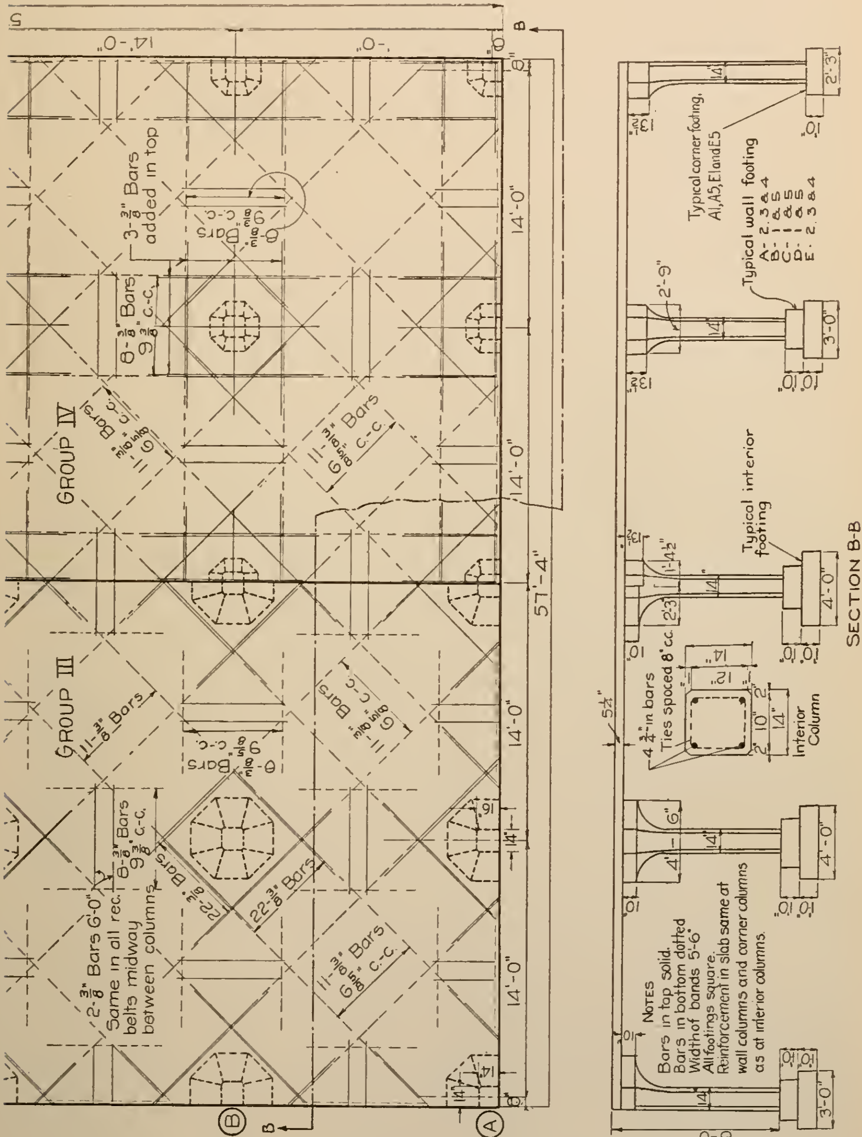


FIG. 58. DIMENSIONS AND REINFORCING

The footings for the interior columns were 5 ft. square, for the corner columns 3 ft. square, and for the columns at the edges 4 ft.



Plan for Worcester Test Floor.

square. It seems likely that the unit pressure on the soil was greater for the interior footings than for the footings at the corners and edges of the slab. Table 9 gives soil pressure calculated on the assumption that the total footing pressures were equal to the sums of the reactions of two systems of four-span, freely supported, continuous beams crossing each other at right angles.

35. *The Test.*—Preparations for testing were started on July 17, 1913, the loading began on July 28, 1913, and the last readings were taken on August 2, 1913. At the time of the test the concrete was about 3½ months old.

Gravel from the bank close at hand was used as loading material. The location of the slab on the side of the gravel hill allowed convenient access to loading material. A runway was built from the test slab to a point on the gravel bank somewhat above the elevation of the slab so that even at the higher loads little elevating of the loading material was necessary.

As explained in the following article, it was necessary to take strain gage readings early in the morning. This made it difficult to obtain readings with the load uniformly distributed over the slab. It was planned to make each increment of load such that it could be completed in a day, but these plans were interfered with by

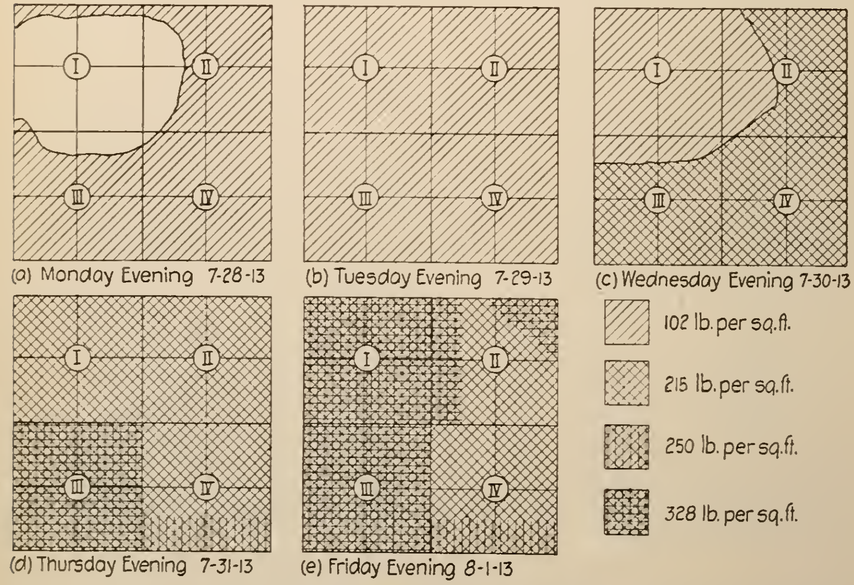

I
II
III
IV
(a) Monday Evening 7-28-13
(b) Tuesday Evening 7-29-13
(c) Wednesday Evening 7-30-13
(d) Thursday Evening 7-31-13
(e) Friday Evening 8-1-13
102 lb. per sq. ft.
215 lb. per sq. ft.
250 lb. per sq. ft.
328 lb. per sq. ft.

Fig. 59. Plan Showing Distribution of Load over Worcester Test Floor at Successive Stages of the Test.

rainy weather, by labor troubles, and by the failure of a portion of the slab at a load between 215 and 250 lb. per sq. ft., a smaller load than it was expected to carry. Fig. 59 indicates the intensity of the load upon the various portions of the slab at various times. After taking the readings at the maximum load the test was discontinued, the slab being left with the load upon it.

Measurements were taken of deformation in the steel and in the concrete, of deflections at the centers of the four interior panels, and of settlement of footings.

Measurements of deformation were taken on 313 gage lines (see Fig. 60, 61, and 72). Of these 281 were on the slab and 32 were on the columns. Of the gage lines on the slab 113 were on the concrete and 168 on the steel. The strain gage observations were made by Professor H. F. Moore of the University of Illinois and Mr. Slater. Great care was exercised in obtaining initial or zero readings. On a predetermined set of gage lines each observer took two independent sets of strain gage readings. Then instruments and assignments of gage lines were exchanged and a third set of zero-load observations was taken. In this way three independent observations were taken on each gage line. Time was taken then to compare the corrected observations before proceeding with the loading. It was found that there was serious lack of agreement not only between the check readings taken by the two observers but also between check readings taken on the same gage line by either observer, the amount of the discrepancy depending somewhat on the time of day the observations were taken.

Evidence was found indicating that temperature variation throughout the day was responsible for the discrepancies between check readings and that the discrepancies could be reduced greatly by taking readings early in the morning. Accordingly, the first three series of zero-load readings were disregarded and one new series was taken by each observer on all the 313 gage lines, beginning at 4:20 a. m. August 28 and finishing a little before 9 a. m. of the same day. The readings thus obtained gave satisfactory checks on accuracy of observation. All subsequent strain gage readings except those used for special purposes were taken between about 4 a. m. and 7 a. m. This experience is given in some detail because it suggests some of the difficulties attending the carrying out of experimental work of this character.

In order to obtain information which might be expected to have the widest range of applicability without undue expenditure of time,

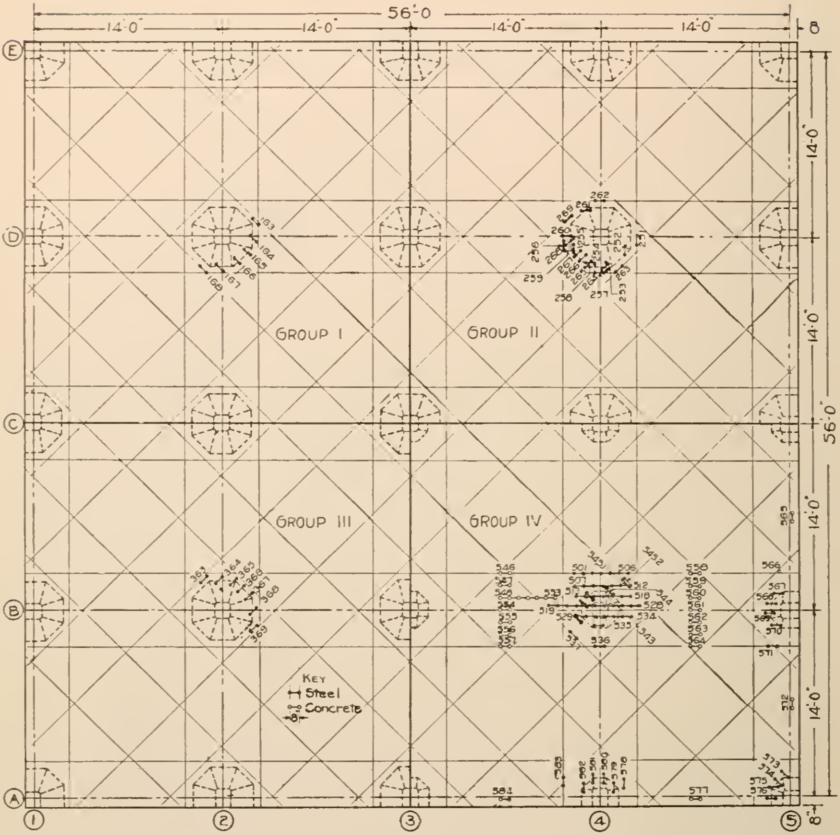


Fig. 60. Location of Gage Lines on Upper Surface of Worcester Test Floor.

it was decided to use a large number of gage lines in one group and a smaller number in each of the other groups. It was expected that the ratios of stresses in interior panels to those at similar positions in wall panels would be about the same for all groups so that a detailed study of one panel would allow the missing parts of the data of the other panels to be filled in. Group IV was chosen as being the most suitable for this purpose because with its smaller capital it would be expected to show higher stresses and because the size of its capital seemed to be nearer general practice than was that of the other capitals. The results of the test seem to indicate that the assumption as to comparative values of stress in similarly located positions was not justified. This probably was due largely to the complications caused by the settlement of columns.

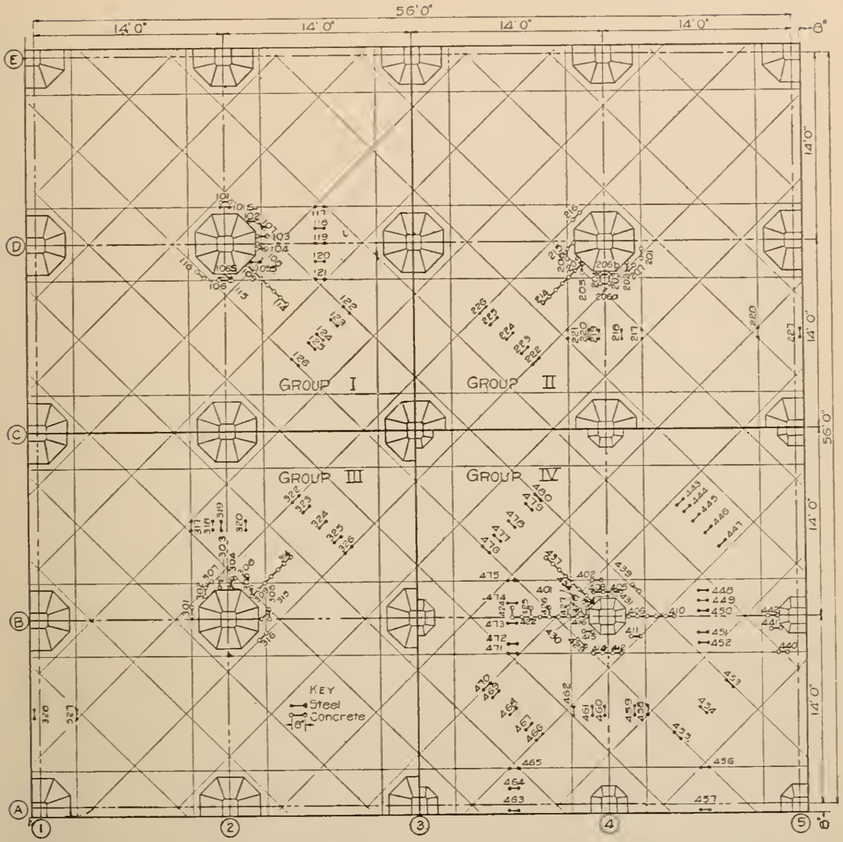


FIG. 61. LOCATION OF GAGE LINES ON UNDER SURFACE OF WORCESTER TEST FLOOR.

Deflections were measured at the center of the inner panel of each of the four groups, but these are so affected by the large settlement of column footings that slight importance is attached to them.

Settlement of footings was measured. A large amount of settlement, of course, was not anticipated and it was thought at first that measurement of settlement of the footings at the corners and at the centers of each of the four groups would be sufficient. These measurements were taken by means of a deflectometer (see Fig. 17a, Bulletin 64 of the University of Illinois Engineering Experiment Station). When a load of 100 lb. per sq. ft. had been applied to the slab it was found that the settlement was so large as not to justify so great refinement of measurement, and the difficulty of keeping the apparatus in adjustment in the presence of so many laborers led to the abandonment of this method and to the use of an engineer's level as a means of measuring further settlement. Average amounts of settlement are shown in Table 9.

36. *Phenomena of the Test.*—The application of load was begun on Monday morning, July 28, 1913. The first increment of load, approximately 100 lb. per sq. ft. over the entire slab, was completed on July 29. Strain gage readings for the load of 100 lb. per sq. ft. were taken from 4:00 to 8:00 a. m. on July 30. At this load a crack was observed across gage line 581 (at the edge of the capital of the wall column), the stress at this place being 9000 lb. per sq. in. No other cracks were recorded for this load, but stresses in the reinforcement were nearly 10,000 lb. per sq. in. in several gage lines at the center column of Group IV on the diagonal band; also at the corner and wall columns of the same group. Therefore, it seems nearly certain that other cracks were present at such places.

At the load of 100 lb. per sq. ft. it was found that there was considerable settlement of the footings.

On July 30 additional load was applied to Groups II, III, and IV, bringing the load on this area up to 215 lb. per sq. ft. while Group I still had only 100 lb. per sq. ft. Strain gage readings were taken on July 31 from 4:00 to 7:00 a. m. with this load in position. This load is referred to later as the load of 215 lb. per sq. ft. and its distribution is shown in Fig. 59.

At this stage of loading cracks appeared on the outer surface of column D5 across gage lines 8 and 9, Fig. 73, which were on the column reinforcement at the lower part of the bell-shaped portion of the column. The unit-deformation in gage line 9 across the lower crack corresponded to a stress of 46,000 lb. per sq. in. In gage line 10, which was at the bottom of the bell-shaped portion of the column and below the cracks, so great an elongation had taken place that the reading was beyond the range of the instrument. A rough measurement indicated a unit-deformation at this place of 0.006. These high elongations indicate that at this stage of the test, however much slip of column bars there may have been, the cracks were in part due to the very high stresses in the reinforcement.

At the same stage of the test, at the central columns of the groups and midway between the columns the stresses in the slab reinforcement quite generally had reached 15,000 to 20,000 lb. per sq. in. The stresses in the slab reinforcement at the wall columns and at the outer corner columns of Group IV were much higher. The deformations at the latter places corresponded to from 40,000 to 50,000 lb. per sq. in. in several instances, a value beyond the yield-point of the steel. The highest deformations in the slab reinforcement were in the diagonal band at column A5, and scaling of one of the bars at this place was noted, indicating that the bar had been stressed to the yield point.

At a load of a little more than 100 lb. per sq. ft. a crack had appeared on the bottom of the slab of Group IV, describing approximately a quadrant of a circle having a radius of about 6 ft. with column A5 in the center of the circle. This crack was fairly large when first noticed and increased in size quite rapidly as the loading progressed. After completing the strain gage readings for the load of 215 lb. per sq. ft. on Group II, III, and IV more gravel was placed upon Group I, bringing the load up to 215 lb. per sq. ft. over the entire four groups. No readings were taken at this stage but another increment was begun by applying load to Group IV in the position indicated in Fig 59d. About one-fourth of Group IV had been loaded to an intensity of 250 lb. per sq. ft. when failure of the slab in this group occurred, allowing the slab at the center of the corner panel to deflect about 8 in. and to settle down upon the 4 by 4-in. post which had been used as a datum for the measurement of deflections. Except for the presence of this post it seems likely that the slab in Group IV would have fallen at this time. Large tension cracks appeared at the middle of the exterior corner panel and over the edges of several of the column capitals of this group, and there were appearances of diagonal tension failure around column B4, but it seems likely that this was a result, or at most a secondary cause, of failure. That this is true is indicated by high deformations at various places under the load of 215 lb. per sq. ft. Some bending of column A5 was visible, although the column had shown no structural defects. Column B5 showed structural defects previous to applying the load and high stresses were developed in the reinforcement of the slab near this column, but the column did not fail. The defects appeared to be due mostly to the first pouring of the concrete having been carried too high on the column. A portion of the bell was filled with the first pouring, and the inability of the concrete in the expanded portion of the column to follow down into the shaft of the column, as shrinkage developed with setting, appears to have left voids in the concrete at the place where the bell begins. Some other columns showed the same features, but to a smaller extent than in the case of column B5.

The area and intensity of the load on Group IV at the time of its failure is shown in Fig. 59d. No more load was applied to this part of the slab and with continued application of load in the other groups the signs of distress in column D5, noted in a previous paragraph, increased. Slipping of the concrete at the top of the column along the column reinforcing bar took place and the crack across gage line 8 opened greatly. At the same time bending in column E5 near the junction of the shaft with the bell became visible and a construction joint opened (see Fig. 73). When about three eighths of Group II was covered with 328 lb. per sq. ft., the remainder of the slab having only 215 lb. per sq. ft., the condition of this portion of the structure became critical and loading was discontinued.

The distribution of the maximum load over the entire slab is shown in Fig 59e. Yielding of column D5 seems to have brought about the critical condition existing in Group II at the maximum load. This column did not fail completely but it had yielded to such an extent that a heavy stress was thrown into the reinforcement extending from this column to column D4.

The settlement of the footings, which was observed to have begun before the completion of the 100-lb. per sq. ft. load, continued, and at the higher loads the difference in elevation of certain parts of the slab could be observed by sighting along the under sides of the column capitals. At the maximum load the settlement of column D4 was 2 1/16 in. Other columns had settled appreciably but no other so much as column D4, and there was no uniformity in the amount of settlement for the various columns.

37. *Usefulness of Results of Test.*—The large amount of settlement of the footings and its unevenness throw serious complications into the interpretation of the results of the test. It seems probable that the distribution of stresses may have been dependent as much upon the relative amounts of settlement of the various footings as upon the variation in distribution of the reinforcement. In the following paragraphs comparisons are given of the stresses and deformations found at various positions on the slab and the discussion of the effect of certain features of the design on the stresses. Such comparisons must be considered to be qualitative and not to show quantitative variations, and further tests may show errors in conclusions based upon such comparisons.

Notwithstanding the limitations to the usefulness of the data it is believed that a presentation of the observed deformations is of value. These are shown in Fig. 62 and 63.

38. *Effect of Variation in the Distribution of Reinforcement.*—An examination of Table 10 shows the same average steel stress in the slab at the central column of Group I as that at the central column of

TABLE 10.

AVERAGE STRESSES IN TENSION REINFORCEMENT; COMPARISON OF GROUPS.

These stresses are based upon an assumed modulus of elasticity of 30 000 000 lb. per sq. in.

Group	Location and Direction of Gage Lines		Gage Lines	Stress in Reinforcement lb. per sq. in.	
				Load 102 lb. per sq. ft.	Load 215 lb. per sq. ft.
I	Midway between Columns	Rect.	117 to 121	5800	24700
		Diag.	122 to 126	1000	5200
	At Central Column	Rect.			
		Diag.	163 to 168	4300	8500
II	Midway between Columns	Rect.	217 to 221	5100	25700
		Diag.	222 to 226	2800	7300
	At Central Column	Rect.	251 to 256	3400	7500
			and 257 to 262	2400	4500
		Diag.	263 to 268	4500	8500
III	Midway between Columns	Rect.	317 to 320	6700	14600
		Diag.	322 to 326	4000	8500
	At Central Column	Rect.			
		Diag.	363 to 368	5600	15400
IV	Midway between Columns	Rect.	*471 to 475	4000	18000
		Diag.	476 to 480	3900	10500
	At Central Column	Rect.	503, 509, 514, 522, 529, 535, 536	4200	23700
		Diag.	537, 538, 539, 540, 542, 542a, 543, 544, 545.1, 545.2	7200	26500
Ratio of stresses in group IV to average stresses in groups I, II, and III	Midway between Columns	Rect.		0.70	0.83
		Diag.		1.53	1.50
	At Central Column	Rect.		1.45	3.86
		Diag.		1.51	2.41

*Area immediately over these gage lines not fully loaded. See observation boxes, Fig. 74.

Group II at 215 per sq. ft., and nearly the same in the two locations at 102 lb. per sq. ft. This does not indicate that the difference in distribution of reinforcement had much effect on the steel stress. Likewise a comparison of the slab stresses and deformations on the upper and under sides of the slab at the columns of Groups I, II, and III (see Tables 10 and 11) shows little difference which it is thought can be attributed to the fact that Group I had compression reinforcement while Groups II and III had none.

39. *Effect of Variation in Size of Capital.*—A comparison of the stresses in the steel and the deformations in the concrete at the central capital of Group II with those at corresponding positions in Group IV may be expected to show the effect of the difference in the size of capi-

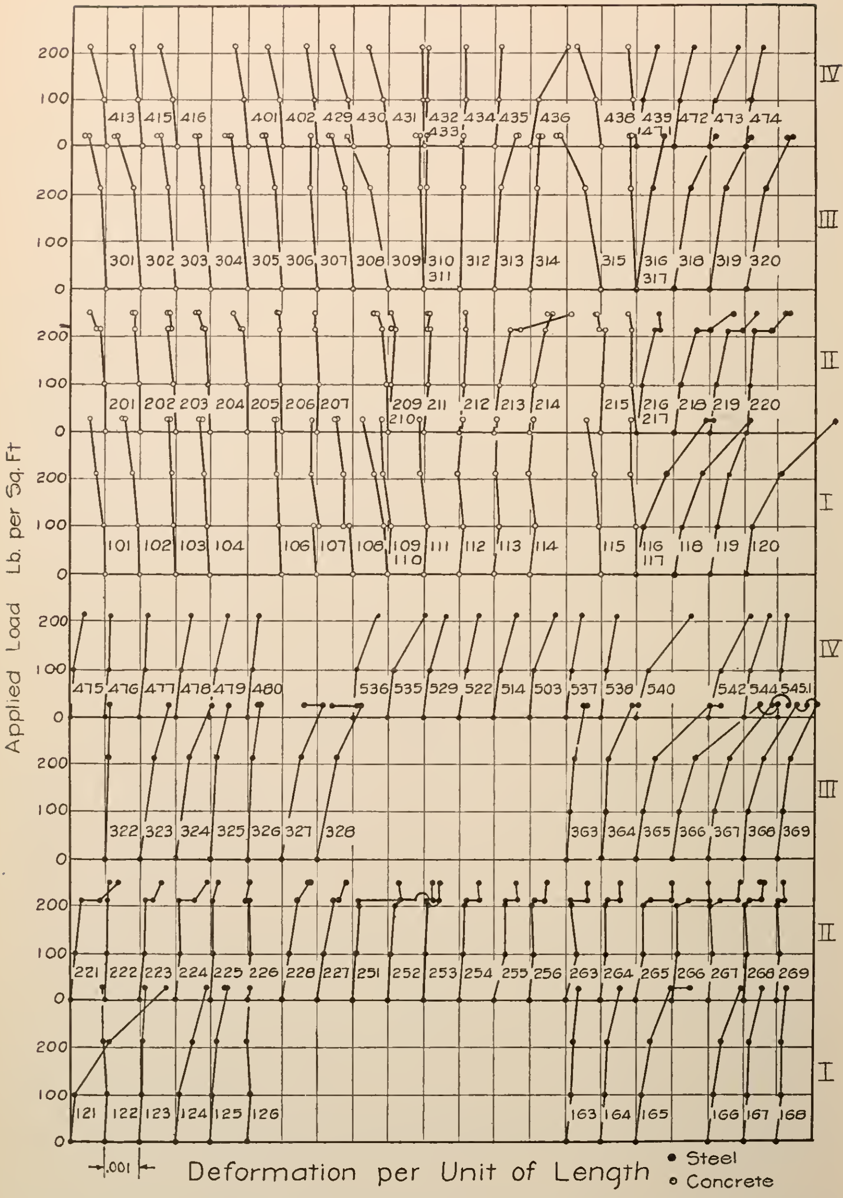


Fig. 62. Load-deformation Diagrams Arranged for Comparison of Groups in Worcester Test Floor

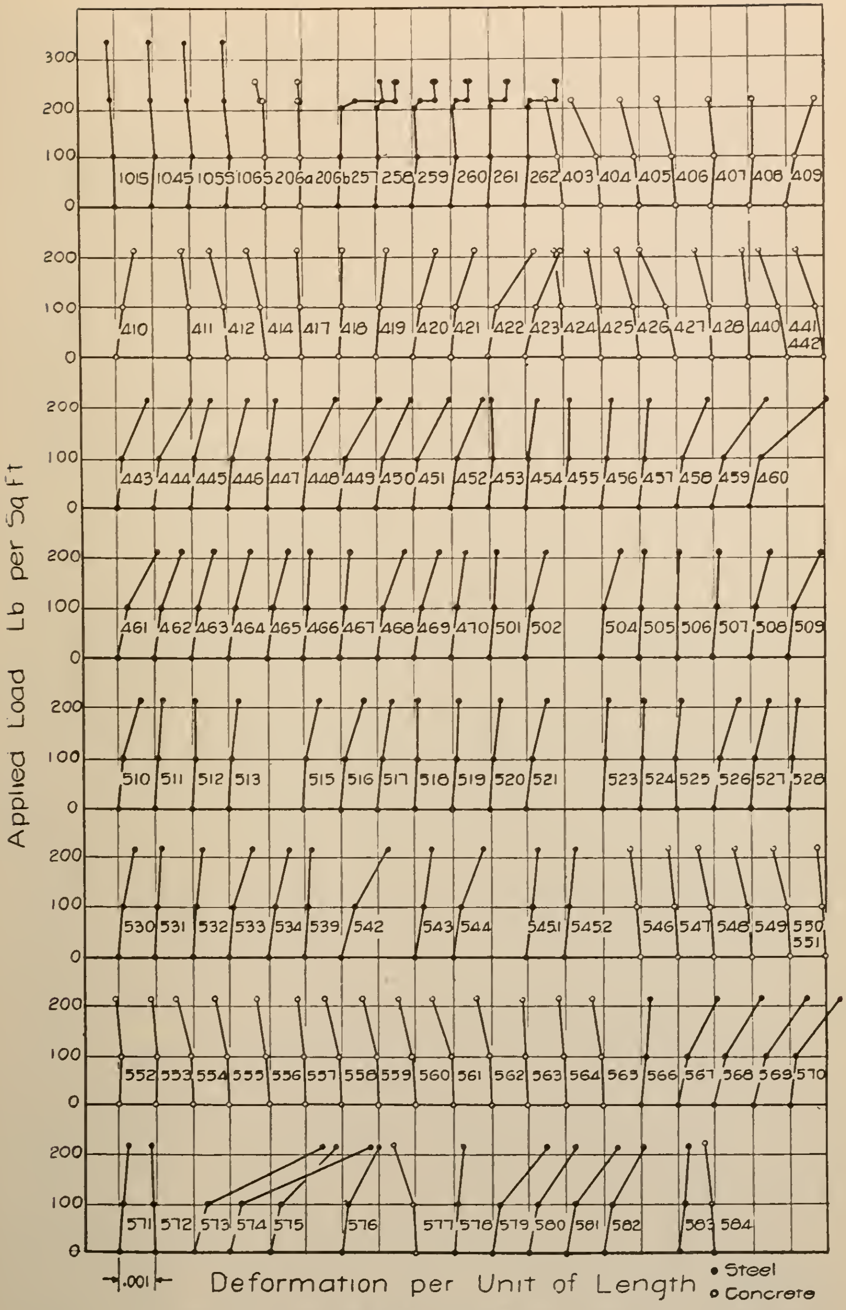


FIG. 63. ADDITIONAL LOAD-DEFORMATION DIAGRAMS FOR GAGE LINES OF WORCESTER TEST FLOOR.

TABLE 11.

AVERAGE COMPRESSIVE UNIT-DEFORMATIONS IN CONCRETE; COMPARISON OF GROUPS.

Group	Location and Direction of Gage Lines		Gage Lines	Unit-deformation	
				102 lb. per sq. ft.	215 lb. per sq. ft.
I	At Central Column	Rect.	101 to 106	.000048	.000187
		Diag.	107, 108, 109, 115, 116	.000045	.000258
II	At Central Column	Rect.	201 to 206	.000038	.000197
		Diag.	207, 208, 209,* 216	.000063	.000099
III	At Central Column	Rect.	301 to 306	.000133	.000287
		Diag.	307, 308, 309, 315, 316	.000210	.000429
IV	Midway between Columns	Rect.	546 to 557	.000067	.000333
	At Central Column	Rect.	404, 401, 416, 415, 413	.000135	.000461
		Diag.	429, 430, 431, 438, 439	.000188	.000568

*215 indicated tension and was omitted.

tals since the design of these groups was the same in other respects. Such a comparison shows that the stresses and deformations were considerably larger in the case of the smaller capital (see Tables 10 and 11 and Fig. 62). A comparison of Groups II and III with Group IV shows nearly the same results.

Attention may well be called to the width and thickness of the large capitals which approach the dimensions sometimes given to the depressed head used over column capitals. It seems evident that the conditions are different from those of the ordinary column capital.

It is seen that the stresses in the steel in Group IV at the 102-lb. load were about 50 per cent greater than the corresponding average stresses in the other groups except at the center of the rectangular band in which case it was 30 per cent less.

The load of 102 lb. per sq. ft. was used for this comparison because at the higher load (215 lb. per sq. ft.) Group IV was on the point of failure, and the excessive stresses at this load are believed to be due partly to a yielding of certain parts of the slab, which apparently had already taken place.

40. *Comparison of Interior Panels with Exterior Panels.*—An examination of Table 12 indicates a smaller stress in the diagonal at the center of the corner panel of Group IV than at the center of the interior panel. In this connection it will be recalled that the corner

Table 12.

Average Stresses in Tension Reinforcement Midway Between Columns in Group IV; Comparison of Interior Panels with Exterior Panels.

By term corner panel is meant a panel having two exterior edges.
By term wall panel is meant a panel having one exterior edge.
By term interior panel is meant a panel having no exterior edges.

Band	Location and Direction of Gage Lines	Gage Lines	Stress in Reinforcement lb. per sq. in.	
			Load 102 lb. per sq. ft.	Load 215 lb. per sq. ft.
Diagonal	Corner panel	453, 454, 455	3100	5600
Diagonal	Wall panel	443 to 447 466 to 470	3700 4600	18600 14900
Diagonal	Interior panel	476 to 480	3900	10400
Rectangular	Between corner panel and wall panel	*448 to 452 458 to 462	3800 8100	29200 40800
Rectangular	Between wall panel and interior panel	*471 to 475	4100	18000
Rectangular	Exterior edge of wall panel	463, 464, 465	5900	17900
Rectangular	Exterior edge of corner panel	456, 457	4500	8500

*Area immediately over these gage lines not fully loaded. See observation boxes, Fig. 74.

panel was the first to show distress by the appearance of a large crack across the diagonal band of reinforcement. The average of the steel stresses in the diagonal bands at the center of wall panels of Group IV was larger than the corresponding stresses in the interior panel for loads of 102 and 215 lb. per sq. ft. In the rectangular bands lying between wall panels and corner panels at points midway between columns the steel stresses were considerably greater than the stresses in the bands between the wall panels and interior panels. These comparisons are for cases where bending of columns at the edge or corner of the slab may be expected to influence the amount of the moment carried by the slab at the center of the span. The conclusion seems justified that the bending of the wall columns was sufficient to allow a material increase in the moment near the center of the rectangular span. The position and size of the cracks indicate that the proportional increase in the moment near the center of the diagonal in the corner panel was still larger though the location of the gage lines was such as not to show that this was true.

41. *Stress at Exterior Edge of Slab.*—Tables 13 and 14 were prepared to show a comparison of stresses and unit-deformation at the exterior edge of a panel with those at similar positions on an interior

TABLE 13.

AVERAGE STRESSES IN TENSION REINFORCEMENT; COMPARISON OF EXTERIOR RECTANGULAR BANDS WITH INTERIOR RECTANGULAR BANDS.

Group	Location of Gage Lines	Gage Lines	Stress in Reinforcement lb. per sq in.	
			Load 102 lb. per sq. ft.	Load 215 lb. per sq. ft.
II	Outer edge wall panel	227, 228	6800	17500
II	Between wall panel and interior panel	217 to 221	5100	26200
III	Outer edge corner panel	327, 328	*1800	18000
III	Between wall panel and interior panel	317 to 320	*3600	14500
IV	Exterior edge corner panel	456, 457	4500	8500
IV	Exterior edge wall panel	463, 464, 465	5900	17900
IV	Between corner panel and exterior panel	448 to 452 458 to 462	3800 8100	29200 40800
IV	Between wall panel and interior panel	471 to 475	4100	18000

*No values tabulated; these obtained from load-deformation curves.

TABLE 14

AVERAGE UNIT-DEFORMATION IN CONCRETE; COMPARISON OF EXTERIOR EDGES WITH INTERIOR EDGES OF PANELS

Plus indicates extension and minus indicates shortening

Location of Gage Lines	Gage Lines	Unit-deformation	
		Load 102 lb. per sq. ft.	Load 215 lb. per sq. ft.
Exterior edge of wall panel	565 583	—.00002 +.00107	—.00031 +.00328
Between wall panel and interior panel	546 to 557	—.00007	—.00033
Exterior edge of corner panel	572 577	—.00002 —.00005	—.00016 —.00064
Between corner panel and wall panel	558 to 564	—.00006	—.00041

edge of the panel. The conditions at an exterior edge of the panel are not shown to be more severe than at an interior edge. This fact is of importance in its bearing on the action of a flat slab built without beams at the exterior edges.

42. *Point in Inflection.*—In Group IV, for purposes of detecting the point of inflection, gage lines were placed on the upper and under surfaces of the slab along the line joining the centers of the columns B4 and B3. Fig 64 shows the positions of the gage lines and the variation in unit-deformation along this line. In using the figure it should be

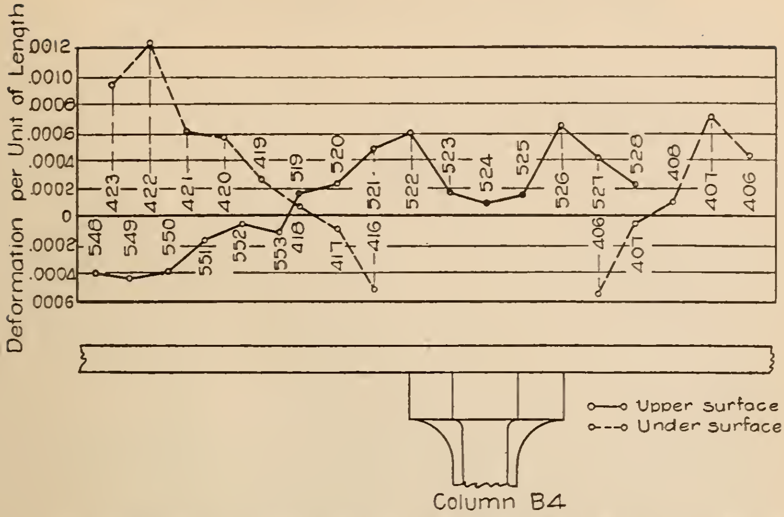


FIG. 64 LOCATION OF POINTS OF ZERO UNIT-DEFORMATION FOR UPPER AND UNDER SURFACES IN GROUP IV OF WORCESTER TEST FLOOR.

borne in mind that settlement of footings probably had much to do with the location of the point of inflection. It appears that the point of zero deformation on the upper side of the slab was farther from the column than that on the under side for both the interior panels and the wall panel. A probable position for the point of inflection, as shown by the intersection of the curves in Fig. 64, is about 33-in. from the column center for both wall panel and interior panels. This corresponds to 17/100 of the panel length from the edge of the column capital and to 20/100 of the panel length from the center of the column.

43. *Locus of Highest Stress.*—In Group IV gage lines were laid out on upper and under surfaces of the slab with a view to determining the locus of the highest stress developed in the elements of the slab parallel to rows of columns. The results for the upper surface are shown in Fig. 65. This diagram indicates that the position of highest stress in a bar crossing the capital is approximately at the edge of the capital. For bars not crossing the capital, the highest stress appears to be at the intersection of the bar with the center line of the row of columns. The locus of highest tensile stress appears to follow rather closely the outline of the capital through 180° and to turn off rather abruptly at the intersection of the capital with the line joining centers of columns. Although it is not possible to get a gage line short enough or close enough to the column to determine what the maximum compressive unit-deformation is, the results of the test indicate that the locus of highest compressive stress is in about the same position as that for tension except that it probably turns off less abruptly at the inter-

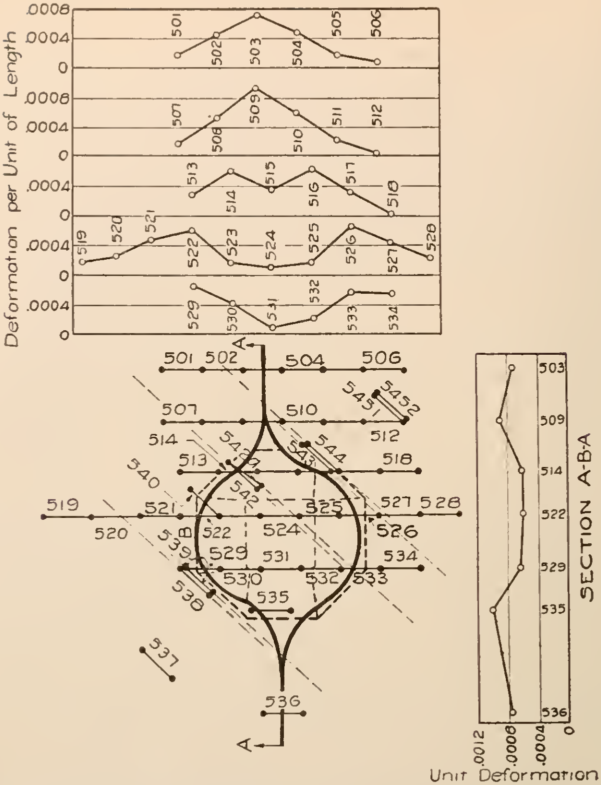


Fig. 65. Locus of Points of Highest Stress in a Rectangular Band of Reinforcement in Top of Slab at Column D4, of Worcester Test Structure.

section with the line joining centers of columns. It will be noted that the compressive deformation in gage line 206b is less than that in 206a (Fig. 63). The same phenomenon has been observed in other tests, and is what may be expected.

It may be added that at the load of 215 lb. per sq. ft. the compressive deformations in the section of maximum negative moment were distributed along the section for the full panel width.

44. *Distribution of Stress over Cross Section of Bands.*—Fig. 66 and 67 show the variation of deformation across the width of several bands of reinforcement at sections midway between columns and close to the column capitals.

Some previous tests have indicated that midway between columns the stress distribution in the diagonal bands may be different from that in the rectangular bands, the unit-deformation being greatest in a bar at the edge of the rectangular bands and at the center of the diagonal bands. Fig. 66 indicates that in general the bars in the central por-

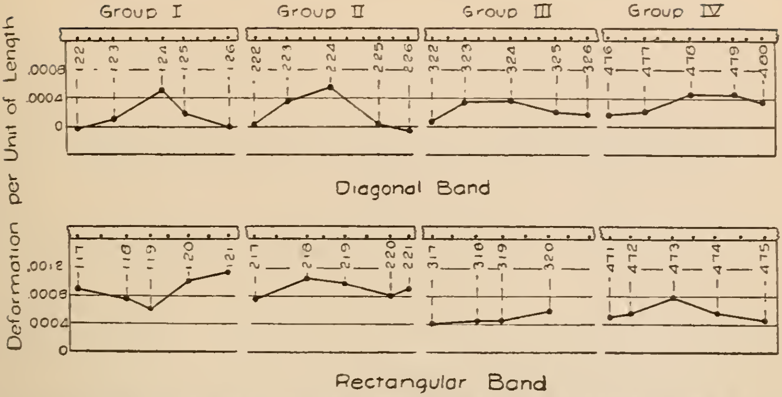


FIG. 66. DISTRIBUTION, MIDWAY BETWEEN COLUMNS, OF DEFORMATION IN DIAGONAL AND RECTANGULAR BANDS OF REINFORCEMENT OF WORCESTER TEST FLOOR FOR LOAD OF 215 LB. PER SQ. FT.

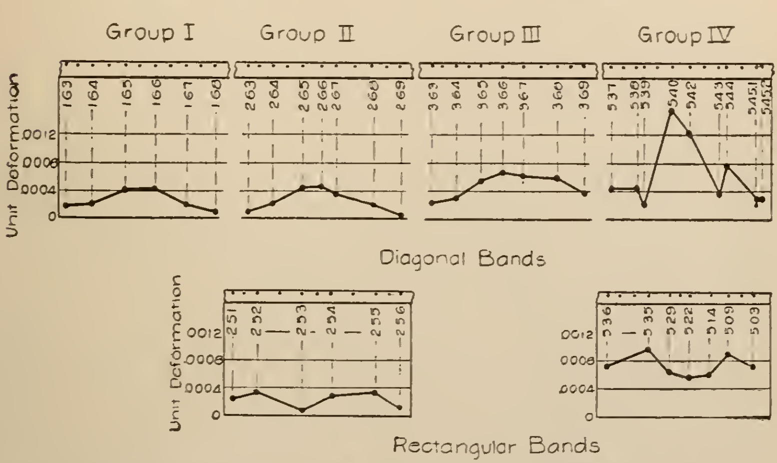


FIG. 67. DISTRIBUTION, NEAR COLUMN CAPITALS, OF DEFORMATION IN DIAGONAL AND RECTANGULAR BANDS OF REINFORCEMENT OF WORCESTER TEST SLAB FOR LOAD OF 215 LB. PER SQ. FT.

tion of the bands show higher stresses than those at the edge. There are one or two cases in which the reverse is true, but there is not sufficient regularity in this to justify the conclusion that this test shows a difference in stress distribution for rectangular and diagonal bands of reinforcement.

45. *Slip of Bars.*—The diagonal reinforcement was lapped at the column capital. The bars of each length extended 12 to 15 in. (32 to 40 diameters) beyond the center line of the column. At several places where the bars were lapped strain gage measurements were taken from one bar across to another for the purpose of determining the

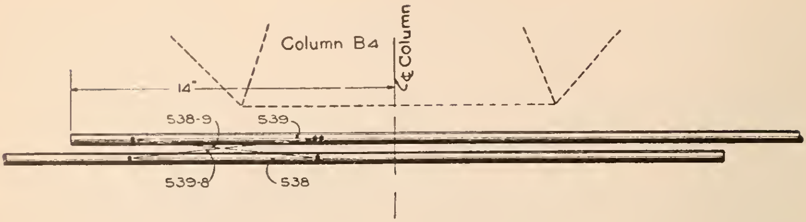


FIG. 68. ARRANGEMENT OF GAGE LINES ON WORCESTER TEST FLOOR FOR MEASUREMENT OF SLIP OF BARS.

amount of slip of the bars relative to each other. Fig. 68 shows the arrangement of gage lines 538, 539, 538-9, and 539-8, used for this purpose, and shows the position of the ends of the bars on which the gage lines were placed. The total change in gage length has been plotted in Fig. 69 as abscissas and the load as ordinates. The movements of the

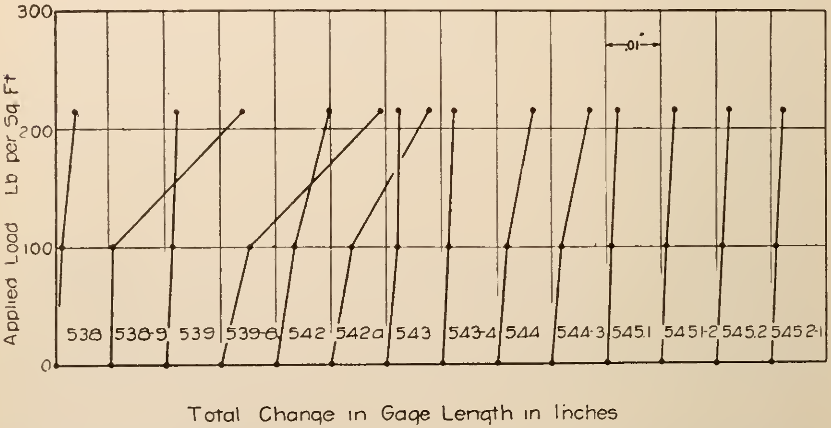


FIG. 69. LOAD-DEFORMATION DIAGRAMS FOR GAGE LINES USED FOR DETERMINING SLIP OF BARS IN WORCESTER TEST FLOOR.

gage points on the two bars relative to each other for loads of 100 and 215 lb. per sq. ft. have been plotted in Fig. 70. In fitting probable curves to the plotted points it was assumed that the curves for the two bars had the same slope at the point where they cross the line joining the centers of the columns A3 and B4 (marked symmetrical axis across bars). This is the same as assuming that the tensile stresses in the two bars are equal at this point, and follows from the symmetry of the arrangement of the bars. If there had been no slip of the bars relative to each other, the curves would touch at the point on the diagram corresponding to the place where the bars cross the line joining the centers of columns A3 and B4. The vertical distance between

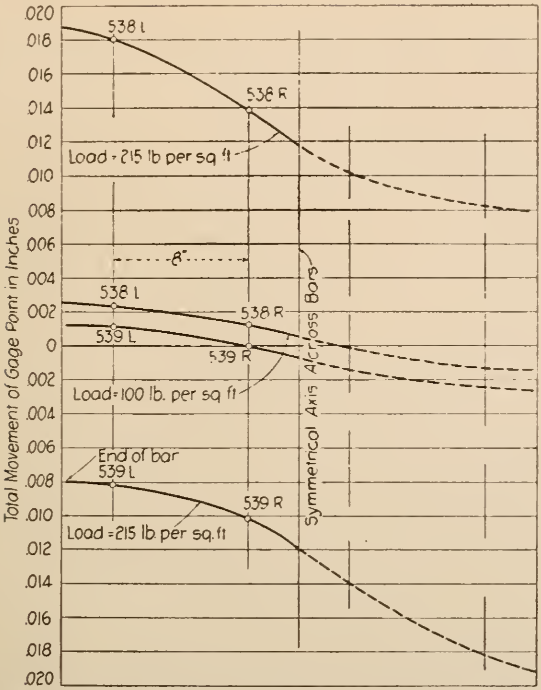


FIG. 70. DIAGRAM INDICATING SLIP OF BARS AT GAGE LINES 538 AND 539 IN WORCESTER TEST FLOOR.

the curves at this point represents the slip of the bars relative to each other.

Assuming that at this place the two bars slipped the same amount relative to a fixed point, but in opposite directions, the zero-line for slip measurements will bisect the vertical distance between curves at their intersection with the symmetrical axis as shown in Fig. 70. Ordinates to points on the two curves indicate the movements of the corresponding points on the bars relative to the line joining columns A3 and B4. Differences between ordinates at any two points on the same curve represent the total deformations taking place in the bar between the points considered. The stress in the bar at any point is proportional to the slope of the curve at the point.

The diagram indicates that at the load of 100 lb. per sq. ft. there was a slip of bars relative to each other of about 0.0013 in. At a load of 215 lb. per sq. ft. the slip of the bars relative to each other had increased to 0.024 in. at the same place. If the two bars did not act in the same way the slip at the place where they cross the center line of the columns may have been all in one bar. This would give a more

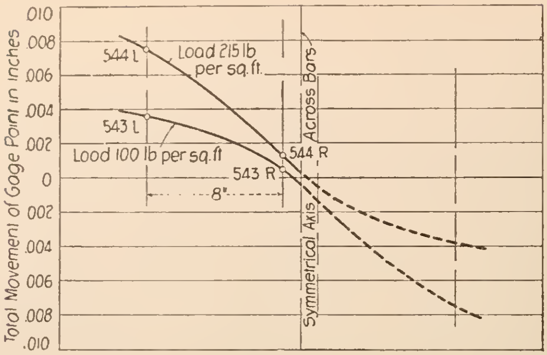


Fig. 71. Diagram Indicating Slip of Bars at Gage Lines 533 and 534 in Worcester Test Floor.

dangerous condition than that in which the slip is the same in both bars at this point.

Fig. 71 was constructed from measurements on gage lines 543, 544, 543-4, and 544-3 in the same manner as Fig. 70 was constructed. It indicates that at a load of 215 lb. per sq. ft. the slip of the bars relative to each other was only about 0.0006 in. It may be due to the smallness of this slip that the difference in stress in gage lines 543 and 544 is so much greater than the difference for gage lines 538 and 539. The difference in stress in the two bars is indicated by the difference in slope of the two curves at the point considered.

From measurements taken in the same way between gage lines 545.1 and 545.2 no slip of the bars relative to each other could be detected at the position of these gage lines, the centers of which were almost on the diagonal passing through the center of the column. At points on the bars near the ends it is likely that there was slip of the bars relative to each other. Measurements on gage lines 542 and 542a indicate at the right hand gage point a slip of one bar past the other of about 0.0078 in. at a load of 215 lb. per sq. ft. Whether this slip extended as far as the center line of the column would be of importance to know, but the observed data do not indicate whether it did or not.

It is not of special importance that these bars showed slip at the ends. This is to be expected if the design includes bars which end in a region of high stress. The significance of the measurements lies in the indications that there was slip at the point where the bar should develop high tensile stress, in the fact that this slip more than doubled with a doubling of the load although the tensile stress in the bar had not reached the yield point, and in the large amount of slip which occurred in some cases. The danger from a small initial slip has been brought out in a series of tests on beams subjected to continued load (*unpublished*).

After the failure of the slab, end slip varying from ½ in. to 1 in. was found in the bars designated by gage lines 539, 542a, and 543, while in gage lines 545.1 and 545.2 no end slip was apparent. It may be of significance that the bars which showed a large amount of slip at failure were the same as those which showed early slip at the point of highest tensile stress, and that the bars (designated by gage lines 545.1 and 545.2), which showed no early slip at the point of highest tensile stress, likewise at the maximum load did not show a slip which was large enough to be observed by the unaided eye.

That the bond stress was high in other bars crossing the column capital is brought out by an inspection of Fig. 64 and 65 which show the varying unit deformations along the lengths of bars. Within a distance of 8 in. (between gage lines 525 and 526) a change in stress of 15,500 lb. per sq. in. took place. This corresponds to a bond stress of 187 lb. per sq. in. if the bond stress be considered to be uniformly distributed. The shape of the curve indicates that the intensity of bond stress at the edge of the capital must have been nearly twice as much as this and it may have been more. This will be well brought out by a consideration of the slopes at various points of any smooth curve which may be passed through this series of points (gage lines 522 to 526 Fig. 64).

46. *Moment Coefficients.*—With the large and irregular settlement of the footings it is not to be expected that the moment coefficients calculated from the observed stresses will have much quantitative value, but a study of the character of the results may be of use.

In a beam or slab loaded in any manner, the total bending moment at any section may be expressed as kWl in which k is a coefficient, W the load on one panel and l the panel length. Values of this coefficient for the positive moment and the negative moment of the reinforcement stresses in Group IV have been computed in the same manner as was done for the Schulze Baking Company Building (see

TABLE 15

CALCULATED MOMENT COEFFICIENTS FOR GROUP IV

Location	Applied Load, lb. per sq. ft.	
	102	215
Positive	.011	.019
Negative	.015	.030
Ratio	.73	.63

article 28). Table 15 gives these coefficients, each value being proportional to the resisting moment of the stresses in one rectangular band of reinforcement plus the components (in a direction parallel to that rectangular band) of the moment of the stresses in one diagonal band. Table 15 shows that these coefficients are much higher for the 215-lb. load than for the 100-lb. load, both around the columns and at sections midway between columns. The difference may be due partly to uneven settlement of footings and partly to partial failure of concrete in tension. In considering the relative values of the negative and the positive resisting moments developed, it should be borne in mind that the slip of the bars at the column would cause a smaller negative bending moment and a larger positive bending moment than would otherwise be the case.

It is seen that the coefficients given in Table 15 are less than those generally used in designing and still less than those found by the more conservative analyses. For the load of 102 lb. per sq. ft. the coefficient of bending moment calculated from the observed stresses was even less than that for 215 lb. per sq. ft. It is probable that even at the higher load the tensile strength of the concrete has played a considerable part.

In Groups I, II, and III, not enough gage lines were read to permit

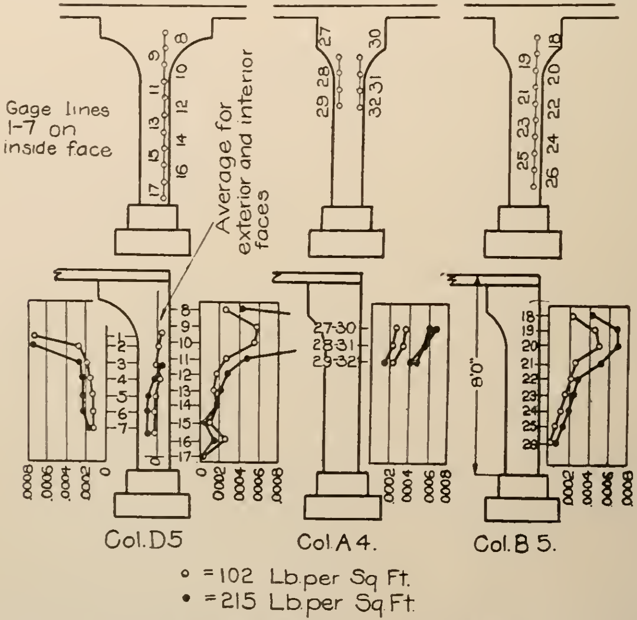


FIG. 72. DIAGRAM SHOWING DEFORMATIONS IN COLUMNS A4, B5, AND D5 OF WORCESTER TEST STRUCTURE.

the calculation of moment coefficients, but it has been shown that the stresses in these groups were lower than those in Group IV.

47. *Deformation in Columns.*—Fig. 72 shows unit-deformations in the reinforcement and in the concrete of columns A4, B5, and D5. These measurements were made for the purpose of detecting bending in the wall columns. The gage lines on the reinforcement were on the bars nearest the outer surface of each column; those on the concrete were on the inner surface of Column D5, that is on the face toward the interior of the panel.

On column D5 (see Fig. 72) deformations were observed on opposite sides of the column. In the lower part of the column where there were no cracks the neutral surface (so far as flexure is concerned) would lie on the center of the column and the average of the unit-deformations measured on opposite sides of the column should be the unit-deformation due to the direct compression. It is found that this average unit-deformation was about 0.00004 at a test load of 215 lb. per sq. ft.

Near the upper end of the column the tension was so large as to

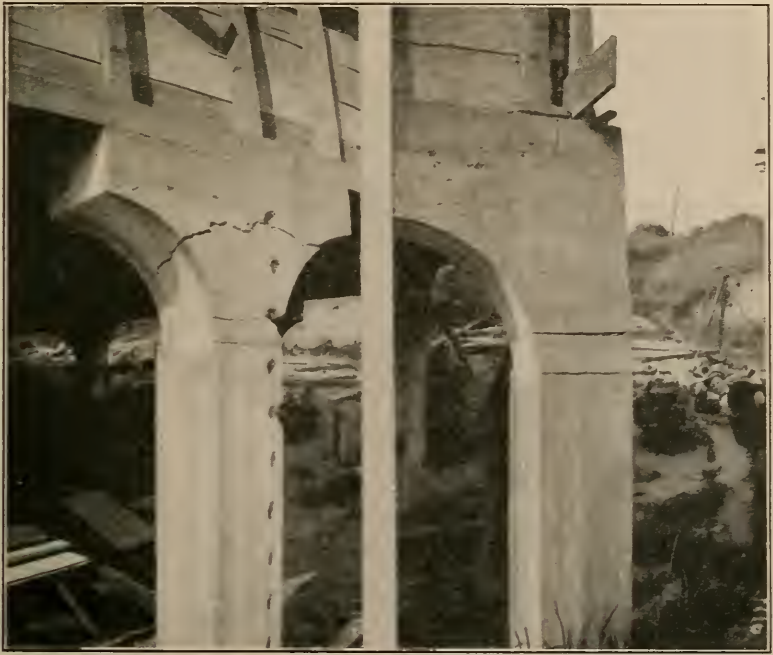


FIG. 73. VIEW OF COLUMNS D5 AND E5 OF WORCESTER TEST STRUCTURE WITH MAXIMUM LOAD IN POSITION.

cause large cracks (see Fig. 73) and slipping of the outer vertical bars, thus throwing the neutral axis closer to the compression than to the tension side of the column. Averaging the measured deformations for this region of the column, therefore, does not eliminate the flexural deformations. A resultant elongation at the center line of this part of the column should be expected and Fig. 72 indicates that elongation occurred at that place.

For a column fixed at the top of a fixed footing and rigidly connected with the slab at the top of the column but having no lateral movement there, the point of inflection is at a point one-third the column height above the top of the footing. The measurements did not extend down far enough on this column to determine the location of the point of inflection of the column, although the series of gage lines covered 0.93 of the distance from the slab down to the top of the footing. This indicates that there must have been sufficient tilting of the footing to modify the condition of restraint of the column at the top of the footing.

Although no strain gage readings were taken on a corner column, bending of the column at the outer corner of Group IV was observable to the unaided eye at a comparatively early stage in the loading, and at the maximum load bending was pronounced (see Fig. 73). The indications were that bending was more severe in the corner columns than in the columns at the side of a loaded area.

An examination of Table 16 indicates that in Group IV at a load of 102 lb. per sq. ft. the stress in the outer reinforcing bars of the wall columns near the bottom of the capital (12000 lb. per sq. in.) was higher than the average stress in the slab reinforcement where it crossed the edge of the capital of the same wall columns (8000 lb. per sq. in). In Group II, which had larger capitals, no measurements were taken of deformation in the slab reinforcement at a wall column, but the stress in the reinforcement of the wall column, D5, at a load of 102 lb. per sq. ft. was higher (16000 lb. per sq. in.) than that in either wall column of Group IV. However, for the load of 215 lb. per sq. ft. the table indicates that in Group IV the average stress in the slab reinforcement at the wall columns (35000 lb. per sq. in.) had become larger than the stress in the outer reinforcing bars of the columns (19000 lb. per sq. in.). At the same time the stress in the reinforcement of a wall column of Group II (Column D5) had passed the yield point.

In Group II the larger capitals afforded additional stiffness to the slab but very little more to the columns. In this group a very severe

TABLE 16.

STRESSES IN TENSION REINFORCEMENT; COMPARISON OF SLAB AND COLUMN AT WALL.

Location of Gage Lines	Gage Line	Stress in Reinforcement lb. per sq. in.		Average Stress lb. per sq. in.	
		Load 102 lb. per sq. ft.	Load 215 lb. per sq. ft.	Load 102 lb. per sq. ft.	Load 215 lb. per sq. ft.
GROUP IV On Slab at Column A4	567 568 569 570	8100 8300 11200 4500	30600 38800 34200 43200	8000	36700
GROUP IV On Slab at Column B5	579 580 581	8000 9500 8900	44300 37200 41000	8300	40800
GROUP IV On Column A4	27 30	10700 7600	18700 19000	9200	18800
GROUP IV On Column B5	20	14200	20000	14200	20000
GROUP II On Column D5	9	16000	Beyond yield point	16000	Beyond yield point

bending of Column E5 and high bending stresses in D5 followed by complete failure of the latter column occurred before the stress in the slab reinforcement at the central column had reached the yield point. In Group IV no column failed completely, although the bending in the slab was marked, the stress in the slab reinforcement at the central column having reached the yield point.

The results of the investigation of columns are not extensive enough or definite enough to furnish a basis for conclusions on the amount of moment carried by the columns. However, using as a criterion the ultimate loads carried and the manner of failure for Groups II and IV, the behavior of the columns relative to the slab was such as would be expected.

48. *Cracks.*—Fig. 74 gives the location of cracks on the under surface of the slab as shown by an examination at the maximum load and also those found on the accessible portions of the upper surface. The first large crack which was observed was in the bottom of the slab in the outer corner panel of group IV (see Fig. 74). It took approximately the form of a quadrant of a circle having a radius of about 6 ft. with column A5 at the center of the circle. This crack appeared soon after the load of 102 lb. per sq. ft. was applied and it opened rapidly as the loading continued. At a load of 215 lb. per sq. ft. the panel failed with the continued opening of this crack. Probably sev-

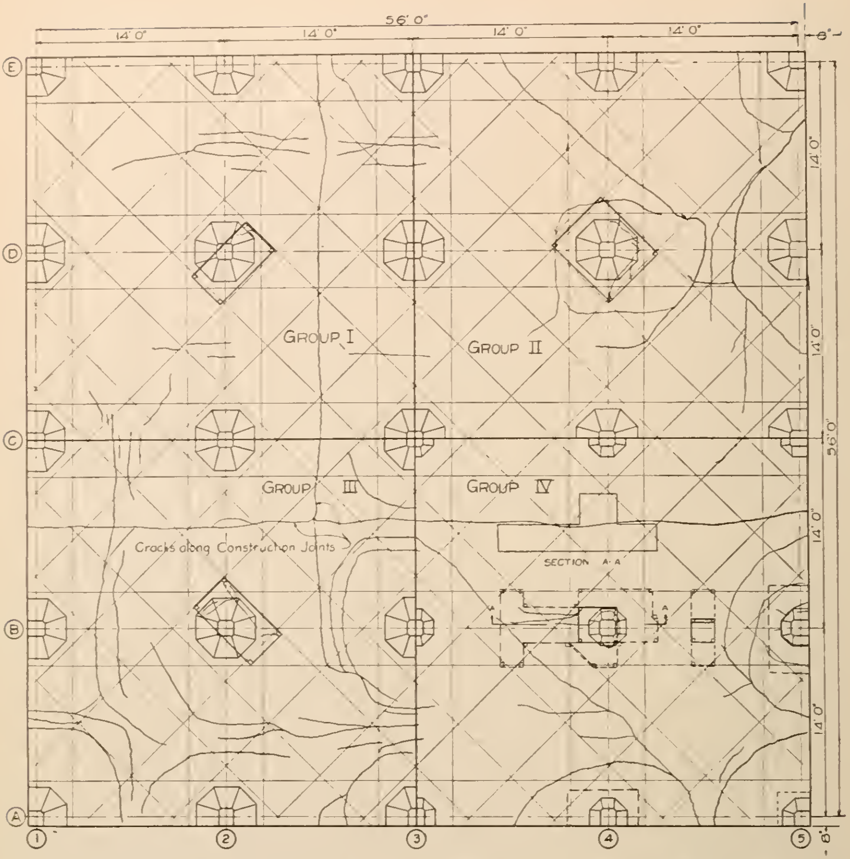


FIG. 74. CRACKS FOUND ON UNDER AND UPPER SURFACES OF WORCESTER TEST FLOOR AT MAXIMUM LOAD.

eral causes contributed to the location of the crack at this place. The unbalanced load caused a bending in Column A5, and it may be expected that this threw the point of maximum positive bending moment outside of the center of the diagonal span. At the short distance from the column at which the crack was located the reinforcement of the diagonal band may not have reached its position in the bottom of the slab. It should be borne in mind that high stresses were observed over the edge of the capital of column A5 in the bars of the diagonal band and that considerable bending of column A5 was visible but that the column did not fail.

In the corner panel of Group II the first prominent crack found took approximately the form of a circle around the corner column

(Column E5) much the same as the crack in Group IV, and a large amount of bending occurred in the corner column, but failure in this group seems to have been due to a weakness in the wall column, D5. In this column the concrete slipped past the upper ends of the vertical reinforcing bars, allowing a crack to form on the outer surface of the column somewhat above the bottom of the column capital, and the column failed in flexure.

The top of the slab was covered with loading material so that the general location of cracks could not be observed. At four interior columns (B2, B4, D2, and D4) the boxes used to give access to gage lines permitted the inspection of the upper surface of the slab over a portion of the column head. Well defined cracks just inside the outline of the capital, as shown within the boxes, were found. At two wall columns (A4 and B5) similar cracks were found.

Uneven settlement may have influenced the formation of cracks. It will be seen from Fig. 74 that many of the cracks are in locations where the reinforcing bars may be expected to be not close to the lower surface. In Group I, having the bars of the rectangular bands carried at the bottom of the slab through to its boundaries, the cracks were comparatively few and small. In this portion of the slab the settlement of the footings was relatively small and regular.

49. *Summary of Results.*—Although settlement of footings puts a serious limitation on the general applicability of the results of the test the following summarized statement of the information obtained is made:

1 In groups having capitals of the same size the variation in stress due to the variation in arrangement and distribution of reinforcement was less, apparently, than that due to uneven settlement of columns.

2 At a load of 102 lb. per sq. ft. the steel stress at the small capital (Group IV) averaged about 50 per cent greater than the stress at the larger capitals. The diameters of the capitals were respectively 0.196 and 0.321 times the panel length. The large stress in Group IV may be due partly to other causes, but it is believed that the small capital is the most important cause.

3 The wall panels and corner panels showed higher steel stresses and generally more pronounced cracks than did the interior panels.

4 This test does not indicate that the wall panels (not the corner panels) suffered because of the absence of wall beams.

5 The point of inflection in Group IV (the group having small capitals) was about two-tenths of the panel length from the center

of the central column, but its exact location is uncertain, since the point of zero unit-deformation on the under surface of the slab was closer to the column than that on the upper surface. The location of the point of inflection probably was influenced by the uneven settlement of the columns.

6 The locus of highest stresses in the bars of a band of reinforcement at a column head followed fairly closely the outline of the column capital through 180°, then branched off and followed the line joining centers of columns. The locus for the compressive stresses on the under side of the slab parallel to a given band occupied a corresponding position so far as may be determined from the data of the test.

7 In few of the bands of reinforcement in which measurements were taken was the stress higher in the bars on the edge of the band than in the central bars. In most cases the stress was highest in the central bars.

8 In cases where bars were lapped as much as 50 diameters beyond the point of maximum stress slipping at that point occurred without the stress having passed the yield-point strength of the steel. The slipping of these bars supports the ruling frequently made that bars should not be spliced at regions of maximum stress. The slipping of bars evidently affected the action of the slab and may have induced failure.

Bars which did not slip were found to have developed a bond stress averaging over the entire gage length 187 lb. per sq. in. At portions of the gage length the bond stress must have been much higher than this.

9 Moment coefficients calculated on the basis of the measured stress in the steel were materially higher at the higher load. Though even at the higher load the coefficients were low, the rapid increase with increased load confirms the view that there is danger in placing reliance on the stresses in the steel measured at ordinary test loads as a basis for determining moment coefficients to be used in design.

10 The bending of corner columns and wall columns was an important feature of the action of the test structure. In certain instances this bending was apparent to the eye.

11 The first large crack on the under surface of the slab was in the corner panel of Group IV near where the diagonal bars were carried from the top of the slab to the bottom.

The location of cracks in the other groups seems to have been influenced by the settlement of the footings, the bending of the outer columns, and the position of the point of carrying the reinforcing bars from the top to the bottom of the slab.

VI. THE TEST OF THE FACTORY BUILDING OF THE CURTIS-LEGER FIXTURE CO.

50. *Description of the Building.*—The floor loaded is in the addition to the Curtis-Leger Fixture Company's factory near Van Buren and Aberdeen Streets, Chicago. This addition is 53 ft. 6 in. by 57 ft. The floor is an 8-in. flat slab of reinforced concrete and is supported on reinforced concrete columns placed 19 ft. and 17 ft. 10 in. apart center to center in the two rectangular directions. It was designed for a live load of 200 lb. per sq. ft. and a dead load of 100 lb. per sq. ft.

A distinctive feature of the Barton Spider Web System of Reinforcement is the use of unit mats as reinforcement for the negative moment at the columns, the mats being independent of the reinforcement for positive moment in the slab. The bars in these mats extend in two directions only, parallel to the sides of the panel. The mats in this building were made up in the shop and consist of 5/8-in. square Havemeyer bars placed as shown in Fig. 75. Certain loose bars were added in this case, and these extend beyond the outlines of the mat. For the positive moment at the center of the panel, four-way reinforcement of 1/2-in. square Havemeyer bars was used. These bars were not bent up to the top of the slab, but extend along the bottom of the slab

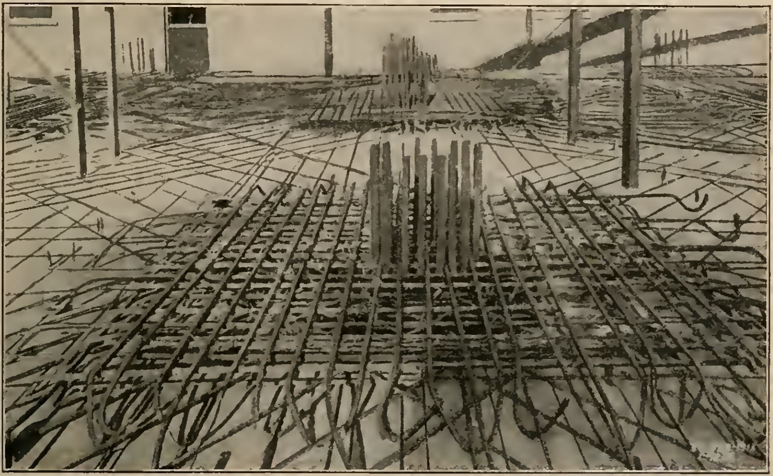


FIG. 75. VIEW SHOWING SLAB REINFORCEMENT IN PLACE IN TEST FLOOR OF CURTIS-LEGER COMPANY BUILDING.

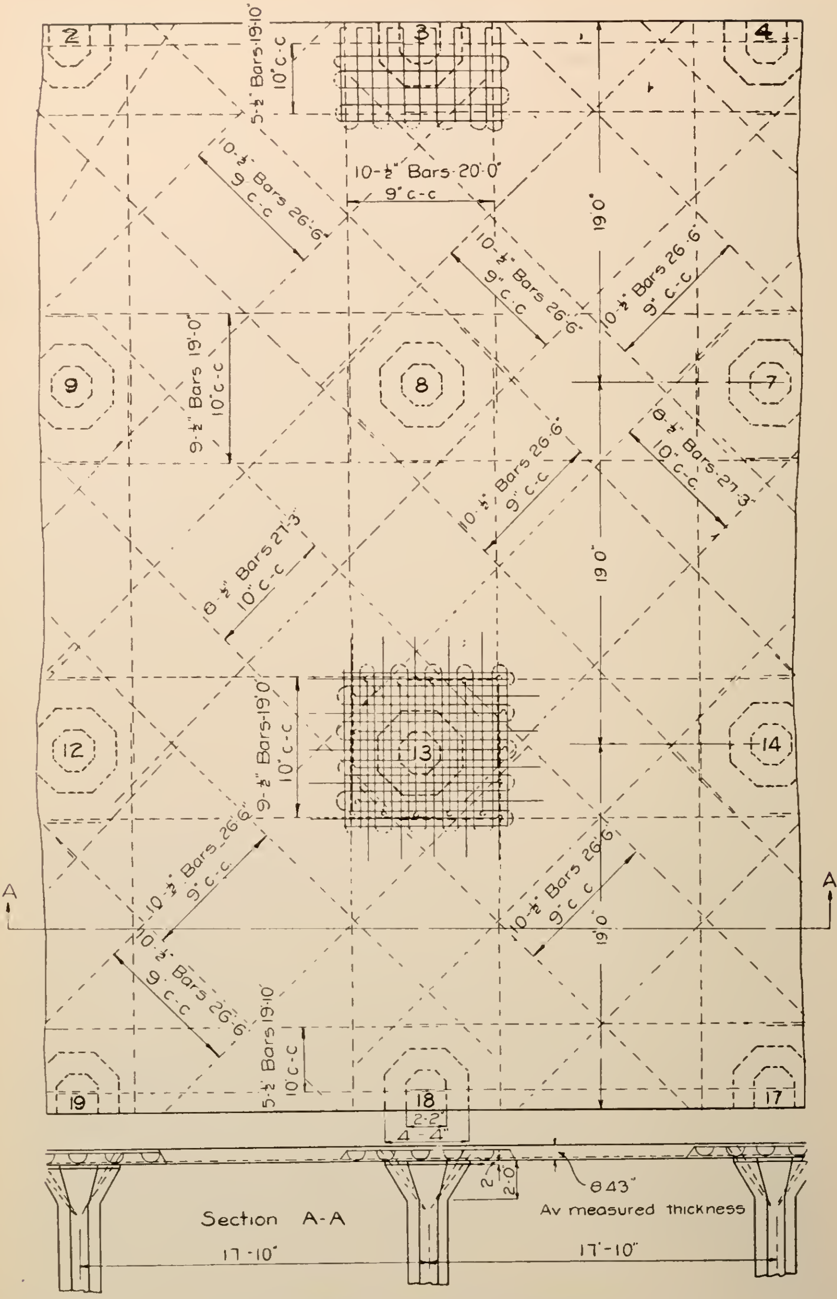


FIG. 76. PLAN SHOWING DIMENSIONS AND REINFORCEMENT OF TEST FLOOR OF CURTIS-LEDGER COMPANY BUILDING.

into the region of compression. They end 6 in. and 7 in. beyond the column center lines in the long and in the short panel lengths, respectively, and 3 in. to 7 in. beyond the column center-line in the diagonal directions. These bars thus serve as reinforcement in compression in the region of the column and in tension at the middle portion of the panel. No drop or depressed head at the column capital was used. The average measured depths to the center of gravity of the reinforcement were 7.04 in. for positions of negative moment and 8.55 in. for positions of positive moment.

The reinforcement for the wall panels is slightly in excess of that for interior panels. Spandrel beams 12 by 22 in. in cross section extend between exterior columns.

The position of the reinforcing bars is shown in Fig. 76.

51. *The Test.*—It was desired to make the amount of load handled as small as possible consistent with developing reasonably high stresses. Instead of covering four entire panels with 200 lb. per sq. ft. and then covering two entire panels with 400 lb. per sq. ft., it was believed to be more satisfactory to use equivalent total areas made up of parts of six panels, as shown in Fig. 77, first with a load of 200 lb. per sq. ft. (boundary dotted in the figure), then rearranging the load to give 400 lb. per sq. ft. over the other parts (boundaries in full line), then adding enough to the latter area to make 500 lb. per sq. ft. This plan gave more complete loading over the principal band of reinforcement and produced negative moments in this region greater than would be obtained under other likely conditions with only two panels loaded. This arrangement of the 500-lb. load also gave easier access to gage lines for the measurement of stress in the vicinity of the columns.

Aside from determining the action of the type of reinforcing, it was expected to obtain information bearing on (a) lateral distribution of stress in bands midway between columns and at the columns and (b) longitudinal distribution of stress along rectangular and diagonal spans. Fig. 78 shows relative location of gage lines on the upper and under surfaces of the test floor.

When the forms were ready and before the concrete had been poured small beveled iron plugs were nailed to the forms in such position that when the forms were removed the smaller base of each plug was in the plane of the under surface of the slab and in proper location for the gage holes used for the measurement of deformation in concrete. Corks were attached to the bars at such places as gage holes in the reinforcement were desired. These steps greatly facilitated the preparation for the test. The concrete was poured on July 30, 1913, and at that time six concrete control cylinders 7½ in. in diameter and 16 in. long were made from concrete taken from the mixer in use on the job. The cylinders were tested at the University of Illinois, November 13 and 14, 1913.

The floor test was made October 22 to 30, 1913. For loading material bags of cement were used. An area equivalent to four panels, shown

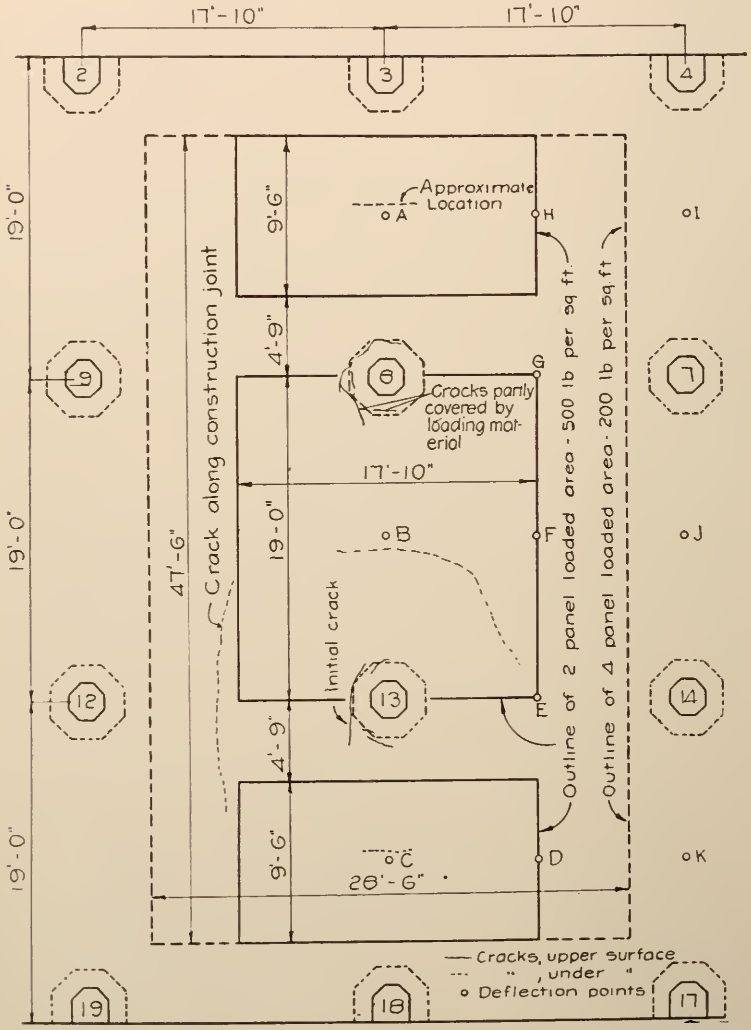


FIG. 77. PLAN INDICATING DISPOSITION OF LOAD AND LOCATION OF DEFLECTION POINTS ON TEST FLOOR OF CURTIS-LEGER COMPANY BUILDING.

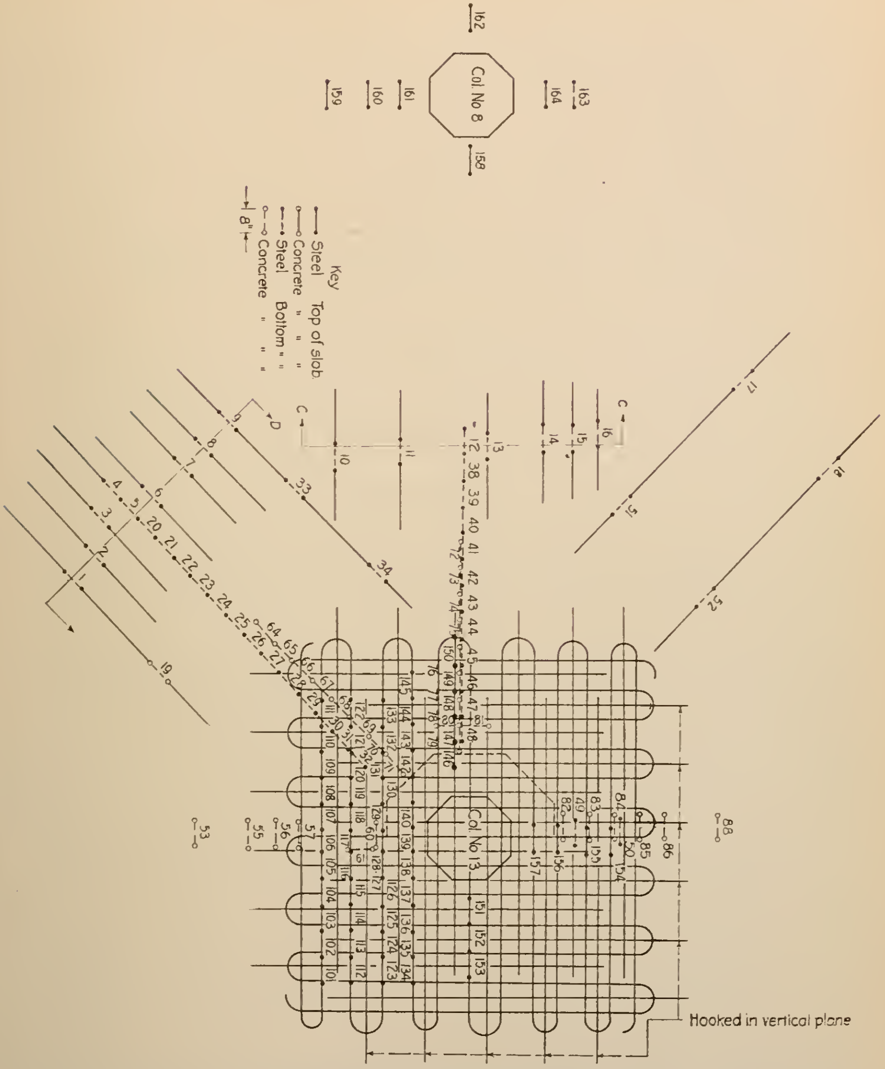


FIG. 78. LOCATION OF GAGE LINES ON UPPER AND UNDER SURFACES OF TEST FLOOR CURTIS-LEGER COMPANY BUILDING.

in Fig. 77 was loaded to 200 lb. per sq. ft., the design live load. The load was then removed from one-half of the area, and the total load was concentrated on an area equivalent to two full panels, shown in Fig. 77. Enough additional load was later applied to the two-panel area to bring the average intensity of the applied load up to 500 lb. per sq. ft. over this area of two panels.

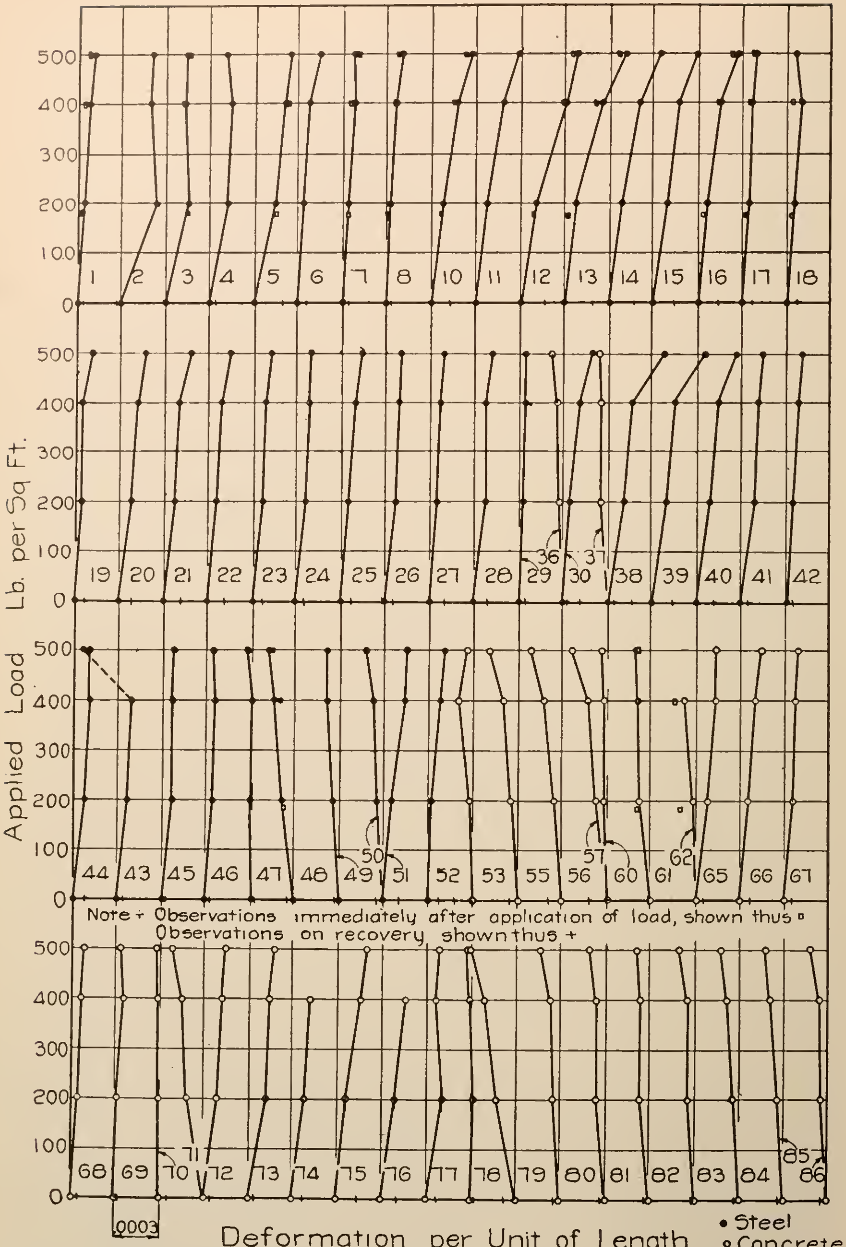


FIG. 79. LOAD-DEFORMATION DIAGRAMS FOR GAGE LINES 1 TO 86 OF TEST FLOOR OF CURTIS-LEGER COMPANY BUILDING.

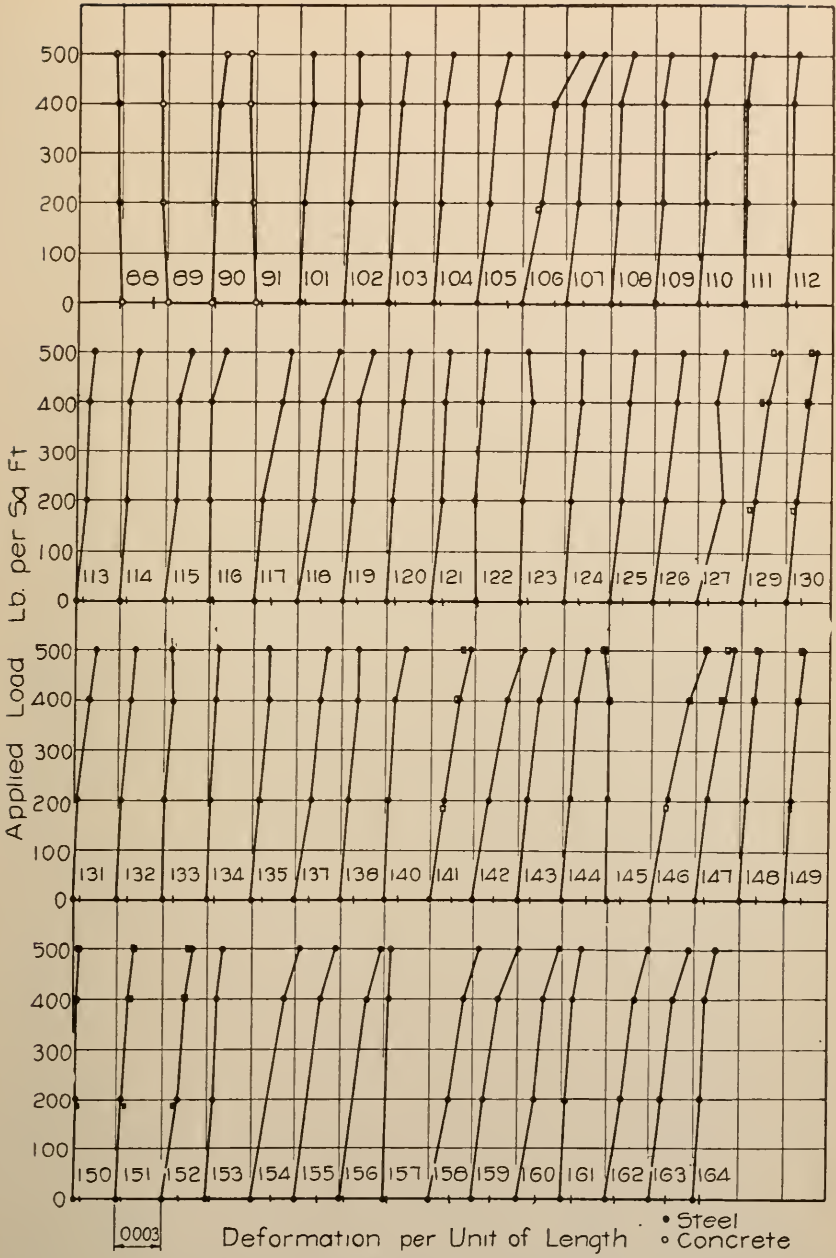


FIG. 80. LOAD-DEFORMATION DIAGRAMS FOR GAGE LINES 88 TO 164 OF TEST FLOOR OF CURTIS-LEGER COMPANY BUILDING.

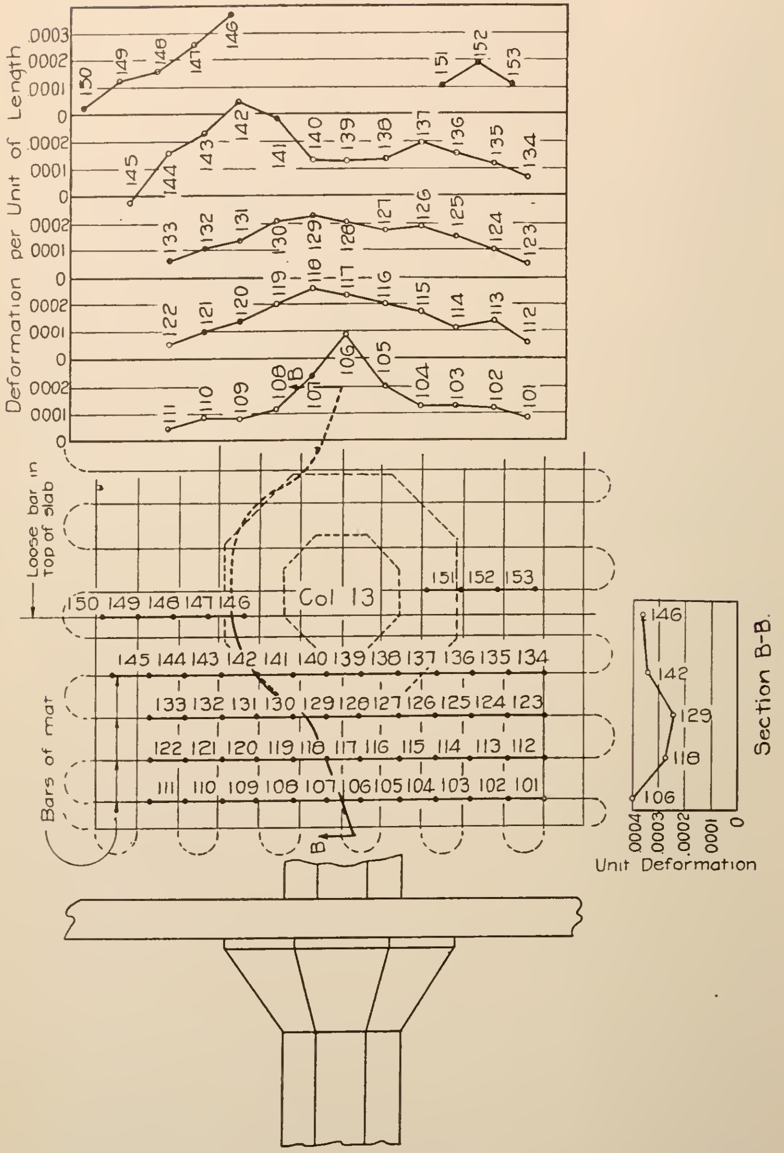


Fig. 81. Locus of Points of Highest Stress in Reinforcement in Top of Slab near Column 13 of Test Floor of Curtis-Leger Company Building.

Deformation readings were taken on a few gage lines in a number of the most significant positions immediately after completing the loadings of 200 lb., 400 lb., and 500 lb. per sq. ft. When each load had been in place 12 hours or more a complete series of readings was taken. For the load of 500 lb. per sq. ft. the complete series of readings was not begun until the load had been in place 46 hours. When the recovery readings were begun, the greater portion of the load had been off the floor for more than 12 hours, but a small portion had been removed less than three hours before.

52. *Control Cylinders.*—The control cylinders gave strengths ranging from 1230 to 1650 lb. per sq. in. and initial moduli of elasticity ranging from about 2,300,000 to 4,700,000 lb. per sq. in. As a fair working value and one which facilitates comparison with the results of other tests, 3,000,000 lb. per sq. in. has been used as the modulus of elasticity in cases in which it is desirable to interpret unit-deformation into stress.

53. *Tension at Capital.*—Load-deformation diagrams for all gage lines are shown in Fig. 79 and 80. Unit-deformations in the reinforcement which extends in the direction of the 19 ft. side of the panel, found at a floor load of 500 lb. per sq. ft. are plotted in Fig. 81 also. From this the locus of the highest stress has been determined and is shown in this figure as section B-B which, for lack of complete information, assumes that gage lines 154, 155, and 156 (see Fig. 78) lie on this locus. The deformations on this section are shown in the right-hand portion of the figure. The highest deformation on this section corresponds to a stress of 11,000, the lowest to 7,000, and the average to 9,300 lb. per sq. in.

54. *Tension Midway Between Columns.*—Fig. 82 shows the distribution of deformation among bars of the rectangular and diagonal bands of reinforcement midway between columns at the applied load of 500 lb. per sq. ft.

Section C-C for the rectangular band lies midway between columns and is cut at right angles by the long side of the panel. The highest deformation was found in one of the central bars of this band, and corresponds to a stress of 11,500 lb. per sq. in. The average deformation corresponds to 9,100 lb. per sq. in.

Section D-D of Fig. 82 lies normal to the diagonal band at the center of the panel. This section lies near the edge of the area covered by the 500 lb. load, and the deformations here probably were smaller than would have been found if the entire area of this panel had been loaded. This is indicated by the fact that the deformations were

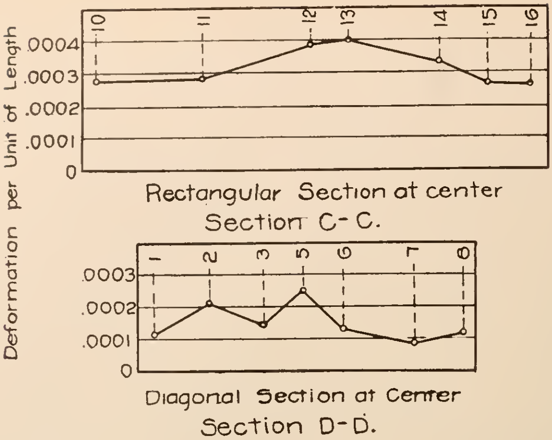


FIG. 82. TENSILE UNIT-DEFORMATION, MIDWAY BETWEEN COLUMNS, IN REINFORCEMENT OF RECTANGULAR AND OF DIAGONAL BANDS IN TEST FLOOR OF CURTIS-LEGER COMPANY BUILDING.

changed only slightly by the shifting of the load from the four-panel area to the two-panel area.

A comparison of the stresses at the gage lines crossed by cracks at construction joints with those at similarly situated gage lines not crossed by cracks shows that considerably larger tensile stress in the reinforcement may be expected at points where the tensile strength of the concrete is impaired than at points where the concrete is intact.

On the short side of the panel no measurements of deformation in the rectangular bands were taken midway between columns because the load was not so placed as to develop representative stresses at that point.

55. *Compression at Column.*—The maximum measured compressive unit-deformation in the concrete was 0.00031. It was found at gage line 79, close to the capital of column 13 on the line of column centers in the direction of the 19-ft. span. The compressive unit-deformation at the corresponding position (gage line 71) on the diagonal line through column centers was 0.00021. If a modulus of elasticity of 3,000,000 lb. per sq. in. be assumed, these unit-deformations correspond to stresses of 930 and 630 lb. per sq. in., respectively. Compressive unit-deformations in these gage lines are shown in Fig. 79, 83, and 84.

To investigate the lateral distribution of compressive stress at the column, measurements of deformation in concrete were taken parallel to the direction of the long side of the panel across the center-line of

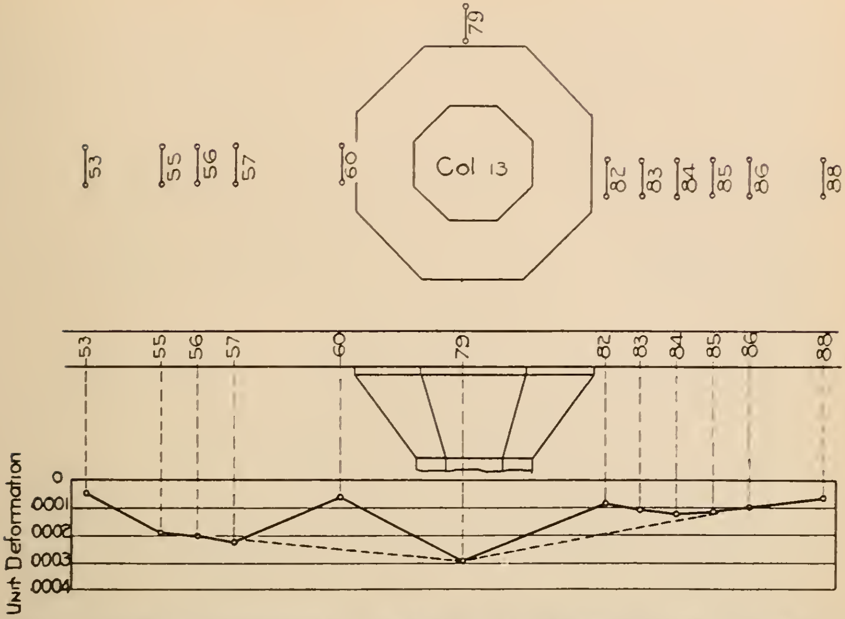


FIG. 83. LATERAL DISTRIBUTION OF COMPRESSIVE DEFORMATION ON BOTTOM OF TEST FLOOR OF CURTIS-LEGER COMPANY BUILDING.

columns, at intervals in the direction of the short side. The extent of area covered by these measurements is shown by the position of gage lines indicated (see Fig. 83). Gage lines 88 and 53 were 84 in. and 76 in. from the center of column 13. The results plotted in Fig. 83 indicate that as far out on either side of column 13 as measurements were taken some compression was developed in a direction normal to the section. If the stress had been uniform and equal to that at gage line 79, a width of about 110 in. would have been required to develop the same total stress as that which was found in the width of about 180 in. It should be noted that gage lines 60 and 82 are close to the column head. By reason of the stiffness of the column head the deformations at these gage lines may not be expected to be as large as may be found farther away from the column head or at points in front or in rear of these positions.

56. *Points of Zero Tension and Zero Compression.*—The manner and amount of variation in deformation along elements of the slab passing over the center of the column in the rectangular and diagonal directions are shown in Fig. 84. The location of the gage lines shown may be found in Fig. 78. An examination of Fig. 84 shows that on the rectangular band the point of zero compression was closer to the

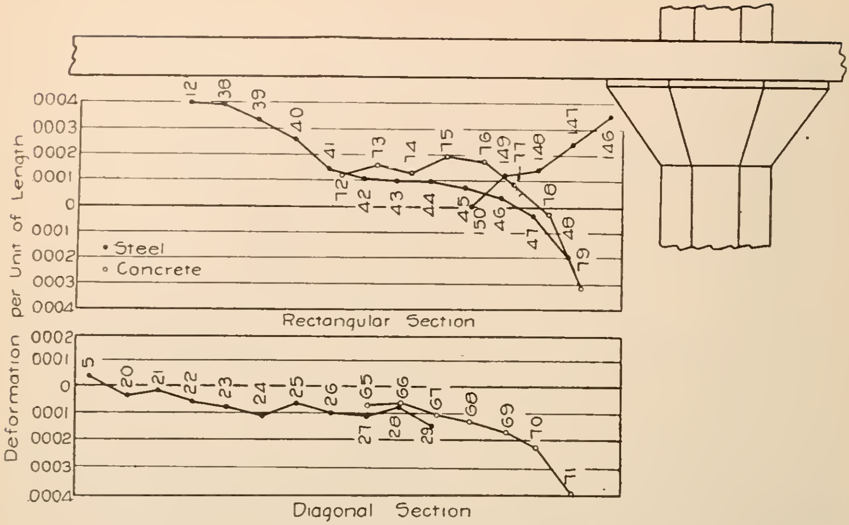


Fig. 84. Location of Points of Zero Unit-deformation on Upper and Under Surfaces of Test Floor in Curtis-Leger Company Building.

edge of the column capital than was the point of zero tension. No explanation of this is offered. For the diagonal direction, section F-F, the point of zero deformation was found on only the under surface. Table 17 gives the positions of the various points of zero stress.

Table 17.

Position of Point of Zero Deformation.

Two panels loaded. Applied load 500 lb. per sq. ft.

Point of Zero Unit-Deformation	Distance from center of column	
	Inches	Ratio to panel length
Bottom; Direct band	42	.184
Bottom; Diagonal band	45	.144
Top; Direct Band, Central Bar	57	.25
Top; Direct Band, other Bars	45	.197
Top; Average, Weighting other bands 3	48	.21

57. *Deflection.*—Load-deflection diagrams for the eleven points shown in Fig. 77 and marked A, B, C, etc., are given in Fig. 85. The maximum deflection at B under the full panel load of 500 lb. per sq. ft. live load was 0.16 in. immediately after the application of this load and 0.17 in. after the load had been in place for 48 hours. These deflections are 1/1425 and 1/1340 of the 19-ft. span.

58. *Cracks.*—In Fig. 77 the locations of cracks found at full load

have been shown by a solid line for the upper surface and by a dotted line for the under surface. These cracks were very fine, the largest being at a construction joint which extended along the direction of the 19-ft. span half way between columns 12 and 13.

59. *Recovery.*—Table 18 gives data on recovery at some of the more important gage lines. This table shows that the recovery was more complete in compression regions than in tension regions whether the measurements were of steel or of concrete deformations. It seems probable that this phenomenon may have been due to the formation of

Table 18.

Principal Full-Load Unit-Deformations and Amounts of Recovery.

Load of 500 lb. per sq. ft. applied over two panels.
Plus indicates extension and minus indicates shortening.

Location of Gage Lines	Gage Line	Full-load Deformation	Residual Deformation	Ratio of Residual to Full-load Deformation
On tension reinforcement midway between columns	10	+.00028	+.00012	.62
	11	+.00028	+.00000	.00
	12	+.00039	+.00016	.43
	13	+.00041	+.00020	.50
	14	+.00033	+.00017	.50
	15	+.00027	+.00014	.53
	16	+.00027	+.00013	.50
	Average			.45
On tension reinforcement at capital	106	—.00040	—.00021	.52
	107	—.00025	—.00012	.47
	117	—.00025	—.00012	.47
	129	—.00024	—.00009	.37
	142	—.00035	—.00014	.40
	146	—.00039	—.00012	.31
	Average			.42
On compression reinment at capital	36	—.00008	+.00001	*.13
	37	—.00005	—.00002	*.40
	48	+.00019	+.00005	.27
	49	—.00010	+.00002	.22
	50	+.00012	+.00004	.31
	Average			.27
On concrete at capital	55	+.00020	+.00004	.20
	56	+.00020	+.00009	.45
	57	+.00023	+.00011	.49
	71	+.00021	+.00004	.20
	79	+.00031	+.00008	.25
	80	+.00014	+.00003	.20
	81	+.00012	+.00006	.50
	83	+.00010	+.00001	.10
	84	+.00012	+.00003	.24
	Average			.29

*Not included in average.

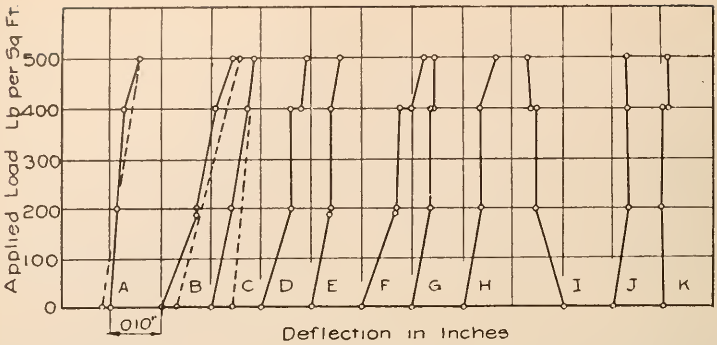


Fig. 85. Load-deflection Diagrams for Test Floor of Curtis-Leger Company Building.

cracks, the fractured surfaces of which could not come together again perfectly after the tensile stress was removed.

60. *Summary.*—The following summary is intended to give the main features of the results of the test:

1 With the load of 500 lb. per sq. ft. distributed over an area equal to that of two panels with a view of making its effect in producing stress as severe as possible, the maximum stress in the reinforcement at the column and midway between columns was about 11,000 lb. per sq. in. Calculations made on the basis of design most commonly used in Chicago give a stress of about 25,000 lb. per sq. in. for this load. However, it seems probable that if a larger area had been loaded to the same intensity the stress developed would have been somewhat larger.

2 The highest unit-deformation observed was near the column on the under side of the slab in the concrete and was measured in the direction of the longer span. Based on a modulus of elasticity for the concrete of 3,000,000 lb. per sq. in. this unit-deformation corresponds to a stress of 930 lb. per sq. in.

3 The point of zero unit-deformation on the under surface of the slab was closer to the column than that on the upper surface. For this reason the location of the point of inflection is not known with certainty, but the indications are that it was at a distance of about two-tenths of the panel length from the center of the column.

4 The deflection under twice the design live load plus the dead load was about 1/1400 of the span.

5 The cracks were very small, the largest being along a construction joint. The stresses in gage lines crossing this crack were enough larger than the tension in similar gage lines not crossing a

crack to indicate that the tensile strength of the concrete adds considerably to the resistance of the slab at this load.

6 The recovery was more complete in regions of compression than it was in regions of tension regardless of whether the measurements were taken on concrete or on steel.

VII. General Comments.

61. *General Comments.*—As was remarked at the beginning, the circumstances surrounding the floor test of a building are unfavorable to securing definite and uniform quantitative results. The distribution of the resistance of the structure to parts beyond the portion which is loaded and the effect of the tensile strength of the concrete, greatly modify the action of the structure. The physical conditions connected with the tests are unfavorable to securing exactness. Conclusions drawn from such tests must be of a general nature and must be confined to the general behavior of the structure. The following comments are given:

1 The stresses measured in the reinforcing steel were relatively low. It is felt that the values of these stresses should not be taken as representative of the stresses which may be developed in the structure when it is loaded over a large area for a considerable time. That this view is not inconsistent with the general practice in designing reinforced concrete may be seen by examining laboratory tests of reinforced concrete beams which have percentages of reinforcement comparable with those in the flat slabs tested. In such beams measured stresses of 5,000 to 20,000 lb. per sq. in. in the steel account for only one-fourth to one-half of the external bending moment. In the tests of flat slabs there is no indication that the tensile resistance of the concrete contributes less to the apparent strength of the structure than is the case with beam construction. It is evident that attention must be given to the mechanics of the structure in determining the requirements for making designs.

There is difficulty in evaluating the compressive deformations of the concrete in terms of stress, since the modulus of elasticity of the concrete in the slab may not agree with the values determined from test specimens. The observations on compression are useful in finding the distribution of compressive stresses.

2 For negative moment the locus of maximum stress in a direction perpendicular to a panel edge was a line which followed the column capital for nearly 180° and merged into the panel edge a little distance away from the column capital. In the Schulze Baking Company

Building the measurements were of compression on the under side of the depressed head. In the Worcester test structure and in the Curtis-Leger Building the measurements were made on the tension reinforcement.

3 In the Shredded Wheat Factory the tensile stresses resisting negative moment across the panel edge would average as high at locations intermediate between columns as exist at points close to the column. It is apparent that the actual distribution along a section of negative bending moment would be affected by the size and spacing of the bars crossing the section.

In the Schulze Baking Company Building also it appears that bars across a panel line at a location midway between columns developed resistance to negative bending moment.

Information having a bearing on the distribution of tensile stresses across panel lines was not obtained in any other test discussed in this bulletin.

4 In the building in which there were depressed heads around the column capitals and in which information was obtained on the distribution of the compressive stresses over the section of maximum negative moment, the Schulze Baking Company Building, there were indications that the compressive stresses of the negative moment were taken almost entirely in the portion of the section within the width of the depressed head and that there was very little compression in the thin portion of the section between depressed heads. In the Worcester slab, which had no depressed heads, at the load of 215 lb. per sq. ft. the compressive stresses in the section of maximum negative moment were distributed along the section for the full width of the panel, although the stress midway between columns was less than that closer to the column. In the Curtis-Leger Building compressive stresses were found in the section of maximum negative moment as far away from the column capital as measurements were taken.

5 In the building having a relatively large thickness of depressed head, the Schulze Baking Company Building, the compressive stresses on the under side of the thin portion of the slab close to the depressed head and perpendicular to its edge were nearly as large as those in the same direction on the depressed head close to the column capital.

6 An increase in the deformations in the section of maximum positive moment was found when the loaded area was changed from a group of panels to a row of panels. This change of loading was made in the Shredded Wheat Factory and in the Soo Terminal. How much of the increase may have been due to a proportionally smaller contri-

bution by the tensile resistance of the concrete is not known, but it is evident that the positive moment must have been increased considerably by this change in loading.

7 High bending deformations, due to eccentric loading, were found in columns located at edges of loaded areas. In the Shredded Wheat Factory, a severe bending moment in a column of the basement story was observed when panels of the first floor on one side of this column were loaded. Even with nine panels loaded bending deformations were found in interior columns, evidently due to difference in the slab moments on the two sides of the column. In this case the bending was in a direction opposite to that found when the column was at the edge of the loaded area. In the Soo Terminal, a one-story structure, marked bending phenomena were observed in columns at the edge of the loaded area, and tensile deformations were found of such amount that even considering the compression due to dead load the tensile resistance of the concrete must have been exceeded. The position of the point of inflection of the elastic curve of flexure was in fair agreement with the usual analysis. In the Schulze Baking Company Building, the bending of columns at edges of the loaded area was an important feature of the action of the structure in the test, the largest bending apparently occurring in a column at a corner of the loaded area. In the Worcester Slab Test, the bending of certain wall columns and corner columns was apparent to the eye, and large tensile deformations were observed in the column reinforcement. Although the bending action was not different from that which may be obtained by analysis, it seems well to call attention to the phenomena observed, since provision for resisting the bending moment produced by the eccentric loading of columns (both wall columns and interior columns) may be overlooked by some designers.

8 In the one building in which load was applied to a wall panel having a lintel beam, the Shredded Wheat Factory, diagonal cracks were found on the interior side of the lintel beam near its ends. None was found on the outside of the beam. The cracks extended upward and away from the ends of the beam. The phenomenon was probably the result of the twisting action produced by bending moment developed in the slab at its edge by the load on the wall panel and transmitted to the lintel beam through the monolithic connection between the slab and the beam.

9 In the two one-story structures tested, the Soo Terminal and the Worcester test structure, the unevenness of settlement of the footings was sufficient to interfere with interpretation of the results. In

a building of several stories the rigidity of the structure may be expected to cause it to settle more as a unit. It is evident that in a one-story structure unusual precautions should be taken to guard against uneven settlement.

10 The tests which have given most definite results and results most useful for comparison with analytical treatment have been made on slabs whose thickness was small in relation to the span.

11 Progress in obtaining experimental knowledge of flat slab structures may best be made through a series of tests on structures designed solely for test purposes and planned systematically to bring out the fundamental differences between different types of design and the effect of varying certain elements of design. Occasional tests of floors may give interesting information, but the differences in design and construction among the different structures may be so unsystematic as to make the results not comparable, rendering them useful mainly for judging of workmanship and the sufficiency of the design.

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THE STRENGTH AND STIFFNESS OF STEEL UNDER BIAXIAL LOADING

BY
ALBERT J. BECKER

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

Bulletin No. 85 April, 1916

THE STRENGTH AND STIFFNESS OF STEEL UNDER BIAXIAL LOADING.*

By Albert J. Becker,

Professor of Applied Mathematics in University of North Dakota, and Formerly Graduate Student in University of Illinois.

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*This bulletin embodies the principal data of the thesis of Albert John Becker, presented in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Engineering in the Graduate School of the University of Illinois, June, 1915, together with further experimental data taken by the author to extend the investigation.

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THE STRENGTH AND STIFFNESS OF STEEL UNDER BIAXIAL LOADING.

I. Introduction.

1. *Scope of Investigation.*—The purpose of this investigation was to determine the laws governing the strength and stiffness of mild steel when subjected to combined stress produced by two tensions at right angles to each other or by a compression combined with a tension at right angles. In order to give a satisfactory basis for comparison of results, the plan of investigation provided that the ratio between the two stresses be kept constant throughout the test of a specimen, and J. B. Johnson's tangent method of determining the "yield point" or "apparent elastic limit" was selected.

The specimens tested were drawn steel tubes of uniform size and practically of uniform thickness. These tubes were subjected to an axial load and to internal pressure. The only variable was the ratio of the circumferential stress to the axial stress. Comparison has been made only in the test results from sets of tubes cut from a single length of seamless drawn tubing. By means of strain gage readings a knowledge of the distribution of stress on the cross section was obtained; no assumptions were made except that of uniform distribution of the circumferential tensile stress throughout the thickness of the tube wall.

The investigations of strength and of stiffness were carried on simultaneously, but the results are discussed separately. The points investigated are:

(a) The change of yield-point stress of the material with increasing ratios of circumferential tensile stress to axial tensile or compressive stress.

(b) Stiffness of the material (strains accompanying stress) for increasing ratios of circumferential tensile stress to axial tensile or compressive stress.

No discussion has been given of the engineering applications, for it is realized that while these applications are important, more work is needed to establish the conclusions reached. When this has been done and all the work has been correlated, it will be a simple matter to make an application of these principles to engineering design.

2. *Acknowledgment.*—All the tests were made in the Laboratory of Applied Mechanics of the University of Illinois, under the supervision of Professors A. N. Talbot and H. F. Moore, to whom acknowledgment is made for their suggestions and criticisms and for the interest they

have shown in the progress of the investigation. Acknowledgment is also made to Mr. J. O. Draffin, research fellow in the Engineering Experiment Station, for his assistance in the conduct of the various tests. It is also desired to make an acknowledgment to the Joint Committee on Stresses in Railroad Track for the use of the new model 4-in. Berry Strain Gage.

3. *General Statement.*—When a steel bar is tested in tension or compression, certain phenomena are observed which have been incorporated as fundamental facts in the theories of the elastic behavior of bodies under stress. In such a test, both the strength and the stiffness of the material are observed, the former by noting the yield point and ultimate strength, the latter by observing the unit-strains corresponding to successive loads and computing the modulus of elasticity. Repeated experiments have shown that for material of the same composition and treatment, these results are practically constant and can be used as a basis of design. The strength of any material of construction cannot be determined by mathematical analysis, neither can its stiffness. Poisson's ratio, modulus of elasticity, yield point, and Hooke's law are experimental results.

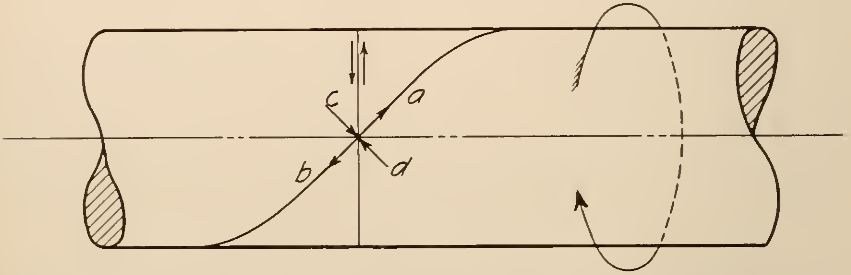

FIG. 1. ILLUSTRATION OF STRESSES PRODUCED ON OBLIQUE PLANES IN A BAR SUBJECTED TO TORSION.

When an investigation of combined stress is attempted, there arises the question of the extent to which the calculations may be based upon the values obtained in the experiments in simple tension, compression, and shear. Constants determined by uni-directional loading cannot be indiscriminately applied to bi-directional loading. Theories have been evolved in which these constants are used by taking account of the interaction of the applied stresses. The analyses for these are correct from the mathematical standpoint, but the soundness of the basic assumptions can be demonstrated only by experiment.

Different combinations of simple stresses are possible, and it may be expected that the same analysis will not apply to all combinations. The presence of shearing stress in a bar subjected to simple tension and the tensile and compressive stresses accompanying the shearing stress due to a torsional load indicate that the governing conditions depend upon the relative strength of the material in shear, tension, and compression. A cast iron bar tested in torsion fails in tension on an oblique plane, because the tensile strength is less than the shearing strength. It is, therefore, logical to suppose that different stress combinations will produce failures differing in character for different materials.

4. *Combined Stress.*—Three types of stress applications are possible, uni-directional or simple stress, bi-directional or biaxial, and tri-directional. The first is illustrated by a specimen subjected to tension or compression in an ordinary testing machine. Bi-directional or biaxial stress is the application of two stresses in the same plane acting in directions at right angles to each other. Tri-directional stress is the application of three stresses at right angles to each other. The condition of biaxial stress is more important, from the point of view of the engineering applications, than that of three stresses at right angles to each other.

The possible combinations of biaxial stress are as follows:

Tension with tension.
Tension with compression.
Compression with compression.
Compression with tension.
Shear (torsion) with tension.
Shear (torsion) with compression.

These may be divided into three classes, tension with tension and compression with compression forming the first, tension with compression and compression with tension the second, and the combination of either tension or compression with torsion forming the third. The third class includes also two special cases of the second class; for a simple torque is equivalent to two equal principal stresses, one compression and the other tension, so that a torque combined with tension or compression can be reduced to the case of tension combined with compression or vice versa. This equivalence will readily be seen by considering a bar of circular cross-section subjected to torsion alone, Fig. 1. The stress on a plane at right angles to the bar is a pure shearing stress, depending in intensity upon the diameter of the bar and upon the torque. But this is not the only plane of stress. As in a bar in simple tension, so in this case there are planes on which both tensile and shearing stresses occur; there are also planes upon which no shearing stresses occur. Referring to Fig. 1, with the torque as shown by the arrow, the stress on the 45° plane CD is tension, and on plane AB at right angles to this plane, the stress is compression. This is equivalent to a biaxial loading which develops a tensile and a compressive stress at right angles to each other and each equal to the shearing stress. It should be noted that there are stresses on oblique planes which may control the strength of the material.

Applications of combined stress are to be found in the familiar examples of the steam boiler for tension combined with tension, and of the crank shaft for tension or compression combined with torque. Biaxial stresses occur in flat plates and in flat concrete slabs or girderless floors.

II. THEORIES OF THE STRENGTH OF MATERIALS UNDER COMBINED STRESS.

5. *The Six Theories.*—The mathematical discussion of stresses and strains in a thin tube under axial load and internal pressure is given in Appendix II, page 58. It follows closely the method used by Love* in his work on the theory of elasticity, to which those who wish to investigate the subject further are referred.

Six theories have been advanced to cover the problems of the strength of material under combined stress. Two of them are empirical, one is developed from a molecular hypothesis, one from the mathematical theory of elasticity, and two from static relations of stresses. Three of these theories have found considerable favor and are given first.

6. *The Maximum Strain Theory.*—In the mathematical theory of elasticity, after the relations between stress and strain are established for simple stress, three equations of the following types are derived:

$$E\epsilon_1 = \sigma_1 - \frac{1}{m}(\sigma_2 + \sigma_3),$$

where σ_1, σ_2, and σ_3 are the three stresses at right angles to each other, E is the modulus of elasticity assumed constant in all directions, ϵ_1 is the unit-strain in the direction of σ_1, and $\frac{1}{m}$ is Poisson's ratio.† Stresses are considered positive if tension, and negative if compression. $E\epsilon_1$ is called by various writers the reduced stress, the true stress, or the ideal stress, but as the term stress is generally used by engineers to mean an internal resisting force which holds external forces in equilibrium it seems best to refer to it merely as $E\epsilon$. Writing two equations similar to the above for $E\epsilon_2$ and $E\epsilon_3$, the three equations for the reduced stress are obtained. The maximum strain theory takes these three equations and assumes that whatever the combination of stresses, the material will fail when the maximum strain (which will be in the direction of the greatest stress) reaches a value equal in magnitude to that at the yield-point stress in simple tension or compression. $E\epsilon$ at the yield-point stress for any combination of stresses must be the same, provided the yield-point stress is the same for tension as for compression. For ductile materials, E is usually assumed to be constant and it follows that ϵ must be the same when the yield-point stress of the material is reached, no matter what combination of stresses is used. But for a brittle material, where E varies, the strain ϵ must vary in an inverse ratio; that is, the product remains constant.

---

*The Mathematical Theory of Elasticity, A. E. H. Love.

†A stress in any direction produces strain in that direction and also strain at right angles to that direction. The numerical ratio between the unit-strain at right angles to the direction of the force and the unit-strain in the direction of the force is called Poisson's ratio.

The maximum strain theory, or St. Venant's theory as it is sometimes called, holds that when a material is subjected to two or three stresses at right angles to each other, its strength is increased if the stresses are of like sign and that its strength is diminished if the stresses are opposite in sign. Thus two tensions or two compressions will produce an increase in the elastic strength of the material, whereas a tension combined with a compression produces a reduction in strength. For a stress ratio of one to one, both stresses tension, the material will be increased in strength 43 per cent if Poisson's ratio is 0.3, while if one stress is tension and the other compression, it will be weakened 23 per cent for the same stress ratio.

If in the equation for reduced stress given above, σ_2 and σ_3 are zero, the case is that of a bar in simple tension (compression is expressed as negative tension) and dividing both sides of the equation by ϵ_1, the result is the equation of the modulus of elasticity.

For combined stress according to this theory, then, the strain accompanying a given stress is changed by the addition of another stress at right angles to the first. It is increased if the stresses have unlike signs and diminished if they have like signs. Also, the strain ϵ is the measure of $E\epsilon$ (the reduced stress) and the material will not reach the yield point until the strain ϵ reaches the value corresponding to the strain obtained in simple tension at the yield point. It should be emphasized that all elastic theory holds only within the elastic limit, or more correctly within the limit of proportionality, where E remains constant for an individual stress-strain diagram. But the slight variation up to the yield point, even though the value of E does change slightly, does not invalidate the theory, and the yield point is commonly taken as the limit of the discussion.

The maximum strain theory is based upon the mathematical theory of elasticity. Temperature effect is neglected and Hooke's law is assumed to hold rigidly. Herein lies its weakness, for the maximum strain theory, like the mathematical theory of elasticity, is dependent upon the accuracy of the relation assumed between stresses and strains. It has been shown* that there is a cooling of a bar of metal as the stress is increased up to the yield-point stress and it is also well known that Hooke's law is only an approximation.† A very good approximation it is, to be sure, for engineering purposes, but lack of isotropy in the materials, cold working and similar causes tend to change conditions, so that a slight deviation from Hooke's law may be observed considerably before the yield-point stress is reached. While the maximum strain theory has a good foundation, it must not be expected that the measured strains upon a body known to be not wholly isotropic, will conform exactly to this theory of stiffness.

The question of strength is quite different, for there is no assurance that the strains are the true measures of strength. Reasonable as the assumption may be, it is an assumption whose correctness must be demonstrated by experiment.

7. *The Maximum Stress Theory.*—The maximum stress theory, or Rankine's theory as it is sometimes called, virtually assumes that whatever the ratio of the stresses in the two directions and whether they are of like or opposite sign, the material will reach the yield point when, and only when, one of the stresses reaches the value corresponding to the yield-point stress in simple tension or in compression, as the case may be. It takes no account of Poisson's ratio as affecting strength and assumes that a material is neither weakened nor strengthened by the addition of a second stress at right angles to the first. If, then, this theory holds, the material should reach its yield point when the greater stress reaches the yield point stress for uni-directional loading.

8. *The Maximum Shear Theory.*—In the preceding theories failure

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*C. A. P. Turner, Trans. Am. Soc. C. E., 1902. Lawson and Capp, Inter. Assn. Test. Mat., 1912. Ew. Rasch, Inter. Assn. Test. Mat., 1909.

†Hedrick, Engineering News, Sept. 16, 1915.

by yielding is considered to take place in tension or compression, whereas the maximum shear theory, or Guest's law as it is sometimes called, holds that all failures are failures by yielding due to shear when the shearing unit-stress reaches the shearing yield-point stress. Therefore, if loads are gradually applied to two specimens developing simple stress in one and combined stress in the other but so as to keep the shearing stresses the same in each specimen, the yielding failure in the two cases will be identical.

The basic principle of the maximum shear theory, that the failure in combined stress is the result of the shearing stress reaching the shearing yield-point stress, when carried to its logical conclusion demands that when two of the principal stresses are zero the failure is still due to shear. A steel bar subjected to axial tension only must therefore fail in shear. The maximum shear in this case occurs on a 45° plane and its intensity is one-half the tensile unit-stress. If the yielding due to shearing stress occurs at the same time as yielding due to tensile stress the yield point unit-stress of the material in shear must be just one-half that in tension, but if the shearing yield-point stress is reached first—as this theory maintains—then the ratio is somewhat less than one-half.

If the stresses which are combined are a compression and a tension, the resulting maximum shearing unit-stress is one-half the sum of the tensile and compressive unit-stresses. When the tensile and compressive stresses are equal, the intensity of the shearing stress is equal to the intensity of the tensile or compressive stresses and failure will take place by shear unless the shearing yield-point stress is equal to or greater than that of either tension or compression. It seems entirely possible, then, that failure may be caused under certain conditions by shear and that in other cases its intensity may be insufficient to cause yielding, the tensile or the compressive yield-point stress being reached first.

Considering compression as negative tension, there are two kinds of elementary stress treated in mechanics—tension and shear. They are quite distinct and have different accompanying phenomena. While a definite relationship may be established between the shearing and tensile stresses, the material may fail either in tension or in shear. This is suggested by the fact that mild steel in torsion gives a square break, a shearing failure, but cast iron tested in torsion breaks along a helicoid, failing in tension because the material is weaker in tension than in shear.

This duality of conditions while not entirely overlooked, has been advanced heretofore solely to form two distinct theories of failure, but these have not been connected. The possibility that both shear and tension may govern, each within certain limits, has apparently not been mentioned in the publications and discussions on this subject. Mallock* has stated a dual law which is quite different from that discussed above. He proposes a volume extension limit and a shear limit, each dependent upon the other, and assumes that the material will fail when the limit of either is reached. This is quite distinct from the simple stresses as controlling factors in the failure of the material, but it recognizes the possibility of dual control.

The usual stress derivation for combined stress given in textbook is based upon the static equilibrium of forces and an application is made to a circular shaft in combined bending and torsion. A solution is given for the maximum normal stress and shearing stress on oblique planes, and safe working stresses are assigned. The assignment of working stresses in shear and tension fixes an arbitrary ratio of shear to tension, and the larger of the two shaft diameters determined by the two formulas is to be taken.

9. *The Internal Friction Theory.*—A short cylinder of brittle material when tested in compression fractures by shearing along a diagonal plane which, if failure be due to shear, should make an angle of 45° with the axis, since this is the plane of greatest shearing intensity. But the angles observed in experiments differ from 45°. In the attempt to explain this variation the theory of internal friction has resulted. When two particles under stress tend to slide over each other, a condition is set up similar to that of ordinary sliding friction. On the supposition that this resistance is similar to sliding friction, one of the laws governing the latter is applied; namely, that the coefficient of internal friction is independent of the load or stress. Therefore, slipping will occur along the surface of the plane inclined at an angle β with the axis of the specimen such that $\beta = 45° - \frac{\varphi}{2}$ for compression and $\beta = 45° + \frac{\phi}{2}$ for tension. ϕ is the angle of friction and $\tan \phi = \mu$, the coefficient of friction. If the limiting friction per unit of surface is the same for tension and for compression, then the normal stress on the surface of slipping, at the instant when yielding begins, must be the same in each case, since this is $\frac{1}{\mu}$ times the limiting friction.

It has been said that the chief difference between the internal friction theory and the maximum shear theory is that the former is based upon a maximum resistance to sliding, while the latter is based upon a maximum shearing stress. If the angle of friction is zero, the internal friction theory becomes the maximum shear theory.

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*Proc. Royal Society of London, 1909.

10. *Mohr's Theory.*[*]

Let k_4 = the shearing yield-point stress.

Let k_3 = the stress in compression and in tension (equal) which together produce a shearing stress equal to the shearing yield-point stress, k_4.

Let k_1 = the tensile yield-point stress.

Let k_2 = the compressive yield-point stress.

Mohr derives the formulas:

$$k_3 = \frac{k_1 \ k_2}{k_1 + k_2} \text{ and } k_4 = \frac{1}{2} \sqrt{k_1 \ k_2}$$

The usual theory developed from the static relation of stresses gives for two equal stresses of unlike sign the following relation for the stress intensities:

Shearing stress = $\frac{1}{2}$ (tensile stress + compressive stress) which is the same as Mohr's theory when the tensile and compressive yield-point stresses are equal. Mohr's theory is an attempt to modify the shearing yield-point stress according to the tensile and compressive yield-point stresses. When these are equal this theory presents nothing new, for it then coincides with the maximum shear theory. If the yield-point stresses are different, Mohr's theory brings in a new relation regarding the shear failure in combined stress. It is virtually an acceptance of the maximum shear theory with the definition of the value of that shear at the yield-point.

11. *Wehage's Theory.*[†]—This theory is based upon a few experiments on cross-shaped pieces of paper submitted to tension in two directions at right angles to each other. If the material has a different yield-point stress in the two directions, the following elliptic relation is given as an empirical deduction:

$$\left(\frac{t_1}{T_1}\right)^2 + \left(\frac{t_2}{T_2}\right)^2 = 1$$

T_1 and T_2 are the yield-point or the ultimate stresses in the two directions (as, for instance, with and across the direction of rolling), and t_1 and t_2 are the applied stresses in the corresponding directions. When $T_1 = T_2$, this elliptic relation becomes a circular one.

This theory assumes that the material is *weakened* by the application of two tensions for the reason that such stresses tend to lessen the cohesion between the fibers. The assertion is also made that a compression combined with a tension should strengthen the material by increasing this cohesion, although no formula is proposed.

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[*] Zeitschrift des Vereines Deutcher Ingenieure, 1900.
[†] Zeitschrift des Vereines Deutcher Ingenieure, 1905.

12. *Graphical Presentation of Three Theories.*—A graphical presentation frequently serves to give a better idea of the working of a theory or formula and for this reason the three most important theories are represented in Fig. 2, 3, and 4, for the four combinations of simple tension and compression. To make the presentation more general, different yield-point stresses in compression and in tension have been assumed where this is possible.

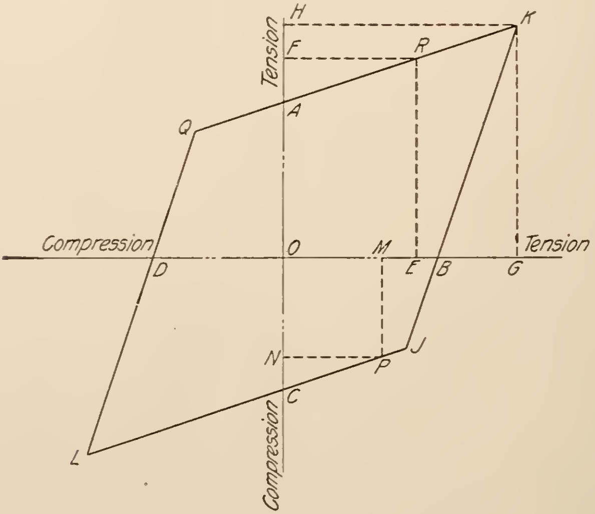

FIG. 2. GRAPHICAL REPRESENTATION OF STRESSES ACCORDING TO THE MAXIMUM STRAIN THEORY.

Maximum Strain Theory. Let OA (Fig. 2) and OB represent the yield-point stress in simple tension and OC and OD that in compression. A tensile stress equal to OE would require a tensile stress equal to OF at right angles to cause yielding. For two equal tensile stresses the condition of yielding would not be reached until each stress attained the value OG, equal to OH. The increase in strength is OG — OB.

For a compression combined with an equal tension, yielding would occur when each stress attained the value O N, equal to O M. The other two quadrants are similar, two compressions producing the same relative effect as two tensions, and a tension and compression producing a corresponding effect to a compression and a tension.

Maximum Stress Theory. Yielding takes place in tension or in compression and since the stress in one direction is not affected by a second stress at right angles to the first the diagram will be a square. The center of the square, however, will not be the origin of co-ordinates since the tensile and compressive yield-points will in general be different. If a tensile stress OB or a compressive stress OD, Fig. 3, equal to the yield-point stress, is applied in one direction, any stress, OE less than the yield-point stress in tension, may be applied at right angles without causing further yielding. In other words a second stress acting at right angles to the first yield-point stress does not change the yield-point stress of the material.

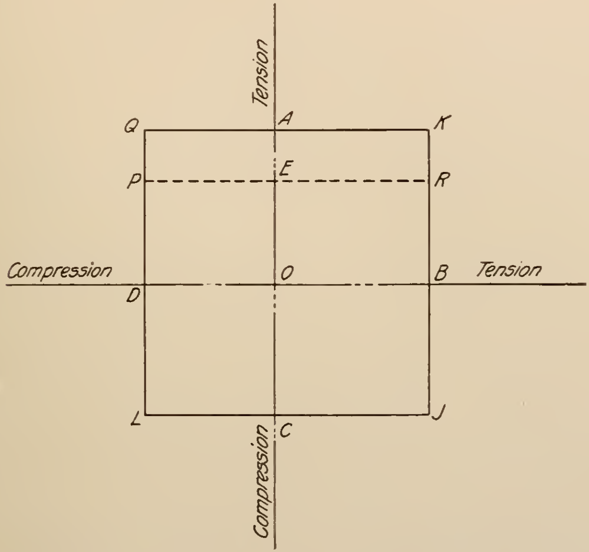

FIG. 3. GRAPHICAL REPRESENTATION OF STRESSES ACCORDING TO THE MAXIMUM STRESS THEORY.

Maximum Shear Theory. The first and third quadrants (Fig. 4) correspond to the maximum stress theory. This follows from the fact that the shearing stress equals one-half the difference between the greatest and the least of the three principal stresses. For biaxial loading one of the three principal stresses is zero and in the first and third quadrants the other two are of like sign, hence the shearing stress will be one-half the greatest stress. But the limiting shearing stress must be constant, therefore the greatest limiting principal stress must be constant and for like stresses (first and third quadrants) the diagram corresponds to the maximum stress theory. For a combination of tension and compression (second and fourth quadrants) the lines CB and AD are inclined at an angle of 45°, because the tensile stress plus the compressive stress is a constant and is equal to twice the shearing stress.

$$t + c = \text{constant.}$$

By setting t and c each equal to zero in turn, it is seen that t must equal c, and this theory demands an equal yield-point stress for tension and compression. Two equal stresses of unlike sign will then cause yielding of the material when each stress equals ON or OM.

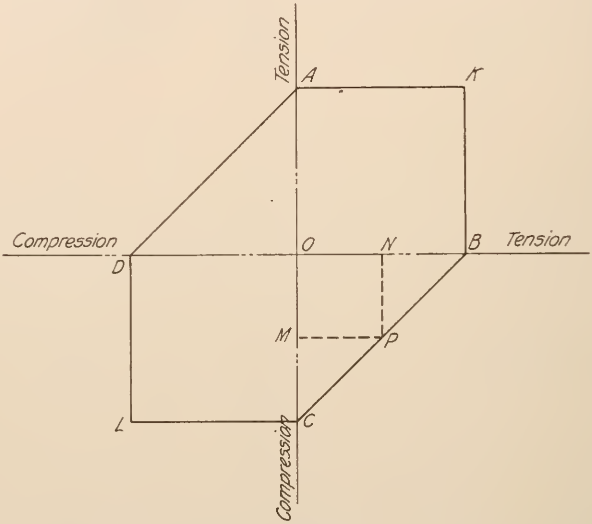

Fig. 4. Graphical Representation of Stresses According to the Maximum Shear Theory.

III. Experimental Work.

13. *Form of Specimen.*—The selection of the type of specimen to be used in the experimental work was a problem of considerable difficulty. Specimens subjected to direct tension or compression in two directions were not considered because of complications produced by the method of application of the load. A cube subjected to compression in two directions could easily have been set up, but the friction between the bearing blocks and surfaces of the cube introduces inequalities and resistance to the change in cross section which could easily vitiate the results.*

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*See Zeitschrift des Vereins Deutscher Ingenieure, 1900, p. 1530.

A large number of short square steel bars, closely spaced to form in effect a bearing block, were considered not to obviate this difficulty sufficiently. Similarly, a tension specimen held at the four edges would not be practicable. Direct stress application seemed out of the question, and recourse was first had to bending to produce stresses in two directions at right angles to each other.

The first biaxial stress experiments in this series of tests were made upon flat cross-shaped specimens subjected to cross bending to produce two compressions or two tensions at right angles to each other. The stress distribution was so far from regular that no safe comparisons could be made. Such difficulties were encountered that this form of test specimen was discarded.

After a preliminary test, thin tubes were adopted as the form of test specimen. They proved satisfactory on account of the certainty with which biaxial stress of known magnitude could be applied by means of an axial load in a testing machine and internal hydrostatic pressure producing a circumferential tension. This method gives two well defined principal stresses at right angles to each other, the stress in the third direction being small since it varies from the intensity of the hydrostatic pressure on the inside to zero on the outside. It is much easier to cover the total range of stress ratios by the use of hydrostatic pressure and axial tension or compression in the tubes, than to use torque and axial load on solid bars. The latter method is inferior to the tube tests since only a small portion of the material is carried to the yield-point stress. The experiments are more successful when as much of the specimen as possible is uniformly stressed, and the best condition is that wherein the entire specimen is uniformly stressed. This is true both on account of the pronounced yield-point effect and on account of the smallness of the strains to be measured. The thinness of the wall and the relatively large tube diameter made the stresses practically uniform throughout the tube. It may be expected that the stress-strain diagrams will show a much sharper break than for solid bar specimens and the yield point is more positively determined. There are no greater eccentricities of application of load when using the tube than when working with a solid bar, and on account of the greater diameter of the tube, this eccentricity is relatively less important.

Strains were measured by means of a Berry strain gage, using a 2-in. gage length in the cross bending tests and a 4-in. gage length in the tube tests. The accuracy and reliability of an instrument of this type has been demonstrated repeatedly and reference is made to the tests by A. N. Talbot and W. A. Slater on reinforced concrete buildings, as given in Bulletin No. 64 of the Engineering Experiment Station of the University of Illinois, to show what results may be achieved with such an instrument. A discussion of the strain gage and its use is given in a paper by Slater and Moore in Vol. XIII of the Proceedings of the American Society for Testing Materials.

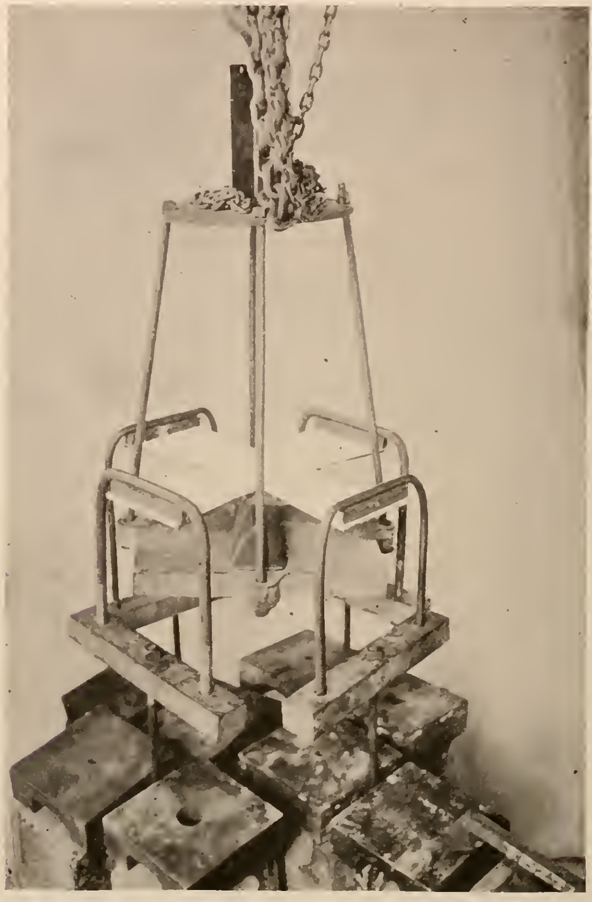

FIG. 5. VIEW SHOWING CROSS-BENDING TEST SPECIMEN UNDER LOAD.

The use of the strain gage marks a decided advance in the measurement of strains. With this instrument it was possible in these tests to take twenty-eight readings on as many gage lines for each increment of load, whereas other investigators have been able to take four at the most and often only two. The advantage of a portable instrument over an attached one is very great and the rapidity of operation and freedom from danger of jarring the instrument as well as the ability to read overlapping gage lines, as was done in these tests, marks a decided step in advance.

14. *Cross-bending Tests.*—The set-up for the bending tests is shown in Fig. 5. Two specimens were prepared from 1/4-in. soft steel plate of the shape shown in the figure. Tension specimens were prepared from the portions cut away. In order to have the upper surface unobstructed for the use of the strain gage, the beam was loaded as an overhung beam with four equal loads placed symmetrically one on each projection of the cross-shaped specimen. The center part of the cross was thus subjected on the top to two tensions at right angles to each other.

Load was applied by placing known weights on the yokes at the ends of the arms of the specimen, thus giving a definite bending moment. The strains were measured over 2-in. gage lines with a Berry strain gage. Instead of a uniform stress over the center portion of the test piece, the readings showed a considerable variation. The effect of the sharp reentrant angles at the corners in changing the lines of stress must have been considerable, for the yield point was reached first at the corners. The lines of yielding spread inward along a line making an angle of approximately 45° with the center lines. As the load was increased these lines divided, curving toward the adjacent corners, gradually changing direction and becoming parallel to the lines of symmetry of the specimen shortly before the lines from adjacent corners joined. New lines formed beside the first ones and others appeared outside the center of the cross. The latter were straight and parallel to the support. The lines are clearly shown in Fig. 6, which is from a photograph of the compression side of the first specimen tested. The lines marking the square from corner to corner and the center lines were used to lay out the specimen and must not be confused with the lines of yielding. The specimen, considered as a beam, widens abruptly for the center four inches, but the effect of this increased width in carrying stress was slight. The places of greatest stress were near each corner and to measure the maximum strain would have required a very short gage line. This stress condition is due to the form of the specimen rather than to combined stress.

15. *Tube Specimens.*—Specimens made from 6-in. tubes with

¼-in. walls were used. Four lengths of seamless drawn tubing were bought in the open market and made into test specimens. A series number was given to the specimens cut from a length of tubing and each specimen was numbered individually. The number of the test specimens in each series cut from each length of tubing and the character of the stresses applied are as follows:

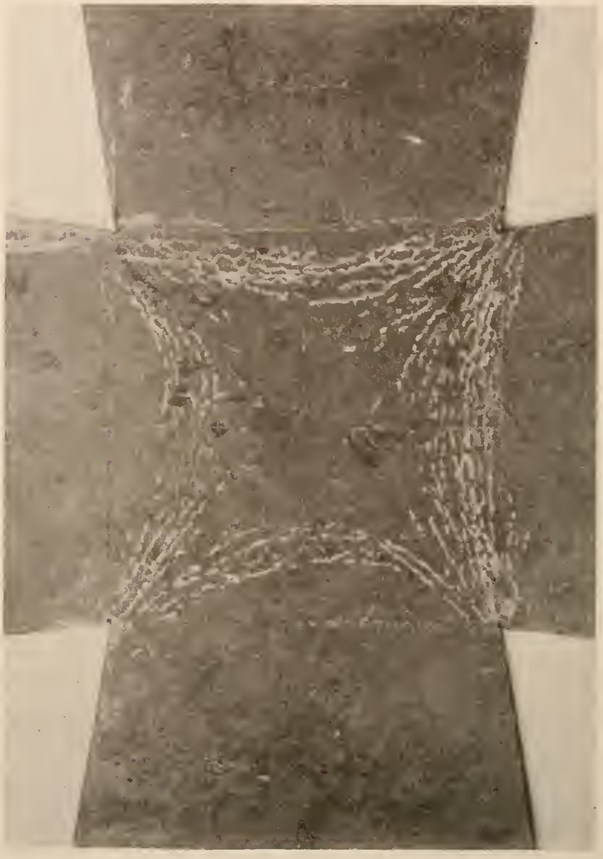

FIG. 6. VIEW SHOWING COMPRESSION SIDE OF CROSS-BENDING TEST SPECIMEN AFTER TEST.

Series	Number of Specimens	Specimen Number	Character of Combined Stress
1	5	1-2-3-4-5	Tension with tension
2	4	6-7-8-9	Tension with tension
3	6	1-2-3-4-5	Compression with tension
4	5	8-9-10	Tension with tension

Tube No. 6 of Series 3 and tube No. 7 of Series 4 were tested in torsion only. There was a marked difference in the physical properties of the material of the four lengths of tubing. This is shown by the stress-strain diagrams of the tensile tests made on specimens cut from the tubes. The yield point stress varied from 21,500 lb. per sq. in. to 50,000 lb. per sq. in., Series 1, 2, 3, and 4 having yield-point stresses of 42,500, 21,500, 24,000, and 50,000 lb. per sq. in. respectively. The tubes were not annealed, but the first three series gave very uniform results for all gage lines, and showed a decided change at the yield point. The specimens of Series 4 showed a much greater variation. The behavior was that of hard, brittle steel of quite irregular composition.

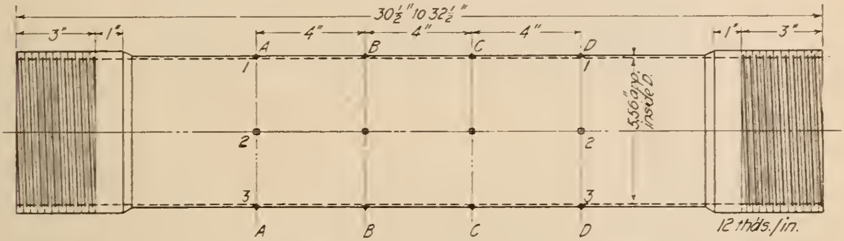

FIG. 7. DIMENSIONS OF TUBE TEST SPECIMEN.

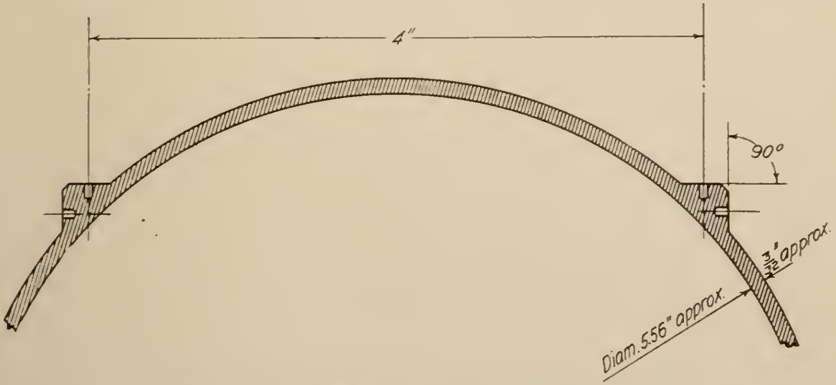

FIG. 8. ARRANGEMENT OF GAGE HOLES ON TUBES.

There was little reduction of area and the rupture was sharp and sudden, both in the tension specimens and in the one tube that broke during testing. The stress-strain diagram for the Series 4 show only qualitative results. These tubes were not suited for a test of this character, the inner and outer circumferences of the tube before machining were not concentric circles, and some gage lines gave diagrams that curved throughout, similar to the diagrams of drawn wire. There was no well-defined yield point. As the stress-strain diagrams did not give positive results, no use will be made of this series.

16. *Preparation of the Tubes.*—The test specimens were first bored out for the entire length on a horizontal boring mill and then turned to the dimensions shown in Fig 7. Each tube was threaded on the two ends with a taper thread of twelve threads per inch over a length

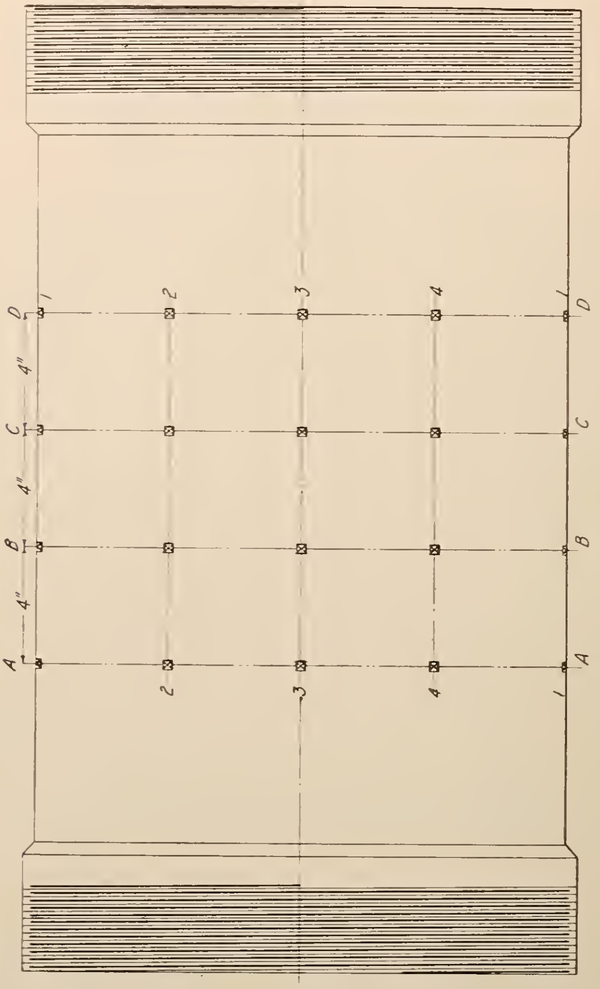

Fig. 9. Location of Gage Lines on Developed Outer Surface of a Tube.

of three inches. The tube was left full thickness for about an inch beyond the threads to furnish a bearing for packing. The remainder of the tube was turned to an approximate thickness of 3/32 in. except for four bands of 1/4-in. width spaced four inches apart along the tube. The greater part of these bands were afterwards milled off leaving four projections on each band for the gage holes. The tube was thus spanned with four circumferential gage lines each four inches long. The axial gage lines used one of the two holes so that the projections could be reduced to the smallest possible size. This gave four rows of three axial gage lines each, twelve in all, and four bands of four circumferential gage lines, sixteen in all, making it necessary to take twenty-eight readings, exclusive of the standard bar and check readings for each increment of load. The standard bar readings are necessary in tests with the strain gage to detect variations in the instrument due to temperature or jarring of the points.

Fig. 9 shows the position of the gage lines on a developed surface of a tube specimen. The circumferential bands were lettered A, B, C, and D; the axial lines were numbered 1, 2, 3, and 4. Thus an axial gage line would take two holes in the same axial line, but in two consecutive circumferential bands. It would, consequently, be called by the letters of the bands, in order, and by the number of the axial line. Thus AB 3 would be an axial gage line spanning the distance between the circumferential bands A and B and lying along the axial line 3. As soon as the tube was machined the numbering was fixed and the projections on the A band marked with small prick punch marks to identify the axial lines. In this way the readings for the thickness of the tube walls could be correlated with the strain gage readings. The gage holes were drilled by hand using a No. 54 drill. They were not reamed.

The boring of the tube caused a slight change of shape of the cross section due to the removal of the inner skin of metal, and after the outside was turned the thickness was uniformly varying, usually having two points of maximum thickness diametrically opposite, and at 90° from these, two points of minimum thickness. This renders the tube slightly elliptical (but not over 0.02 in. in 5.50 in.) and of varying thickness. While the variation in thickness was as high as 15 per cent in some cases, it apparently did not affect the averages of the readings, although the individual circumferential curves show the effect of this variation and the effect of the water pressure in making the tube more nearly cylindrical.

17. *Determination of the Thickness of Tube Walls.*—The principle of the apparatus adopted for measuring the thickness of the tube walls is that a micrometer caliper with a very deep throat. Fig. 10 shows the apparatus with the tube in position for a zero reading. A 4½ by 2½ by 7/16-in. T-bar was clamped at one end to a support and a stiff wooden bar was bolted to it. At one end of the wooden bar an Ames Dial reading to thousandths of an inch was fastened so that the plunger rested on a steel ball (*a* Fig. 10) embedded in the stem of the T-bar. To determine the thickness of the tube wall the plunger of the dial was raised, the tube was slipped over the T-bar and rested on the steel ball. Two other steel balls (*b* and *c* Fig. 10) were embedded in the stem of the T-bar, one on each side of the ball under the plunger at such a distance from it that the tube always swung free on the center ball and one of the others. The ball under the plunger was slightly higher than either of the others to insure a bearing on it at all times. When the plunger of the dial was in contact with the tube, the thickness of the tube was the difference between the reading then taken and the zero reading. Zero readings were obtained by suspending the tube in two fine wire slings in such a manner

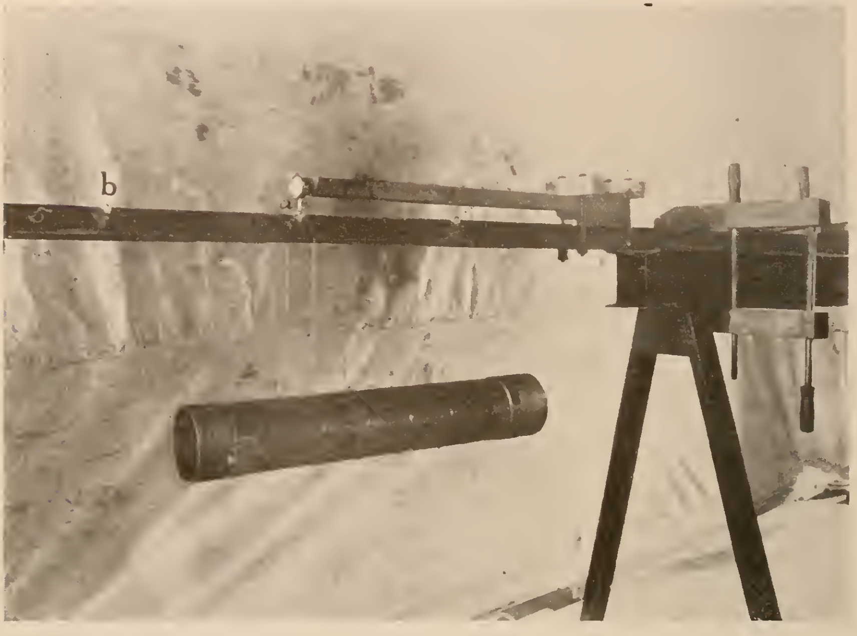

Fig. 10. View of Apparatus Used in Measuring Thickness of Tube Walls.

that its weight came on the T-bar in the same way as when the tube swung on the steel balls. With the plunger of the dial in contact with the steel ball, the initial or zero reading was taken for every position of the tube along an axial line. As the T-bar was a cantilever with two point loading, this gave slightly different zero readings for the various positions of the tube, but any error arising on account of the deflection of the apparatus was removed. With the tube in a given position and with the plunger on the ball, a reading was taken after a traverse of two axial lines. These readings were taken to detect any possible change in the apparatus and are not zero readings. They correspond to the standard bar readings when using a strain gage. A set of check readings was taken and the average of the two readings was used. Readings were taken to tenths of a division (ten-thousandths of an inch) and tube thicknesses are given in thousandths of an inch.

It is thought that this method of measurement is accurate and the check results obtained with a micrometer after the tube had been cut, have borne out this conclusion. The tube must be of relatively large diameter to apply this method, but with 6-in. tubes no difficulty was experienced.

18. *Method of Testing.*—Two steel castings were designed to fit over the ends of a tube. The stresses carried by these heads were comparatively low, for the maximum load was but 167,000 lb., and the material was about ¾ in. thick at the thinnest part. The castings were machined all over and threaded internally, at one end to receive the tube and at the other to receive a 4-in. bar which served to apply the tension. The two threaded portions were separated by about an inch of metal which served to retain the water under pressure in the tube. These castings are shown in Fig. 11.

To withstand the water pressure, two layers of ⅜-in. hydraulic packing were used in an ordinary four-screw stuffing box. The heads were recessed to receive the packing and the gland, while the tube walls were left nearly full thickness for about an inch beyond the threads to furnish a firm bearing for the packing. After the packing was adjusted to position there were no perceptible leaks although pressures up to 1,800 lb. per sq. in. were used. Fig. 11 shows the general arrangement of the apparatus for the tension tests.

All the tests except the torsion tests were made in the 600,000-lb. Riehle machine of the Laboratory of Applied Mechanics of the University of Illinois. By using spherical seats with careful centering of the specimens in the machine, the eccentricity of loading was reduced

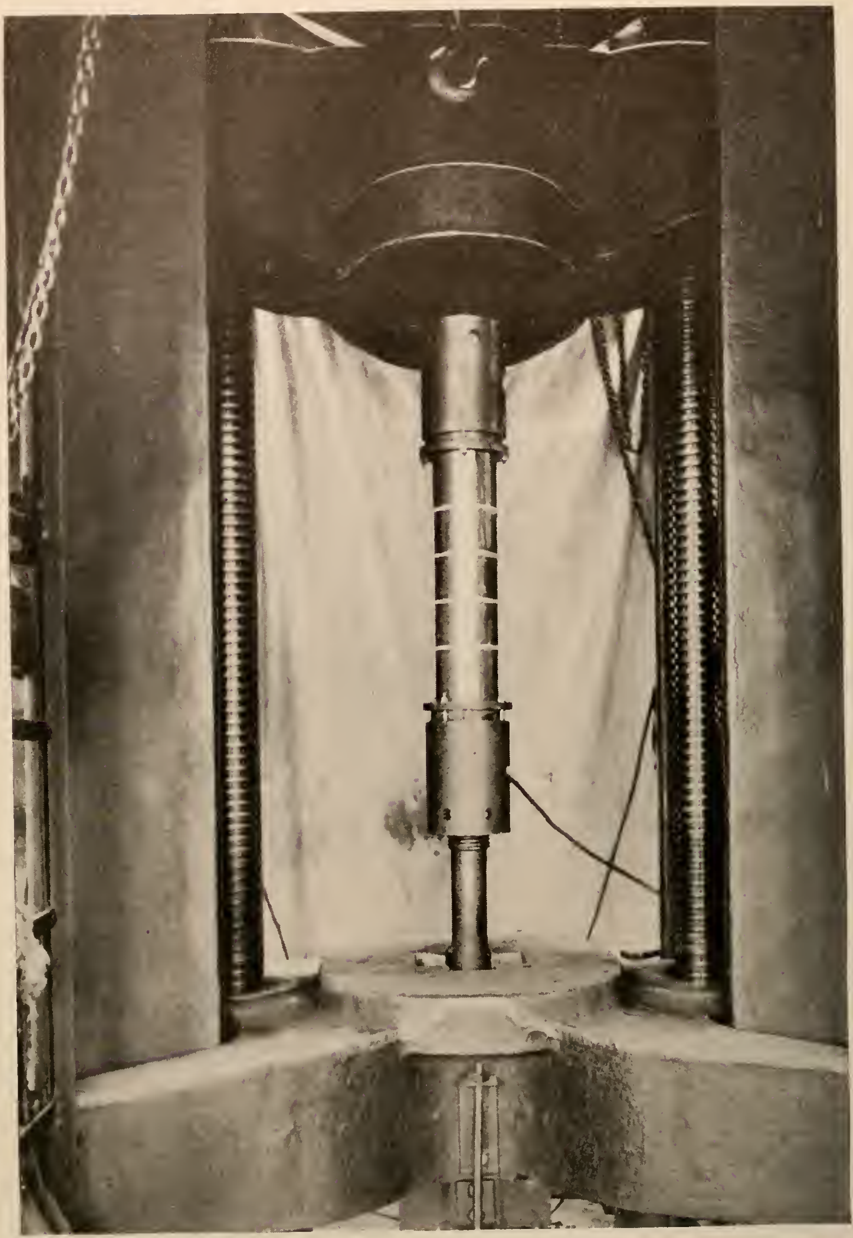

FIG. 11. VIEW SHOWING ARRANGEMENT OF APPARATUS FOR TENSION TEST.

to a minimum and the bending stresses were low as shown by the uniformity of the individual stress-strain diagrams. In the tension tests the nuts of the 4-in. bars bore directly against the spherical seats, the lower one being inverted. For the compression tests the bars were removed and the tube heads bore directly against the spherical seats and the upper spherical seat was inverted. The length of thread on the specimens tended to give a good distribution of load and the distance of the first gage hole from the end of the thin part of the wall (6½ in.) together with the thinness of the wall itself, were sufficient to insure a high degree of uniformity of stress. Holes were drilled into each head to connect into the interior of the tube; the hole in the lower head was for connection to the pump and the hole in the upper head was for the purpose of filling the tube with water. Each hole was tapped with a ½-in. pipe tap.

The torsion tests were made in a 230,000-lb. in. Olsen Torsion Machine. The heads were screwed on the specimens as in the other tests. and short steel bars threaded on one end transmitted the torque from the machine to the steel heads. Two fixed wooden clamps with 40-in. arms, one of which carried a pointer and the other a scale, were used to measure the angle of torsion over a known gage length.

19. *Character and Sequence of the Tests.*—Three tests in each of Series 1 and 2 were first made. These were the tests in direct tension, the tests with the ratio of tensile stresses equal to 0.475 and with this ratio equal to 0.92. The results of these tests were worked up before the remainder of the tests in the series were made, so that the other ratios could be chosen to the best advantage. As the difference in the strength of the steel in the direction of drawing and across it would complicate the problem, and as it was not intended to raise the question of the variation in strength in different directions throughout the specimen, the highest ratio of circumferential stress to axial stress used was made less than 1.0, being 0.92.

The average area of the inner cross section of the tubes was about 24.50 sq. in., and the axial load due to the water pressure was 2,450 lb. per 100 lb. per sq. in. water pressure. To produce a ratio of circumferential tension to axial tension equal to 0.50 required a machine load of 4 × 2,450 — 1 × 2,450 = 7,350 lb. per 100 lb. per sq. in. water pressure, since the water pressure acts with the machine load. For axial compression combined with circumferential tension, the two quantities would be added instead of subtracted, since the water pressure tends to reduce the machine load. Dividing the net axial load (9,800 lb. per

100 lb. per sq. in. water pressure) by the cross-sectional area of the tube gives the unit axial stress. A slight error is introduced by using the inside diameter of the tube rather than the mean diameter, for in order that the circumferential tension shall be exactly twice the axial tension when the water pressure alone is acting, the mean diameter must be used to compute the axial tension. This error is about 1½ per cent, which represents the variation of the circumferential tensions from the mean. The stress ratios for Series 3 and 4 were planned complete and carried out as planned. The stress ratios used in the four series are given in Table 1.

The strain gage used was a 4-in. Berry strain gage made for the Joint Committee on Stresses in Railroad Track and loaned by that Committee. It has invar steel sides and shows a negligible correction for temperature. Two standard bars were used to detect any variation of the instrument due to jarring or striking the fixed point. All data have been corrected for variation in the standard bar readings. To avoid variation due to change of temperature of the tubes, they were usually

TABLE 1.

OUTLINE OF TEST SPECIMENS AND TESTS.

Series No.	Tube No.	Ratio of Circumferential to Axial Stress	Stress Combination
1	5	0.00	Axial tension only
	1	0.24	Tension with tension
	2	0.475	" " "
	4	0.69	" " "
	3	0.92	" " "
2	9	0.00	Axial tension only
	7	0.475	Tension with tension
	8	0.92	" " "
	6	0.92	" " "
3	4	0.00	Axial compression only
	2	0.20	Compression with tension
	5	0.30	" " "
	3	0.60	" " "
	1	0.90	" " "
	6	1.00	Torsion only
4	9	0.00	Axial tension only
	10	0.30	Tension with tension
	8	0.50	" " "
	11	0.80	" " "
	7	1.00	Torsion only

filled with water in the evening and by the time the test began the next day the tube and water were at a temperature that scarcely changed during the entire test.

20. *Test Operations.*—The initial load in all cases was small, producing an average axial unit-stress of approximately 4,000 lb. per sq. in. This load was applied after the specimen had been carefully centered and the spherical seats tried. A load sheet was prepared for each test which gave the required machine loads, the approximate yield point, the water pressure, and unit-stress. When the load was increased the water pressure was increased first and then the machine load.

The record of a test was a combination of the ordinary record and a graphical one. Co-ordinate paper was used and was divided into a series of rectangles, one for each standard bar and gage line. Along one side of this rectangle the instrument reading was noted and this reading was then plotted against the machine load. In this way the progress of the test was very evident and any doubtful reading was checked. When the nature of the curve is well known, it is advisable to see that the results for any gage line that do not show some systematic sequence of plotted points are checked to insure their accuracy. If this is not done false breaks may sometimes be obtained in the curve. If the error is experimental, the check reading will correct it, and if the stress suddenly departs from the straight line law, the check reading will be a repetition of the first reading and will give greater confidence in the result. Though but few errors were discovered and corrected, the result justifies the method employed. Whenever there are variations from the straight line in the stress-strain diagram, these are indications of a change in the rate of taking stress. As the load changes, the distribution of stress over a given cross section often changes, so that at one point there may be a rapid increase in the elongations for one increment of load, while in an adjoining gage line the change is slight. The next load increment may bring about a complete reversal of the conditions shown by the previous instrument readings.

Whatever variation occurs in one gage line, usually it is reflected in one or more of the others, so that the average takes out all these peculiarities. This is especially true of the circumferential readings.

It will be seen that the circumferential gage line readings give the correct unit-strain, the chord length being used and not the arc length. Circumferential readings are subject to the tendency of the tube to become truly cylindrical under water pressure. For low water pressures

TABLE 2.

DATA OF TUBES.

Tube No.	Inside Diameter Inches	Location	Tube Walls. Average Thickness, Inches	Sectional Area, Sq. In.
		Series 1.		
1	5.564	AB	0.089	1.590
		BC	0.088	1.570
		CD	0.086	1.527
2	5.558	AB	0.087	1.543
		BC	0.087	1.543
		CD	0.088	1.570
3	5.561	AB	0.087	1.543
		BC	0.087	1.543
		CD	0.087	1.543
4	5.563	AB	0.085	1.508
		BC	0.084	1.481
		CD	0.083	1.472
5	5.554	AB	0.091	1.623
		BC	0.092	1.641
		CD	0.092	1.641
		Series 2.		
6	5.579	AB	0.083	1.467
		BC	0.083	1.467
		CD	0.080	1.422
7	5.588	AB	0.091	1.623
		BC	0.091	1.623
		CD	0.091	1.623
8	5.560	AB	0.094	1.678
		BC	0.094	1.678
		CD	0.094	1.678
		Series 3.		
1	5.573	AB	0.106	1.891
		BC	0.106	1.891
		CD	0.112	2.001
2	5.561	AB	0.114	2.040
		BC	0.111	1.981
		CD	0.111	1.981
3	5.566	AB	0.108	1.934
		BC	0.107	1.912
		CD	0.106	1.889
4	5.581	AB	0.116	2.076
		BC	0.114	2.041
		CD	0.112	2.004
5	5.622	AB	0.094	1.688
		BC	0.094	1.688
		CD	0.096	1.724
6	5.634	AB	0.084	1.509
		BC	0.084	1.509
		CD	0.083	1.490

this was sufficient in some cases to change the stress from a tension to a compression or vice versa.

21. *Diagrams and Tables.*—Stress-strain diagrams representing the general average results of the axial and circumferential gage lines are given in Fig. 17, 18, and 19, while sample diagram showing the average results at different sections of the tube for both the tension-tension and the compression-tension experiments are given in Fig. 13 to 16. Diagrams of the experimental results of Series 1 and 2 are to be found in Fig. 22, and those of Series 3 in Fig. 23. A comparison of the theories of the strength of materials under combined stress is made in Fig. 24, while Fig. 26 and 27 illustrate some of the work of other investigators. An outline of the test specimens and tests and the principal data of the tubes are given in Tables 1 and 2. Table 3 is given as a sample of the data for a single tube, tube No. 4 of Series 1. These data have been reduced and corrected for standard bar readings. All the original and reduced data as well as the stress-strain diagrams are on file at the Laboratory of Applied Mechanics of the University of Illinois.

IV. Discussion of Results.

22. *The Criterion of Strength.*—There are three possible stress limits any one of which may be the criterion of the strength of material —limit of proportionality, yield point, and rupture or ultimate strength. It is recognized that there may be a sharp distinction between the laws governing ductile materials and the laws governing brittle materials, for such a distinction is observed in the stress-strain diagrams and in compression and torsion failures. Since this discussion is limited to ductile materials, conditions will be treated only as they apply to such materials.

It would appear at first thought that the limit of proportionality would be the proper basis upon which to determine the relative strength of material. The mathematical theory of elasticity is based upon Hooke's law generalized, engineering practice bases its computations largely upon this same law, and several investigators have used the limit of proportionality (which they called the elastic limit) as their criterion, notably Hancock* and Turner.†

The limit of proportionality, or p-limit, is defined as the stress at which the constancy of the ratio of stress to strain ceases; that is, the modulus of elasticity is a constant up to this stress. It is often stated

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*American Society for Testing Materials, 1905, '06, '07, '08.
†Engineering, London, February 5, 1909.

TABLE 3.

TEST DATA OF AXIAL GAGE LINES TUBE NO. 4, SERIES 1.

Ratio of Circumferential Tension to Axial Tension 0.69.
Inside Diameter of Tube 5.563.

AB Gage Lines
Average Thickness of Tube .085in. Area of Section 1.508sq.in

Water Pressure lb.per sq.in		Axial Load due to W.Pressure	Machine Load pounds	Total Axial Load pounds	Av. Axial Unit Stress lb.per sq.in	Reading on Gage Line				Differences				Av. Diff.	Av Unit Elongation
Gage Rdg.	Corrected					AB1	AB2	AB3	AB4	AB1	AB2	AB3	AB4		
100	100	2430	5100	7500	4950	80.1	10.9	28.0	625	0	0	0	0	0.	0
300	300	7290	14100	21400	14200	77.0	5.9	2.9	581	3.1	5.0	6.1	44	47	00024
500	500	12150	23050	35200	23300	72.5	00	17.1	527	76	109	109	98	9.8	00049
700	700	17000	32100	49100	32500	661	940	11.9	478	140	169	161	147	154	.00077
900	800	21900	41100	63000	41700	61.0	869	50	41.9	19.1	24.0	23.0	206	217	00109
950	950	23100	43400	66500	44000	603	841	3.5	409	198	268	245	216	23.2	00116
1000	1000	24300	45600	69900	46200	590	820	1.8	397	211	289	262	228	248	00124
1050	1050	25500	47900	73400	48500	57.3	790	990	379	221	319	290	246	269	00135
1100	1090	26500	50100	76600	50900	552	764	977	361	249	345	303	264	29.0	00145
1150	1140	27700	52400	80100	53100	531	733	95.3	340	270	376	327	285	31.5	00158
1200	1190	29200	54500	83500	55300	50.9	690	93.0	321	292	419	35.0	304	341	00171
1250	1240	30100	56800	86900	57600	470	653	880	282	331	456	400	343	383	00192
1300	1290	31300	59100	90400	60000	30.9	579	747	161	492	530	53.3	464	50.5	00253

BC Gage Lines
Average Thickness of Tube .084in Area of Section 1481 sq in.

Water Pressure lb per sq.in		Axial Load due to W Pressure	Machine Load pounds	Total Axial Load pounds	Av Axial Unit Stress lb per sq.in	Reading on Gage Line				Differences				Av Diff.	Av Unit Elongation
Gage Rdg	Corrected					BC1	BC2	BC3	BC4	BC1	BC2	BC3	BC4		
100	100	2430	5100	7500	.5060	92.1	795	35.0	63.5	0	0	0	0	0	0
300	300	7290	14100	21400	14400	87.0	748	299	584	5.1	47	5.1	5.1	50	00025
500	500	12150	23050	35200	23800	81.9	690	24.7	536	102	10.5	103	9.9	102	00051
700	700	17000	32100	49100	33100	76.0	639	19.1	480	161	15.6	159	155	158	.00079
900	900	21900	41100	63000	42500	689	57.2	.30	429	232	22.3	220	20.6	22.0	00110
950	950	23100	43400	66500	44800	672	550	11.0	412	24.9	24.5	240	22.3	239	00120
1000	1000	24300	45600	69900	47200	65.0	535	103	400	271	260	247	235	254	.00127
1050	1050	25500	47900	73400	49400	62.5	51.5	84	385	296	28.0	266	25.0	273	00137
1100	1090	26500	50100	76600	51900	599	492	69	366	32.2	30.3	28.1	269	294	.00147
1150	1140	27700	52400	80100	54100	563	474	48	343	358	321	30.2	292	318	.00159
1200	1190	29200	54500	83500	56400	523	448	1.5	303	398	34.7	33.5	332	353	00177
1250	1240	30100	56800	86900	58600	478	41.5	97.3	231	443	380	37.7	404	401	00201
1300	1290	31300	59100	90400	61100	300	300	738	910	62.1	492	612	725	613	.00307

CD Gage Lines
Average Thickness of Tube .083in Area of Section 1.472 sq.in

Water Pressure lb per sq.in		Axial Load due to W.Pressure	Machine Load pounds	Total Axial Load pounds	Av. Axial Unit Stress lb per sq.in.	Reading on Gage Line				Differences				Av. Diff.	Av Unit Elongation
Gage R'dg.	Corrected					CD1	CD2	CD3	CD4	CD1	CD2	CD3	CD4		
100	100	2430	5100	7500	5100	682	449	950	409	0	0	0	0	0	0
300	300	7290	14100	21400	14600	64.1	39.5	898	349	4.1	54	5.2	6.0	5.2	.00026
500	500	12150	23050	35200	23900	59.0	347	840	30.9	92	10.2	11.0	10.0	10.1	.00051
700	700	17000	32100	49100	33300	53.0	290	789	250	15.2	15.9	16.1	15.9	15.8	00079
900	900	21900	41100	63000	42900	473	232	740	19.4	20.9	21.7	21.0	21.5	21.3	.00107
950	950	23100	43400	66500	45100	461	22.0	730	183	22.1	229	22.0	226	22.4	.00112
1000	1000	24300	45600	69900	47500	439	200	71.3	17.0	24.3	249	237	23.9	24.2	.00121
1050	1050	25500	47900	73400	49700	410	171	683	152	272	27.8	25.7	25.7	26.6	.00133
1100	1090	26500	50100	76600	52200	388	141	670	139	29.4	308	28.0	270	29.0	.00145
1150	1140	27700	52400	80100	54400	354	111	65.0	118	328	33.7	300	291	31.4	00157
1200	1190	29200	54500	83500	56800	320	74	625	90	362	375	32.5	319	345	.00173
1250	1240	30100	56800	86900	59000	26.9	4.0	550	45	41.3	407	40.0	36.4	39.6	.00198
1300	1290	31300	59100	90400	61500	8.0	860	472	90.0	60.2	58.9	47.8	50.9	545	.00273

that the distinction between yield point and p-limit is very slight and that it really makes no material difference which is used. But a glance at the stress-strain diagrams in Fig. 13 to 16, will show that in some cases the modulus of elasticity changes and that the diagram consists of a broken line instead of a straight line nearly up to the yield point. This fact, due to the lack of isotropy in the material and to the mechanical work done upon it, makes it difficult to get consistent results by using the p-limit as a criterion. When the material has been cold worked, the stress-strain diagram often curves away from a straight line slowly and the exact point of departure is not easily located. Special treatment of the material usually affects the yield point in the same way in different specimens, but not the p-limit.

The use of rupture or ultimate strength as a criterion of the strength of ductile materials still persists in the case of simple stresses, and specifications ordinarily require that the ultimate strength of the material shall have a certain value. But this is an indirect measure of the toughness rather than of the strength, and in the best specifications the yield point (or elastic limit as it is frequently but incorrectly called) is specified as well. Conditions at rupture give no indication of those existing at the yield point and whatever value a knowledge of the conditions attending rupture in a ductile material may have, no conclusions can be drawn from them which may safely be applied to the period preceding the yield point. As engineering design deals principally with stresses within the yield-point stress, rupture cannot be considered as the criterion, even though Bridgman* in his tests on thick cylinders uses it and decries the use of the yield point. When the distribution of stress is unknown and no extensometers are used to measure strains, rupture is the only criterion available.

For ductile material that has not been worked cold, the stress-strain diagram shows a very decided change in character when the material passes the yield point. When the material has been cold-rolled or cold-drawn, the yielding is more gradual and the curve, instead of breaking sharply, departs more gradually from a straight line. If the specimen of the cold-rolled or cold-drawn material is tested in simple tension with an extensometer, and the load is slowly but steadily applied, the roll of the curve is apparent a short time before the yield point is registered by the drop of the beam.

As all the investigations hereinafter described were made with instruments to measure the strains, some criterion must be adopted that is applicable to a stress-strain diagram. The first deviation from a straight line (p-limit) is an indefinite point to locate, and, after considering everything that has been noted above, the method proposed by the late J. B. Johnson was adopted. This is called by him the "apparent elastic limit," although it is here taken as the yield point. This method empirically locates a point at which there is evidently some plastic action and furnishes a very convenient method for comparison of results. It is defined as the unit-stress at which "the rate of deformation is 50 per cent greater than it is at zero stress." Fig. 12 shows the application to a stress-strain diagram. Let O B E be a stress-strain diagram drawn in the usual manner. Then A O B is the angle determining the slope at zero stress. At any point K lay off horizontally a distance K F equal to 1.50 times K B. Then O F is the slope 50 per cent greater than the slope at zero stress. A parallel to O F drawn tangent to the curve B E, locates the point of tangency L and the corresponding stress is the yield-point stress.

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*Phil. Mag., July, 1912.

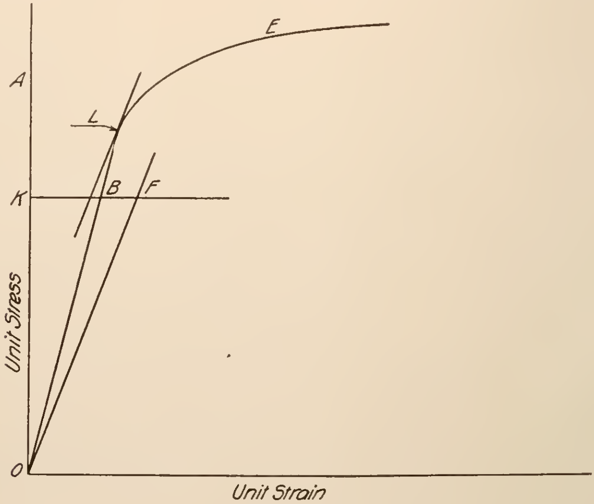
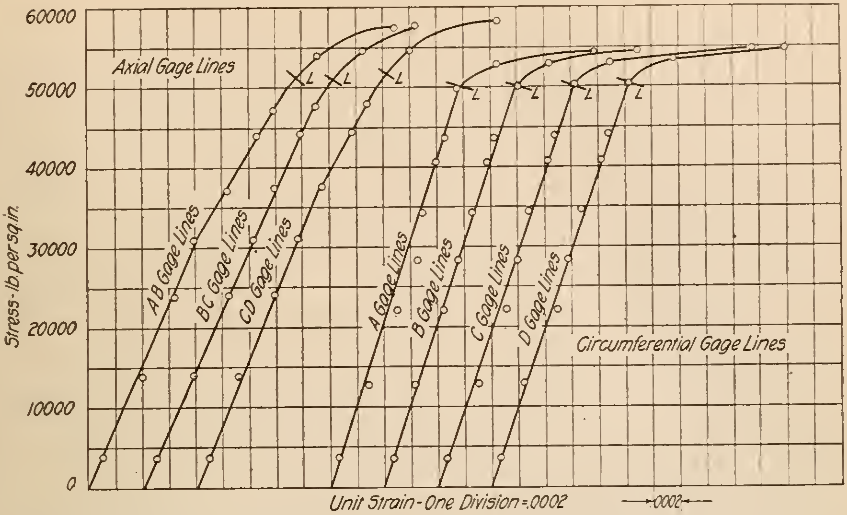


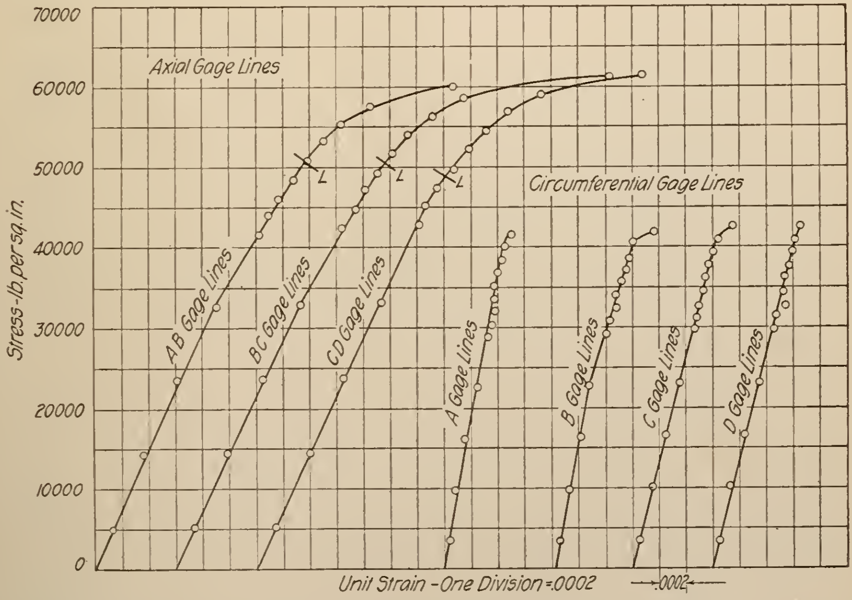
FIG. 12. STRESS-STRAIN DIAGRAM SHOWING JOHNSON'S APPARENT ELASTIC LIMIT.

23. *Strength.*—In the tabulation of the results of the tests of tubes under biaxial stress, the average of the strains measured on the four gage lines intersected by any cross section was taken as the strain at that section of the tube. Thus, the strains for the axial gage lines, A B1, A B2, A B3, and A B4 of a tube (for notation see page 23 and



L denotes yield point

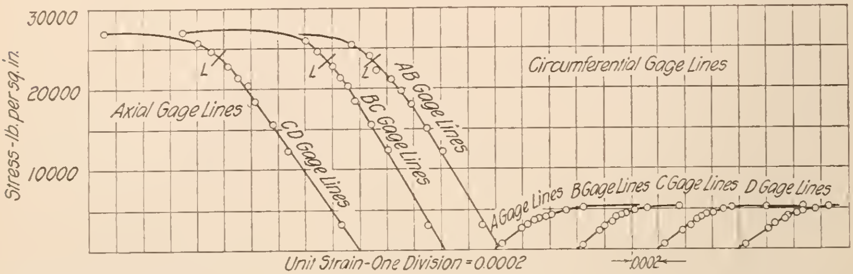
FIG. 13. STRESS-STRAIN DIAGRAMS FOR TUBE NO. 3, SERIES 1. RATIO OF CIRCUMFERENTIAL TO AXIAL TENSION, 0.94.



L denotes yield point

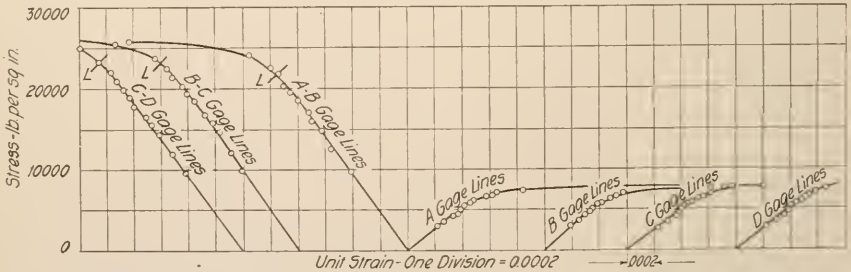
FIG. 14. STRESS-STRAIN DIAGRAMS FOR TUBE NO. 4, SERIES 1. RATIO OF CIRCUMFERENTIAL TO AXIAL TENSION, 0.69.

Fig. 9) were averaged, and this average is taken as the strain of the A B gage lines of that tube. Likewise for the B C and C D gage lines. For the circumferential gage lines, 1–2–A, 2–3–A, 3–4–A, and 4–1–A were averaged; that is, the four gage lines made a complete traverse of the circumference. There are then three sets of average results for the axial gage lines and four for the circumferential gage lines of each tube. The curves formed from these average results (see Fig. 13 to 16 for samples) were then used to obtain the general average results for each of the tubes. That is, the general average results for the circumferential strains represent the average obtained from all the circumferential gage lines in any one tube, and the general average results for the axial strains the average obtained from all the axial gage lines. The only exception is in the case of tube No. 1, Series 1, where the averages of the A B gage lines are omitted in the general average. Each general average curve represents the average results of twelve axial gage lines or sixteen circum-



L denotes yield point

FIG. 15. STRESS-STRAIN DIAGRAMS FOR TUBE NO. 2, SERIES 3. RATIO OF CIRCUMFERENTIAL TENSION TO AXIAL COMPRESSION, 0.20.

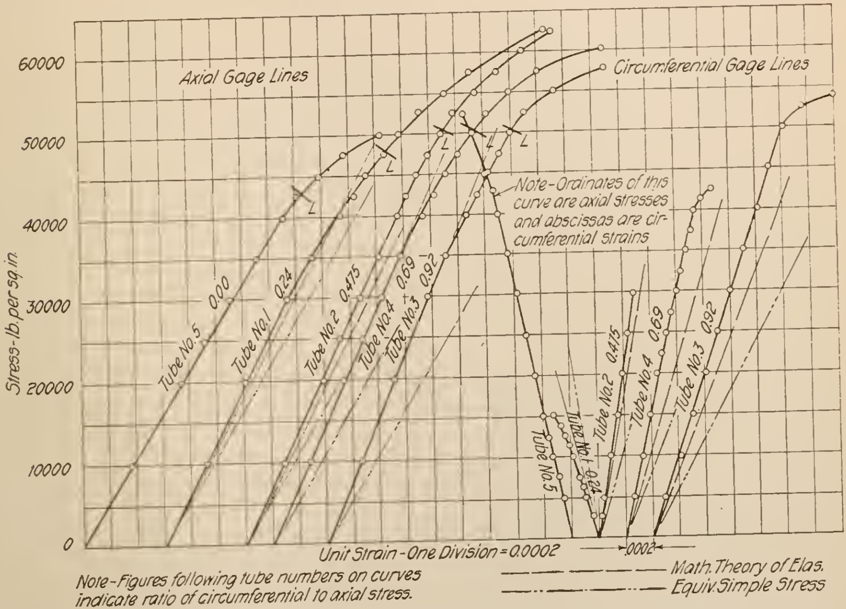


L denotes yield point

FIG. 16. STRESS-STRAIN DIAGRAMS FOR TUBE NO. 5, SERIES 3. RATIO OF CIRCUMFERENTIAL TO AXIAL TENSION, 0.30.

ferential gage lines. These general average stress-strain diagrams are given in Fig. 17 to 19.

The yield-point stresses are quite uniform for the different sets of gage lines and in close agreement with those of the general average curves. Because of this uniformity, the use of the general average curves as a basis of comparison seems justified. The circumferential strains are plotted with the apparent circumferential tensile stresses as ordinates, except in the case of the tubes where no internal pressure was applied. In these cases the ordinates are the axial stresses, so that it is easy to determine Poisson's ratio, which is given in Fig. 19 by the ratio of abscissas, corresponding to the same stress, on the two curves of tube 4, such as r to r'.



Note-Ordinates of this curve are axial stresses and abscissas are circumferential strains

Note-Figures following tube numbers on curves indicate ratio of circumferential to axial stress.

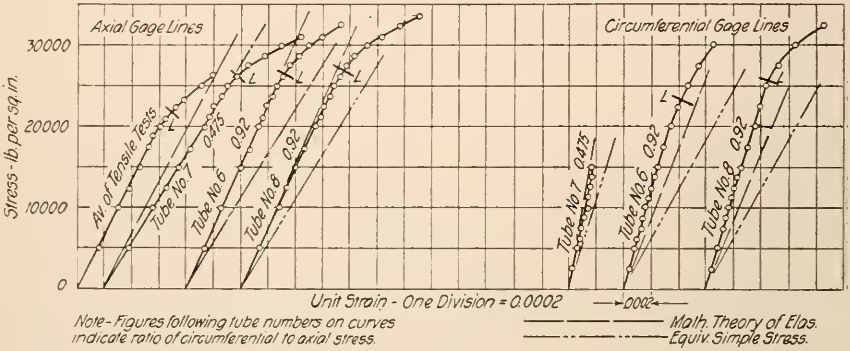
L denotes yield point

FIG. 17. STRESS-STRAIN DIAGRAMS SHOWING GENERAL AVERAGES FOR SERIES 1. TENSION WITH TENSION.

If diagrams are drawn having the yield-point unit-stresses as ordinates and the ratio of the circumferential tension to axial tension or axial compression as abscissas, a comparison can be made with the results reached by the different theories. For the combination of tension with tension, the maximum stress theory and the maximum shear theory demand that the yield-point stress shall be constant for all ratios.

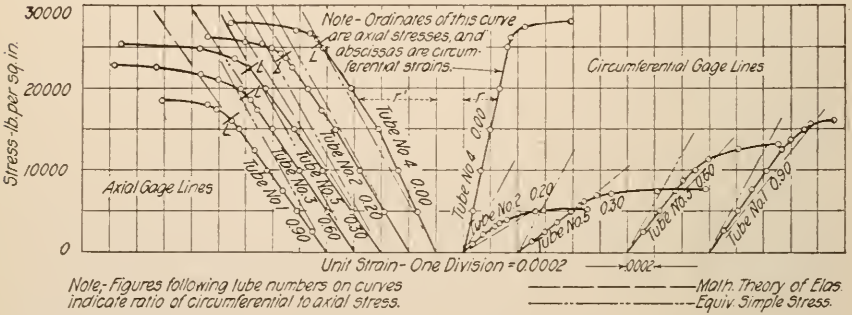
Mohr's theory and the internal friction theory have the same requirements; Wehage's theory demands a reduction in the yield-point stress and the maximum strain theory demands an increase in proportion to the increase of stress ratio.

For the combination of compression with tension the maximum stress theory demands that the yield-point stress shall be constant for all stresses, while the maximum strain theory, the internal friction theory, the maximum shear theory, and Mohr's theory demand a decrease in the yield-point stress as the stress ratio increases.



L denotes yield point

FIG. 18. STRESS-STRAIN DIAGRAMS SHOWING GENERAL AVERAGES FOR SERIES 2. TENSION WITH TENSION.



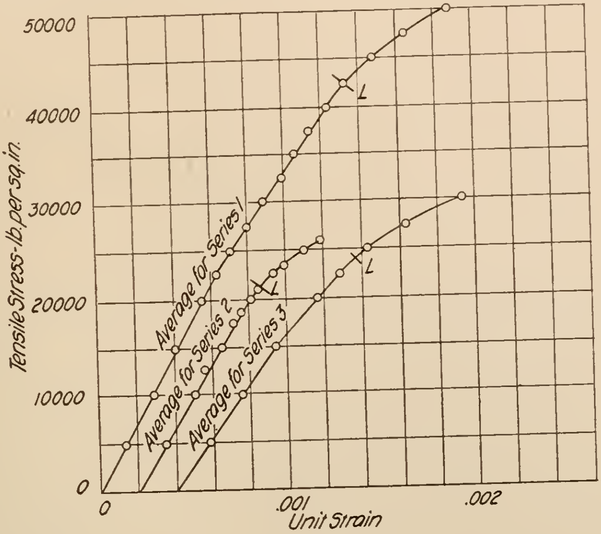
L denotes yield point

FIG. 19. STRESS-STRAIN DIAGRAMS SHOWING GENERAL AVERAGES FOR SERIES 3. COMPRESSION WITH TENSION.

What is the law that governs? Referring to the stress-strain curves of the general averages of the axial gage lines, Fig. 17, 18, and 19, it will be seen that for Series 1 and 2 as the stress ratio increases the yield-point stress rises unmistakably until the value of the stress ratio of

0.50 is reached. Beyond this the yield-point stress remains constant, no matter what the stress ratio. For Series 3 the yield-point stress steadily diminishes as the stress ratio increases.

Since for the case of compression combined with tension all the theories except one demand a decrease of the yield-point stress as the stress ratio increases, while for tension combined with tension the maximum strain theory is the only one which calls for the increase that



L denotes yield point

FIG. 20. TENSION TESTS OF SMALL SPECIMENS FROM TUBES OF SERIES 1, 2 AND 3. RESULTS OF TEN TESTS FOR EACH SERIES.

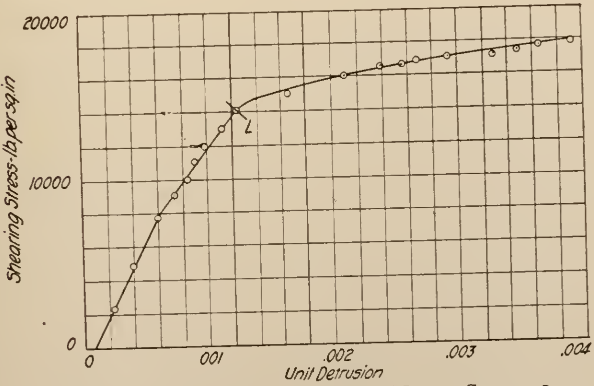


FIG. 21. TORSION TEST OF TUBE NO. 6, SERIES 3.

has been observed. Series 1 and 2 will be discussed first and the results of Series 3 compared with the conclusions drawn from the results of Series 1 and 2.

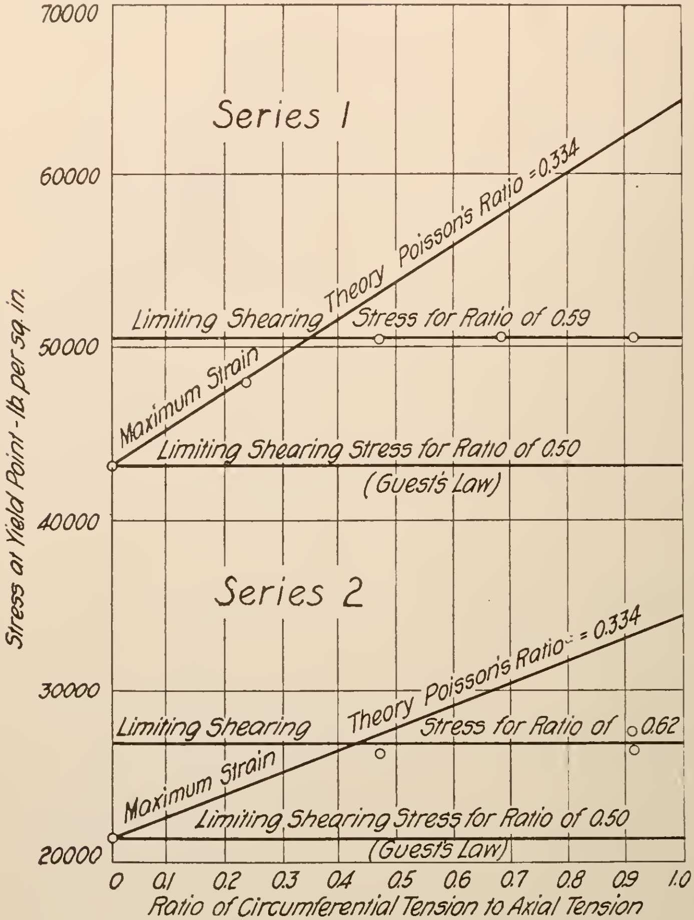


FIG. 22. DIAGRAM GIVING YIELD POINT STRESSES AND STRESS RATIOS FOR SERIES 1 AND 2.

In Fig. 22 the yield-point stresses of Series 1 and 2 are plotted against the ratio of circumferential tension to axial tension. The line of the maximum strain theory is then drawn through the yield-point stress determined in simple tension (stress ratio zero), the inclination being determined by Poisson's ratio (0.334). For Series 1 the yield-point stress was taken from the test of tube No. 5 and for Series 2 the average of the tension tests of twenty small specimens cut from tubes of Series 2 was taken.*

The determination of Poisson's ratio is discussed on page 46. A line of constant yield-point stress is drawn which best fits the experimental points for stress ratios of 0.50 or above. It is seen that the line of the maximum strain theory fits the experimental points up to its intersection with the line of constant yield-point stress, and that thereafter the line of constant yield-point stress well fits the points. This line of constant yield-point stress may also be a line of constant shearing stress. If, as Fig. 22 seems to indicate, tension ceases to be a governing factor and the shearing stress becomes dominant, two things must be true for the line of constant yield-point stress:

(a) The shearing unit-stress must actually reach the shearing yield-point stress as determined by tests in pure shear, and
(b) Since the shear is one-half the maximum principal stress, this maximum principal stress must remain a constant.

The first condition is important only in so far as showing that the shearing yield-point stress must be greater than one-half the tensile yield-point stress; otherwise the shear would be dominant at all times. The latter is the contention of the maximum shear theory. Counting compression a negative tension and with the principal unit-stresses numbered in the order of their magnitude, p_1, p_2, p_3, the criterion for shearing stress is:

$$\text{Shearing unit-stress} = \tfrac{1}{2}\,(p_1 - p_3),$$

but as the third principal stress is zero, this reduces to $\tfrac{1}{2}\,p_1$. The water pressure inside the tube does not constitute a third principal stress (compressive), for all the readings of the strains were taken on the outside of the specimen where the third principal stress was undoubtedly zero, if the atmospheric pressure is neglected.

The maximum shear theory carried to its logical conclusion requires that the yield-point stress of the material subjected to two stresses of like sign at right angles shall not vary from that reached in simple tension, for the shear is the determining factor at all times. If the theory holds in this form, a horizontal line drawn through the yield-point stress in simple tension should pass through all the points. Instead, it touches only the initial point. Experiments* have shown that for ductile material the ratio of the shearing yield-point stress, obtained by torsion tests, to the tensile yield-point stress varies with the material, but usually lies between 0.55 and 0.65, in the majority of tests ranging near 0.60, which is the commonly accepted value. A few tests show a ratio less than 0.50, but they are relatively small in number. With a ratio of 0.60, the shearing yield-point stress line would lie above the line through the yield-point stress in simple tension by an amount equal to 0.20 of the latter stress. The exact location of the line will

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*The tension test of tube No. 9, Series 2, the first test made, did not furnish the necessary data on account of an unexpectedly low yield point. It is thought that the use of the yield-point stress obtained from the average curve for the specimens from the tubes of Series 2 (see Fig. 20) is justified because the break of the curve of the small specimens from the tubes of Series 1 agrees closely with the break in the curve obtained from tube No. 5 of that series (axial load only), 42,500 lb. per sq. in. and 43,000 lb. per sq. in. respectively. The yield-point stress obtained from the average curve of the specimens from the tubes of Series 2 (21,500 lb. per sq. in.) has been taken as the yield-point stress in simple tension of Series 2 and the value of Poisson's ratio obtained from Series 1 has been used for Series 2.

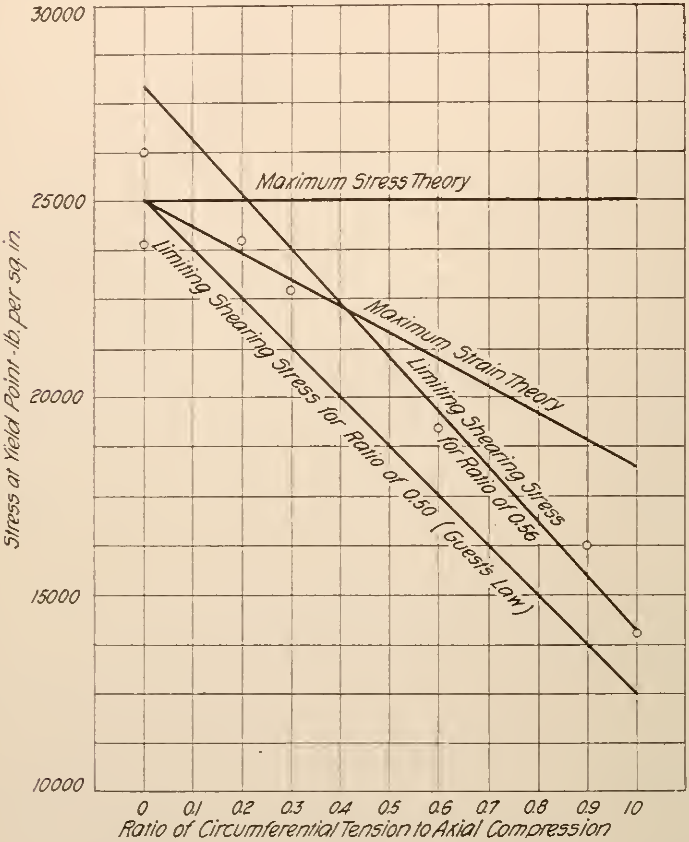

FIG. 23. DIAGRAM GIVING YIELD POINT STRESSES AND STRESS RATIOS FOR SERIES 3.

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*L. B. Turner, Engineering, London, February 5, 1909.

vary with the material, but as long as the ratio of the yield-point stresses is above 0.50, there is the hiatus between this condition and that demanded by the above form of the maximum shear theory.

The horizontal line through the experimental points in Fig. 22 is evidently the limit of the shearing strength. It corresponds to a ratio of shearing yield-point stress to tensile yield-point stress of 0.59 for Series 1 and 0.62 for Series 2, which values agree well with the majority of experiments.

These tests indicate that there are two laws covering the case of combined stress when the stresses are both tension and act in two directions at right angles. Apparently the point at which the change in law occurs depends upon the ratio of the yield-point stress in shear to that in tension and the change from one law to the other may occur at different ratios of the principal stresses for different materials. It is important to establish this ratio of yield-point stresses, for if it is not approximately constant the use of combined stress formulas will require a knowledge of such a ratio for all materials.

Before discussing Series 3, the two laws just referred to will be applied to the other combinations of stress and a comparison made with the maximum stress theory, the maximum strain theory, and the maximum shear theory. Assuming the ratio of the shearing and tensile yield-point stresses to be 0.60 and the tensile and compressive yield-point stresses equal, the co-ordinates of the rectangle A B C D (Fig. 24) represent the maximum stress theory, the rhombus Q K J L the maximum strain theory, and the figure A K_1, B C L_1, D A the maximum shear theory. The line A M K_2, N B represents the two laws in the tension-tension quadrant, while B R S C represents them in the tension-compression quadrant. The lines M K_2 and K_2N are parallel to the axes and at such a distance from them that the ordinate of M K_2 and the abscissa of K_2N are each 1.20 times O A or O B, the tensile yield-point stress. R S is parallel to B C and at such a distance from it that one-half the sum of the ordinate and abscissa of any point between R and S is equal to 0.60 of O B or O C. The construction of the other two quadrants is such that the figure is symmetrical about the bisectors of the quadrants. The diagrams, Fig. 22, showing the comparison of theory and experiment for Series 1 and 2 correspond to A M K_2 in the tension-tension quadrant.

In Fig. 23 the yield-point stresses of Series 3 are plotted as ordinates and the stress ratios of circumferential tensile stress to axial compressive stress as abscissas. Before discussing the various theories in connection with the experimental results, the starting points of the theoretical lines must be fixed. The maximum shear theory demands the same yield-point stress in tension and in compression; the maximum strain theory and the maximum stress theory do not. From the average curve of ten specimens cut from a ten-inch remnant of the original tubing from which the tube specimens of Series 3 were cut, (Fig. 20) the tensile yield-point stress was found to be 24,000 lb. per sq. in. The compressive yield-point stress obtained from tube No. 4 (no internal water pressure) was 26,250 lb. per sq. in. With the demand of the maximum shear theory for equal yield-point stress in tension and in compression it seems correct to take as the initial point of the line of that theory the average of these values, or 25,100 lb. per sq. in. The lines of the maximum strain theory and of the maximum stress theory were also drawn through

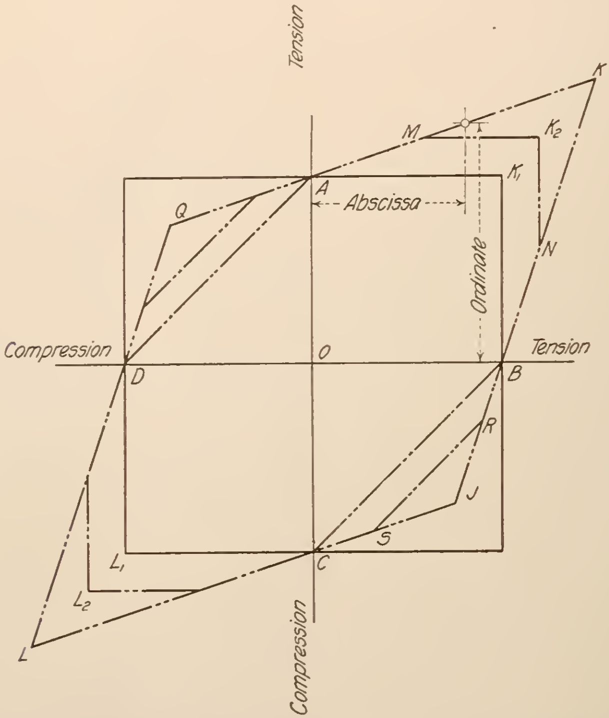

Fig. 24. Representation of Yield Point Strengths for Combined Stresses According to the Maximum Stress Theory, the Maximum Strain Theory, and the Maximum Shear Theory.

this average value of the yield-point stresses and compressive yield-point stresses The use of this average value is believed to be justified by the observed fact that the yield-point stress of low-carbon steel in tension is found in nearly all cases to have the same numerical value as the yield-point stress in compression. In determining the line for the maximum strain theory a value of Poisson's ratio of 0.395 was used. This value was obtained from the test of tube No. 4, Series 3 (Fig. 19). A line of constant shearing stress has been drawn through the yield-point stress (14,000 lb. per sq. in.) obtained from the torsion test, since simple torsion produces tensile and compressive stresses of equal intensities and hence corresponds to a stress ratio of unity (see Fig. 21). The lines of internal friction theory and of Mohr's theory practically coincide with the maximum shear theory.

An inspection of Fig. 23 shows that for Series 3 as well as for Series 1 and 2, the experimental results follow the maximum strain theory up to a certain stress ratio and then follow a line of constant shear which is the maximum shear developed. The ratio of the shearing yield-point stress from the torsion test to the average of the tensile and compressive yield-point stresses is 0.56. The question of the neglect of water pressure as a third stress does not enter in this series, for taking the stresses in the order of their magnitude the compression due to the water pressure becomes intermediate between the circumferential tension and the axial compression, so that the maximum shearing stress is equal to one-half the sum of the axial compressive stress and the circumferential stress. This series leads to the same conclusions as the other two, although the ratio of the shearing and tensile yield-point stresses is somewhat lower.

The net result of this investigation as it affects the strength of steel under combined stress in two directions at right angles to each other—biaxial loading—is that instead of a single law, whatever its nature, as has heretofore been assumed, there are two distinct laws governing the strength of the material, each law dominant within its limits. These two laws are the maximum strain theory and the maximum shear theory; the first governs until the shearing yield-point stress of the material is reached, after which the shear theory holds. The exact point of the change from one law to the other depends upon the ratio of the shearing yield-point stress to the yield-point stress in simple tension and compression.

24. *Stiffness.*—Although strains have been measured in many tests

heretofore made, no attempt seems to have been made to determine the law of stiffness. It has been taken for granted that the deductions of the mathematical theory of elasticity, as embodied in St. Venant's theory, hold, or else no attention has been paid to strains except as related to the strength of the material in the determination of the yield point or so-called elastic limit. The weakness of the mathematical theory of elasticity lies in its generalization of Hooke's law and the neglect of the temperature changes, so that the strains obtained in tests of isotropic materials will only closely approximate the computed values. The effect of shear in producing strain has been neglected and is small before the yield-point stress is reached, but the variation of shearing strength in different directions throughout the specimen, the possibility of a change in Poisson's ratio with increasing stress, the possibility of a different Poisson's ratio and modulus of elasticity with and across the direction of rolling or drawing, enter to complicate the problem. The material experimented upon is not the isotropic substance assumed in the theory. Lines have been drawn on the stress-strain curves of the general averages of the axial gage lines, Fig. 17, 18, and 19, giving the strains as computed by the mathematical theory of elasticity using the values of Poisson's ratio* and modulus of elasticity obtained from tests in simple tension and in compression. These lines agree quite closely with the observed values except in the case of tube No. 1 of Series 1, and tube No. 7 of Series 2, the former showing lower strains and the latter greater strains than the computed values. Apparently within the range of application of the mathematical theory of elasticity, where E is constant, the strains follow the theory with sufficient exactness to say that the theory holds. Lines have also been drawn to represent the strains corresponding to a simple tensile or compressive stress equal to the greater principal stress.

For the circumferential lines there have been drawn on the stress-strain curves of the general averages, Fig. 17, 18, and 19, lines giving the strains computed by the mathematical theory of elasticity and also the strains accompanying simple tensile stresses equal to the circumferential stresses. The values of Poisson's ratio and the modulus of elasticity are taken the same as for the axial lines. The lines of the mathematical theory of elasticity do not fit as well as in the case of the axial strains. It can be seen that to fit the experimental points of Series 1 and 2, it is necessary to use a higher value for both the modulus of elasticity and Poisson's ratio, the latter requiring the greater change. An increase in Poisson's ratio will increase the strains of tube No. 1 and lower those of the other tubes of these two series. An increase of the modulus of elasticity will diminish all the strains proportionally. It will be recalled that the value of Poisson's ratio obtained in compression tests was high, 0.395. For Series 3, where Poisson's ratio is higher than for Series 1 and 2, the modulus of elasticity alone need be increased. With a higher modulus the computed circumferential curves fit the experimental curves quite closely except for tube No. 5. There is a strong probability that both Poisson's ratio and the modulus of elasticity vary in the two directions with and across the direction of drawing. It is scarcely probable that the law changes, and the close agreement between the computed and observed values for the axial strains gives strong support to the belief that all the strains follow the requirements of the mathematical theory of elasticity. The indications are that the modulus of elasticity and Poisson's ratio may be different in different directions throughout the steel, in much the same way that Bauschinger has shown that the shearing strength of rolled steel varies in different directions.

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*The values of Poisson's ratio for the tubes tested in simple compression and in simple tension were obtained by dividing the circumferential unit-strain taken from the general average curves of these tubes (which is the same as the diametral unit-strain) by the corresponding axial unit-strain. For Series 1 this ratio for tube No. 5 is 0.334; for Series 3, obtained from tube No. 4, it is 0.395. The modulus of elasticity of Series 1 is 27,200,000 lb. per sq. in., and for Series 3 it is 29,500,000 lb. per sq. in. An examination of the axial and circumferential stress-strain diagrams of tube No. 5, Series 1, Fig. 17, and of tube No. 4, Series 3, Fig. 19, shows that in the first case (tension) Poisson's ratio remains practically constant, diminishing about 6 per cent after the yield-point stress has been passed, but that in the second case (compression) this ratio increases to almost 0.50 after the yield-point stress has been passed. There is no reason why Poisson's ratio should be constant for all kinds of steel, and it may well be that tension and compression tests on the same material will show different results. It is not known what the effect of the hollow specimen is in changing this ratio for tension or compression tests, but it is thought that the method used to obtain Poisson's ratio is accurate and reliable. It is to be noted that for the compression tests the value of both Poisson's ratio and the modulus of elasticity are higher than for the tension tests.

In Series 1 and 2, Fig. 17 and 18, for the tubes tested with a stress ratio of 0.92, the yield-point stress in the circumferential direction was practically the same as the yield-point stress in the axial direction, but the circumferential curves show a more sudden yielding of the material. In Series 3, Fig. 19, for the tube tested with a stress ratio of 0.90, the circumferential yield-point stress was lower than that in an axial direction. All the circumferential stress-strain curves of Series 3 show a sharp, sudden break when the yield-point stress in the axial direction is reached, no matter what the circumferential stress was. Granting that for Series 3 the value of Poisson's ratio increases to 0.50 above the yield-point stress, this is not sufficient to account for the great increase in the strains. It must be that the shearing stresses, which have passed the shearing yield-point stress, produce shearing strains of sufficient magnitude to account for this increase in the circumferential strains. This explanation is more strongly suggested by the stress-strain curves of Series 1 and 2, where Poisson's ratio remains nearly constant. The circumferential stress-strain diagrams that continue to show an increase in strain after the axial yield-point stress has been passed are those from the tubes whose axial yield-point stresses lie on the line of constant shear of Fig. 22. Those that do not show an increase at this time are from the tubes whose axial yield-point stresses lie on the line of the maximum strain theory.

That the shearing strains accompanying the axial stress can affect the circumferential strains is shown by the stress-strain diagrams for the circumferential lines of tube No. 4, Series 1, Fig. 14. The circumferential stress-strain diagram continues straight for a short distance after the yield-point stress has been passed in the axial direction, the circumferential stress corresponding to the axial yield-point stress being 34.500 lb. per sq. in., approximately. This is during the stage intermediate between the elastic and plastic conditions. When the axial curve breaks sharply, the circumferential curve changes direction also. If Poisson's ratio were the only factor, all the diagrams, with the possible exception of those of tubes No. 3, 6, and 8, where a high stress ratio was used, would show diminishing strains with increasing stress after the yield-point stress in the axial direction had been passed. This means that the tendency to reduce the diameter of the tube, due to the rapidly increasing axial strains, would be greater than the tendency to increase the diameter produced by the increase of the water pressure. But the curves of tubes No. 2 and 4 show an increasing strain (increasing tube diameter) even though the circumferential stresses were well below the yield-point stress of the material. These two tubes are the ones whose yield-point stresses lie on the line of constant shear, Fig. 22, and without the assistance of the shearing strains in producing circumferential strains, the curves of these two tubes would show a diminishing circumferential strain as the circumferential stress increased after the axial yield-point stress had been passed. The shear which causes yielding in an axial direction is on a different plane from that causing yielding in a circumferential direction. The former shear acts along a plane which passes through the direction line of the circumferential tension and cuts the axis of the tube at an angle of 45°. The latter shear acts on a plane which passes through the direction line of the axial tension and is parallel to the axis of the tube making an angle of 45° with the direction line of the circumferential stress. These shearing stresses are of different magnitudes, according to the ratio of the stresses, and each is equal to one-half the principal stress cut by its plane at an angle of 45°.

Apparently the strains follow the requirements of the mathematical theory of elasticity for all stress ratios, but there may be different values of Poisson's ratio and the modulus of elasticity for the axial and circumferential directions. After the yield-point stress in one direction has been passed the shearing strains have a considerable influence upon the deformations in the second direction.

25. *Comparison With the Methods and Results of Other Investigations.*—Attention is called to several points of difference between the method of investigation here recorded and the methods used by others. The greatest difference lies in the use of a portable extensometer to measure strains, the strain gage, whereby a large number of measurements were taken, both along the specimen and around it. Previous investigations used a fixed extensometer which measured strains along one or two gage lines, or, in some cases, used no strain measurements. No assumptions of uniform stress distribution were made, in the present series, for the strain gage records the variations and the gage length can be varied to suit the needs. With readings taken on a large number of gage lines for every load increment, a certain positiveness of result is attained which is impossible with attached instruments and few gage lines. Local effects are thus minimized. Another difference lies in the larger size of the specimens tested and in the smaller ratio of thickness of tube wall to diameter. Because of the form of specimen and the method of applying load, the stress was nearly uniform throughout the specimen; there was no "helping" effect by understressed material, no point of maximum stress to be located. The use of Johnson's apparent elastic limit method for determining yield-point stress gives a definite point for comparison.

An attempt was made to keep a definite ratio between circumferential and axial stresses throughout the test of each tube, so that comparison might be made later for these ratios. As far as possible, it was intended with a set of specimens cut from a given length of tubing to cover the entire range of stress ratio within tension-tension or compression-tension quadrants. The experiments reported by others and referred to in this section show generally a haphazard ratio of stresses, and the loads used were such that a definite stress was produced in one direction and then the other stress was increased until yielding took place.

The earliest important investigation of this subject was that reported by J. J. Guest* in 1900. The tests were made upon small steel, copper, and brass tubes about 1¼ in. outside diameter and varying in thickness from 0.025 in. to 0.034 in. Tests were made in combined torsion and axial tension, in torsion and circumferential tension, and in axial and circumferential tension. The strains were measured by a two-point extensometer, and although it was attached to the outside of the tube, the full hydrostatic pressure was counted as a third principal stress (compressive). Other than the tests on tube No. 1 of Guest's investi-

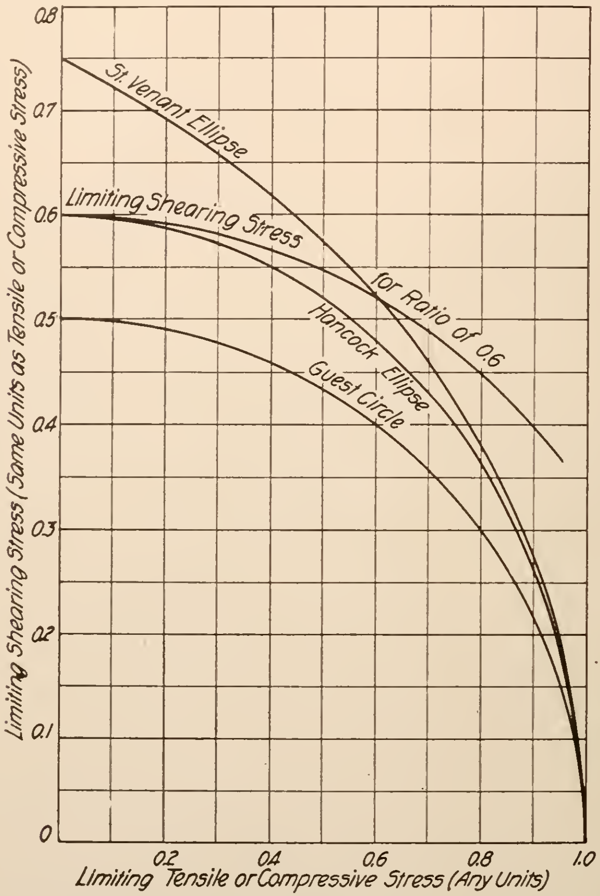

FIG. 25. RELATION BETWEEN SHEARING STRESSES DUE TO TORSION AND THE TENSILE OR COMPRESSIVE STRESSES DUE TO AXIAL LOAD OR BENDING.

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*Phil. Mag., 1900.

gation, there are but two tests where the stress ratio of circumferential tensile stress to axial tensile stress is 0.50 or less, and test No. 1 and one of the others follow the maximum strain theory closely. The yield-point stress was used as the basis of comparison, and each test was carried just beyond the yield point. Criticism may be made of the repeated use of the same specimen, since the yield-point stress is raised by repeated loading beyond the yield-point stress of the first test. It is not stated whether the tubes were annealed between tests. The results are taken to justify the maximum shear theory, and in the main they do within the field investigated, since the majority of the tests had a stress ratio between 0.50 and 1.00 within which limits the shear theory undoubtedly holds. The tests also show that the maximum shear developed is greater than one-half the yield-point stress in simple tension.

Following Guest comes the work of C. A. M. Smith,* W. A. Scoble,† E. L. Hancock,‡ and Wm. Mason* on bars and tubes in torsion and tension or compression and on small tubes in compression and internal pressure. All these results are used to justify the maximum shear theory which demands that the shearing yield-point stress is equal to one-half the yield-point stress in simple tension. With one exception, however, that of Scoble's tests reported in 1906, the maximum shear developed is greater than one-half the yield-point strength in tension, which, as noted above, was also found in Guest's tests. The majority of these tests—like Guest's—are in the region where the stress ratio is greater than 0.50. These tests cover the entire four quadrants of combined stress.

The tests of Professors Smith and Hancock will be shown on diagrams similar to Fig. 25 (Fig. 26 and 27), in which the ordinates represent the shearing stress due to torque and the abscissas represent the tensile or compressive stress due to axial load or bending. The diagram of Fig. 25 will be discussed before the tests are taken up. The shearing stress is plotted to twice the scale of the tensile or compressive stress. If a circle with a radius equal to the tensile yield-point stress is drawn with O as a center, it will represent the relation between the shearing and the direct stress which produces a combined stress causing yielding required by the maximum shear theory. It will be observed that the shearing yield-point stress must therefore equal exactly one-half the tensile yield-point stress. A circle with radius equal to the shearing yield-point stress obtained from tests in simple torsion (0.6 the tensile yield-point stress) is shown; also, the ellipse representing the St. Venant or maximum strain theory, beginning at the tensile yield-point stress. The two laws as advanced in this bulletin require that the maximum strain theory hold to the intersection of the St. Venant ellipse and the circle for limiting shearing stress for ratio of 0.6, and that then the shear shall govern. Hancock's ellipse has been added to show how closely he came to the results here advanced.

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*Inst. Mech. Engrs., 1909.

†Phil. Mag., 1906

‡Am. Soc. for Testing Materials, 1905, 6, 7, 8.

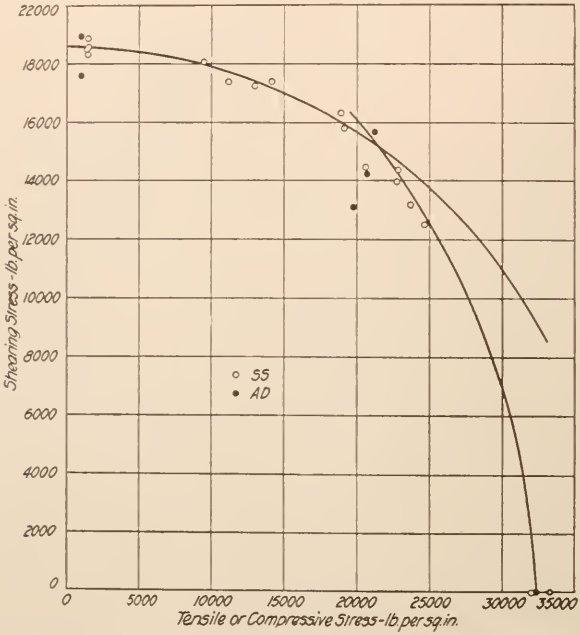

Fig. 26. Results of Tests, by C. A. M. Smith.

The results of Smith's and Hancock's tests have been plotted in Fig. 26 and 27. Both compression and tension have been plotted on the same side of the diagrams (symmetry permitting this), and the results of the different tests have been changed proportionally in order to compare them with a single set of theoretical curves. A comparison of the two laws herein proposed with the experimental results of these investigations show that the experimental results fit these laws better than the maximum shear theory which the tests were taken to prove.

Fig. 26 shows the results of C. A. M. Smith's tests on S. S. and

A. D. steel. Professor Smith maintains that the shearing yield-point stress of steel is one-half the tensile yield-point stress within small limits and quotes Turner's tests* to prove his point. His own tests do not bear out his contention and Turner's tests show considerable variation, averaging about 0.54 for this ratio. Professor Smith's tests are examples of careful work, but the interpretation of the tests as an unqualified endorsement of the maximum shear law cannot be accepted.

Mr. Scoble's tests seem to indicate that the shearing yield-point stress is lower than half the tensile yield-point stress. This result may possibly be accounted for by the way the shearing yield-point stress was located. This stress was taken at the intersection of the straight line of the elastic portion of the stress-strain diagram with a line drawn through the diagram beyond the yield point. Since a stress-strain curve for torsion breaks more quickly than a tension curve, it may be that the determination of the shearing yield-point stresses are affected by this. Scoble's method of measuring the bending moment by means of the deflection of the beam may be in error, for the law of deflection under the combined stress would be influenced by the very law he was seeking to determine.

The results of Professor Hancock's tests† are shown in Fig. 27. The curves of the maximum shear theory and the maximum strain theory have been drawn as well as his ellipse. Hancock used the p-limit as his criterion. He alone of these investigators realized the shortcomings of the maximum shear theory and endeavored to remedy them by fitting an ellipse to the experimental results. The ellipse fits quite closely, but while it is a close approximation, it does not fit the results as closely as do the curves representing the two laws herein proposed. His ellipse is empirical, while the combination of the maximum strain theory with the maximum shear theory has a foundation in the theory of the strength of materials.

Since torsion combined with compression or tension can be resolved into a case of tension combined with compression, Smith's and Hancock's tests fall in the fourth quadrant and show the applicability of the two laws there.

Mason's tests on tubes in compression and internal pressure show that the maximum shearing stress developed is greater than the shearing stress developed in simple compression. The average of all his tests in which a constant stress ratio of one to one was used, gives this maximum shearing stress as 0.60 of the compressive yield-point stress. As all the tests had the one stress ratio, it is not possible to make a comparison with theories, but the point thus located falls on the line of the two laws.

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*Engineering, London, February 5, 1909.

†Proceedings of the American Society for Testing Materials, 1908.

Minor inconsistencies are to be expected in experimental work of this nature, both on account of the variation in the material tested and on account of the apparatus used. The number of tests made by these investigators is insufficient to establish completely any theory, but a careful study of the published data will lead to the conclusion that the two laws, the theory here advanced, conform more closely to the experimental results than any single law.

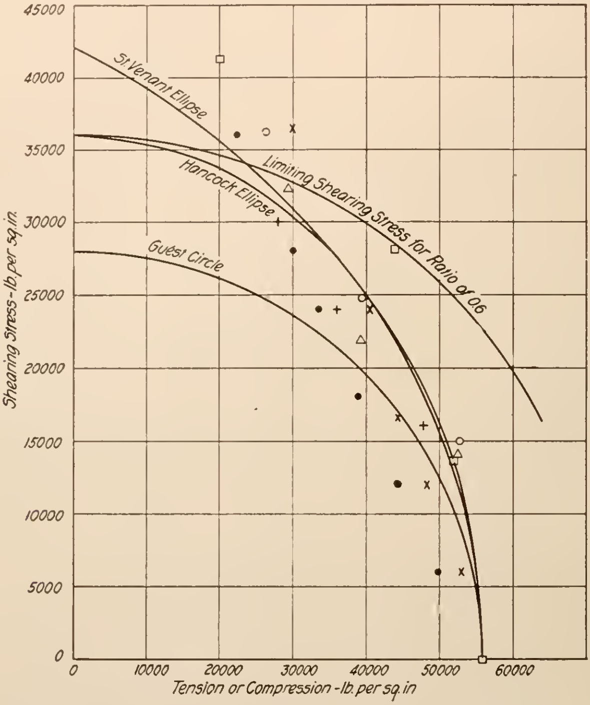

FIG. 27. RESULTS OF TESTS, BY E. L. HANCOCK.

26. *Summary and Conclusions.*—The following summary deals with the method of investigation and with the deductions which have been made from the data. As this is the first investigation of combined stress wherein a portable strain measuring instrument—such as the strain

gage—has been used, it is felt that considerable emphasis may be laid upon this fact. The size of the specimen is much larger than any heretofore used. These conditions tend to give more trustworthy results.

The experimental conclusions are:

1. The use of a portable strain measuring instrument is a decided advantage since it makes it possible to take measurements on a large number of gage lines for each increment of load, obviating to a large extent the effect of local variations in the test specimen.
2. The use of large tubes with thin walls gives quite uniform stress distribution, the yield-point stress is more positively determined, and the effect of eccentricity of loading is less than with solid bars on account of the larger diameter of the tube.
3. With large tubes the thickness of the tube walls can be accurately determined.
4. Flat plates in cross bending give uneven distribution of stress and are not satisfactory for biaxial loading tests.

The deductions which have been made from the experimental data are:

5. With increasing values of the ratio of the biaxial stresses the yield-point strength follows the maximum strain theory until the value of the shearing stress reaches the shearing yield point, then the shearing stress controls according to a maximum shear theory. There are thus two independent laws each dominant within proper limits instead of some single law as has heretofore been assumed.
6. Because these two laws govern the strength of ductile materials under biaxial loading, the ratio for simple stresses of the shearing yield-point stress to the tensile yield-point stress is important.
7. The stiffness follows the requirements of the mathematical theory of elasticity for all stress ratios, but the values of Poisson's ratio and the modulus of elasticity may be different in the two directions, with and across the rolling and drawing of the steel.
8. The results of the tests reported by previous investigators conform better to the two laws of strength than to any single law.

APPENDIX I.

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APPENDIX II. MATHEMATICAL TREATMENT.

1. *Stresses and Strains.*—The analysis of stress and strain in elastic materials known as the mathematical theory of elasticity embodies the most complete and elaborate theory of the action of elastic bodies under stress. The following brief presentation of the mathematical theory of elasticity as it applies to the problem of the investigation follows largely the treatment of Love.* It will be desirable to outline briefly the work leading up to the derivation of the general equations of the mathematical theory of elasticity connecting stress and strain before taking up the derivation of the equations of stress and strain in a cylinder under internal pressure and an axial load.

In the theory of elasticity the relations between three sets of magnitudes must be considered.

1. The displacements of the points of the strained body. If the ordinary rectangular system of coordinates is used for reference, the displacement s of a point due to the strain is resolved into components u, v, w parallel respectively to the X, Y and Z axes.

2. The strain components. Let ϵ_1, ϵ_2, ϵ_3 denote the strains in the directions of the X, Y, Z axes, respectively; then

$$\epsilon_1 = \frac{\partial u}{\partial x}, \epsilon_2 = \frac{\partial v}{\partial y}, \epsilon_3 = \frac{\partial w}{\partial z} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The components of shearing strain are defined as follows:

$$\epsilon_{23} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}, \epsilon_{31} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \epsilon_{12} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \quad \ldots\ldots\ldots (2)$$

Here ϵ_{23} denotes the shearing strain in the plane YZ, etc. Along with the strain components may be included the components of the rotation of an element of the body. If the displacement involves a rotation ω of the element as a whole and this rotation be resolved into X, Y and Z components, then these components are given by the relations

$$\omega_1 = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \omega_2 = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \omega_3 = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad \ldots\ldots\ldots (3)$$

3. The stress components. The six stress components may be denoted by σ_1, σ_2, σ_3; σ_{23}, σ_{31}, σ_{12}. σ_1 is the stress in the direction of the X-axis on a plane perpendicular to the X-axis; similarly for σ_2 and σ_3. σ_{23} is the stress in the direction of the Y-axis over a plane perpendicular to the Z-axis; therefore it is a shearing stress.

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*The Mathematical Theory of Elasticity, A. E. H. Love, 1904.

The stress components must satisfy certain conditions of equilibrium, which are expressed by three equations of the following type (assuming that the body forces, such as gravity, may be neglected, and that the body is at rest).

$$\frac{\partial \sigma_1}{\partial x} + \frac{\partial \sigma_{12}}{\partial y} + \frac{\partial \sigma_{13}}{\partial z} = 0 \quad \dots\dots\dots\dots\dots\dots\dots (4)$$

The six strain components ϵ_1, ϵ_2, etc., and the six stress components are connected by certain relations. Hooke's law, the linear relation between stress and strain, is the basis of these relations. Each stress component is taken as a linear function of the six strain components; thus

$$\left.\begin{aligned} \sigma_1 &= a_1\epsilon_1 + a_2\epsilon_2 + a_3\epsilon_3 + a_4\epsilon_{23} + a_5\epsilon_{31} + a_6\epsilon_{12} \\ \sigma_2 &= b_1\epsilon_1 + b_2\epsilon_2 + b_3\epsilon_3 + b_4\epsilon_{23} + b_5\epsilon_{31} + b_6\epsilon_{12} \end{aligned}\right\} \dots\dots\dots (5)$$

etc.

A consideration of the work done in deforming a body leads to the conclusion that there must exist a so-called strain-energy function V, such that

$$\sigma_1 = \frac{\partial V}{\partial \epsilon_1},\ \sigma_2 = \frac{\partial V}{\partial \epsilon_2},\ \text{etc.}$$

It follows that the function V must be a homogeneous quadratic function of the six strain components and must have therefore 21 terms. The number of coefficients apparently 36 in eq. (5) is thereby reduced to 21 by relations of the form $a_2 = b_1$, $a_3 = c_1$, $a_4 = d_1$, etc.; that is, V is the symmetric determinant of the quadric $\sum_{1}^{6} c_{ij}\,\epsilon_{ij}$.

If the body is isotropic, these 21 coefficients can be reduced to two. Denoting by λ one of these remaining coefficients and by μ one-half the difference between the two coefficients, the following relations between stresses and strains are established:

$$\left.\begin{aligned} \sigma_1 &= \lambda\Delta + 2\mu\epsilon_1 \\ \sigma_2 &= \lambda\Delta + 2\mu\epsilon_2 \\ \sigma_3 &= \lambda\Delta + 2\mu\epsilon_3 \end{aligned}\right\} \dots\dots\dots\dots\dots\dots\dots (6)$$

where σ_1, σ_2 and σ_3 are the stresses along the X, Y and Z axes respectively and ϵ_1, ϵ_2 and ϵ_3 are the corresponding strains. Δ is the dilatation and is equal to the sum of ϵ_1, ϵ_2 and ϵ_3.

Let $\frac{1}{m}$ = Poisson's ratio

E = the modulus of elasticity.

Applying the relations between stress and strain to a bar in simple tension, the following relation between Poisson's ratio and the modulus of elasticity is established. Both $\frac{1}{m}$ and E are to be determined from tests in simple tension or compression.

$$E=\frac{\mu(3\lambda+2\mu)}{\lambda+\mu} \quad \ldots\ldots (7)$$

$$\frac{1}{m}=\frac{\lambda}{2(\lambda+\mu)} \quad \ldots\ldots (8)$$

$G =$ the shearing modulus of elasticity $= \mu$.

$$G=\frac{1}{2}\left(\frac{m}{m+1}\right)E] \quad \ldots\ldots (9)$$

The values of λ and μ may now be established in terms of E and $\frac{1}{m}$.

$$\lambda=\frac{Em}{(m+1)\,(m-2)} \quad \ldots\ldots (10)$$

$$\mu=\frac{E\,m}{2\,(m+1)} \quad \ldots\ldots (11)$$

Adding the second and third equations of (6)

$$\begin{aligned}\sigma_2+\sigma_3 &= 2\lambda\,\Delta+2\mu\,(\epsilon_2+\epsilon_3)\\ &= 2\,\lambda\,\Delta+2\mu\,(\Delta-\epsilon_1)\end{aligned}$$

$$\Delta=\frac{\sigma_2+\sigma_3+2\mu\epsilon_1}{2\,(\lambda+\mu)}$$

$$\lambda\,\Delta=\frac{1}{m}(\sigma_2+\sigma_3+2\mu\epsilon_1)$$

$$\sigma_1-2\mu\,\epsilon_1=\frac{1}{m}(\sigma_2+\sigma_3)+\mu\epsilon_1\left(\frac{\lambda}{\lambda+\mu}\right)$$

$$\left.\begin{aligned}\sigma_1&=\frac{1}{m}(\sigma_2+\sigma_3)+E\epsilon_1\\ \text{Similarly}\quad\sigma_2&=\frac{1}{m}(\sigma_1+\sigma_3)+E\epsilon_2\\ \sigma_3&=\frac{1}{m}(\sigma_1+\sigma_2)+E\epsilon_3\end{aligned}\right\} \quad \ldots\ldots (12)$$

Rearrangement of Eq. (12) gives

$$\left.\begin{aligned} E\epsilon_1 &= \sigma_1 - \frac{1}{m}(\sigma_2+\sigma_3) \\ E\epsilon_2 &= \sigma_2 - \frac{1}{m}(\sigma_1+\sigma_3) \\ E\epsilon_3 &= \sigma_3 - \frac{1}{m}(\sigma_1+\sigma_2) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

These are the three fundamental equations connecting stress and strain. $E\epsilon_1$, $E\epsilon_2$ and $E\epsilon_3$ are called by various writers the reduced stresses, the true stresses, or the ideal stresses.

2. *Stresses and Strains in a Thin Tube.*—For bodies of cylindrical form it is convenient to use cylindrical coordinates r, θ and z instead of the rectangular system x, y, z. The z coordinate is measured parallel to the axis of the tube, r denotes the radial distance from the axis, and θ the angle of an axial plane from some chosen initial plane.

Denoting by u, v and w the displacement components, as before (u radial, w axial and v perpendicular to a radius r) the three strain components ϵ_r, ϵ_θ, ϵ_z are given by the relations

$$\epsilon_r = \frac{\partial u}{\partial r},\ \epsilon_\theta = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r},\ \epsilon_z = \frac{\partial w}{\partial z} \quad \ldots\ldots\ldots\ldots (14)$$

and the corresponding stress components are given by the equations

$$\left.\begin{aligned} \sigma_r &= \lambda\Delta + 2\mu\epsilon_r \\ \sigma_\theta &= \lambda\Delta + 2\mu\epsilon_\theta \\ \sigma_z &= \lambda\Delta + 2\mu\epsilon_z \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Expressions for the shearing strain and stress components may be deduced, but they are not needed in the present investigation.

In the case of a hollow cylinder under internal pressure, conditions of symmetry require that the displacement v shall be zero; hence the expression for ϵ_θ in (14) reduces to $\epsilon_\theta = \frac{u}{r}$. Furthermore, it is permissible in the case under consideration to assume a condition of plane strain, in which all points in a cross section of the cylinder experience the same displacement w in the z direction. With this assumption, $w = az$, where a is a constant, and therefore $\epsilon_z = a$. With these simplifying assumptions, the expression for the dilatation takes the form

$$\Delta = \frac{\partial \mu}{\partial r} + \frac{u}{r} + a$$

If now general expressions for the displacements u and w are found, eq. (14) will give the strain components and eq. (15) the stress components. The conditions of equilibrium must, however, be satisfied, that is relations analogous to (4) must be introduced. It is possible, however, to eliminate the stress components by the aid of eq. (6) or eq. (15) and thus to express the equilibrium conditions in terms of displacements only. Thus from (4) and (3) may be derived the relation

$$(\lambda+2\mu)\frac{\partial\Delta}{\partial x}-2\mu\left(\frac{\partial\omega_3}{\partial y}-\frac{\partial\omega_2}{\partial z}\right)=0 \quad \dots\dots\dots\dots (16)$$

with two similar; and in cylindrical coordinates a similar process leads to the relation

$$(\lambda+2\mu)\frac{\partial\Delta}{\partial r}-2\mu\left(\frac{1}{r}\frac{\partial\omega_3}{\partial\theta}-\frac{\partial\omega_2}{\partial z}\right)=0 \quad \dots\dots\dots\dots (17)$$

In the case under consideration the rotation components ω_2 and ω_3 must be zero (ω_2 may have a small finite value at the extreme ends of the tube), hence (17) reduces to

$$(\lambda+2\mu)\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{u}{r}+a\right)=0 \quad \dots\dots\dots\dots (18)$$

Integration of this equation leads to the following relation for the displacement:

$$u=Cr+\frac{D}{r}$$

in which C and D are constant.

Introducing this expression for u in the expression for ϵ_r, ϵ_θ and Δ, the result is

$$\epsilon_r=\frac{\partial u}{\partial r}=C-\frac{D}{r^2}$$

$$\epsilon_\theta=\frac{u}{r}=C+\frac{D}{r^2}$$

$$\Delta=2C+a$$

Hence the relations (15) become

$$\begin{aligned}\sigma_r&=\lambda(2C+a)+2\mu\left(C-\frac{D}{r^2}\right)\\&=2C(\lambda+\mu)-2\mu\frac{D}{r^2}+\lambda a \quad \dots\dots\dots\dots (20)\end{aligned}$$

$$\begin{aligned}\sigma_\theta\,\lambda&=(2C+a)+2\mu\left(C+\frac{D}{r^2}\right)\\&=2C(\lambda+\mu)+2\mu\frac{D}{r^2}+\lambda a \quad \dots\dots\dots\dots (21)\end{aligned}$$

$$\begin{aligned}\sigma_z&=\lambda(2C+a)+2\mu a\\&=2C\lambda+(\lambda+2\mu)a \quad \dots\dots\dots\dots (22)\end{aligned}$$

To determine the constants C and D, we have the conditions
$\sigma_r = -p_1$ the internal pressure, when $r = r_1$, the internal radius
$\sigma_r = -p_0$ the external pressure, when $r = r_0$, the external radius
From (20)

$$-p_1 = 2C(\lambda+\mu) - 2\mu\frac{D}{r^2} + \lambda a$$

$$-p_0 = 2C(\lambda+\mu) - 2\mu\frac{D}{r^2} + \lambda a$$

whence

$$2\mu D = (p_1 - p_0)\frac{r_1^2 r_0^2}{r_0^2 - r_1^2} \qquad (23)$$

$$2C(\lambda+\mu) = \frac{p_1 r_1^2 - p_0 r_0^2}{r_0^2 - r_1^2} - \lambda a \qquad (24)$$

Equations (23) and (24) may be written

$$2\mu D = T \qquad (23a)$$

$$2C(\lambda+\mu) = S - \lambda a \qquad (24a)$$

It will be observed that $S = \frac{p_1 r_1^2 - p_0 r_0^2}{r_0^2 - r_1^2}$ gives the mean intensity of tensile stress in a cross-section of a closed tube due to the internal fluid pressure p_1, with external pressure p_0. In the test the axial stress was in part applied by the testing machine; hence its value may be denoted by kS, where k is a constant that becomes equal to 1 when the axial stress is one-half of the hoop tension. In the test an axial stress was applied by the testing machine and this must be added to the axial stress due to internal pressure. Hence the *total* axial stress may be taken as kS. Putting kS for σ_z in (22) and combining with (24a), we have two equations for the determination of a and C, namely:

$$kS = 2\lambda C + (\lambda + 2\mu)a$$

$$2C(\lambda + \mu) = S - \lambda a$$

From these the following results are readily obtained:

$$a = \frac{S}{E}\left(k - \frac{2}{m}\right) \qquad (25)$$

$$C = \frac{S}{E}\left[1 - \frac{1}{m}(1 + k)\right] \qquad (26)$$

Also from (23a)

$$D = \frac{T}{2\mu} = \frac{T}{E}\,\frac{1+m}{m} \qquad (27)$$

The strain component ϵ_θ is now found.

$$\epsilon_\theta = C + \frac{D}{r^2} = \frac{1}{E}\left\{ S\left[1 - \frac{1}{m}(1+k)\right] + \frac{T}{r^2}\,\frac{1+m}{m}\right\} \tag{28}$$

The value of ϵ_θ at the outer surface of the cylinder, where the strain was measured, is found by taking $r = r_0$. Substituting now the proper expressions for S and T, and putting $r = r_0$, (28) becomes

$$(\epsilon_\theta)_0 = \frac{1}{E}\left\{ \frac{p_1 r_1^2 - p_0 r_0^2}{r_0^2 - r_1^2}\left[1 - \frac{1}{m}(1+k)\right] + (p_1 - p_0)\,\frac{r_1^2}{r_0^2 - r_1^2}\,\frac{1+m}{m}\right\} \tag{29}$$

Since p_0 is small compared with p_1, the terms $p_1 r_1^2 - p_0 r_0^2$ and $(p_1 - p_0)\ r_2$ may be considered equal. With this approximation (29) becomes

$$(\epsilon_\theta)_0 = \frac{1}{E}\left\{ \frac{p_1 r_1^2 - p_0 r_0^2}{r_0^2 - r_1^2}\left(2 - \frac{k}{m}\right)\right\} \tag{30}$$

If we consider a closed cylindrical tube with internal hydrostatic pressure p_1 and external pressure p_0, the net load producing axial tension is

$$\pi(p_1 r_1^2 - p_0 r_0^2)$$

and the area of the cross section of the tube is

$$\pi(r_0^2 - r_1^2)$$

Denoting the load by P and the area by A, we have

$$S = \frac{p_1 r_1^2 - p_0 r_0^2}{r_0^2 - r_1^2} = \frac{P}{A}$$

$$(\epsilon_\theta)_0 = \frac{P}{EA}\left(2 - \frac{k}{m}\right) \tag{31}$$

$$\epsilon_z = a = \frac{P}{EA}\left(k - \frac{2}{m}\right) \tag{32}$$

In the test the axial load applied was $kP = L$; hence

$$(\epsilon_\theta)_0 = \frac{L}{EA}\left(\frac{2m - k}{k\,m}\right) \tag{33}$$

$$\epsilon_z = \frac{L}{EA}\left(\frac{k\,m - 2}{k\,m}\right) \tag{34}$$

The corresponding values of $E\epsilon$ (reduced stresses) are

$$E\,(\epsilon_\theta)_0 = \frac{L}{A}\,\frac{2m - k}{k\,m} \tag{35}$$

$$E\epsilon_z = \frac{L}{A}\,\frac{km - 2}{k\,m} \tag{36}$$

The actual stresses are

$$\sigma_z = k\,S = \frac{k\,P}{A} = \frac{L}{A}$$

$$\sigma_\theta = S + \frac{T}{r^2}$$

$$\sigma_r = S - \frac{T}{r^2}$$

In the preceding discussion the results have been obtained in terms of p_1 and p_0 the absolute internal and external fluid pressures. Evidently p_0 is the pressure of the atmosphere. In the test the internal pressure p_1 was measured by the gage, and no account was taken of the external pressure p_0. This procedure is justified by the following results:

Let $p' = p_1 - p_0$ = internal gage pressure.

Then
$$S = \frac{p_1 r_1^2 - p_0 r_0^2}{r_0^2 - r_1^2} = \frac{p_1 r_1^2 - p_0 r_0^2 - p_0 r_1^2 + p_0 r_1^2}{r_0^2 - r_1^2}$$

and
$$= (p_1 - p_0)\frac{r_1^2}{r_0^2 - r_1^2} - p_0 \frac{r_0^2 - r_1^2}{r_0^2 - r_1^2} = \frac{p' r_1^2}{r_0^2 - r_1^2} - p_0$$

$$T = (p_1 - p_0)\frac{r_0^2 r_1^2}{r_0^2 - r_1^2} = p' \frac{r_0^2 r_1^2}{r_0^2 - r_1^2}$$

From (38) the hoop tension at the outer surface is

$$(\sigma_\theta)_0 = S + \frac{T}{r_0^2} = \frac{p' r_1^2}{r_0^2 - r_1^2} - p_0 + \frac{p' r_1^2}{r_0^2 - r_1^2} = 2\,\frac{p' r_1^2}{r_0^2 - r_1^2} - p_0$$

Since p_0 is entirely negligible in comparison with S, we may take

$$(\sigma_\theta)_0 = 2S = \frac{2L}{k\,A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

For the corresponding stress at the inner surface, we have

$$(\sigma_\theta)_1 = S + \frac{T}{r_1^2} = p' \frac{(r_1^2 + r_0^2)}{r_0^2 - r_1^2} = 2S + p' = \frac{2L}{k\,A} + p' \ldots\ldots (41)$$

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STRENGTH OF WEBS OF I-BEAMS AND GIRDERS

BY
HERBERT F. MOORE
And
W. M. WILSON

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 86 MAY, 1916

THE STRENGTH OF WEBS OF I-BEAMS AND GIRDERS

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THE STRENGTH OF THE WEBS OF I-BEAMS AND GIRDERS

I. INTRODUCTION.

1. *Preliminary.*—In designing beams and girders it is usual to consider that the bending action is resisted by the flanges, and the shearing stress by the web. There is a tendency, however, for webs of deep girders to fail by buckling; and there are complex stresses set up at the junction of the web and flange. There are also crushing stresses in the web over the supports of the girder or under the points of application of concentrated loads.

When a girder is subjected to flexure the material on one side of the neutral axis is subjected to longitudinal tensile stresses and the material on the other side is subjected to longitudinal compressive stresses. The material at any point in the girder is also subjected to longitudinal and to transverse shearing stresses of equal intensity. The longitudinal tensile and compressive stresses are equal to zero at the neutral axis and increase to a maximum at the outer edges of the flanges. The shearing stresses are equal to zero at the outer edges of the flanges and increase to a maximum at the neutral axis. Since the shearing stress is equal to zero at the outer edge of the girder the flange carries little and the web practically all of the shear on a transverse section.

In the design of a girder the longitudinal stress in the outer edge of the flange is limited to a safe value for the material in tension or compression, and the average shearing stress in the web (obtained by dividing the maximum total shear upon a transverse section by the cross-sectional area of the web) is kept within safe limits for the material in shear. Although, according to the elastic theory of beams, points intermediate between the neutral axis and the outer edge of the flange are subjected to both longitudinal tension or compression and to transverse and longitudinal shear, and although it is known that these combined stresses result in diagonal tensile (or compressive) and shearing stresses which are greater than the component stresses producing them, these diagonal stresses are not considered in the design of the girder. The view has been held that the diagonal tensile or compressive stresses do not materially exceed the simple longitudinal stress at the outer edge of the flange, and that the diagonal shearing stress does not materially exceed the shearing stress at the neutral axis.

It has been shown by tests that a tensile (or compressive) stress produces a strain in a direction at right angles to the the line of action of the stress, the term strain being here used to designate deformation and not stress which is used to designate an internal resisting force. Recent tests, notably those of Dr. Becker,* indicate that the tendency of a material to fail depends, within certain limits, upon the *strain,* and that a stress in one direction, while it does not set up lateral *stress,* does set up lateral *strain,* and affects the strength of the material. An analysis of diagonal stresses in a girder shows that, in general, a load which sets up a stress in a diagonal direction sets up a second stress at right angles to that direction. Hence, in considering the strength of a girder to resist a diagonal stress any stress at right angles to that diagonal stress must be taken into account.

The tests reported in this bulletin were made to study the web strains in I-beams and girders so designed that the primary failure would be a web failure. The test data obtained were used in conjunction with a mathematical analysis made to determine the importance of the diagonal strains and the methods of failure of girders.

2. *Previous Tests of the Web Strength of I-Beams and Girders.*—Not many tests of I-beams and girders in which the primary failure was web failure have been made. Table 1 (reprinted from bulletin 68 of the Engineering Experiment Station of the University of Illinois) gives the results of a few such tests.

3. *Acknowledgment.*—This Investigation was a part of the research work of the Department of Theoretical and Applied Mechanics and of the Department of Civil Engineering, and was conducted under the general supervision of Professor A. N. Talbot, of the Department of Theoretical and Applied Mechanics, and Professor I. O. Baker, of the Department of Civil Engineering. Wherever reference has been made to the work of other investigators, credit has been given in the text.

II. MATHEMATICAL ANALYSIS OF STRESS AND STRAINS IN WEB MEMBERS.

4. *Notation and Units.*—The following notations are used:

S=tensile or compressive stress (lb. per sq. in.); (various subscripts are used to denote stresses in various parts of the web).

S_s=shearing stress (lb. per sq. in.); (shearing stresses in various parts of the web are denoted by S'_s, S''_s, etc.).

---

*Bulletin No. 85, Engineering Experiment Station, University of Illinois.

P = force acting upon a girder or any small element (pounds).

ϕ = angle between the diagonal plane and the plane of one side of a element of a girder.

E = modulus of elasticity in tension or compression (lb. per sq. in.).

F = modulus of elasticity in shear (lb. per sq. in.).

ϵ = unit-strain in inches per inch (an abstract number).

M = bending moment at any section of a girder (pound-inches).

I = moment of inertia of the cross-section of a girder (inches)^4.

c = distance from the neutral axis of a girder to the outer fibers (inches).

V = vertical shear at any section of a girder (pounds).

t = thickness of the web of a girder (inches).

$(a_1\, c_1)$ = "static moment" of any portion of the cross-sectional area of a girder about the neutral axis (inches)^3.

$\left(\frac{l}{r}\right)$ = slenderness ratio for the web of a girder acting as a long column under compression (an abstract number).

h = clear depth of web of girder between flanges (inches).

b = length of a bearing block over a support or under a concentrated load (inches).

Δ_f = deflection of a girder due to direct flexure (inches).

Δ_s = deflection of a girder due to shear (inches).

Δ = deflection of a girder due to both flexure and shear (inches).

a = area of the cross-section of a girder (square inches).

l = length of span of a girder (inches).

l_1 = distance from support of a girder to nearest concentrated load (inches).

λ = Poisson's ratio, the factor of lateral strain (an abstract number).

Throughout this bulletin the term *strain* is used to denote the deformation caused by stress; it is not used as a synonym for stress.

5. *Common Formulas of Girder Design.*—The tensile or com-

TABLE 1.

WEB FAILURE OF I-BEAMS

Reprint from Bulletin 68 of the Engineering Experiment Station of the University of Illinois

1. Beam	12-in., 31.5-lb. I-Beam	12-in., 31.5-lb. I-Beam. Web Planed Thin	12-in., 31.5-lb. I-Beam. Web Planed Thin	12-in., 31.5-lb. I-Beam. Web Planed Thin	Beth'm Girder Beam 30-in., 175-lb.	20-in. Special* Built-up Girder
2. Span, ft.	2.92	3.00	3.00	3.00	20	15
3. Loading	Two Points 4 ⅞ in. Each Side of Center	Two Points 4 ⅞ Each Side of Center	Two Points 4 ⅞ Each Side of Center	Two Points 4 ⅞ Each Side of Center	Quarter Points	One Load at Quarter Point
4. Material	Steel	Steel	Steel	Steel	Steel	Steel
5. Number tested	2	1	1	1	3	1
6. Tested by	Univ. of Illinois	Univ. of Illinois	Univ. of Illinois	Univ. of Illinois	Marburg (Univ. of Pa.)	Turneaure (Univ. of Wis.)
7. Fiber stress due to direct flexure, lb. per sq. in.	33,500	28,200	19,300	12,700	31,000	Sidewise buckling prevented by bracing
8. Vertical distance between flanges, inches (h)	10.52	10.52	10.52	10.52	26,8	14
9. Thickness of web, inches (t)	0.35	0.28	0.19	0.16	0.69	0.14
10. Slenderness ratio for web $\frac{l}{r} = \sqrt{12}\,\frac{h \sec 45^\circ}{t}$	148	184	272	324	190	490
11. Load at failure, pounds	190,100	160,500	109,600	72,100	538,400	106,000 (Approx.)
12. Computed fiber stress (shear and also compression) at middle of web, lb. per sq. in.	25,800	27,200	27,400	21,400	14,800	26,500
13. $\frac{4\pi^2 E}{(\frac{l}{r})^2}$	53,900	34,900	16,000	11,300	32,700	4,900
14. Length of block under support, inches	6	6	6	6	12	....
15. Computed fiber stress (compression) in web adjacent to support, lb. per sq. in.	45,300	47,800	48,200	37,600	32,500	Stiffeners used at ends and under load
16. Yield-point strength of material at root of flange, lb. per sq. in.	31,700	33,100	34,000	32,200	28,200	37,700

*Test reported in full in the Journal of the Western Society of Engineers for 1907, p. 788. Failure by sidewise buckling was prevented by bracing. The load at failure is given by Dean Turneaure as that at which very great distortion had taken place and noticeable buckling in the web occurred. Excessive compressive stress in the web adjacent to reactions and concentrated loads was prevented by using stiffeners, well fitted to the flanges.

pressive stress in the outer fibers of the flanges of a girder is denoted by the equation

$$S=\frac{M\,c}{I} \quad \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

in which S is the stress in the outer fiber, M the bending moment, I the moment of inertia of the cross sectional area of the girder and c the distance from the neutral axis to the outer fiber. The longitudinal or the transverse shearing stress at any point in a girder is given by the formula

$$S_s=\frac{V}{It}(a_1\ c_1) \quad \dots\dots\dots\dots\dots\dots\dots\dots (2)$$

in which S_s is the shearing stress, V the total shear on a vertical section through the point, I the moment of inertia of the cross-sectional area of the girder, t the thickness of the beam at the point considered, a_1 the area of that part of the cross-section between the point and the extreme fiber, and c_1 the distance from the center of gravity of the area a_1 to the neutral axis of the beam.

For I-beams and built-up girders it is customary to use an approximate method for obtaining the shearing stress at any transverse section of the web; the total shear V is divided by the cross-sectional area of the web, and the quotient is taken as the shearing stress.

For the derivation of the above formulas the reader is referred to any standard author on the mechanics of materials, such as Merriman, Boyd, Murdock, Slocum, or Morley.

6. *Discussion of the Theory of Web Stress.*—The stresses on a transverse section of a beam in flexure consist of shearing stresses parallel to the section and tensile and compressive stresses normal to the section. The stress on a longitudinal section normal to the force plane consists of a shearing stress parallel to the section. Fig. 1 (a) represents a small element of a beam in flexure. Before the beam was subjected to flexure the element was rectangular. The plane *ab*, normal to the paper, is a transverse plane, and *cb* is a longitudinal plane normal to the force plane. The forces acting upon the element will produce a stress upon the oblique plane *ac* which makes an angle ϕ with *bc*. This stress may be resolved into two components, one parallel and the other normal to *ac*. The relation between the stresses on the diagonal plane and the stresses on the faces of the element depends upon the value of ϕ. Let Fig. 1 (a) represent any element having a length *dx*, a width *dy*, and a thickness normal to the paper equal to

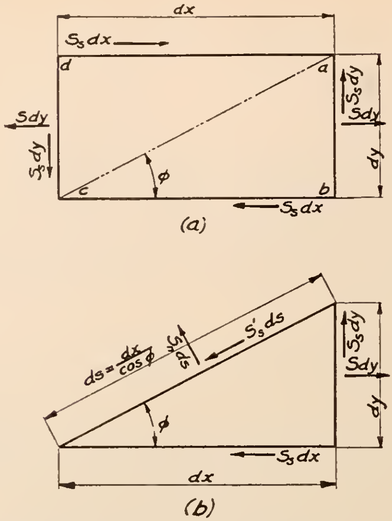

FIG. 1. STRESSES ACTING ON AN ELEMENT OF A GIRDER

unity. The normal stress per unit area on the ends of the element is represented by S and the shearing stress per unit area on the sides of the element, which are normal to the paper, is represented by S_s. The normal stress per unit area on the oblique plane (see Fig. 1 (b)) is represented by S_n and the tangential stress is represented by S'_s. It can be proven that*

$$S_n = \tfrac{1}{2}S\,(1 - \cos 2\phi) + S_s \sin 2\phi \tag{3}$$

$$S'_s = \tfrac{1}{2}S \sin 2\phi + S_s \cos 2\phi \tag{4}$$

The stresses S_n and S'_s are functions of ϕ. Their maximum values are given by the equations*

$$\text{Max } S_n = \tfrac{1}{2}S \pm \sqrt{S_s^2 + (\tfrac{1}{2}S)^2} \tag{5}$$

$$\text{Max } S'_s = \sqrt{S_s^2 + (\tfrac{1}{2}S)^2} \tag{6}$$

It should be noted that Max S_n and Max S'_s are not in the same direction.

The stress S may be either tension or compression. When S is tension the plus sign before the radical is used to find the maximum tensile unit stress S_n, and the minus sign before the radical, to find the maximum compressive unit-stress S_n. The latter is always normal to the former.

If stresses S_1 and S_2 at right angles to each other are acting on small particles, S_1 causes strain at right angles to its direction and thus influences the total strain of the particle in the direction of S_2. If S_1 and S_2 are both tensions or both compressions, the strain in the direction of either stress is *diminished* by the lateral strain due to the other stress; if S_1 is a tension and S_2 a compression (or vice versa) the strain in the direction of either stress is *increased* by the lateral strain due to the other stress. Following what they believe to be the best practice, the writers use the term *stress* to mean an internal resisting force set up by the action of external forces. A stress in any direction causes a lateral *strain* at right angles to that direction, but, accepting the above definition of stress, does not cause a lateral stress.

---

*Merriman, "Mechanics of Materials," 10th ed., p. 264.

Whether or not the strain at right angles to a stress influences the *strength* of a material has for a long time been a subject of discussion among students of the mechanics of materials. Three theories of the strength of materials under combined stresses have been advanced:

(a) The strength of a material depends only on the normal *stress*. If this theory is accepted, the transverse strain produced by a stress does not affect the strength, and a particle under the action of two stresses at right angles to each other is just as near failure (and no nearer) as it would be under the action of the greater of the two stresses acting alone. This is called the "maximum stress theory."

(b) The strength of the material is dependent upon the *strain*. If this theory is accepted, a particle under the action of stress in one direction is in greater danger of failure than it would be under the action of two stresses of no greater magnitude, alike in sign, and acting in two directions at right angles to each other. This is known as the "maximum strain theory."

(c) The strength of the material depends on the maximum *shearing* stress set up by the action of the various stresses. This is known as the "maximum shear theory."

The latest, and in some respects the most conclusive, tests which bear on this subject are those of Becker (University of Illinois Engineering Experiment Station, Bulletin No. 85). Becker studied the behavior of thin tubes in which axial tension or compression was produced by means of a testing machine and transverse tension was set up by means of internal water pressure. He found that for steel the second theory, the maximum strain theory, held unless the maximum shearing stress developed was greater than about 0.60 of the tensile or compressive stress. Within certain limits the strength of the material seems to depend on the strain, and beyond those limits it seems to depend on the shearing stress.

For expressing the value of a strain in this bulletin, the numerical value of $E\epsilon$ (the product of the unit strain and the modulus of elasticity, 30,000,000 lb. per sq. in. for steel) is given instead of the numerical value of the strain. Thus, in speaking of the safe working strain for a bar in tension, instead of saying that the safe unit strain is 0.000533 inches per inch length, the writers have used the expression, the strain corresponding to a value of $E\epsilon$ of 16,000 lb. per sq. in. or, more briefly, the $E\epsilon$ value of 16,000 lb. per sq. in. It is thought that this method of expressing the strain enables the engineer, who is not in the habit of thinking in terms of strain, to compare more easily the combined strains in girders with the strains resulting from the simple stresses with which he is familiar. $E\epsilon$ is not a stress, but a simple stress in one direction only with a magnitude equal to $E\epsilon$ would produce the same structural damage to the material as is produced when the strain ϵ occurs.

In the following equation S_1 and S_2 are two stresses at right angles to each other* and λ is Poisson's ratio

$$E\epsilon=S_1-\lambda S_2 \quad \text{.........} (7)$$

$$E\epsilon_2=S_2-\lambda S_1 \quad \text{.........} (8)$$

If a stress is compression it is to be taken as negative; if the strain is positive it is an elongation; if negative, a contraction. The stresses S_1 and S_2 produce shearing stresses along planes oblique to their lines of action, and it can be proven that the maximum shearing stress S'_s set up by the two stresses is given by the equation.†

$$S''_s=\tfrac{1}{2}(S_1-S_2) \quad \text{.........} (9)$$

III. TESTS

7. *Specimens.*—The specimens of the series of tests reported in this bulletin (1914 series) comprised six 12-inch I-beams and two 24-inch built-up girders. The webs of the I-beams were planed thin and the webs of the girders were made of thin plates. The webs in all of these test specimens were thinner than those used in standard practice. Had standard practice been followed, the webs would have been so strong in comparison with the flanges that the primary failure would probably not have been a web failure, but a failure due to some other cause, such as direct flexure or sidewise buckling of the compression flange. To investigate the web strength of beams it was

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*Lanza, "Applied Mechanics," pp. 868-869.
†Merriman, "Mechanics of Materials," 10th ed., p. 363.

necessary to use specimens in which web failure would be the primary failure. Hence, thin-webbed specimens were used.

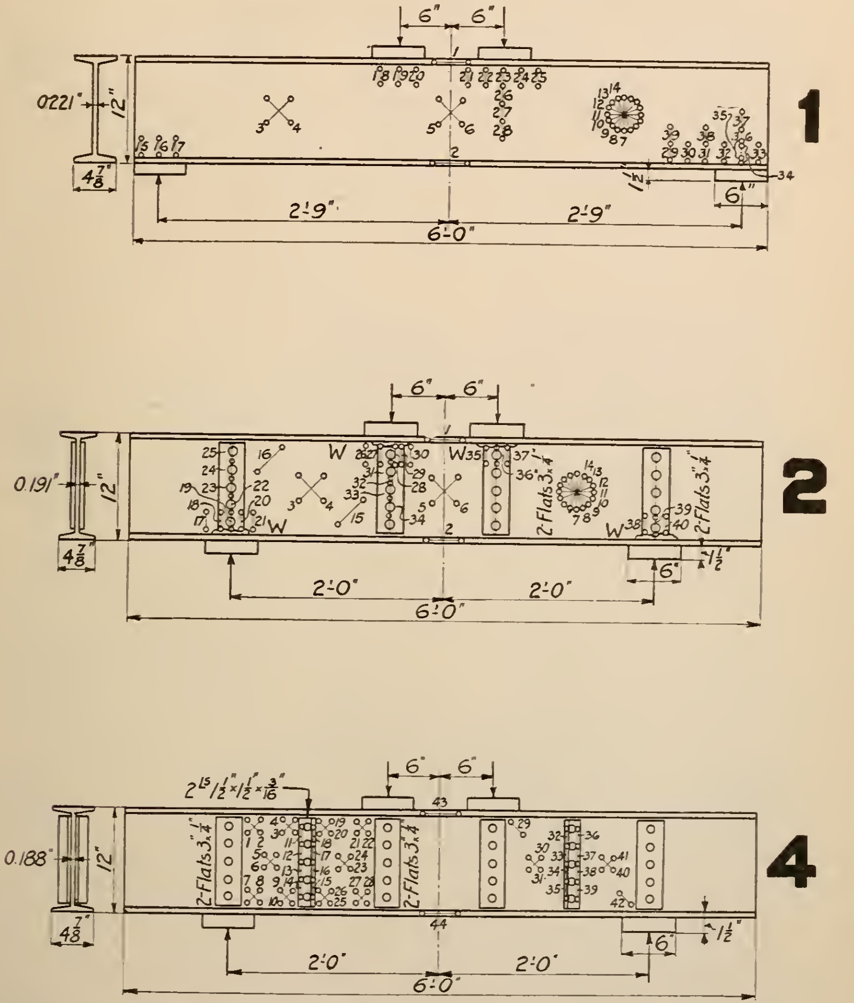


FIG. 2. I-BEAMS 1, 2, AND 4 SHOWING LOCATION OF GAGE LINES (W DENOTES AN OXY-ACETYLENE WELD)

Fig. 2 and 3 show the shape and size of the I-beam specimens. Beams 1 and 5 had no stiffeners; beams 2 and 6 had flat stiffeners on the web adjacent to the bearing blocks. Flats were used instead of angles so that there would be very little resistance to the buckling of

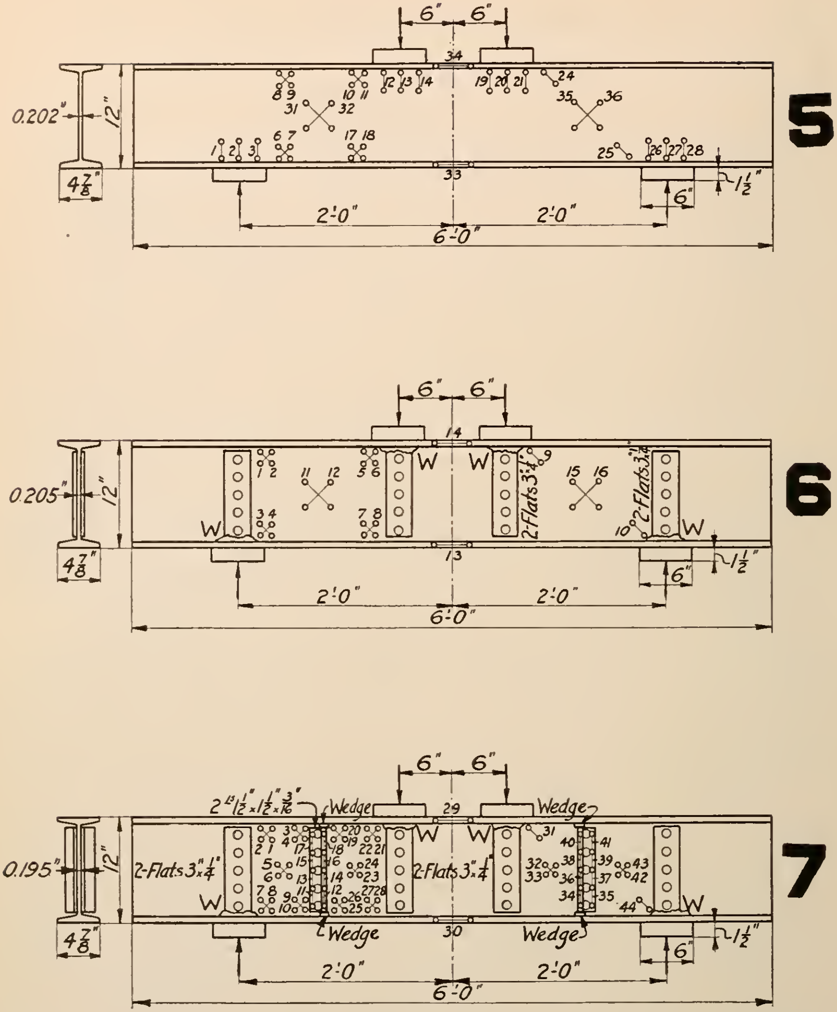
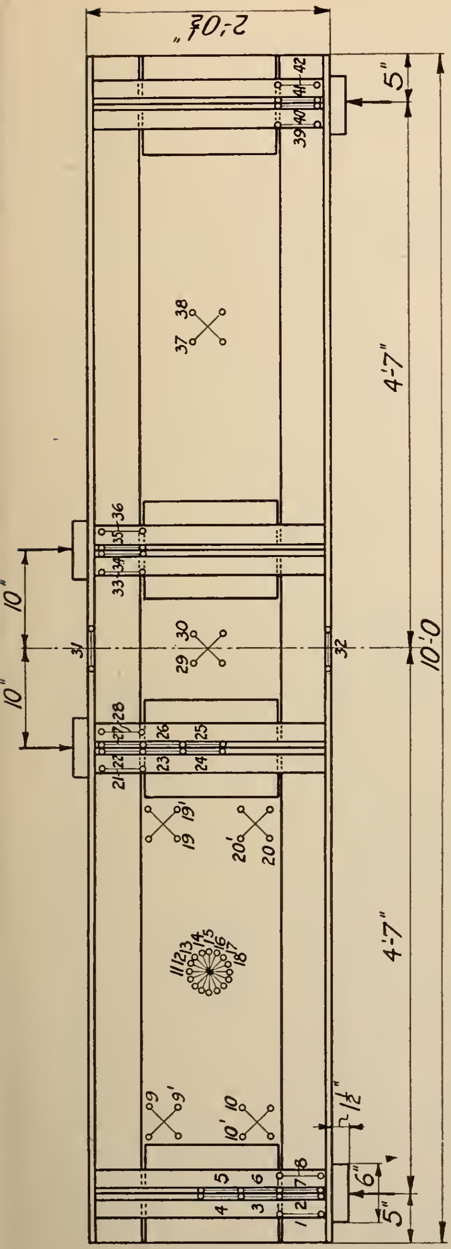


FIG. 3. I-BEAMS 5, 6, AND 7 SHOWING LOCATION OF GAGE LINES (W DENOTES AN OXY-ACETYLENE WELD)

the web. The flats were welded to the flanges to insure a good bearing. For beams 4 and 7 there were flat-stiffeners at the bearing blocks and angle stiffeners between bearing blocks. Fig. 4 and 5 show the built-up girders. Girder 8 had angle-stiffeners at the bearing blocks and at intermediate points; girder 3 had angle-stiffeners at the bearing blocks only.



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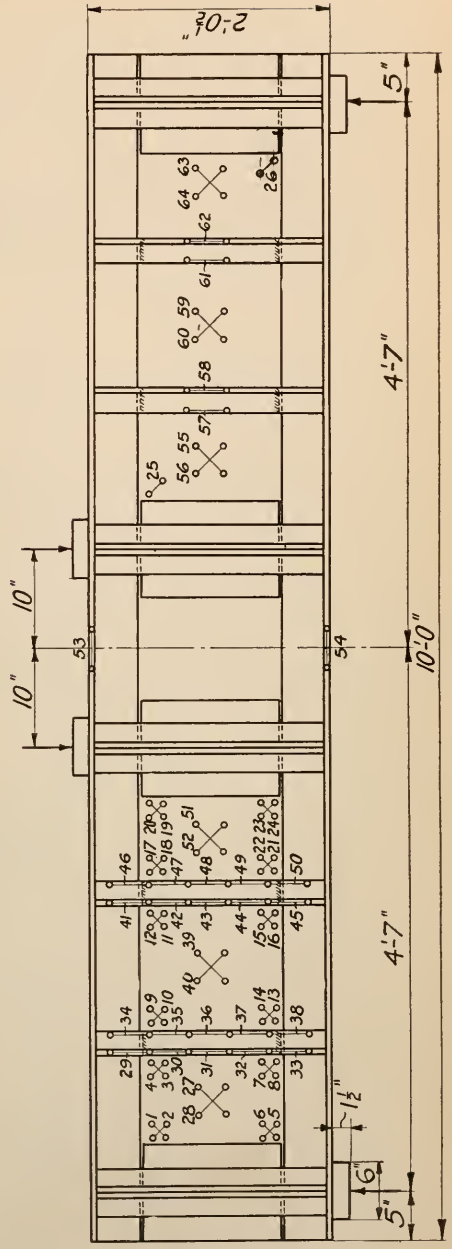

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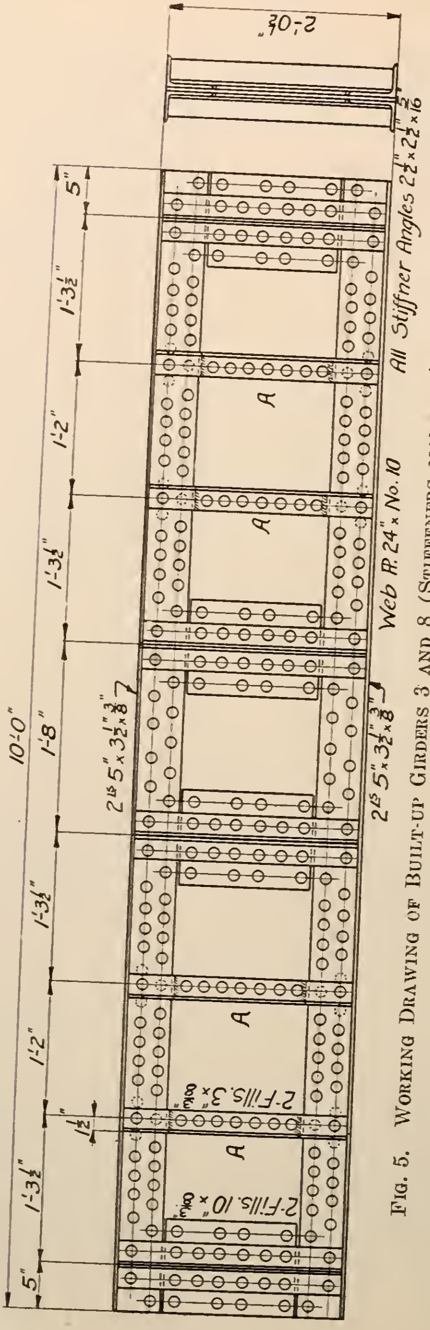

FIG. 5. WORKING DRAWING OF BUILT-UP GIRDERS 3 AND 8 (STIFFENERS MARKED A OMITTED FOR GIRDER 3)

For the purpose of determining the quality of the material in the beams, test pieces were cut from some portion of each beam which had not been under heavy stress during the test. For the I-beams the specimens were cut from the portion overhanging the end bearing. Tension specimens were cut from the webs, from the flanges, and from the points at which the webs join the flanges. Shear specimens were cut from the webs. For the girders these specimens were cut from the central portion of the webs where the shear was zero. A typical tension specimen is shown in Fig. 6 and a typical shear specimen in Fig. 7.

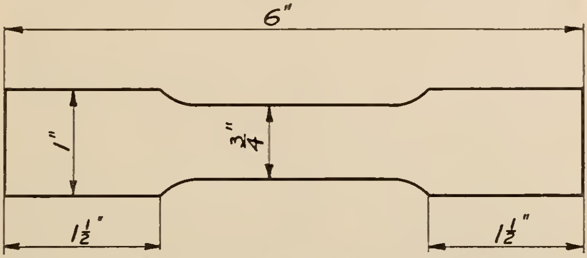

FIG. 6. FORM OF TENSION SPECIMEN CUT FROM GIRDER

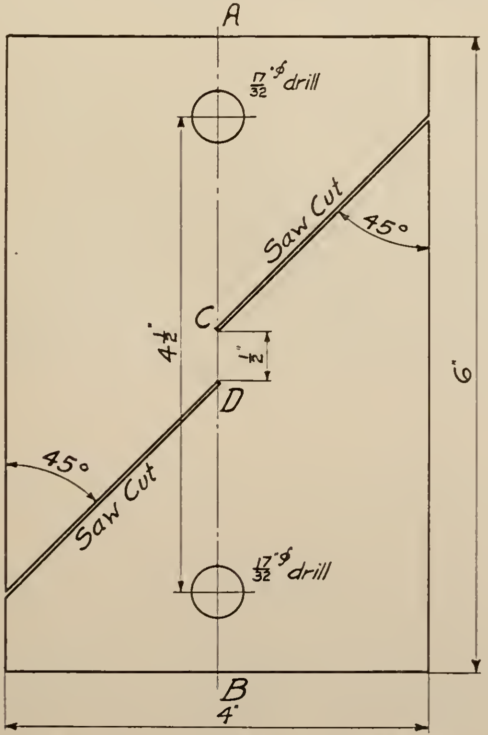

FIG. 7. FORM OF SHEAR SPECIMEN CUT FROM WEB OF GIRDER

The form of the shear specimen was suggested by Mr. Malcolm Westergaard. When axial pull is applied to the shear specimen along the line *AB* (Fig. 7) shear occurs along the line *CD*.

8. *Apparatus.*—Beam 1 was tested in a 200,000-lb. Olsen vertical-screw testing machine. The other tests were made in a 600,000-lb. Riehle vertical-screw testing machine. In all tests, loads were applied at two points equidistant from the center of the span. Fig. 8 shows a specimen in the testing machine.

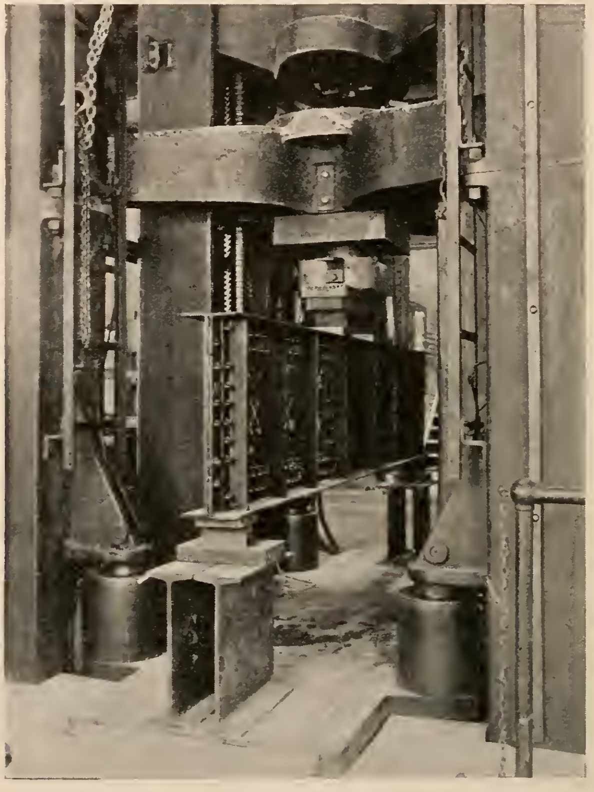

Fig. 8. Girder in Testing Machine in Position for Testing

Strains were measured by means of Berry strain gages. The locations of the gage lines for the different strain measurements are shown in Fig. 2, 3, and 4. Each gage line is denoted by two small circles, one for each end of the line, joined by a straight line. The gage lengths used were two inches and four inches, and, as Fig. 2, 3, and 4, are reproduced to scale, the length of any gage line is apparent. Each gage line was given an identifying number as shown in the figures. For each gage line on one side of the specimen there was a mating line directly opposite on the specimen. The lines for the opposite sides of the specimen were distinguished by the letters *E* or *W*, *N* or *S*, following the line number.

The deflections of beams 3, 4, 5, 6, 7, and 8, were measured at the middle of the span. Several forms of deflectometer were used, the most convenient being the level bar shown in Fig. 9. The point *A* of the instrument is set on the beam over the end bearing, and the point *B*, which is at the end of a micrometer screw *M*, is set at mid-span. With zero load on the beam the micrometer screw is adjusted, raising or lowering the point *B*, until the level bubble *L* is in mid-position. With any load on the beam the same process is repeated, and the difference between this micrometer reading and the reading for zero load gives the deflection at mid-span. The point *A* is placed in a prick-punch hole, and the point *B* in a cold-chisel mark made along the flange of the beam. The sensitiveness of the level bubble was such that twenty seconds change of angle from the horizontal caused the bubble to move one division of its scale.[*]

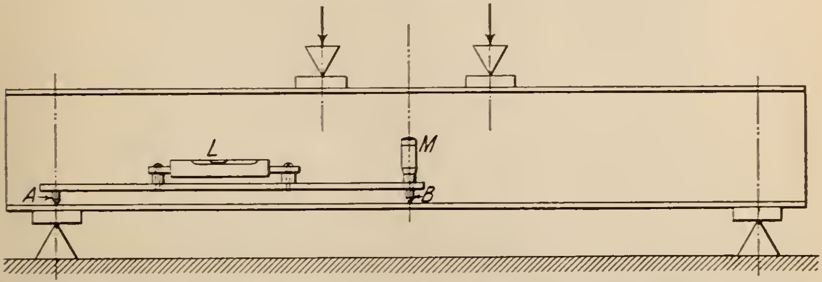

FIG. 9. LEVEL BAR FOR MEASURING DEFLECTION OF GIRDER. (THE POINT *A* CAN BE SCREWED INTO VARIOUS HOLES ALONG THE BAR FOR DIFFERENT LENGTHS OF SPAN)

9. *Data and Results.*—Readings of the strain gages and readings of the deflectometer were taken for various loads up to the ultimate. The strain gage readings were corrected for variation of temperature by means of an unstressed standard bar.[†] The value of $E\epsilon$ along

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[*] Since these tests were made a later form of level bar has been constructed in the shops of the Laboratory of Applied Mechanics. In this new level bar, designed by Mr. H. R. Thomas, the screw micrometer shown in Fig. 9 has been replaced by a leveling screw which actuates the plunger of a direct reading dial gage micrometer. This later form of level bar is much quicker in operation than the earlier form.

[†] For detailed discussion of the use of the strain gage see: Proceedings of the American Society for Testing Materials, 1913, Slater and Moore on "The Use of the Strain Gage in Testing Materials"; also, Bulletin No. 64 of the Engineering Experiment Station of the University of Illinois, "Tests of Reinforced Concrete Buildings under Load," by Talbot and Slater.

each gage line was determined from the strain gage readings assuming a modulus of elasticity of 30,000,000 lb. per sq. in. Curves were plotted for each beam with values of applied loads as ordinates and values of $E\epsilon$ along the gage lines, determined from the strain page readings, as abscissas. These curves are given in Fig. 14-22. The load at failure for each beam is given in Table 2. Fig. 10, 11, and 12 are reproduced from photographs of the beams after failure.

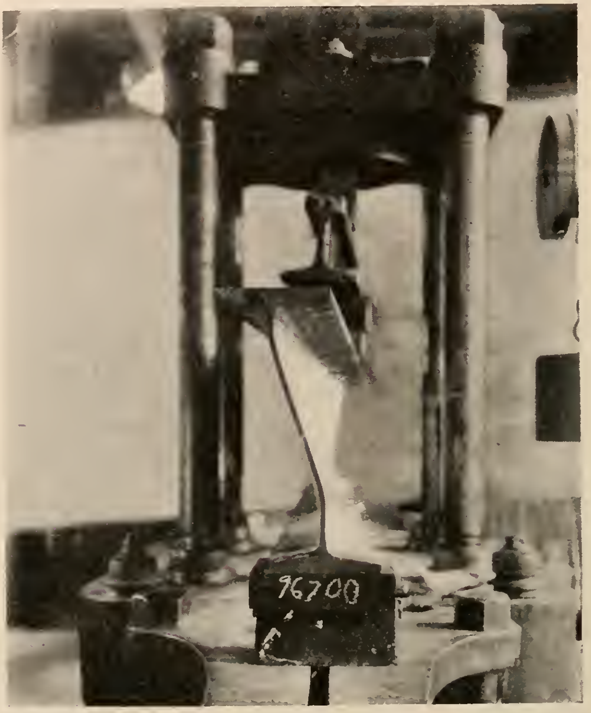

FIG. 10. I-BEAM 1 AFTER FAILURE

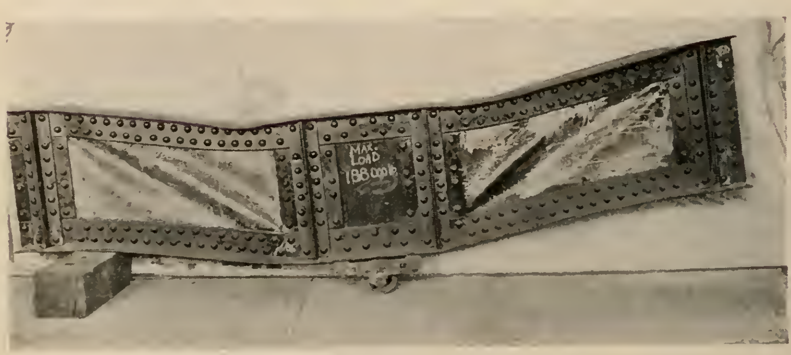

FIG. 11. GIRDER 3 AFTER FAILURE

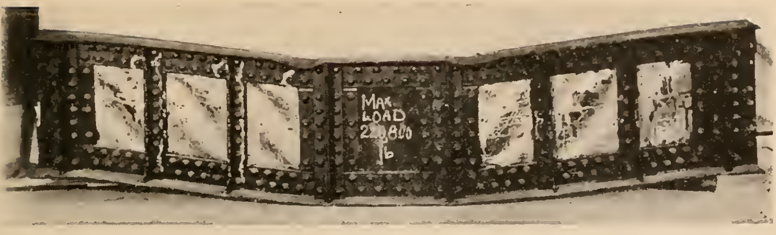

FIG. 12. GIRDER 8 AFTER FAILURE

The local yielding of the built-up girders is shown in Fig. 11 and 12. During the progress of the tests local yielding was made evident by the flaking of the mill scale and the paint on the specimen.

During a test three stages of structural failure were noted: (1) Local overstress, shown by flaking of paint and high local strain gage readings. (2) Signs of general yielding of the girder as a whole. (3) Final collapse under the ultimate load.

10. *Determination of the Yield Point of Girders.*—The writers believe that the second stage of failure, as given in the preceding paragraph, gives the best indication of the limit of load-carrying capacity of girders for static loads. The term "yield point of the girder" is used to designate this stage of general yielding. The general yielding of an I-beam or girder is shown by the "knee" of the load-deflection curve, and an examination of the load-deflection curves shown in Fig. 16-22 shows that the departure of a curve from a straight line is fairly well marked. In determining the yield point of a girder from the load-deflection curve the method of J. B. Johnson was used.* The yield

TABLE 2.

FAILURE OF GIRDERS (1915 TESTS)

Girder Number	Load at Yield Point of Girder, pounds.	Load at Ultimate, pounds.	Manner of Failure.
1	89,500	96,700	Local compression over bearing block followed by twist of web.
2	85,500	138,000	Wrinkling of web due to diagonal compression.
3	101,500	188,000	Wrinkling of web due to diagonal compression.
4	73,500	141,500	Wrinkling of web due to diagonal compression.
5	82,000	104,500	Local compression over bearing block followed by twist of web.
6	86,500	120,500	Wrinkling of web due to diagonal compression.
7	82,000	144,000	Wrinkling of web due to diagonal compression.
8	112,500	220,800	Wrinkling of web due to diagonal compression.

*J. B. Johnson's method consists in finding, by means of drawing a tangent line, the point on any stress-strain curve (or load-deflection curve) at which the strain is increasing 50 per cent more rapidly than its initial rate of increase. See J. B. Johnson, "The Materials of Construction," pp. 18-20.

point obtained from the load-deflection curve does not differ greatly from a yield point obtained from a curve showing the average web-strains at mid-web. The yield point for those girders for which no deflection was measured was determined from the average web-strain (along a 45-degree gage line) at mid-web. The yield point of the girder occurs under loads greater than those causing the first evidence of local structural distress (shown by scaling of paint or excessive strain along individual gage lines) and at loads less than the ultimate. The yield point seems to be a fairly reliable criterion of the beginning of serious structural damage to the girder as a whole. The loads at the yield points, the ultimate loads, and the manner of failure are given in Table 2.

IV. DISCUSSION OF RESULTS.

11. *Relation between the Actual and the Theoretical Strains.*—Equation (7) gives the theoretical value of $E\epsilon$ for any gage line in a girder. In this equation S_1 is the computed unit stress along a gage line, and S_2 the computed unit stress normal to the gage line and in the plane of the web. The values of S_1 and S_2 can be determined from equation (3).

To illustrate the use of these equations consider gage line 19 of girder 3 (see Fig. 4 and 16). A total load of 1,000 lb. on the girder will produce on the transverse section at the middle of gage line 19, a compressive stress S of 70.7 lb. per sq. in. as computed by equation (1), and a shearing stress S_s of 153.4 lb. per sq. in. as computed by equation (2). $\phi = -45$ degrees. Substituting these qualities in equation (3) gives

$$S_1 = \tfrac{1}{2}\ (-70.7)\ [1-\cos(-90^\circ)] + 153.4 \sin\ (-90^\circ) =$$
-188.75 lb. per sq. in.

This is compression along gage line 19. S_2, the stress at right angles to S_1, is also given by equation (3). S and S_s are the same as for S_1, and $\phi = +45^\circ$. Substituting these values in equation (3) gives $S_2 = \tfrac{1}{2}(-70.7)\ (1-\cos 90^\circ) + 153.4 \sin 90^\circ = 118.05$ lb. per sq. in. This is tension normal to gage line 19.

λ (Poisson's ratio) for steel is about $\tfrac{1}{3}$. Substituting values of S_1, S_2, and λ in equation (7) gives

$E\epsilon = -188.75 - \tfrac{1}{3}(118.05) = -228.10$ lb. per sq, in. for 1,000 lb. load on the girder.

This is slightly less than the value given by the average of the strain gage readings on the two sides of the web at gage line 19, as shown in Fig. 16.

A comparison of the strains as computed by the above method with the strains measured by the use of the strain gage may be made by means of the diagrams shown in Fig. 14-22. In these diagrams the full lines represent values of $E\epsilon$ corresponding to the strain as determined from the readings of the strain gage on the two sides of the girder and the dot and dash lines give the values of $E\epsilon$ as calculated by the formulas given in the preceding pages.

Comparison of measured strain with computed strain was made on 163 gage lines for the eight test pieces. Of these 163 gage lines, 40 were near bearing blocks where local compression materially modified the strains. For 107 of the remaining 123 gage lines the measured strain and the computed strain agreed closely, and for most of the other gage lines there were evidences of local bending action which might explain the discrepancy. The good general agreement between measured strain and computed strain furnishes an experimental confirmation of the theory on which the computation is based. This theory is therefore used as the basis of the computation of the maximum strains, and the shearing stresses in the girder webs.

12. *Maximum Shearing Stress in the Web.*—The longitudinal and transverse shearing stresses in the web of a girder are a maximum at the neutral axis at which the tensile and compressive stresses are zero. Equation (6) shows that it may be possible to have a diagonal shearing stress greater than the longitudinal and transverse shearing stress at the neutral axis. The longitudinal and transverse shearing stresses vary inversely as the thickness of the girder; hence they are much smaller in the flange than in the web adjacent to the flange. The tensile and compressive stresses vary directly as the distance from the neutral axis. Therefore, in the case of the girders under consideration, the maximum diagonal shearing stress is either at the neutral axis or at the inner edge of the flange and under a load. The maximum diagonal shearing stress can be determined by the use of equation (6). The shearing stresses in girders 1 to 8, calculated by the above methods, are given in Table 3.

In all I-beams, except No. 1 and No. 5, there are stiffeners under the loads. The stresses given in Table 3 are at the outer edges of these stiffeners. The moment is slightly greater immediately under the load, but the stiffener helps the web to resist the shear at this point.

The quantities used in the calculation of the maximum shearing stresses are also given in Table 3; the longitudinal tensile or compressive stress in the web at the inner edge of the flange is given in Column 2; the longitudinal and transverse shearing stresses at the same

TABLE 3.

SHEARING STRESSES IN WEBS OF GIRDERS

All stresses are given in pounds per square inch

No. of Test (1)	Stress for 1000 Pounds Load on the Girder				Maximum Shearing Stress in Web at Yield Point of Girder. (6)	Maximum Shearing Stress in Web at Ultimate of Girder. (7)	Yield Point in Shear of Material in Web (from Tests of Specimens). (8)
	Longitudinal Stress at Inner Edge of Flange (Tension or Compression) (2)	Longitudinal and Transverse Shearing Stress at Edge of Flange. (3)	Maximum Diagonal Shearing Stress in Web. (4)	Longitudinal and Transverse Shearing Stress at Neutral Axis‡ (5)			
1	353	172	246*	206	22,000	23,800	24,400†
2	219	203	230	236*	20,200	32,600	26,500†
3	108¶	151	161*	158	16,300	30,300	25,700
4	220	206	234	240*	17,600	34,000	25,400†
5	236	190	224*	224*	18,300	23,400	25,100†
6	217	190	219	224*	19,400	27,000	26,100†
7	219	198	226	232*	19,000	33,400	23,700†
8	108¶	151	161*	158	18,100	35,500	26,100

*Maximum shearing stress in the web of the girder.
†Taken as 0.60 of the yield point in tension. The yield point in shear was not well defined for these particular specimens.
‡Computed by Equation (2).
¶Stress on center line of inner row of rivets.

point are given in Column 3; the maximum shearing stresses in the web at the inner edge of the flange in Column 4; and the shearing stresses at the neutral axis are given in Column 5. The maximum shearing stress in the web, the greater of the quantities in Columns 4 and 5, is indicated by an asterisk. All of the above stresses are given in lb. per sq. in. per 1000-lb. load on the girders. The maximum shearing stresses in the web when the beam is loaded to the yield point are given in Column 6.

A comparison of Columns 3 and 4 of Table 3 shows that, except for I-beam 1, the diagonal shearing stress in the web at its junction with the flange is not materially greater than the longitudinal and transverse shearing stress at the neutral axis. In the case of I-beam 1 the high diagonal shearing stress is due to the fact that the longitudinal tensile (or compressive) stress at the point at which the flange joins the web is much greater than the transverse and longitudinal shearing stress at the same point.

To illustrate the possible importance of diagonal shearing stresses consider the following numerical example. A girder is supported upon end supports 70 ft. apart. It carries two concentrated loads of 450,000 lb. each, one located 9 ft. 8 in. from each end support. The girder is made up of one web plate 90 in. x ½ in., and (for each flange) two 6 in. x 6 in. x ¾ in. angles, and two cover plates 14 in. x ¾ in.

Neglecting the weight of the girder, the shear is constant between a load and the adjacent support, and is equal to 450,000 lb. The maximum bending moment occurs under a load and is equal to 450,000 x 116=52,200,000 pound-inches.

The web area equals 90×0.5 or 45 sq. in. and the average shearing stress as usually figured is 10,000 lb. per sq. in. The moment of inertia of the net section is 153,100 $(\text{in.})^4$ and the distance from the neutral axis to the outer edge of the flange is 46.75 in. The longitudinal stress in the outer fiber, as given by equation (1), is, therefore, 15,930 lb. sq. in.

The maximum diagonal stress occurs at the point at which the flange is connected to the web. For shear consider this point to be on the center line of the inner row of rivets, a distance of $45.25 - 4.75 = 40.50$ in. from the neutral axis. The longitudinal stress varies directly as the distance from the neutral axis and at the point in question is equal to 13,800 lb. per sq. in. Consider the shear at the neutral axis on a transverse section under a load. The unit shearing stress at any point is given by equation (2). For the girder consider, $V = 450{,}000$ lb., $I = 183{,}200$ $(\text{in.})^4$, $t = \frac{1}{2}$ in., and $a_1c_1 = 2{,}204$ $(\text{in.})^3$ for the gross area. Then from equation (2) S_s equals 10,800 lb. per sq. in. Consider next the shear on the same transverse section on the gage line of the angles, a distance of 40.50 in. from the neutral axis. For this point the value of a_1c_1 is 1,794 (in.)^3, and V, I, and t have the same values as in the previous case. S_s equals 8,800 lb. per sq. in.

The maximum diagonal shearing stress at the edge of the angles, S_s^1, given by equation (6), is 11,200 lb. per sq. in. This value is 400 lb. per sq. in. or 3.7 per cent greater than the value of the shearing stress at the neutral axis (10,800 lb. per sq. in.).

In this girder the longitudinal tensile and compressive stresses are as high at a section at which maximum shear occurs as good practice will permit, and therefore the excess of the diagonal shearing stress at the inner edge of the flange over the transverse and longitudinal shearing stress at the neutral axis should, if ever, be of importance. Since this difference is only 3.7 per cent it would seem that the excess of diagonal shearing stress over shearing stress at the neutral axis is not, in general, of much importance. It is true that if the shearing stresses had been low at the inner edge of the flange while the longitudinal tensile and compressive stresses had been high, the diagonal shearing stress might have been materially greater than the shearing stress at the neutral axis. But the very supposition on which this condition is based; namely, that the shearing stresses should be low, makes it certain that they will not be the criterion of strength of the girder, and that the difference between the two low shearing stresses will be of no importance.

The shearing stress at the neutral axis, as computed by equation (2) is 10,800 lb. per sq. in.; whereas the value obtained by dividing the total shear by the gross area of the web (the method usually used in girder design) is 10,000 lb. per sq. in. The excess of the former value over the latter is about 8 per cent. For the I-beams and built-up girders tested the difference between the values obtained by the two methods of calculation is also about 8 per cent. The relation between the transverse and longitudinal shearing stress at the neutral axis, as obtained by equation (2), and the approximate value obtained by dividing the total shear upon a section by the area of the cross-section of the web depends upon the relation between the area of the cross-section of the web and the area of the cross-section of the flange. In the example given above, in which the area of cross-section of the web is not large compared with the area of cross-section of the flange, equation (2) gives values for the shearing stress about 8 per cent higher than those obtained by approximate method. Calculations for girders, in which the area of the web is large compared with the flange area, show that the difference between the sharing stresses as computed by the two methods may be as great as 20 per cent. Therefore, in the design of a girder, if the value obtained by dividing the total shear by the area of cross-section of web is more than 80 per cent of the allowable stress for the material in shear, a check computation of the shearing stress should be made, using the more exact formula, equation (2). The computing of the diagonal shearing stresses at the inner edge of the flange seems unnecessary.

13. *Maximum Tensile and Compressive Strains in the Web.*—The longitudinal tensile and compressive stresses increase with the longitudinal distance of a point from the support and vary as the transverse distance from the neutral axis of the girder. The longitudinal and transverse shearing stresses decrease as the distance from the neutral axis increases, but they are nearly as great at the point at which the flange joins the web as at the neutral axis. However, after this point is passed the decrease is very rapid. The maximum diagonal tensile and compressive stresses depend upon the shearing stress and upon the longitudinal tensile (or compressive) stress. In the case of the beams under consideration the maximum tensile and compressive diagonal stresses occur at the inner edges of the flanges, under the load point for beams which have no stiffeners, and at the outer edge of the middle stiffeners for beams which have stiffeners.

The maximum diagonal tensile or compressive stress is given by equation (5). The values of S and S_s to be used in this equation for a load of 1,000 lb. on the girder are given in Columns 2 and 3 of Table 4. A diagonal tensile or compressive stress at a given point in the web in one direction is accompanied by a second diagonal stress at the same point normal to the first. If the first is the maximum diagonal stress, the second is the minimum. If the longitudinal stress is tension, the maximum diagonal stress is given by equation (5) if the upper or plus sign is used, and the minimum diagonal stress is given when the lower or minus sign is used. In considering the strength of the web it is necessary to get the value of $E\epsilon$ corresponding to the maximum strain. The value of $E\epsilon$ is given in equation (7). Substituting in equation (7) the values of the maximum and the minimum diagonal stress for S_1 and S_2, as given by equation (5) there is obtained

$$E\epsilon=\frac{S}{2}(1-\lambda)+(1+\lambda)\sqrt{S_s^2+\left(\frac{S}{2}\right)^2}\ \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

TABLE 4.

DIAGONAL TENSILE AND COMPRESSIVE STRESSES IN WEBS OF GIRDERS

All stresses are given in pounds per square inch

No. of Test (1)	Longitudinal Stress in Web Adjacent to Flange for 1000 lb. Load on Girder. (2)	Shearing Stress In Web Adjacent to Flange for 1000 lb. Load on Girder. (3)	Maximum Value of $E\epsilon$ along a Diagonal for 1000 lb. Load on Girder. (4)	Maximum Value of $E\epsilon$ along a Diagonal at Yield Point of Girder. (5)	Compressive Stress along 45-degree Line at Neutral Axis at Yield Point of Girder. (S_y) (6)	Ratio of Clear Vertical Depth to Thickness of Web. (7)	Ultimate Buckling Strength of 45-Degree Strip by Euler's Formula for Fixed-Ended Columns (S_b) (8)	$S_y:S_b$ (9)
1	353	172	447	40,000	18,700	48	21,500	0.87
2	219	203	381	32,500	19,500	55	16,200	1.20
3	104*	151	250	25,400	20,000	101	4,800	4.16
4	220	206	385	28,300	17,300	56	15,800	1.09
5	236	190	377	30,800	17,100	53	17,400	0.98
6	217	190	364	31,400	18,300	52	18,200	1.00
7	219	198	375	30,700	18,800	54	16,900	1.11
8	104*	151	250	27,900	22,200	101	4,800	4.63

*Stress on center line of inner row of rivets.

The values of $E\epsilon$ corresponding to a load of 1,000 lb. on the girder are given in Column 4 of Table 4. The values of $E\epsilon$ at the yield point of the girders are given in Column 5 of Table 4, and are repeated in Column 3 of Table 5. The longitudinal stresses at the outer edges of the flanges are given in Column 2 of Table 5. Since the transverse stress at the outer edge of the flange is zero, the values in Column 2 of Table 5 represent also values of $E\epsilon$ for the outer fibers of the flanges, and since the values in Column 2 are for the same transverse section of the girder as those in Column 3, a direct comparison of the values of the two columns can be made. The ratios of the values in Column 3 to those in Column 2 are given in Column 4 of Table 5. An examination of these ratios shows that the maximum value of $E\epsilon$ along a diagonal in girder 1 is only slightly greater than the maximum longitudinal stress at the outer edge of the flange, but that for the other girders the values of $E\epsilon$ are much greater than the longitudinal stress at the outer edge of the flange. An examination of equation (5) shows that the relation between the two quantities depends on the relation between the transverse shearing stress and the longitudinal stress. This fact is apparent from Table 5 also, which shows that the values of $E\epsilon$ along a diagonal are not much greater numerically than the longitudinal stresses for cases in which the shearing stress is low compared to the longitudinal stress, but that for cases in which the shearing stress is high compared with the longitudinal stress the values of $E\epsilon$ along a diagonal are decidedly higher than the longitudinal stress at the outer edge of the flange. However, for the girders tested the ratio of the shearing stress to the longitudinal stress in the outer fibers is higher than ordinarily occurs in practice, and this fact tends to exaggerate the excess of the values of $E\epsilon$ over those of the longitudinal stress.

TABLE 5.

COMPARISON OF LONGITUDINAL STRESSES IN EXTREME FIBER WITH MAXIMUM VALUES OF $E\epsilon$ FOR GIRDERS

No. of Test	Longitudinal Stress in outer Fiber of Girder at Yield Point of Girder* lb. per sq. in.	Maximum Value of $E\epsilon$ along a Diagonal at Yield Point of Girder lb. per sq. in.	Ratio of $E\epsilon$ along a Diagonal to Longitudinal Stress
1	36,100	40,000	1.11
2	21,400	32,500	1.52
3	15,800	24,600	1.56
4	18,400	28,300	1.53
5	22,200	30,800	1.39
6	21,400	31,400	1.46
7	20,600	30,700	1.49
8	17,500	27,000	1.54

*Computed by the commonly used flexure formula, Equation (1).

A consideration of the girder described in section 12 will illustrate the possible importance of the diagonal strain in a girder designed in accordance with current practice. The maximum diagonal strain occurs at a point at which the flange is connected to the web. For tensile and compressive strains this point is taken on the center line of the inner row of rivets, a distance of 4.75 in. from the backs of the angles, and 45.25–4.75=40.50 in. from the neutral axis. The longitudinal stress varies directly as the distance from the neutral axis, and at the point in question is equal to $15{,}930 \times \frac{40.50}{46.75} = 13{,}800$ lb. per sq. in. In computing the shearing stress at this point the numerical values to be used in equation (2) are the same as those used on p. 26, except the value of a_1c_1 which in this case is 1,794 $(\text{in.})^3$. Using equation (2), $S_s = 8{,}800$ lb. per sq. in.

The maximum value of $E\epsilon$ obtained from equation (10), is 19,530 lb. per sq. in. This is 3,600 lb. per sq. in. or 22.6 per cent greater than 15,930 lb. per sq. in., the longitudinal stress at the outer edge of the flange.

In this example the excess of the value of $E\epsilon$ along a diagonal over the longitudinal stress at the outer edge of the flange is so great (22.6 per cent) that it should have been considered in the design of the girder. This indicates that in the design of structural steel girders in which maximum shear and maximum moment occur *at the same transverse section of the web,* it is not safe to allow an average transverse shearing stress of 10,000 lb. per sq. in. and at the same time to allow a longitudinal stress of 16,000 lb. per sq. in. in the outer fibers of the flanges. It is evident that the diagonal strains in such girders should be given special attention.

14. *Buckling of Web.*—The tendency of webs of girders to fail by buckling is well illustrated by the action of the webs of girders 3 and 8 as shown in Fig. 11 and 12. The exact analysis of buckling action in the web of a girder would be extremely complicated, and the following approximate analysis is in common use: A narrow strip of web making an angle of 45 degrees with the longitudinal axis of the girder is regarded as a column carrying an average stress over its cross-section equal to the shearing stress at the neutral axis (which is equal to the compressive stress on a 45 degree line at the neutral axis—the maximum compressive stress at that point). The length of this column is taken as $h\sqrt{2}$, in which h is the clear depth of web. The column is regarded as fixed-ended. Since the web is thin

the slenderness ratio l/r is large for the strip, and Euler's column formula, which gives good results for very slender columns, may be used. Applying Euler's formula for fixed-ended columns, the computed web stress at failure by buckling S_c, becomes

$$S_c=\frac{1.64\ E}{\left(\frac{h}{t}\right)^2} \qquad (11)$$

The compressive stress at the neutral axis at the yield point of the girders tested is given in Column 6 of Table 4; the ultimate strength of a 45-degree strip as obtained by Euler's formula in Column 8; and the relation between these two quantities is given in Column 9.

It will be seen for all girders except 3 and 8 that the load at the yield point of the beam corresponds quite closely to the load for the failure of the web-column as given by Euler's formula. In the case of girder 3 the stiffeners at the ends apparently helped stiffen the whole web, though the unsupported length of web was much greater than is allowable under standard specifications. In the case of girder 8 the web was supported against buckling by stiffeners at intermediate points. It would seem that the webs of girders are capable of developing a shearing stress at the neutral axis equal to the ultimate stress on a 45-degree strip considered as a column, figured by Euler's formula for columns with fixed ends, unless the stress given by that formula is higher than the yield-point strength of the material in shear. An examination of Tables 1 and 3 shows that before the *ultimate* of any test girder had been reached the yield-point strength in shear of the web material was developed (unless failure was due to local compression of the web over bearing blocks), but that the *yield point* of the girder was reached before the web material was stressed to its yield point in shear. It should be kept in mind that the yield point of a girder is the practical limit of its load-carrying capacity.

15. *Local Web Compression Adjacent to Bearing Blocks.*—The exact determination of the stresses in the web of an I-beam or girder adjacent to a bearing block would involve a consideration of the combined effect of the shearing stresses and the longitudinal stresses in the web adjacent to a bearing block. It would also involve a knowledge of the law of variation of the transverse stress from the flange towards the neutral axis and of the distribution of the pressure along

the bearing block. Since exact knowledge of these conditions is lacking, it is thought that a simple, approximate treatment of the local stress adjacent to bearing blocks will serve the structural engineer as well as a more elaborate analysis, especially since the simple analysis gives results in fair agreement with tests.

The determination of the local compressive stress in the web of an I-beam or girder, adjacent to a bearing block is discussed in Bulletin No. 68 of the Engineering Experiment Station of the University of Illinois. The following is quoted from that bulletin, with a few changes in notation:

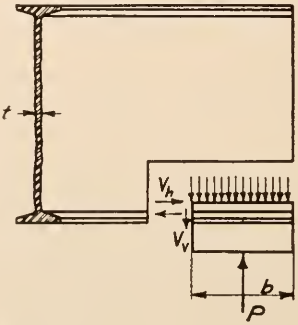

FIG. 13. DIAGRAM OF COMPRESSIVE STRESS IN WEB OF GIRDER OVER A BEARING BLOCK

"There may be an excessive compressive stress near the junction of web and flange and adjacent to a concentrated load or reaction. What has been referred to previously as failure by twisting of ends of I-beams is in most cases primarily caused by excessive local compression at the root of the flange.

"An approximate method of computing the compressive stress at the root of the flange adjacent to a concentrated load or an end reaction, has been given by C. W. Hudson as follows:* Imagine a small piece cut from the flange and web of an I-beam immediately over a bearing block (Fig. 13), and imagine this piece to be held in equilibrium by the elastic forces which act on it while it is in its place in the beam. The forces are (1) the pressure of the reaction at the bearing block P; (2) the compression in the web which equals $S_w t b$, when S_w=the average intensity of compressive stress, t= the thickness of web, and b=the length of bearing block; (3) a horizontal shearing force V_h; (4) a vertical shearing force V_v and (5) horizontal tensile or compressive stresses. Very little of the total shear would be balanced by the small internal shearing stress in the flange of an I-beam, and if the section considered be taken at the root of the flange, an equation may be written without serious error as follows:

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*Engineering News, Dec. 9, 1909.

$$V_v = V_h = 0$$

Then, the compressive stress on the web is balanced by the reaction on the bearing block. The compressive stress may be regarded as uniformly distributed, and an equation may be written as follows:

$$S_w = \frac{P}{bt} \quad \text{.......................................(12).}$$

"In the above discussion the case considered is for the compressive stress adjacent to an end reaction. The reasoning for the compressive stress in the web adjacent to a concentrated load would be similar.

"It is unwise to regard the ultimate compressive fiber stress in the web adjacent to a bearing block as higher than the yield-point strength of the material at the root of the flange. Moreover, the fact should be borne in mind that the material at the root of the flange of an I-beam usually has a yield-point strength somewhat lower than that of the material in the flange or in the web. In the absence of special tests the yield-point strength of the structural steel at the root of the flange of an I-beam may be taken as about 30,000 lb. sq. in."

In comparing Hudson's analysis with the results of tests either of two methods of procedure may be used: (1) The fiber stress at failure, computed by Hudson's formula for beams which failed by local web compression, may be compared with the yield-point strength of the material at the root of the flange. (2) Strain gage measurements directly over bearing blocks may be compared with values given by Hudson's formula. Table 6 gives a statement of the results following the first method of procedure. From this table it is evident that for the beam tested by Marburg and for the 1913 series of beams tested at the University of Illinois, the fiber stress, computed by Hudson's formula, was slightly greater than the yield-point strength of the material at the root of the flange of the beams; and for the 1914 tests at the University of Illinois the stress, computed by Hudson's formula was slightly less than the yield-point strength of the material at the root of the flange.

Tests 1 and 5 of the 1914 series furnish the available data for a comparison of the results of strain gage measurements with the fiber stress as computed by Hudson's formula. It is necessary to make al-

TABLE 6.

FAILURE OF I-BEAMS BY LOCAL COMPRESSION IN THE WEB ADJACENT TO A BEARING BLOCK

I-beam	Tested by	Thickness of Web in.	Length of Bearing Block in.	Load at Failure lb.	Local Compressive Stress Adjacent to Bearing Block by Hudson's Formula lb. per sq. in.	Yield Point of Material at Root of Flange lb. per sq. in.	Remarks
30 in. 175 lb. Beth. Girder	Marburg	0.69	12	538,400	32,200	28,200	
12 in. 31.5 lb. with planed web	U. of Ill.	0.35	6	190,100	45,300	31,700	1913 series
12 in. 31.5 lb. with planed web	U. of Ill.	0.28	6	160,500	47,800	33,100	1913 series
12 in. 31.5 lb. with planed web	U. of Ill.	0.19	6	109,600	48,200	34,000	1913 series
12 in. 31.5 lb. with planed web	U. of Ill.	0.16	6	72,100	37,600	32,300	1913 series
12 in. 31.5 lb. with planed web	U. of Ill.	0.22	6	96,700	36,500	38,000	Test 1 1914 series
12 in. 31.5 lb. with planed web	U. of Ill.	0.20	6	104,500	41,600	44,000	Test 5 1914 series

lowance for the fact that the strain gage readings are taken over a gage length of which the center is necessarily some distance from the root of the flange. The approximate relation between the strain and the distance from the root of the flange can be obtained by comparing the strains indicated by the readings on gage lines 34, 35, 36, 37, and 23, 26, 27, and 28 of Test 1 shown in Fig. 14.

Assuming that the strain varied directly as the distance from the root of the flange, the stress at the root of the flange would be about 1.25 times as great as the stress indicated by the strain gage readings nearest the flange. Table 7 gives the compressive stress in the web directly over the center of a bearing block as determined from the strain gage readings (corrected for distance from root of flange), and as computed by Hudson's formula. The results show an actual fiber stress over the center of the bearing blocks slightly greater than that given by Hudson's formula.

TABLE 7.

COMPRESSIVE STRESS IN WEBS OF I-BEAMS ADJACENT TO A BEARING BLOCK

I-beam	Thickness of Web in.	Length of Bearing Block in.	Gage Line (See Figs. 2 and 3)	Stress for a Load of 1000 lb. lb. per sq. in.	
				From Strain Gage Measurements	Computed by Hudson's Formula
1– 1914 series	0.221	6	16 at end	426	376
			35 at end	400	376
5–1914 series	0.202	6	2 at end	500	412
			27 at end	472	412

Considering both the results of tests to failure and results of strain gage tests, Hudson's formula seems fairly reliable.

The writers wish to call especial attention to the importance of considering the local compression over a bearing block for an I-beam or a girder. If there is not sufficient bearing area in the web, stiffeners must be provided to prevent the flange from folding over towards the web (Fig. 10). Tendencies toward such folding action were observed in several tests, and the inequality of the strains on opposite sides of the web (test 1, gage lines 18, 19, 20, 21, 22, 23, 24, test 5, gage lines 12, 13, 14, 19, and 20) indicates bending action as well as compression. Angle stiffeners placed over the supports as shown in Fig. 8 check this tendency to bend. It is of the highest importance that such stiffeners fit closely against the flange. An excellent illustration of this point is furnished by the behavior of the built-up girders during the tests. The stiffeners at points at which concentrated forces are applied, for girders 3 and 8 are angles having their outstanding legs ground to fit the outstanding legs of the flange angles, whereas the corresponding stiffeners for the I-beams are flats. Girders 3 and 8 maintained a vertical position much better than did the I-beams, indicating that it is an essential feature at points at which concentrated forces are applied, to have the stiffeners support the outstanding legs of the flange as much as possible.

16. *Functions of Stiffeners.*—Stiffeners, as commonly used on plate girders, perform two functions. Those placed at frequent intervals along the girders prevent the web from buckling as a column because of diagonal compression. Stiffeners placed under concentrated loads or over supports distribute the concentrated force, which would otherwise be delivered directly to the flange, over a considerable portion of the depth of the web. These latter stiffeners assist also in preventing the buckling of the web.

The spacing of stiffeners is usually determined more or less arbitrarily. The General Specifications of the American Railway Engineering Association for Steel Railway Bridges, 1910 edition, state, "There shall be web stiffeners generally in pairs, over bearings, at points of concentrated loading, and at other points where the thickness of the web is less than 1/60 of the unsupported distance between flange angles. The distance between stiffeners shall not exceed that given by the following formula, with a limit of six feet (and not greater than the clear depth of the web):

$$d = \frac{t}{40}(12{,}000 - s).$$

in which d=clear distance between stiffeners of flange angles, t=thickness of web, s=shear per sq. in."

According to these specifications the spacing of the intermediate stiffeners is in a general way a function of the shearing stress; but stiffeners are required, no matter how low the stress may be, if the ratio of the unsupported width to the thickness of the web exceeds 60.

The ratio of the unsupported width to the thickness of the web for the I-beams tested was less than 60 in all cases. For girders 3 and 8 this ratio was 100, and therefore, according to the specifications quoted, intermediate stiffeners were required. Girder 8 was equipped with intermediate stiffeners in accordance with the specifications; whereas girder 3 was provided with stiffeners only at points at which concentrated forces were applied. The load at the yield point for girder 3

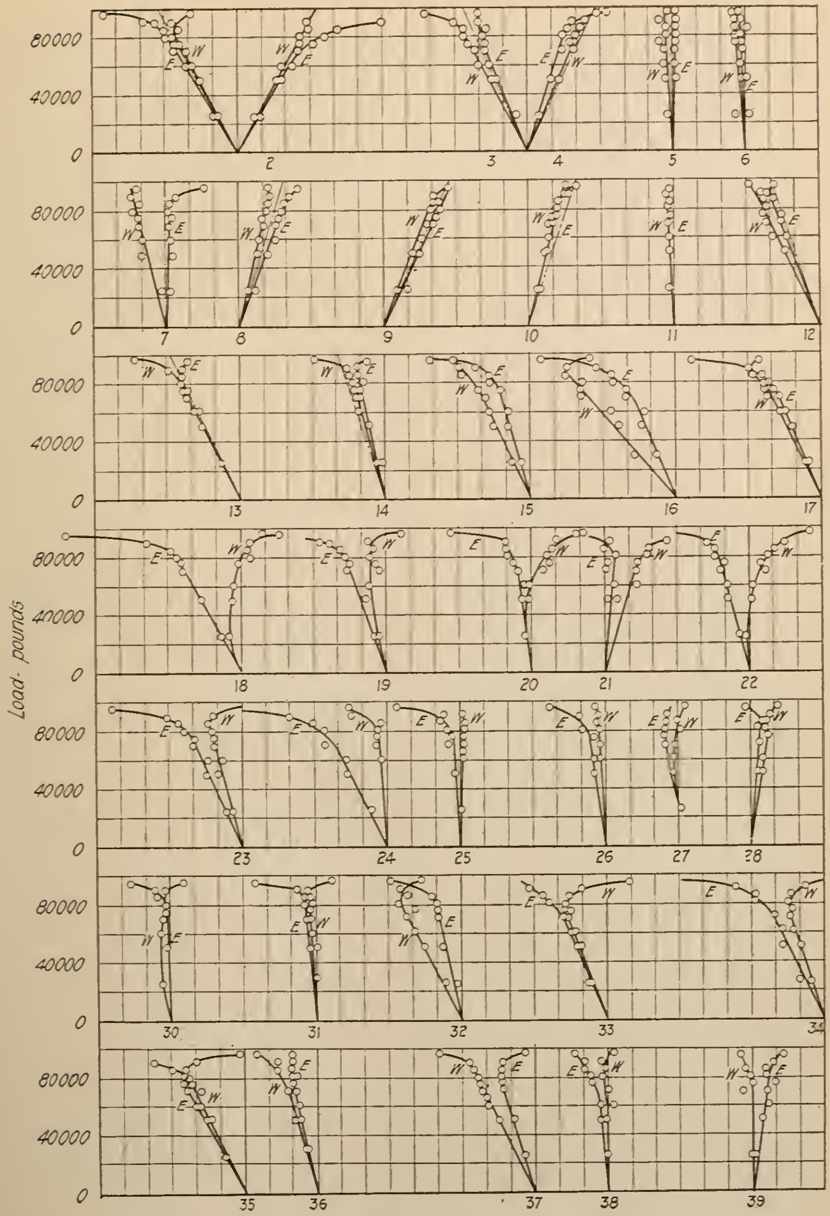

FIG. 14. STRAIN DIAGRAMS FOR I-BEAM 1

Note: 1 horizontal division=10,000 lb. per sq. in. as indicated by the strain gage. Numbers refer to gage lines as given in Fig. 2. Tension plotted to right; compression to left. Measured quantities ————; computed quantities —— — —— — ——.

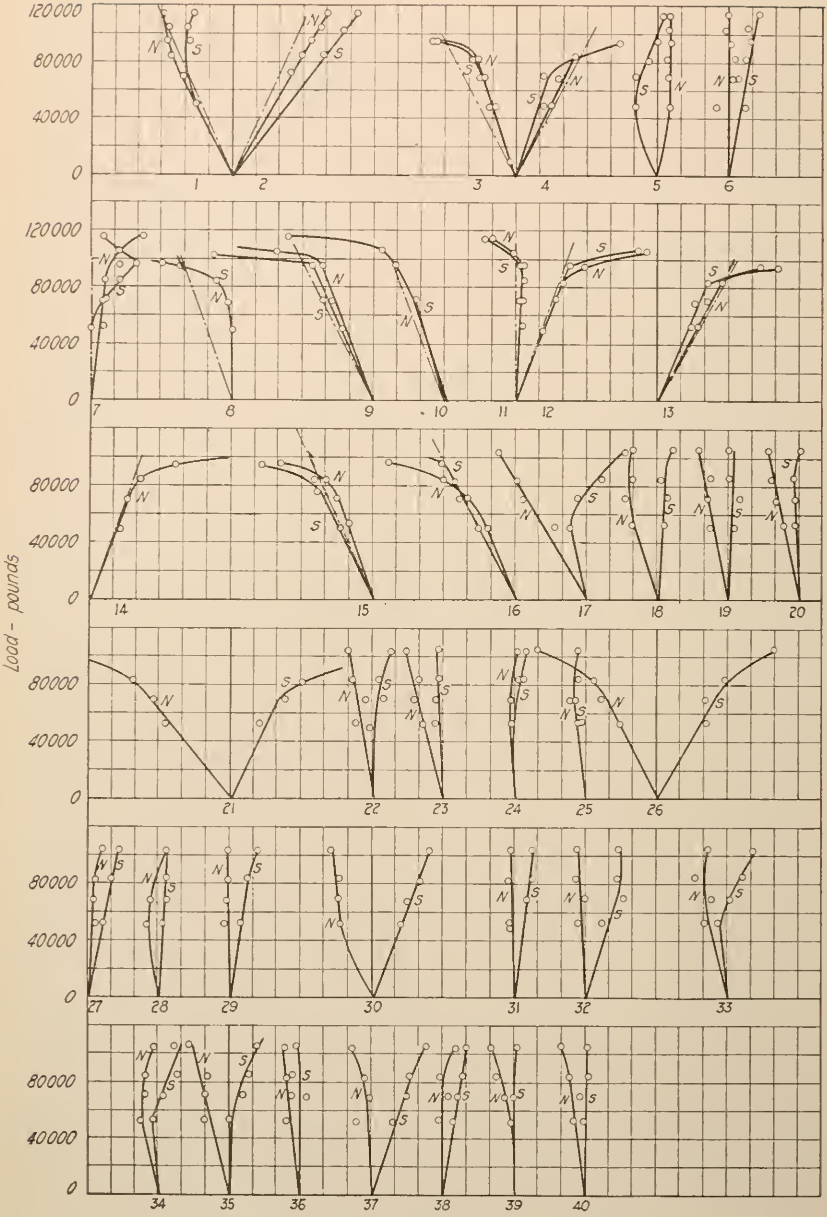

FIG. 15. STRAIN DIAGRAMS FOR I-BEAM 2

Note: See Fig. 14.

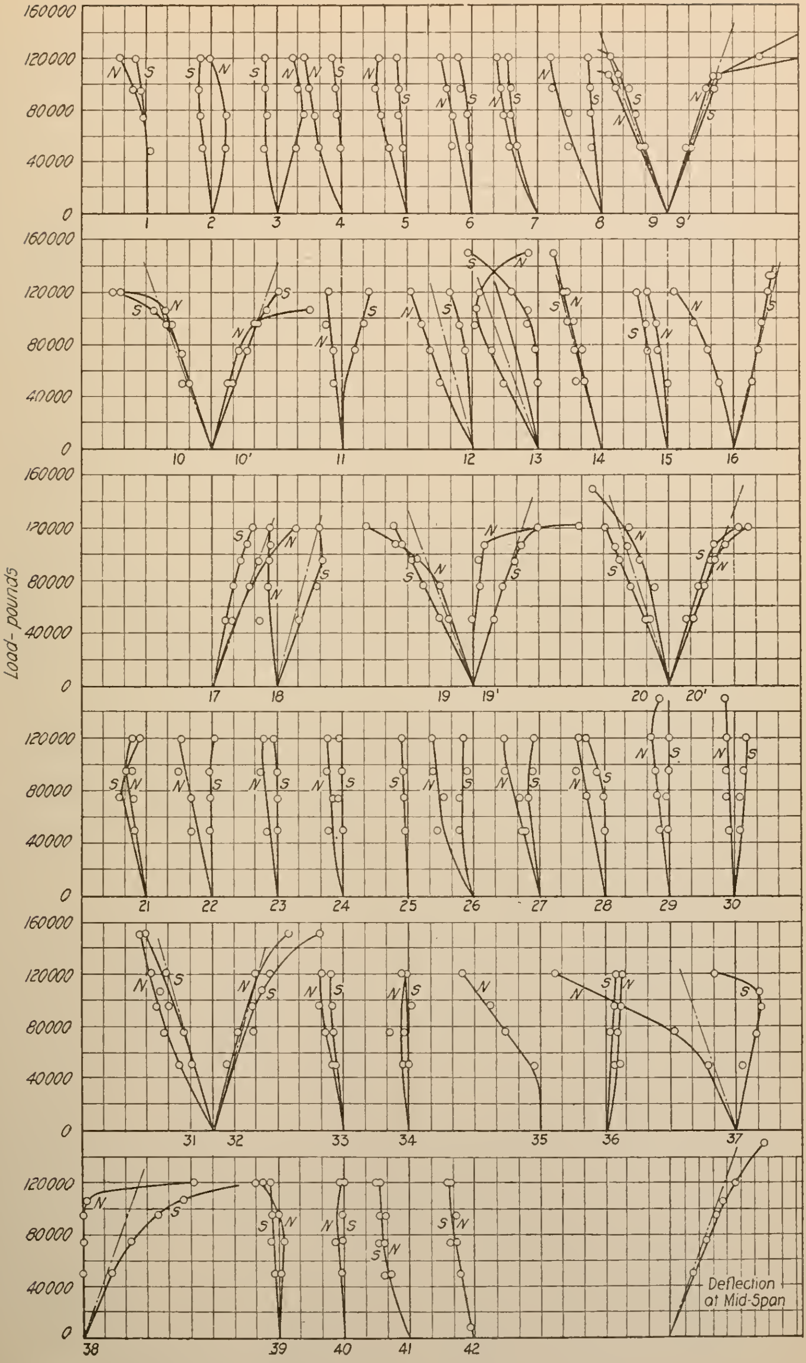

FIG. 16. STRAIN DIAGRAMS FOR BUILT-UP GIRDER 3

Note: See Fig. 22.

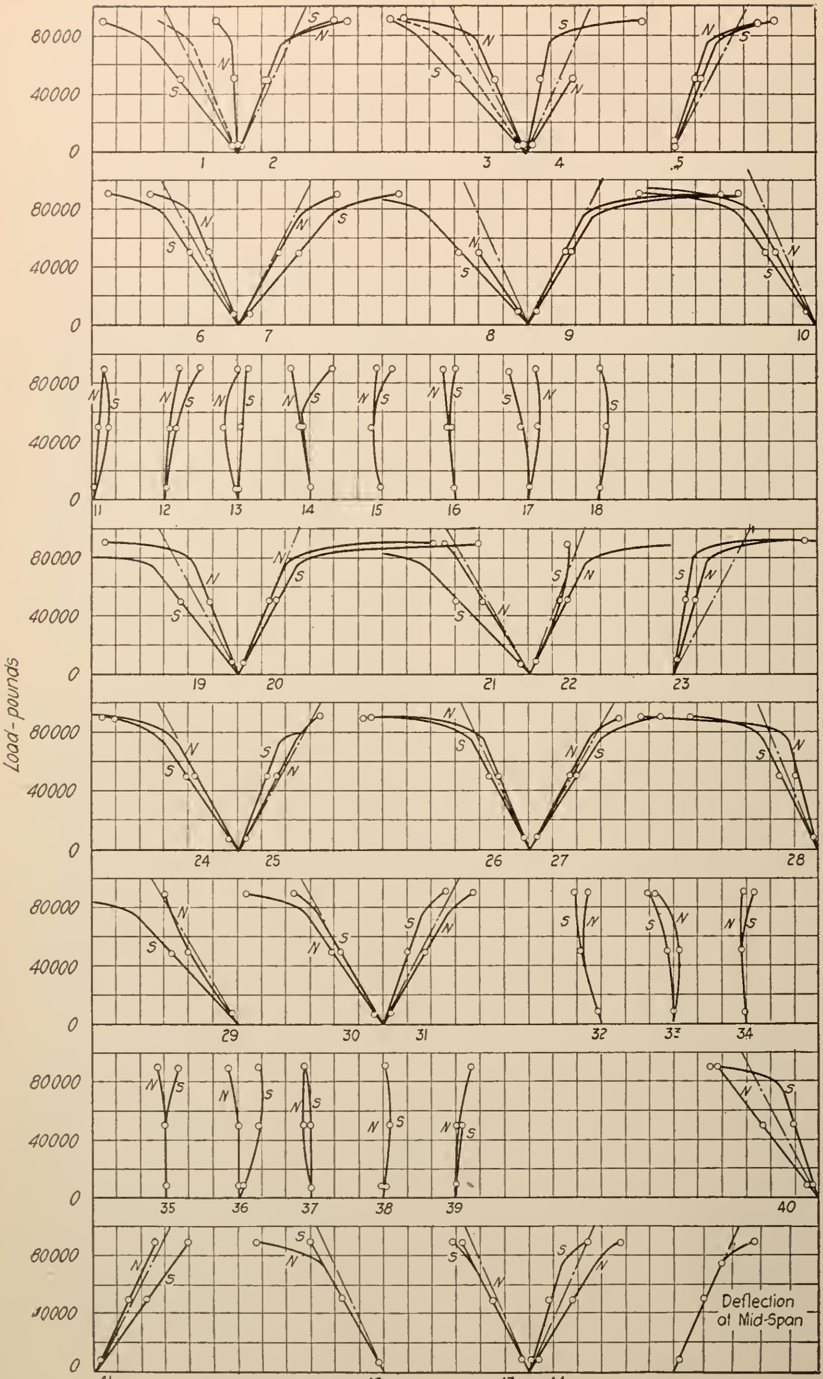

FIG. 17. STRAIN DIAGRAMS FOR I-BEAM 4

Note: See Fig. 14.

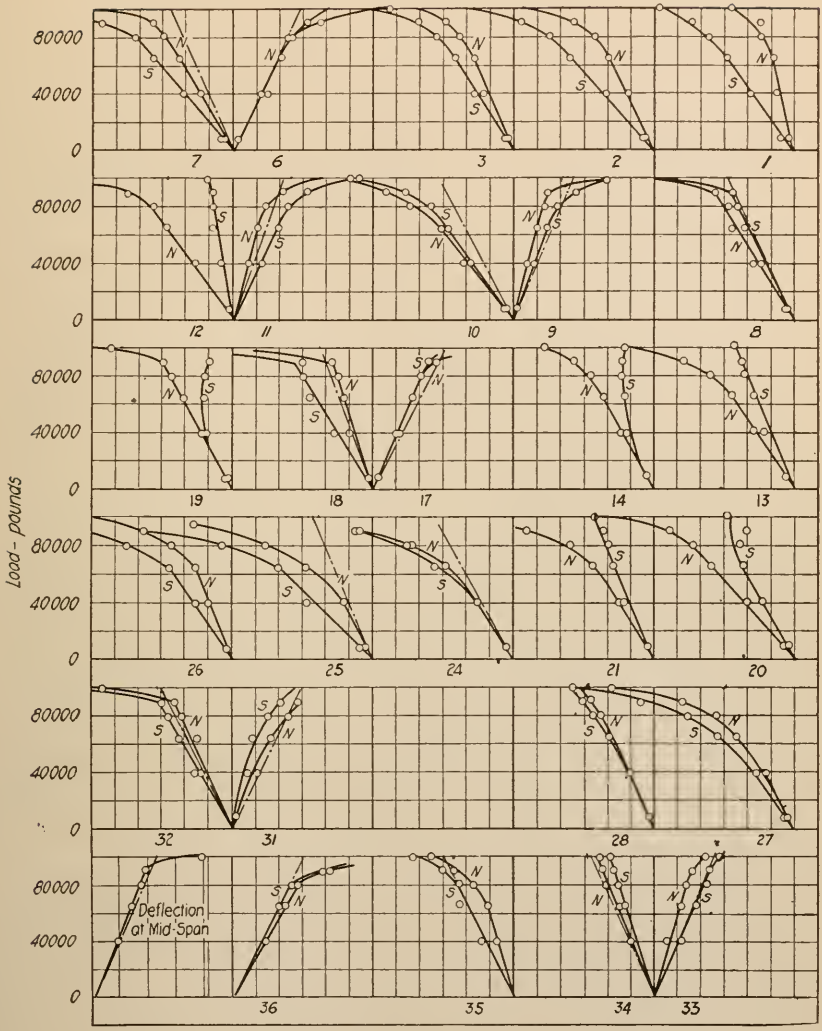

FIG. 18. STRAIN DIAGRAMS FOR I-BEAM 5

Note: 1 horizontal division= 10,000 lb. per sq. in, as indicated by the strain gage.
1 horizontal division=0.02 inch (for deflection curve only)
Numbers refer to gage lines as given in Fig. 3.
Tension plotted to right; compression to left.
Measured quantities ———; computed quantities —— — —— — ——.

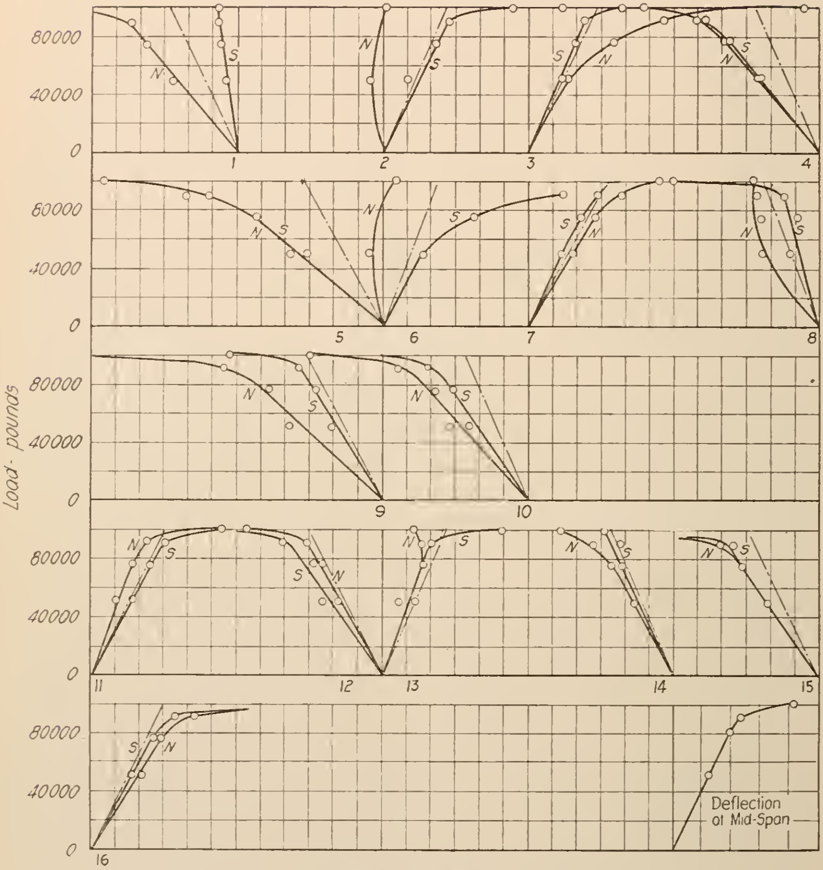

Fig. 19. Strain Diagrams for I-Beam 6

Note: See Fig. 18.

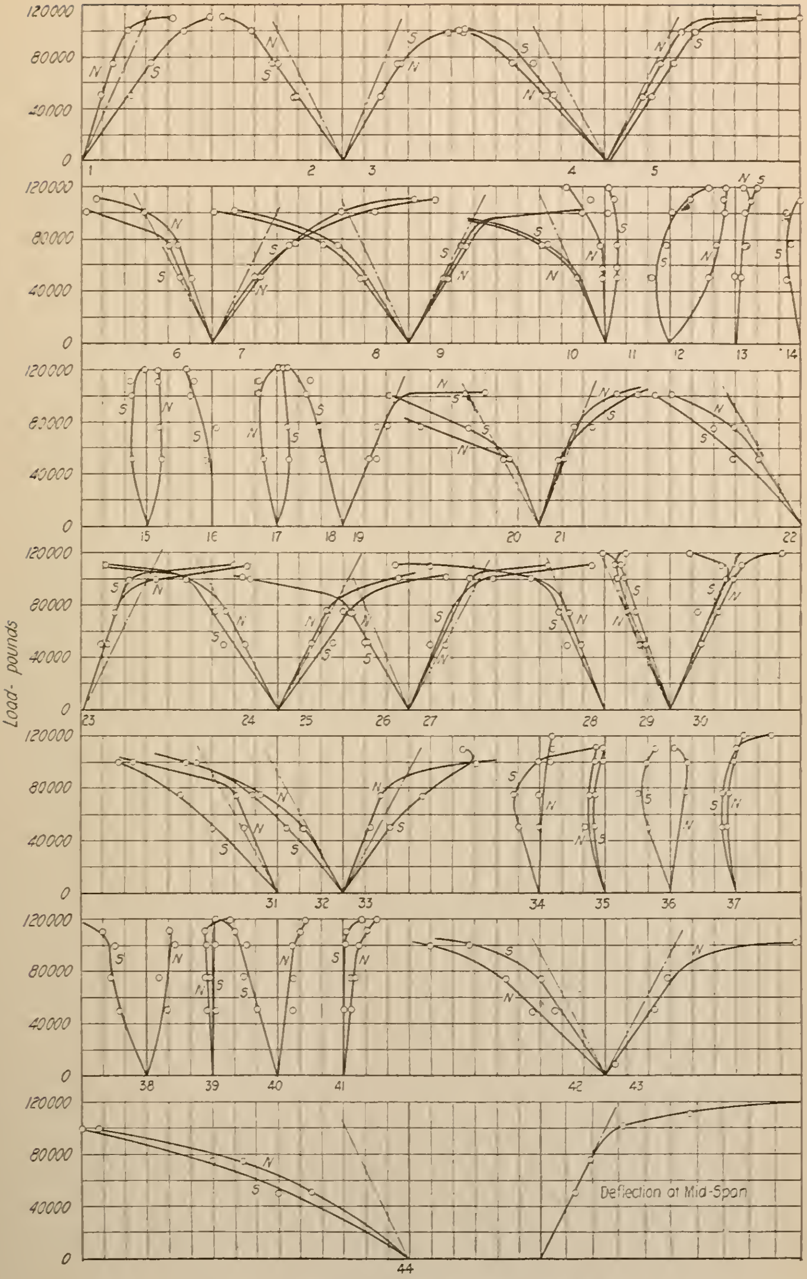

FIG. 20. STRAIN DIAGRAMS FOR I-BEAM 7

Note: See Fig. 18.

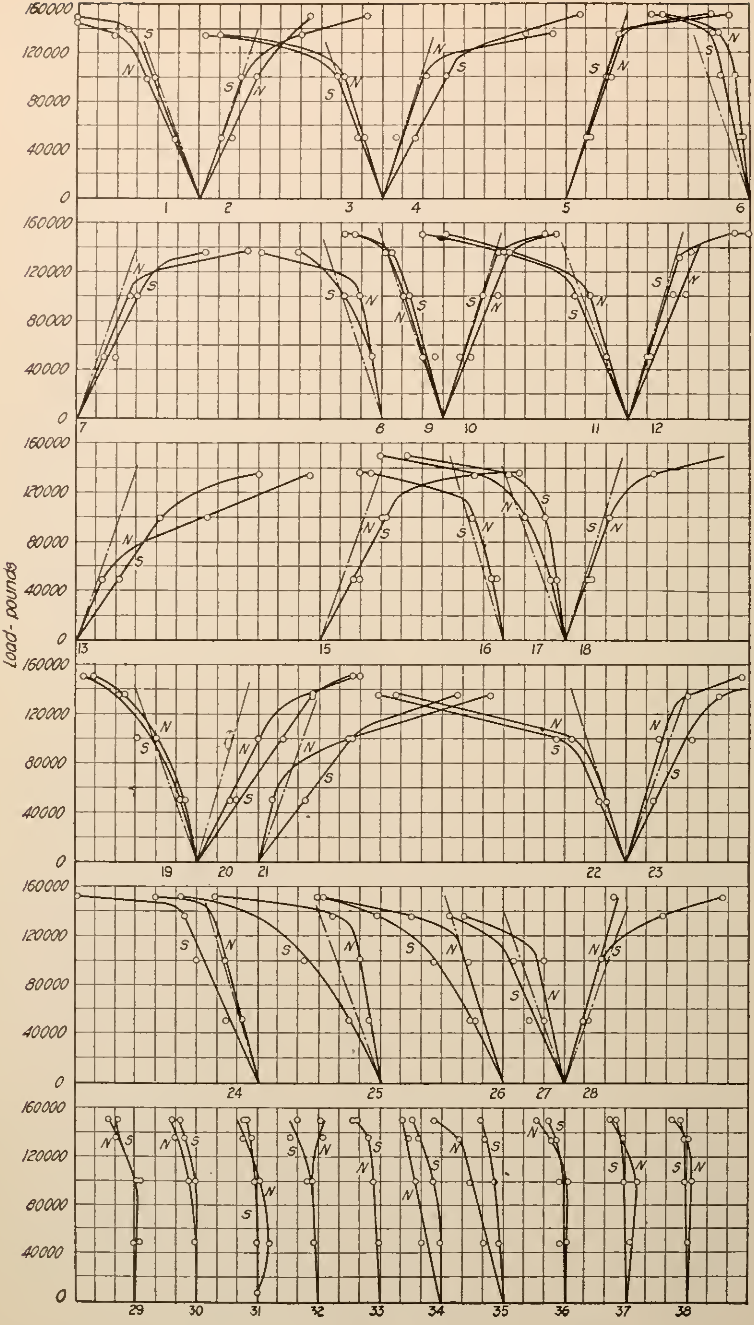

FIG. 21. STRAIN DIAGRAMS FOR BUILT-UP GIRDER 8,—I

Note: See Fig. 22.

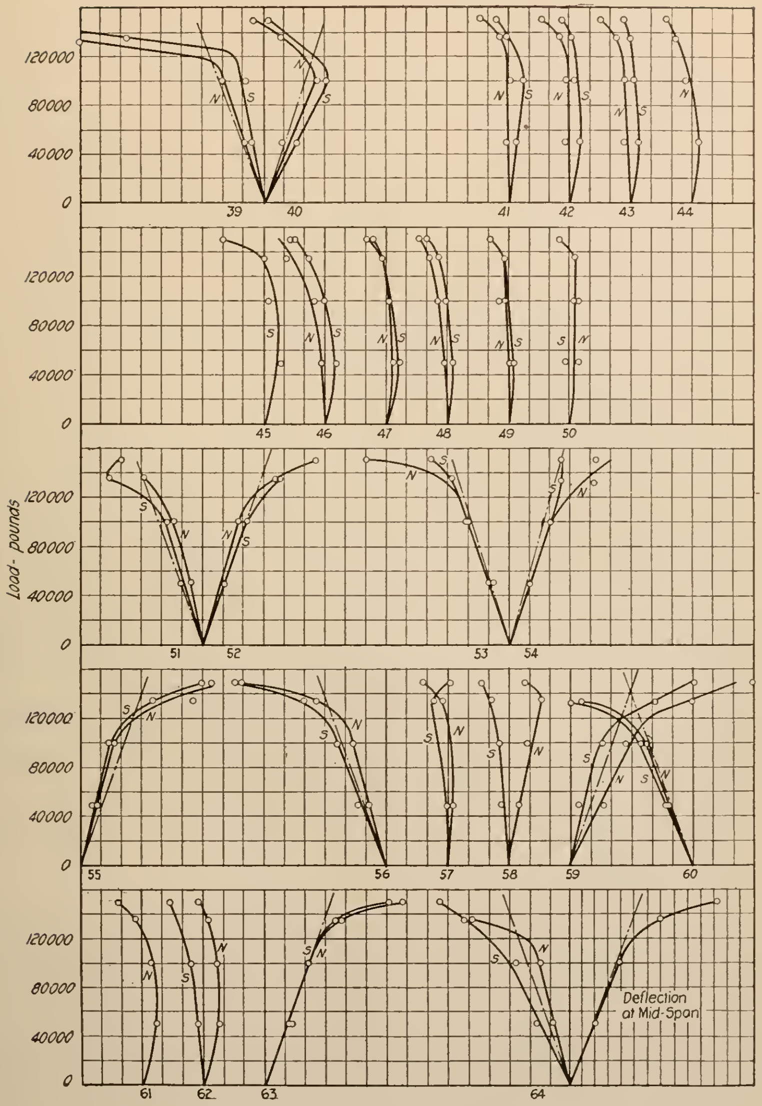

FIG. 22. STRAIN DIAGRAMS FOR BUILT-UP GIRDER 8,—II

Note: 1 horizontal division=10,000 lb. per sq. in. as indicated by the strain gage.
1 horizontal division=0.02 in. (for deflection curve only)
Numbers refer to gage lines as given in Fig. 4.
Tension plotted to right; compression to left.
Measured quantities ———; computed quantities —— — ——· — ——

was 101,500 pounds, and for girder 8, 112,500 pounds. That is, the addition of intermediate stiffeners to a girder for which the ratio of the unsupported width to the thickness of the web was 100, increased the yield point of the girder only about 10 per cent. This is remarkable in view of the fact that the proportions of the girders were such that the web received a relatively higher stress than the flanges, a condition which is usually reversed in practice. While the girder without stiffeners does not have a yield point quite as high as the girder with stiffeners, the fact that the deflection curve is practically straight up to the yield point indicates that it may be safe to build girders without intermediate stiffeners if the ratio of the unsupported width to the thickness of the web exceeds 60. However, it is necessary to decrease the working stress allowable in the web as this ratio becomes greater.

For the determination of the proper working stress for webs without intermediate stiffeners three calculations should be made: (1) The maximum shearing stress (which may be taken equal to the shearing stress at the neutral axis) should not be greater than the safe working stress in shear for the web material. (The strength in shear of web material is discussed in the next section "Strength of Materials in I-beams and Girders".) (2) The value of $E\epsilon$ corresponding to the maximum diagonal strain should not exceed the value of the safe tensile or compressive working stress for the material. (3) The maximum diagonal compressive stress at the neutral axis should not exceed the safe stress as given by Euler's formula for fixed-ended columns, which when applied to webs of I-beams and girders becomes

$$S_w = \frac{1.64\ E}{f\left(\frac{h}{t}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11a)$$

in which S_w equals the average working stress in the web, f=the factor of safety, E=the modulus of elasticity (30,000,000 lb. per sq. in. for steel), h=the unsupported depth of the web between flanges, and t=the thickness of the web.

It may be necessary to provide stiffeners for girders having relatively thin webs, irrespective of the stress, in order that the girder may not be injured in handling.

17. *Strength of Material in the I-beams and Girders.*—Specimens for tension tests and shear tests were cut from the beams and girders as described in section 7 "Specimens." The results of the tension tests are given in Table 8, and the results of the shear tests are given in Table 9. In all tension tests the yield point was determined by the

TABLE 8.

TENSION TESTS OF MATERIAL IN I-BEAMS AND GIRDERS

Each value is the average of two or more tests

Girder (or I-beam)	Specimens from	Yield Point lb. per sq. in.	Ultimate lb. per sq. in.	Elongation in 2 inches per cent
1	Flange	40,200	66,800	35.0
	Web	40,600	67,500	29.5
	Root of flange	38,000	65,700	27.7
2	Flange	42,600	67,600	27.5
	Web	44,200	68,200	27.7
	Root of flange	39,200	67,700	31.5
3	Web	41,800	54,100	35.7
4	Flange	37,500	66,900	36.1
	Web	42,300	68,900	29.7
	Root of flange	45,800	70,100	27.0
5	Flange	36,200	66,900	34.2
	Web	41,800	68,900	28.2
	Root of flange	46,000	69,200	27.0
6	Flange	39,300	67,400	33.5
	Web	43,500	66,500	27.0
	Root of flange	49,200	69,800	28.7
7	Flange	41,400	67,700	35.5
	Web	39,500	65,300	28.7
	Root of flange	43,800	68,200	30.2
8	Web	36,900	53,700	36.0

TABLE 9.

SHEARING TESTS OF MATERIAL FROM WEBS OF I-BEAMS AND GIRDERS

Each result is the average of two or more tests

Girder (or I-beam)	Yield Point lb. per sq. in.	Ultimate lb. per sq. in.
1	Not well defined	53,300
2	do.	53,300
3	25,700	49,000
4	Not well defined	52,300
5	do.	54,400
6	do.	50,500
7	do.	53,100
8	26,100	47,900

drop of the beam of the testing machine, and in shear tests by the first noticeable stretch as shown by a pair of dividers. The results of the tension tests indicate that the material at the root of the flanges of the I-beams was well rolled, since the specimens cut from this part of the I-beams showed fully as high strength as did the specimens from the webs and from the flanges. The yield-point strength of the shear specimens cut from the webs of the I-beams was not clearly defined, but definite yield points in shear were obtained for the specimens cut from the webs of the two built-up girders. Several specimens from each built-up girder were tested. These shear tests indicate a ratio of yield-point strength in shear to yield-point strength in tension of about 0.65, a ratio not widely different from the value 0.60 which is commonly used. If a fiber stress of 16,000 lb. sq. in. is allowable for structural steel in tension, a stress of about 10,000 lb. per sq. in. would be allowable for steel in shear. This agrees with the usual practice.

18. *Deflection of Test Girders.*—As a matter of interest, though secondary in importance to the determination of strength properties, the observed deflection at mid span of the girders under load has been compared with the deflection computed by means of the formulas commonly given in texts on the mechanics of materials. The girders tested had such short spans that the deflection due to shear, about 20 per cent of the total, is important. This deflection is calculated from the formula

$$\Delta_s = \frac{P}{2}\ \frac{l_1}{aF} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

in which Δ_8 is the deflection at mid span due to shear, P is the total applied load, l_1 is the distance from support to the near of the two symmetrically spaced loads, F the modulus of elasticity in shear (taken as 12,000,000 lb. per sq. in. for steel) and a is the total area of cross-section of the beam. This formula is readily derived from the discussion given in Merriman's "Mechanics of Materials," 10th ed. p. 320.

The deflection due directly to flexure is *

$$\Delta_s = \frac{Pl_1}{EI}\left(\frac{l^2}{16} - \frac{l_1^2}{12}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)\,..$$

in which l is the total length of span of the beam, E is the modulus of elasticity in tension and compression (taken as 30,000,000 lb. per sq. in. for steel), I the moment of inertia of the cross-section of the beam, and other symbols are the same as given for equation 14.

The total deflection Δ is then $\Delta_s + \Delta_f$. In the deflection curves of Fig. 16-22, the deflection computed by the preceding formulas is shown by the dot and dash line, and the observed deflection by solid lines. The computed and observed values agree very closely.

The theoretical curve of deflection due to shear alone for a girder loaded at two symmetrical points is made up of two inclined straight lines for the end portions, joined by a horizontal straight line for the

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**Boyd, "Strength of Materials," p. 115.

middle portion. The general tendency of the girders to assume such a shape under excessive shearing strain is well shown in Fig. 10 and 11.

19. *Summary.*—The following summary is given:

(1) The measured strains in various parts of the six I-beams and two built-up girders agree closely with the strains as computed by the ordinary elastic theory if due allowance is made for the lateral strain (Poisson's ratio effect).

(2) The maximum shearing stress in an I-beam or a built-up girder is in some cases the shearing stress at the neutral axis, and is in other cases the diagonal shearing stress caused by the combined stresses in the web at its junction with the flange. However, the two are usually nearly equal, and, in general the shearing stress at the neutral axis may be used in designing girders.

(3) A common approximate method of computing the shearing stress in the web of a girder is to divide the total shear upon a transverse section by the area of the cross-section of the web. If the value given by this method is more than 80 per cent of the allowable stress in shear for the material, a check computation for shearing stress should be made, using the more precise formula, equation (2).

(4) The yield point (not the ultimate strength) of the material in shear should be regarded as the ultimate shearing stress which can be developed in the webs of girders. The ratio of the yield point in shear to the yield point in tension for structural steel is about 0.6, and the ratio of the allowable shearing stress to the allowable tensile or compressive stress may be taken at the same value.

(5) The maximum tensile or compressive strain is an I-beam or built-up girder is in some cases the longitudinal strain in the extreme fibers of the flange, and is in other cases the diagonal strain in the web adjacent to the flange. The diagonal strain may be enough greater than the longitudinal strain to make it desirable to consider the former in the design of a girder. The value of $E\epsilon$ corresponding to the maximum strain should not in any case exceed in magnitude the safe working stress of the material in tension. (The safe stress in tension for structural steel is usually taken at 16,000 lb. per sq. in.)

(6) In the case of girders having no stiffeners except at points at which concentrated forces are applied, the web is capable of developing the lowest of the following critical values: (1) the yield-point strength of the material of the web in shear; or (2) the compressive strength of the web as computed by Euler's formula, considering a 45-degree strip as a fixed-ended column subjected to a compressive stress equal to the transverse shearing stress at the neutral axis; or

indicate a ratio of yield-point strength in shear to yield-point strength in tension of about 0.65, a ratio not widely different from the value 0.60 which is commonly used. If a fiber stress of 16,000 lb. sq. in. is allowable for structural steel in tension, a stress of about 10,000 lb. per sq. in. would be allowable for steel in shear. This agrees with the usual practice.

18. *Deflection of Test Girders.*—As a matter of interest, though secondary in importance to the determination of strength properties, the observed deflection at mid span of the girders under load has been compared with the deflection computed by means of the formulas commonly given in texts on the mechanics of materials. The girders tested had such short spans that the deflection due to shear, about 20 per cent of the total, is important. This deflection is calculated from the formula

$$\Delta_s = \frac{P}{2} \; \frac{l_1}{aF} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

in which Δ_8 is the deflection at mid span due to shear, P is the total applied load, l_1 is the distance from support to the near of the two symmetrically spaced loads, F the modulus of elasticity in shear (taken as 12,000,000 lb. per sq. in. for steel) and a is the total area of cross-section of the beam. This formula is readily derived from the discussion given in Merriman's "Mechanics of Materials," 10th ed. p. 320.

The deflection due directly to flexure is *

$$\Delta_s = \frac{Pl_1}{EI}\left(\frac{l^2}{16} - \frac{l_1^2}{12}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)..$$

in which l is the total length of span of the beam, E is the modulus of elasticity in tension and compression (taken as 30,000,000 lb. per sq. in. for steel), I the moment of inertia of the cross-section of the beam, and other symbols are the same as given for equation 14.

The total deflection Δ is then $\Delta_s + \Delta_f$. In the deflection curves of Fig. 16-22, the deflection computed by the preceding formulas is shown by the dot and dash line, and the observed deflection by solid lines. The computed and observed values agree very closely.

The theoretical curve of deflection due to shear alone for a girder loaded at two symmetrical points is made up of two inclined straight lines for the end portions, joined by a horizontal straight line for the

---

**Boyd, "Strength of Materials," p. 115.

middle portion. The general tendency of the girders to assume such a shape under excessive shearing strain is well shown in Fig. 10 and 11.

19. *Summary.*—The following summary is given:

(1) The measured strains in various parts of the six I-beams and two built-up girders agree closely with the strains as computed by the ordinary elastic theory if due allowance is made for the lateral strain (Poisson's ratio effect).

(2) The maximum shearing stress in an I-beam or a built-up girder is in some cases the shearing stress at the neutral axis, and is in other cases the diagonal shearing stress caused by the combined stresses in the web at its junction with the flange. However, the two are usually nearly equal, and, in general the shearing stress at the neutral axis may be used in designing girders.

(3) A common approximate method of computing the shearing stress in the web of a girder is to divide the total shear upon a transverse section by the area of the cross-section of the web. If the value given by this method is more than 80 per cent of the allowable stress in shear for the material, a check computation for shearing stress should be made, using the more precise formula, equation (2).

(4) The yield point (not the ultimate strength) of the material in shear should be regarded as the ultimate shearing stress which can be developed in the webs of girders. The ratio of the yield point in shear to the yield point in tension for structural steel is about 0.6, and the ratio of the allowable shearing stress to the allowable tensile or compressive stress may be taken at the same value.

(5) The maximum tensile or compressive strain is an I-beam or built-up girder is in some cases the longitudinal strain in the extreme fibers of the flange, and is in other cases the diagonal strain in the web adjacent to the flange. The diagonal strain may be enough greater than the longitudinal strain to make it desirable to consider the former in the design of a girder. The value of $E\epsilon$ corresponding to the maximum strain should not in any case exceed in magnitude the safe working stress of the material in tension. (The safe stress in tension for structural steel is usually taken at 16,000 lb. per sq. in.)

(6) In the case of girders having no stiffeners except at points at which concentrated forces are applied, the web is capable of developing the lowest of the following critical values: (1) the yield-point strength of the material of the web in shear; or (2) the compressive strength of the web as computed by Euler's formula, considering a 45-degree strip as a fixed-ended column subjected to a compressive stress equal to the transverse shearing stress at the neutral axis; or

(3) a diagonal strain equal to the strain at the yield point of the material in tension.

(7) It would seem that the ability to resist buckling of thin webs without intermediate stiffeners had been underestimated.

(8) Stiffeners at supports and under concentrated loads are very necessary. These should be well fitted to the flanges.

(9) The local compressive stress in the web of a girder when no stiffeners are used at points at which concentrated forces are applied may be computed with a fair degree of accuracy by the use of Hudson's formula (see p. 32 for detailed discussion). Even if this stress is low, the use of stiffeners at points at which concentrated forces are applied diminishes the danger of lateral bending of the beam at the junction of the web and the flange.

(10) The deflection of the girders as measured and as computed by the ordinary elastic theory agrees closely when the deflection due to shear is considered (see p. 48 for discussion of formulas). For short-span beams the deflection due to shear may be as much as 20 per cent of the total.

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BULLETIN No. 87

CORRECTION OF ECHOES AND REVERBERATION IN THE AUDITORIUM, UNIVERSITY OF ILLINOIS

BY
F. R. WATSON
And
JAMES M. WHITE

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 87 MAY, 1916

THE CORRECTION OF ECHOES AND REVERBERATION IN THE AUDITORIUM AT THE UNIVERSITY OF ILLINOIS

BY

F. R. WATSON, Associate Professor of Experimental Physics, at the University of Illinois, and

PROFESSOR J. M. WHITE, Supervising Architect

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THE CORRECTION OF ECHOES AND REVERBERATION IN THE AUDITORIUM AT THE UNIVERSITY OF ILLINOIS

I. Introduction.

1. Preliminary.—The work described in this bulletin may be considered as a continuation of an earlier investigation on "Acoustics of Auditoriums."*

Figure 1 shows the floor plans of the auditorium under investigation. The interior approximates a sphere cut off on the lower surface by the sloping floor of the room. There is a balcony, but no gallery. The balcony projects 12 feet over the main floor at the sides and 34 feet in the rear. The stage is built out into the room instead of being set back behind a proscenium arch as originally designed, the stage house having been omitted to reduce the cost of the building.

The domed ceiling is supported on four equal arches, and the side walls above the gallery are double curved surfaces. The limited appropriation for the building made it impossible to embellish the surfaces of the walls and ceiling, and therefore, they were left practically plain, which increased their power to reflect sound and cause echoes. There are no windows in the room, the daylight lighting being exclusively through a ceiling light 30 feet in diameter in the center of the dome.

The results set forth in the previous bulletin are briefly as follows. A systematic investigation of the acoustical properties of the Auditorium at the University of Illinois was carried on for several years. "Cut and try" methods of cure were avoided. It was shown by theory and experiment that the usual acoustical faults in a room are due first, to a reverberation, or undue prolongation of sound, and second, to echoes; both of these defects being caused by the reflection of sound from the walls. Various methods of cure were considered,—the effect of padding and paneling the walls, the possible advantage of installing wires† and sounding boards,‡ and finally, the action of the ventilating system.* The conclusion was drawn that the most effective cure lay in padding the walls with materials which absorb sound.

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*Bulletin No. 73 of the Engineering Experiment Station, University of Illinois.

†"Inefficiency of Wires as a Means of Curing Defective Acoustics of Auditoriums." Science, Vol. 35, p. 833. 1912.

‡"The Use of Sounding Boards in an Auditorium." Physical Review, Vol. 1 (2), p. 241, 1913. Also The Brick Builder, June, 1913.

An experimental diagnosis of the acoustical properties of the Auditorium was made. This was done by tracing the path pursued by a small bundle of sound when it was sent in a definite direction and noting what became of it after reflection. Several methods of tracing sound were tried before a suitable one was found. A ticking watch backed by a reflector, or a metronome enclosed in a box having a directed horn gave definite data. However, a hissing arc light with a parabolic reflector was much more satisfactory and gave conclusive results. Enough data were secured in this way to show the general behavior of the sound in the room and also to indicate how the chief echoes were set up.† Attempts were then made to secure satisfactory acoustics by hanging curtains and draperies at critical points suggested by the diagnosis. This result was finally secured by suspending four large pieces of canvas in the dome.

From the acoustical standpoint, the Auditorium was then in a much improved condition. The canvas, however, was very unsightly and did not accord with the architectural features of the room. It was therefore proposed that the materials used to correct the acoustics be installed in such manner as to remedy this fault. It was also proposed at this time to install a pipe organ, to decorate the interior of the room, and to change the lighting system.

2. *Object of the Bulletin.*—The object of this bulletin is to describe the changes that were made in the Auditorium to carry out the proposals just mentioned, and especially to show how the acoustical properties were modified.

II. PRELIMINARY ACOUSTICAL INVESTIGATION.

It was desired that the materials used to correct the acoustics be installed in such manner as to conform with the architectural features of the Auditorium. This introduced a new problem since in the provisional cure the canvas sheets in the dome hung with very little conformity to the curvature of the walls. A further complication appeared when it was found by calculation that the amount of material necessary to correct the reverberation was insufficient to pad all the

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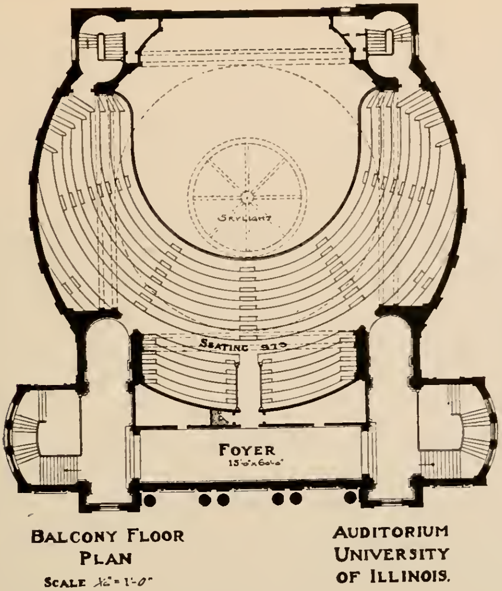
*"Air Currents and Acoustics of Auditoriums." Engineering Record. Vol. 67, p. 265, 1913.

†"Echoes in an Auditorium." Physical Review, Vol. 32, p. 231, 1911.



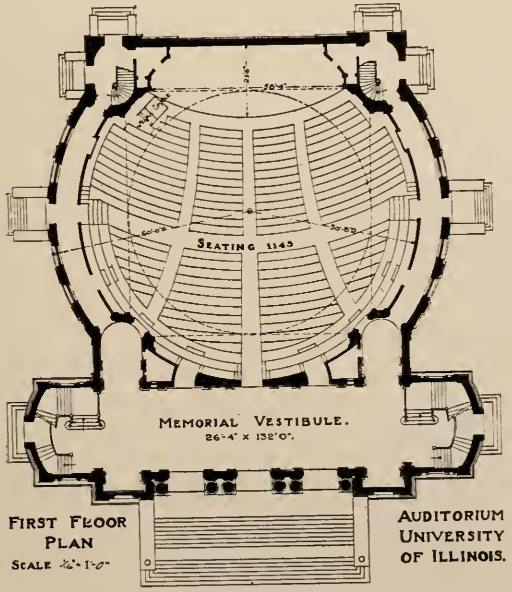


FIG. 1. FLOOR PLANS SHOWING INTERIOR OF THE AUDITORIUM AT THE UNIVERSITY OF ILLINOIS WHICH WAS CORRECTED FOR ECHOES AND REVERBERATION.

walls that produced echoes. It was desirable to eliminate the echoes, but it was regarded as risky to install too much sound absorbing material, owing to the danger of making the Auditorium too dead for sound.

Because of these difficulties it was decided to carry on further experiments and to secure more data before deciding on the final cure. Accordingly, one large curved wall was covered with strips of one-inch hair felt, 30 inches wide, placed vertically and 30 inches apart so as to leave bare spaces between them. This arrangement was satisfactory for several reasons; it did not change the curvature of the wall; it used only half the amount of material necessary to cover the entire wall; and because of diffraction and interference effects, it was theoretically more efficient in breaking up the reflected sound than if the same material were spread continuously over the whole surface. Although encouraging, the results were not so marked as expected in diminishing the echoes.

On the basis of this experiment, plans were made for covering other walls in a similar way, except that the hair felt was to be mounted on wooden ribs built out from the wall surface. Such an installation seemed more likely to break up the incident sound than the first plan of mounting the hair felt snugly against the wall. The sound wave on striking these outer felt strips would suffer partial reflection and change of phase, while the remaining portion of the sound would pass through the open spaces and be spread out by diffraction and reflection from the walls. The hair felt strips would oppose the incident and reflected waves, thus breaking up the original sound and diminishing its intensity and possibility of producing echoes.

Because the scaffolding erected for the use of the workmen interfered with the passage of sound waves, the efficiency of this method of placing the felt could not be tested step by step as the material was mounted. The test was deferred, therefore, until the installation was completed. In the meantime the pipe organ was installed, the interior was redecorated, and the lighting system changed, so that only the combined effect of all these factors on the acoustics could be investigated.

III. Modifications of the Interior of the Auditorium.

3. *Installation of the Pipe Organ.*—The organ was mounted in a unique way by dividing it into two parts and placing them in lofts 24 feet above the ends of the stage with a distance of 75 feet between centers. This arrangement placed the organ at a considerable distance

above the audience. The absence of any vertical surface between the lofts and the audience room prevented any visible arrangement of the organ pipes, but the necessary free exit of the sound was provided for by the construction of ornamental plaster grills covering the pendentives on either side of the stage. (See Fig. 2.)

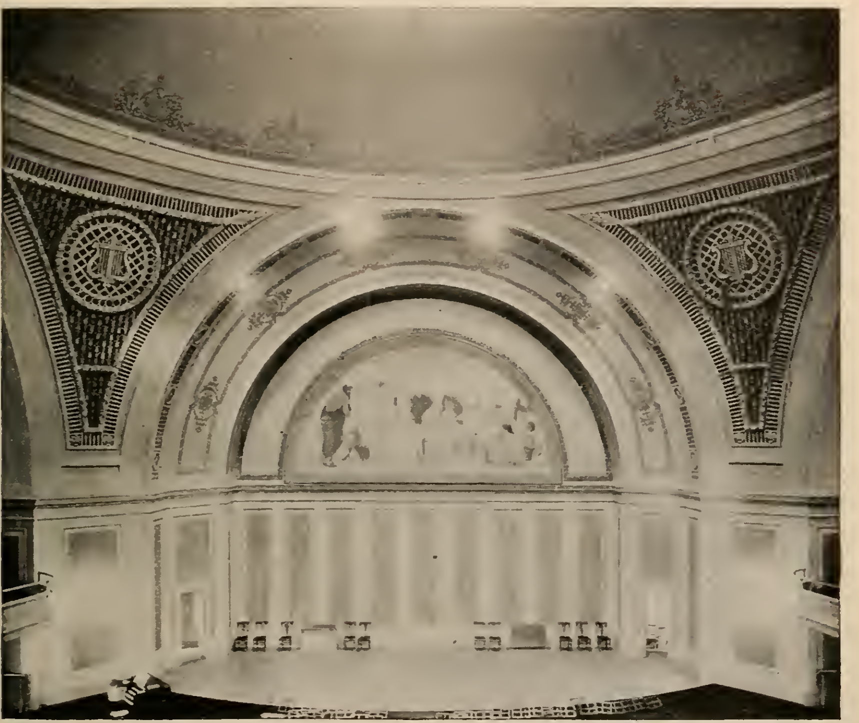


FIG. 2. VIEW TOWARD THE STAGE SHOWING THE GRILL WORK FOR FREE PASSAGE OF SOUND FROM THE CONCEALED ORGAN. THE ORGAN CONSUL IS SHOWN TO THE LEFT. CARPET IS REMOVED FROM STAGE IN PREPARATION FOR AN ORCHESTRA CONCERT

4. *Method of Mounting Hair Felt.*—The hair felt was mounted on thin furring strips which were bent to fit the curvature of the surfaces. The dome above the arches and the double curved side walls and single curved rear wall above the balcony were padded in this way. The felt was mounted in vertical strips on the west side wall as shown in Fig. 3. Fig. 4 shows the wall after the material was installed and decorated.

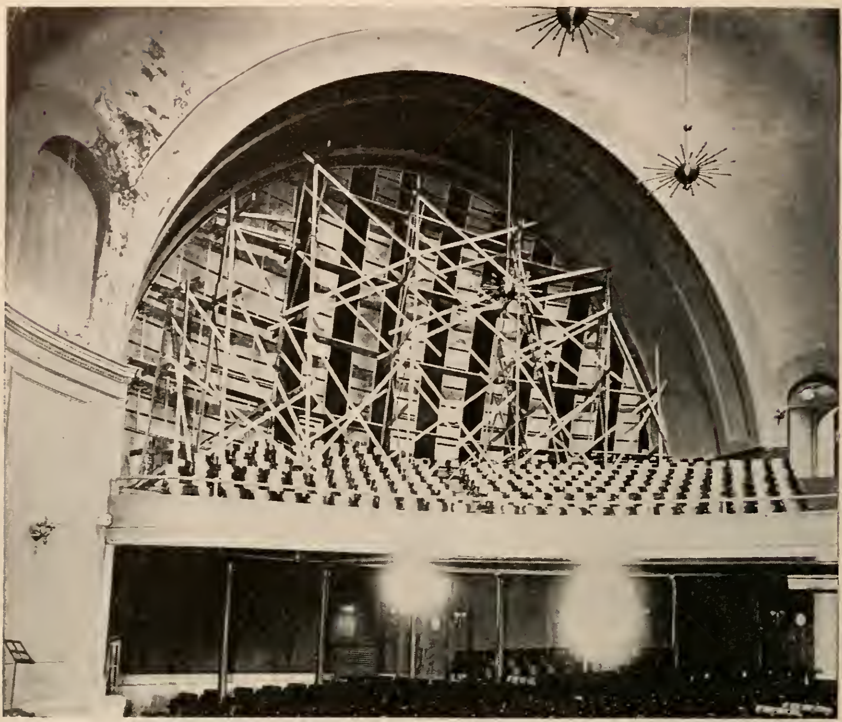


FIG. 3. PHOTOGRAPH SHOWING THE MOUNTING OF THE HAIR FELT IN VERTICAL STRIPS. THE MATERIAL WAS FASTENED TO THIN FURRING STRIPS WHICH COULD BE BENT TO CONFORM TO THE CURVATURE OF THE SURFACE

On the east balcony wall the felt was mounted on wooden ribs so that it stood concentric with the plaster surface at a distance of one foot. Eighteen inches below the edge of the skylight in the dome radial strips of felt which approached the wall until they touched at the crown of the arches, were mounted on wooden ribs. (Fig. 5.) The hair felt used was the Akustikos Felt developed especially for correction of acoustical faults by the H. W. Johns-Manville Company under the direction of Professor Sabine.

Before the changes were made in the Auditorium, Professor Sabine visited the building at the invitation of President James. After this visit, he wrote to President James as follows: "If such confirmation of the results of Professor Watson's investigation is necessary, please permit me to assure you that you will obtain an excellent effect from following out his suggestions in all detail." The final installation was

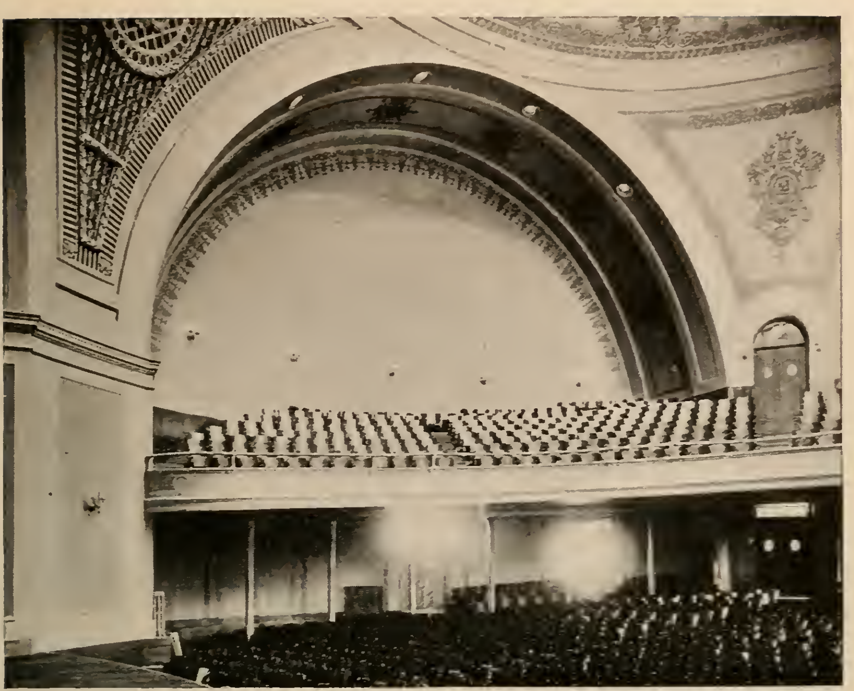


FIG. 4. PHOTOGRAPH SHOWING THE SIDE WALL OF FIG. 2 WHEN COMPLETED. A REP OF SUITABLE COLOR WAS STRETCHED OVER THE ENTIRE SURFACE AND DECORATED. IT WAS NECESSARY FOR THE FREE PASSAGE OF THE SOUND THAT THE MATERIAL USED IN DECORATING SHOULD NOT CLOSE THE PORES OF THE REP

modified somewhat from the original plans, but the general features were maintained.

5. *The Decoration and the Lighting System.*—The modification of the lighting system involved the elimination of the suspended fixtures. The wall brackets were retained, but the main lighting was changed to a semi-indirect system with reflectors above the arches and around the skylight. An ivory tone was selected for the basic color in the redecoration. Ornamentation was stenciled and painted on the various walls and surfaces to give a unified effect. With the exception of the ornamental borders the rep covering the padded surfaces was left its natural color. The difference between the old and new interiors is shown in Figs. 6 and 7. The modifications relieved the auditorium of its cheerless. barn-like interior.

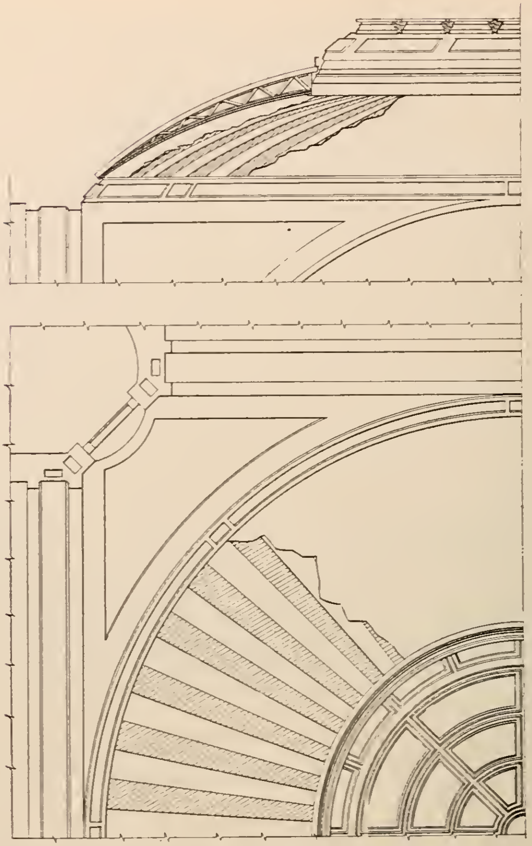


FIG. 5. DIAGRAM SHOWING THE DOME SURFACE WITH WOODEN RIBS SEPARATING THE HAIR FELT IN RADIAL STRIPS. THIS FALSE CEILING ARRANGMENT WAS THOUGHT MORE EFFECTIVE FOR ABSORPTION OF SOUND THAN IF THE MATERIAL WERE MOUNTED SNUGLY AGAINST THE SURFACE

IV. FINAL ACOUSTICAL INVESTIGATION.

The remodeled Auditorium has been tested under varied conditions for music and speaking, and popular opinion has pronounced the acoustics satisfactory. A speaker with a moderate voice can be heard distinctly by auditors in the most distant seats. The music of the new pipe organ, according to experts, is satisfactorily rendered. The room is also suited for orchestra music, though for this case, it has been found advantageous to follow the usual custom of leaving the wooden floor of the stage bare of carpet so as to reenforce the sound from the instruments.

While the Auditorium has proved to be generally satisfactory, a detailed investigation of the acoustical effects secured by the modification of the room was thought desirable. A request was made, accordingly, that auditors report any echoes or acoustical disturbances however slight they might be. About a dozen replies were received, and on the basis of these and other considerations, a systematic investigation was undertaken.

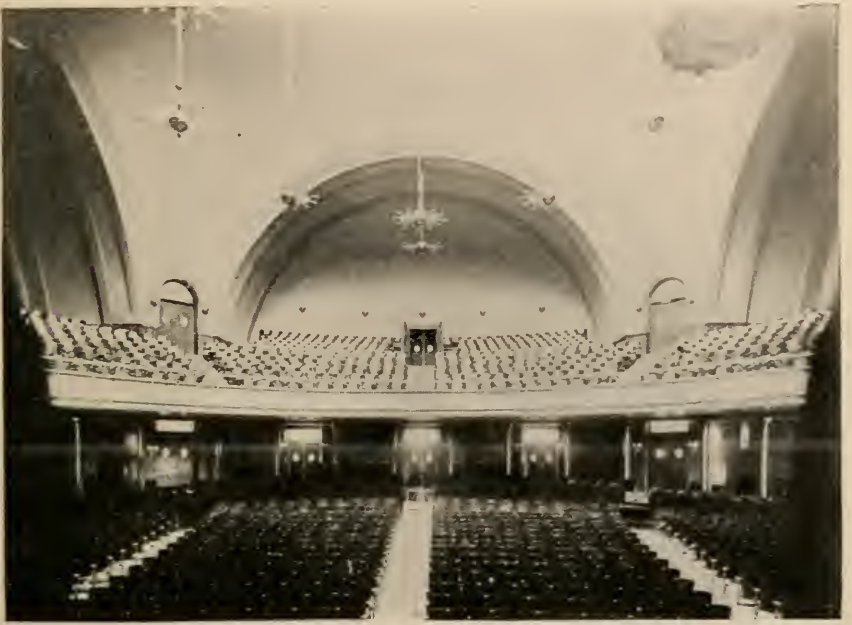


Fig. 6. Photograph showing Auditorium before changes were made

The acoustical results, beneficial and otherwise, may be anticipated by considering the changes made. According to Sabine, the hair felt installed would reduce the reverberation. This would also eliminate echoes if installed on certain surfaces in accordance with the analysis; but, since the amount of material used to correct the reverberation was insufficient to cover all the walls, acoustical defects might still be set up by the unpadded surfaces, especially by the pendentives. The pipe organ, by generating musical sounds that emerged through the pendentives in the dome, might introduce new acoustical disturbances. The openings made in the surfaces of two of the pendentives for the passage of the organ music would reduce the general

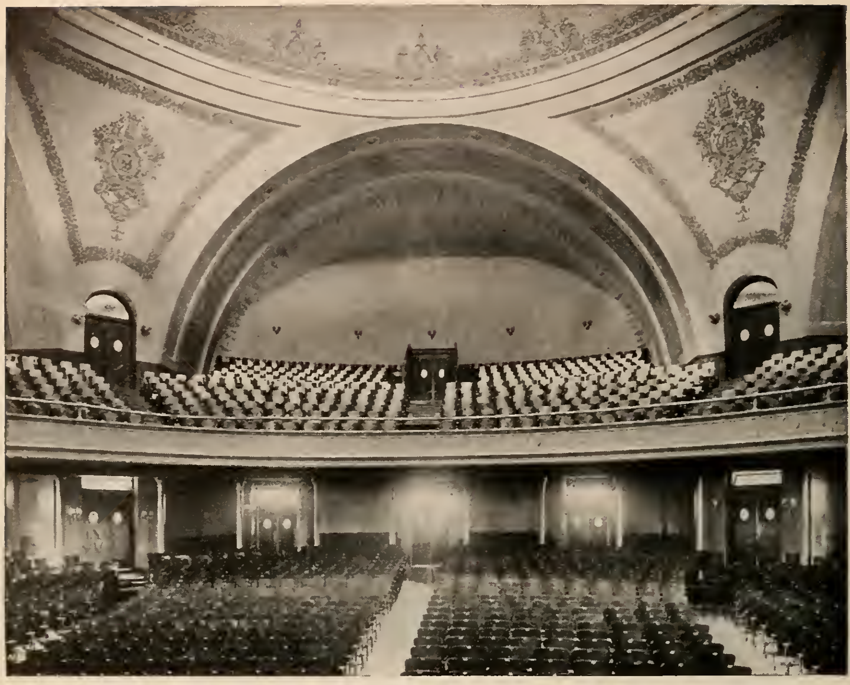


FIG. 7. PHOTOGRAPH SHOWING NEW INTERIOR OF AUDITORIUM. THE SUSPENDED LIGHTING FIXTURES WERE REMOVED, THE INTERIOR REDECORATED, AND THE REAR WALL IN THE ALCOVE PADDED

reverberation and would also diminish echoes. The changes in the decoration and in the lighting system would produce little effect.

6. *Investigation of Echoes.*—Tests were made in several ways to determine the presence of echoes. The opinion offered by auditors that the echoes had generally disappeared was, of course, the most satisfactory evidence. One test was made by talking through a megaphone toward different walls (Fig. 8). The sound was generated inside a small house and its direction of propagation controlled by two megaphones, one being pointed toward an observer and the other toward a wall which previously gave echoes. No distinct echo could be obtained by speaking simultaneously into the two megaphones. The ticks of a metronome produced very little additional effect, but when a sharp intense metallic sound was tried, echoes were obtained from the unpadded walls but only faint responses from the padded walls. The intense hissing sound of an arc light backed by a parabolic reflector gave more pronounced results. It showed that the padded walls produced a marked effect in reducing the intensity of the sound.

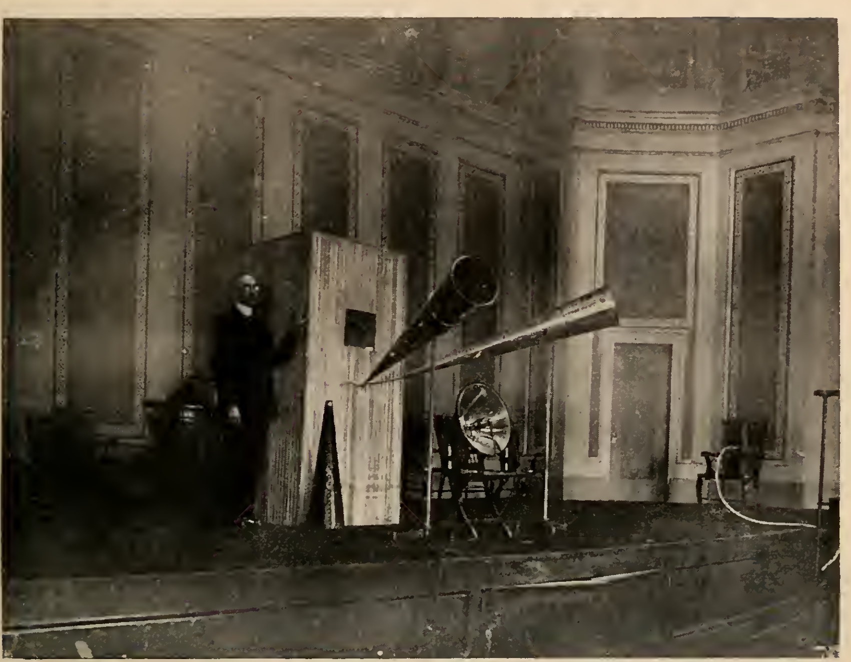


FIG. 8. PHOTOGRAPH OF STAGE SETTING SHOWING EXPERIMENTAL HOUSE WITH MEGAPHONES USED FOR TESTING ECHOES

The effect of the unpadded pendentives in the rear dome surface is shown in Fig. 9. The cone of incident sound received by each pendentive is small and, after reflection, spreads over a large area. It was therefore anticipated that little disturbance would result. This prediction was not entirely correct since the echoes reported by auditors, so far as could be ascertained, came from these two walls. An echo was perceptible when the speaker faced directly toward one of these pendentives so that the profile of his face was seen by an auditor seated at one side of the auditorium. The direct sound coming to the auditor was then diminished while the reflected sound was augmented, thus producing an echo.

Other unpadded walls, notably the side walls under the balcony, still set up concentrations of sound. Thus, an observer at A, Fig. 10, can hear not only the direct sound from the speaker, but also the portion that is concentrated by reflection from B. He does not hear an echo because the time interval between the direct and reflected sounds is too short to enable his ear to detect them separately. The result is

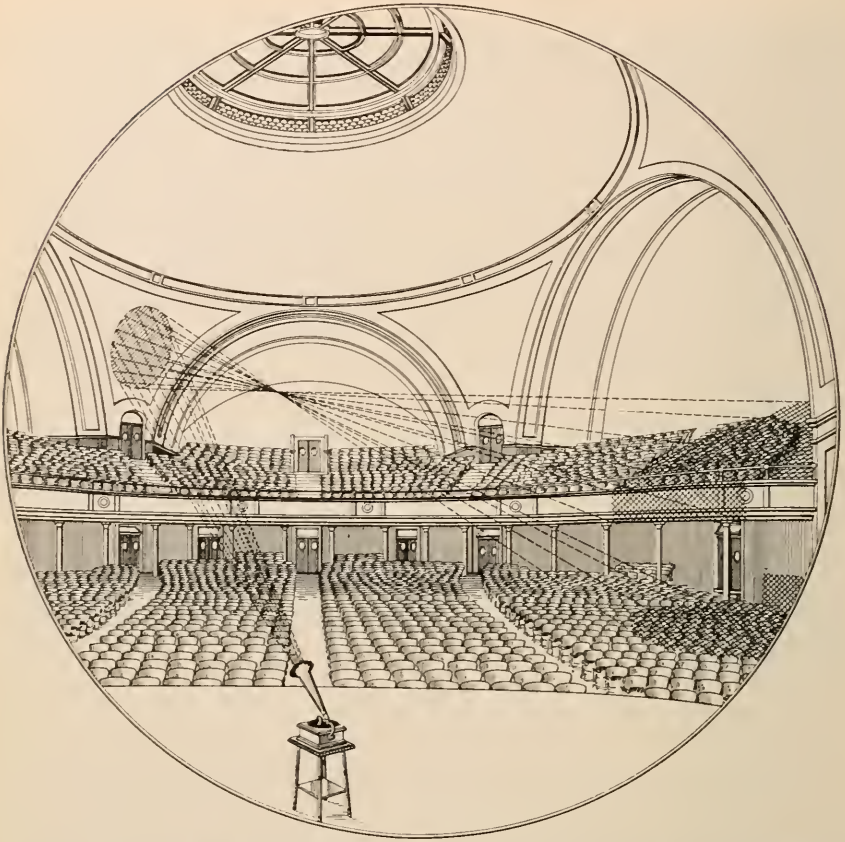


FIG. 9. DIAGRAM SHOWING THE REFLECTION OF SOUND FROM THE UNPADDED PENDENTIVE IN THE REAR WALL. ECHOES SET UP BY THIS WALL CAN OCCASIONALLY BE NOTED

much the same as if his neighbor on the side toward the wall were to say the words of the speaker in his ear at the same time that he received them from the speaker. The auditor realizes that something is peculiar about the sound but usually does not understand the cause of the trouble. An auditor at C, however, may get an echo when the speaker faces the point D.

7. *Investigation of the Reverberation.*—By means of Sabine's formula and coefficients of absorption* the time of reverberation of the Auditorium was found and a calculation was made to determine the amount of sound absorbing material necessary to correct the fault. The following tabulation shows the method employed:

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*American Architect, 1900.

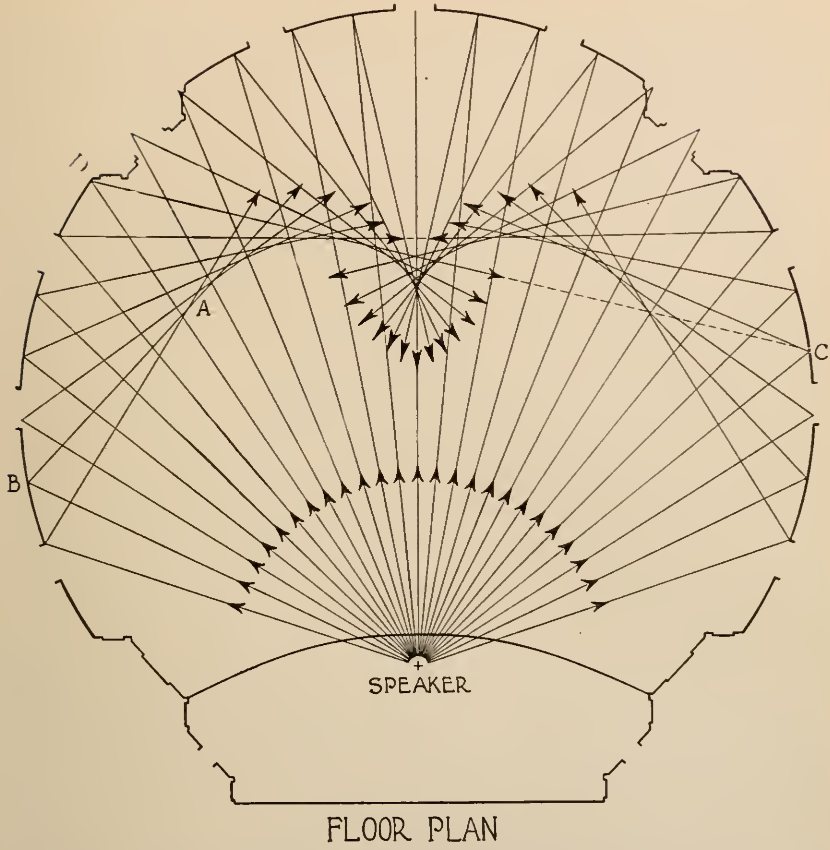

FIG. 10. PLAN OF AUDITORIUM SHOWING CONCENTRATION OF SOUND BY THE WALLS UNDER THE BALCONY

Material	Exposed Area in sq. meters	Coeff. of Absorption	Total Absorption
Plaster on lath	2000	0.0330	66.0
Plaster on tile	510	0.0250	13.0
Wood work	1630	0.0610	99.0
Glass	83	0.0270	2.3
Cocoa matting	145	0.0200	2.9
Wood seats	2150 seats	0.0082	17.7
			201
Average audience	1200 people	0.44	527
		Total	728

Volume of room..........12000 cubic meters.

Substituting these data in the formula $t=0.164\ V \div a$, in which t is the time of reverberation, V the volume of the room and a the total absorbing power, the following equation for the empty room is obtained:

$t=0.164 \times 12000 \div 201=9.8$ seconds.

When an audience of 1200 people is present,

$t=0.164 \times 12000 \div 728=2.7$ seconds.

This value is too great for good acoustics and a reverberation results. To correct the fault, absorbing material should be added until the time of reverberation is reduced to about 1.8 seconds; this value having been found satisfactory for halls as large as the Auditorium when used for both music and speaking.

The amount of Akustikos Felt needed to carry out the plans already described was 3315 square feet. This was less than the area necessary for felt mounted snug against the wall since the coefficient of absorption is greater when the felt is mounted out from the wall.* Calculations, which allowed for the sound absorbing power of the felt and the other alterations in the Auditorium indicated that the time of reverberation would be reduced to about 1.90 seconds with 1200 people present.

V. DISCUSSION AND CONCLUSIONS.

The Auditorium fulfilled the theory held many years ago by Lord Rayleigh* that a large room with hard, non-porous walls and with few windows has a prolonged resonance, and that the best chance of improvement lies in padding the walls and ceiling with sound absorbing materials. Thus, the installation of hair felt in the Auditorium reduced the reverberation; the amount of reduction being calculated in advance by Sabine's† formula and constants of absorption.

The amount of hairfelt necessary to correct the reverberation was insufficient to cover all the walls, and it was found that some of these unpadded surfaces still produced echoes. This action was anticipated in part from the general considerations discussed by Rayleigh‡ in which the possibility of reflection of sound was shown to depend on the positions of the source and receiver of sound, and also upon the size and form of the wall compared with the wave length of the incident sound.

---

*Sabine. Architectural Quarterly of Harvard University, p. 22, March, 1912.

*Theory of Sound, Vol. 2, pp. 287 and 351.

†American Architect, 1900.

‡Theory of Sound, Vol. 2, p. 28?.

The installation in an auditorium of considerable sound absorbing material eliminates the objectional condition of satisfactory reverberation being wholly dependent on the sound absorbing power furnished by an audience. This means that rehearsals without an audience can be conducted satisfactorily and that a speaker addressing a small audience is not obliged to contend with a distressing reverberation.

The theoretical advantages in absorbing and breaking up sound waves when hair felt is mounted out from a wall instead of placed snugly against the surface do not appear to be so great as expected. Observers listened to sounds reflected from both types of surface and concluded that a surface having the hair felt mounted out from the wall was more efficient. The conclusions, however, should be checked by quantitative, instrumental measurements since the ear is inaccurate in its estimation of the comparative intensities of different sounds.* It appears that the felt is more effective when mounted out from the wall, but there is some question whether or not the advantages secured justify the additional expense of installation and the greater risk of fire.

The music of the pipe organ emerging in large volume from the pendentives in the dome introduced concentrations of sound different from those set up when the source of sound was on the stage. This made it desirable to pad other walls in addition to those requiring padding for the single source of sound.

The effect of the organ music confirmed one conclusion set forth by Jäger†; namely, that the strength of the source of sound for good acoustics should be in correct proportion to the volume of the room. It appears that the Auditorium is too small for loud organ music since the sound in this case becomes unpleasantly intense. On the other hand, it appears that the volume is fairly well suited for softer organ music and for a weak source of sound, such as a speaker with a moderate voice. In this connection Jäger contends that an auditorium is limited in its acoustical possibilities; that if a room is too large, it is impossible to make it satisfactory for weak sources of sound. He points out also that the problem of correcting faulty acoustics must include a consideration of intensity of sound as well as of reverberation; that is, the variable factors at command, the volume and absorbing power of the room and the source of sound, must be so proportioned as to give not only a suitable reverberation but also an acceptable intensity of sound. He discusses the limitations in obtaining this desired result.

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*Rayleigh, Scientific Papers, Vol. II, p. 132.

†"Zur Theorie des Nachhals," Sitzungsberichten der Kaiserl. Akademie der Wissenschaften in Wien. Matem-naturw. Klasse; Bd. CXX, Abt. IIa, Mai, 1911.

Another deduction made by Jäger which applies rather directly to the Auditorium is that the ratio S/W should be large for good acoustics, in which S is the total surface of walls, furniture, and fixtures struck by the sound and W is the volume of the interior. Theoretically, this ratio is smallest for a sphere, and, since the Auditorium approximates a hemisphere, the excessive reverberation might have been predicted.

Reverberations and echoes were corrected simultaneously by installing a suitable amount of hair felt on the walls which produced echoes. To locate these walls, a new method was developed in which the source of sound was an arc light as explained earlier in this bulletin.

The investigation showed that curved walls are worse acoustically than plane walls since they produce undesirable concentrations of sound and echoes. It also appears that the openings in the pendentives for the organ music and the ventilation openings act similarly to open windows and thus reduce reverberation and diminish echoes.

One acoustical disturbance which was not corrected was that due to talking and walking in the foyer and on the stairs immediately outside the Auditorium. The sounds of footsteps and the reverberation caused by loud talking and accidental noises in the foyer could be reduced by covering the stairs and foyer with a yielding material, such as cork and by padding some of the walls.

It is apparent from this discussion that the means employed to correct the acoustics, as exemplified by this complex problem, were based upon established scientific principles and this investigation and others of like nature have served, to a large extent, to dispel the mystery surrounding the action of sound in auditoriums.

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(This is an addition to the bibliography published with Bulletin No. 73 on "Acoustics of Auditoriums.")

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DRY PREPARATION OF BITUMINOUS COAL AT ILLINOIS MINES

BY

E. A. HOLBROOK

BULLETIN No. 88

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 88 JUNE, 1916

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BY E. A. HOLBROOK

ASSISTANT PROFESSOR OF MINING ENGINEERING

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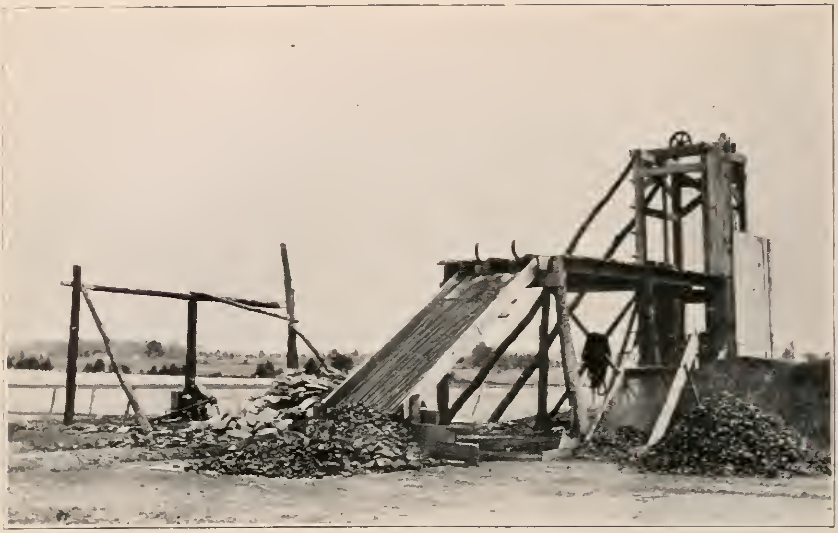

(PHOTOGRAPH BY E. C. LEE.)

FIG. 1. HORSE WHIM AT A LOCAL MINE.

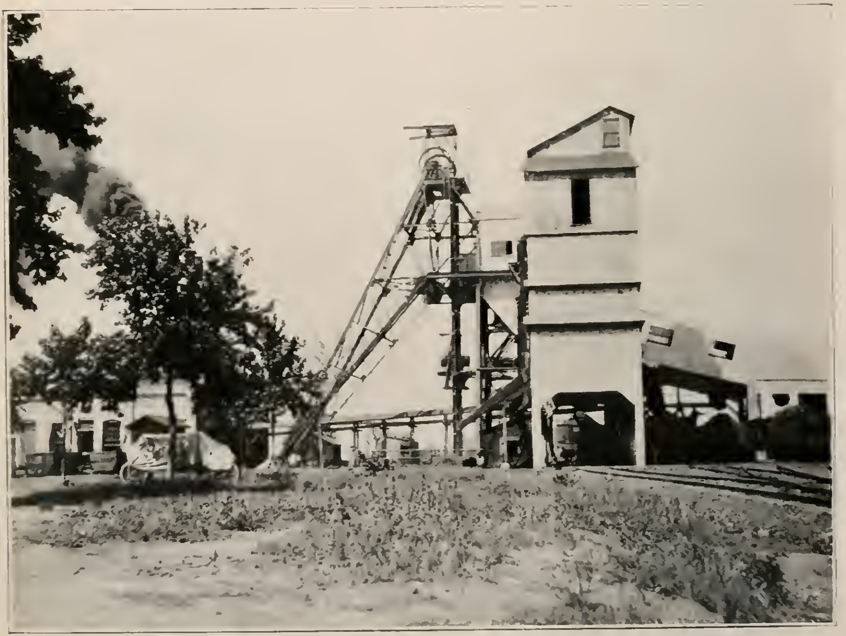

FIG. 2. A MODERN STEEL TIPPLE WITH CONCRETE RESCREENER.

DRY PREPARATION OF BITUMINOUS COAL AT ILLINOIS MINES.

INTRODUCTION.

Coal, as it comes from the mine, is not usually in condition for direct delivery to the consumer, but must be first subjected to treatment or preparation in order to remove impurities and to secure the sizes suitable for the different markets.

"Preparation" of coal is understood generally to include that set of operations which begins after the coal is delivered to the mine tipple, and ends when the loaded railroad cars at the mine are weighed and ready for shipment. It is, however, often difficult to state the exact points at which preparation of coal begins and ends. On the one hand, preparation encroaches on the domain of the miner because much of the impurity may be separated from the coal at the face underground and because of the natural production there of various sizes of coal. On the other hand, coal is subjected to breakage and inspection after being shipped to market, and therefore repreparation may be necessary after delivery and before consumption. Thus, preparation may enter the province of the mechanical or fuel engineer.

In 1909 a committee of the International Railway Fuel Association, appointed to inquire into the difficulties encountered in producing clean coal, reported the causes for poor coal as follows:*

(1) The physical conditions of the seam, mine, and mine equipment.

(2) The class of labor that produces and handles the coal.

(3) The conditions surrounding the sale of the coal, including the prices obtained.

Any or all of these conditions may seriously affect the preparation required by Illinois coal.

In this bulletin the mining and marketing of the coal are discussed from the standpoints of contained impurities and breakage only as these affect preparation in the tipple and auxiliary buildings. Preparation of coal may be divided into two separate and distinct processes: (1) Wet preparation, called coal washing or jigging. (2) Dry preparation, including weighing, screening, dry cleaning, and loading. The first division of the subject has been covered by F. C. Lincoln in Bulletin No. 69 of the Engineering Experiment Station of the University of Illinois, under the title "Coal Washing in Illinois," and will not be considered here.

In the early days of coal preparation in the anthracite regions of Pennsylvania coal was frequently washed or rinsed with a spray of water in order to make easier the detection of intermixed refuse and to give it a better market appearance. Afterwards, cleaning of coal by jigging was introduced and called coal washing, and thus the same term was applied to two different processes of coal preparation, giving rise to some confusion. Since in this state, in at least one instance, coal has been rinsed by a spray of water before hand picking, this process will be called rinsing and not washing, in order to avoid confusion with true coal washing or jigging.

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*Proceedings of the First Annual Convention, 1909, p. 13.

The subject of dry preparation has been divided into two parts, the first of which considers the development, the standard methods, and the products of present practice and is treated in this bulletin. The second part, which will be an engineering discussion of the machinery, appliances, and costs of preparation, will be covered by future work.

The subject matter of the present bulletin is subdivided as follows:

Chapter I. Evolution of present preparation practice.

Chapter II. Standard types of Illinois coal mine tipples or preparation plants.

Chapter III. Impurities and breakage of the coal, making preparation necessary.

Chapter IV. Sizing and sizes of Illinois coal.

This bulletin is not intended as a reference for all the machinery and appliances used in dry coal preparation in this state, but merely outlines the standard practice; therefore the inclusion and description of certain appliances do not imply preferences, but rather indicate that data were available concerning them, that they illustrate the principle of a process, or that they are in common use.

Data for this bulletin were gathered by an inspection of about fifty mine tipples in Illinois during the summer of 1914, followed by visits or letters to the offices of many of the producing coal companies in this field. Access was also had to the data of the Illinois Co-operative Coal Mining Investigation.

The author wishes to thank Prof. H. H. Stoek, in charge of the Department of Mining Engineering, of the University of Illinois, without whose cooperation the bulletin could not have been written. Various engineering firms, including the Allen and Garcia Company, The Link Belt Company, Roberts and Schaefer Company, and the Wisconsin Bridge and Iron Company, all of Chicago, aided freely by suggestions and by drawings. Acknowledgment is due the various mining companies visited for their interest and the uniformly courteous treatment received from their staffs.

CHAPTER I.

EVOLUTION OF PRESENT PREPARATION PRACTICE.

DEVELOPMENT OF COAL PREPARATION.

During the year ended June 30, 1915,* the mines of Illinois produced 57,601,694 short tons of coal, valued at the mines at about $1.14 per ton or at a total value of $65,665,931.16. Of this total about 4,000,000 tons were treated by washing, while 53,600,000 tons were prepared to a greater or less degree in the dry state, principally by screening or sizing, and by picking. The run of mine coal produced amounted to more than 10,000,000 tons or 17.3 per cent of the total; lump coal amounted to 19,200,000 tons or 33.3 per cent; egg coal, 8,700,000 tons or 15.1 per cent; nut coal, 3,800,000 tons or 6.7 per cent; and pea, screenings, and slack coal, 15,900,000 tons or 27.6 per cent. The exact significance of these terms for designating sizes, is explained fully on page 103. Briefly, "run of mine" coal refers to coal shipped as it is mined, all sizes being mixed together. Lump coal refers to the largest sizes of coal, from which the fines have been taken by screening; thus, lump is made over screens ranging in size from a ¾-inch round hole up to 8-inch round hole. If the coarser sizes of screens, with 5-inch or 6-inch holes are used, the coal is passed over another set of screens having perhaps 3-inch round holes, the oversize produced being called egg coal. Nut coal refers usually to sizes made between 1-inch and 3-inch round hole screens. Pea coal is somewhat smaller, generally below 1 inch, and having only the finest sizes, say, under 5/16 inch round or square hole taken out. The smallest sizes of coal remaining after these processes are performed are usually called slack. The term screenings usually refers to sizes of coal passing 2-inch or 1¼-inch round hole screens from which no smaller sizes have been taken. The washing preparation in Illinois is confined entirely to coal under three inches in size.

The growth of the coal mining industry in Illinois, and the increasing importance of coal preparation are shown in Fig. 3, which is based on production figures compiled from the annual Illinois coal reports from 1882 to 1915. It should be remembered in considering these figures that during the earlier years a larger percentage than at present of the total tonnage came from longwall mines, which make less fine coal than the pillar and room mines.

In most bituminous coal fields the first seams mined were thick and contained the best quality of coal with a minimum of impurities. At first only lump coal was considered of value. Later it became necessary to mine relatively impure seams or thinner seams which, owing to the admixture of roof and floor impurities with the smaller amount of clean coal, might produce a coal which is relatively impure and which in either case must be cleaned and otherwise prepared for the market. Consumers also become more exacting as to size and quality of the fuel and the general increasing value per ton permits more careful preparation of the impure coal and of the smaller sizes.

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*Illinois Coal Report, 1915.

Germany, forced to use her lower grades of coal and to mine the thinner seams, led the way by introducing bituminous coal preparation

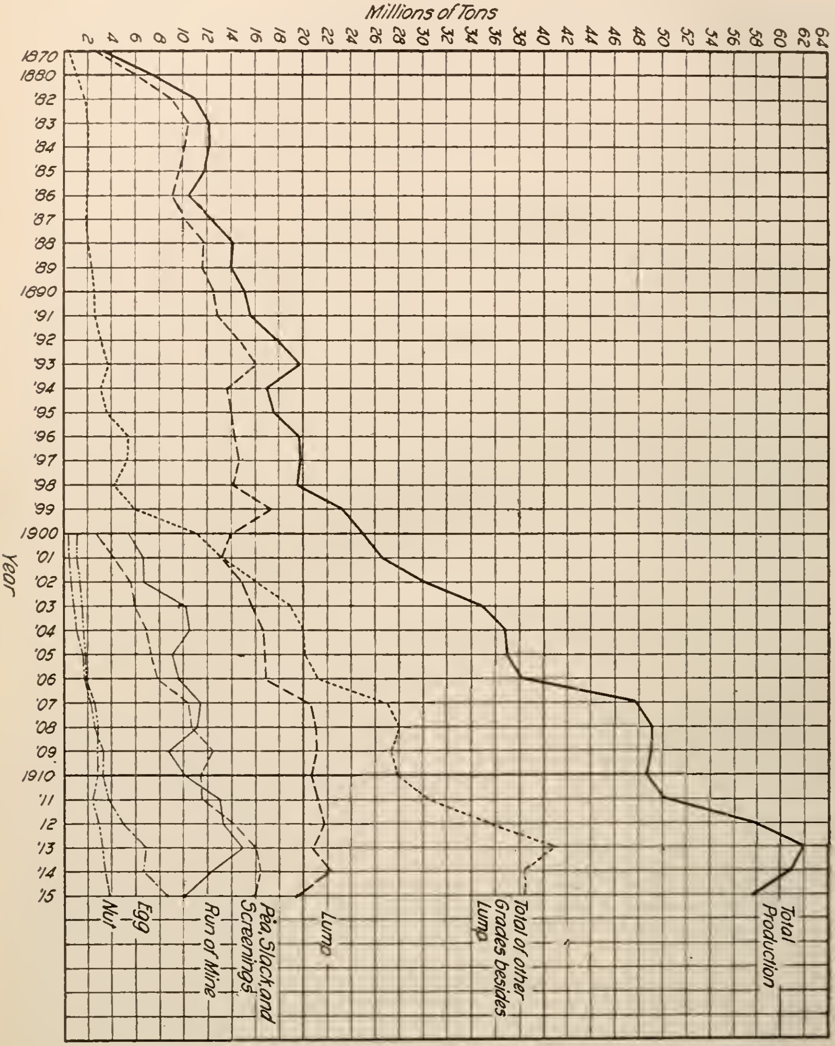

FIG. 3. YEARLY PRODUCTION OF DIFFERENT SIZES OF COAL IN ILLINOIS (1880-1915).

in the decade, 1870-1880. France and England followed as like pressure was felt, until by 1895 preparation by sizing and cleaning was highly developed in these countries. One German colliery at the present time prepares twenty sizes and grades of bituminous coal. Belgium divides her small production into five degrees of quality and into twelve standard sizes. In general, European technique in bituminous coal preparation is more advanced than in this country.

In America the close sizing and extensive preparation of Pennsylvania anthracite have been notable for nearly half a century. Probably the first bituminous region in the United States in which close attention was given to preparation was the Fairmount region in West Virginia, in which sorting and cleaning had become general by 1900.

In Illinois close preparation has been developed although scarcely ½ per cent of the coal resources have been extracted. The attention given to careful preparation has been increasing since the early nineties, the progress during the past seven years having been especially marked. The causes for this development are:

(1) The introduction of state laws regulating the weighing of coal and the basis of payment to the miner for his coal.

(2) The various struggles and consequent agreements between operators and miners dealing chiefly with payment for mining and with the cleaning of coal.

(3) The demand of the consumers, who, having become educated by the publicity given during the past few years to the purchasing of coal on specifications, are no longer content with the grades of coal they received a decade ago.

(4) The campaigns waged by the cities to abate the smoke nuisance.

(5) The excessive competition among producers, caused not only by the operation of too many mines and the consequent desire to keep these in constant operation; but also by the maintenance of highly organized selling departments, which have a tendency to introduce new sizes and trade names for coal.

(6) The general introduction of improved machinery used in coal mine tipples to prevent breakage and facilitate preparation.

HISTORICAL DEVELOPMENT OF ILLINOIS PREPARATION PRACTICE.

Early History and Methods.—Before the general advent of railroads in the middle of the 19th century coal from Illinois mines was transported largely by boat on the Illinois and Mississippi rivers. Gordon Buchanan* states that the early railroads hauled coal into Chicago with engines that burned wood. A large proportion of the tonnage in these early days came from the northern or longwall districts in which the proportion of large clean lump or chunk coal is high. This coal requires little preparation. The earlier U. S. Census Reports, particularly those of 1870 and 1880, give statistical information concerning Illinois production, but, since little weighing was done at the mines at that time, such information probably is not so accurate as that contained in later statistical records.

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*Black Diamond, May 17, 1913, p. 16.

Little information concerning the preparation of Illinois coals prior to 1882 is available. The second biennial report of the Illinois Bureau of Labor Statistics, published in 1882, contains the first annual report on the coal industry of the state, although the first biennial report, published in 1880, contains the reports of coal mine inspectors to the governor of the state for the years 1879 and 1880. From 1882 until 1911 the Bureau issued each year an annual report or compilation of statistics on the coal mining industry of the state. After the first two years it became the duty of the various mine inspectors to gather data, which was in turn compiled by the secretary of the Bureau. Since 1911 these annual coal reports have been issued by the State Mining Board.

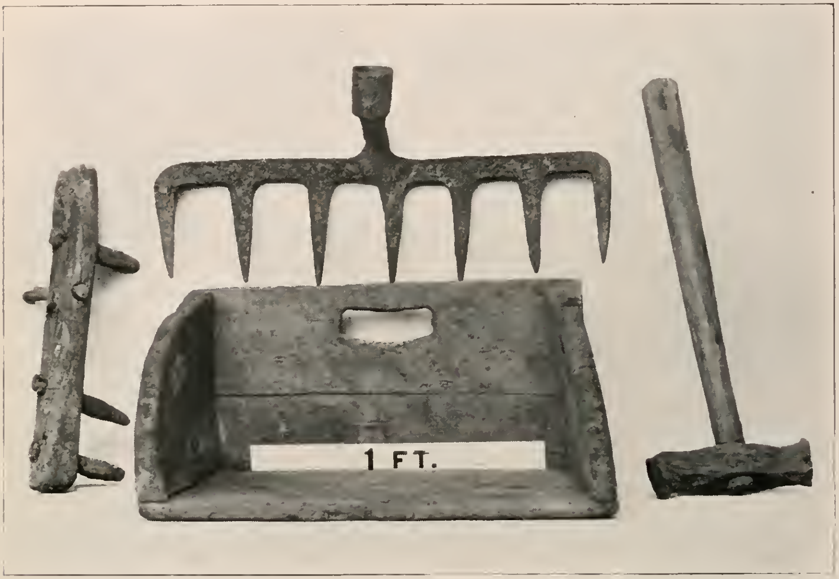

FIG. 4. THE MINER'S RAKE (WOODEN AND IRON). PAN AND SLEDGE. MUSEUM, DEPARTMENT OF MINING ENGINEERING, UNIVERSITY OF ILLINOIS.

The expression "coal is coal" might appropriately be used in speaking of the early periods of mining in Illinois, since the practice of selling coal as brought to the surface with no attempt at preparation before shipment was generally followed. A rough separation of the fines from the lump coal took place underground at the face and during the period of loading the coal into the mine cars. Instead of the customary miner's shovel of today the coal was loaded with a tined fork, having spaces between the tines of from ¾ inch to 1½ inches. This allowed the finer coal to pass through the fork, after which it was thrown back into the gob and left in the mine. While such a fork is still used at coke ovens to free the coarse coke from the "breeze" during the loading process and is also used by retail coal dealers for the purpose of delivering clean coarse coal when necessary, it has passed out of use in the preparation of Illinois coals, excepting in some of the small local and coöperative mines.

The common round hand screen, or coal miner's riddle, with square mesh wire cloth, having perforations varying from ½ inch to 2 inches in size, was also used by the loaders underground for screening out the smaller coal which was to be left in the mine and for carrying the coarser coal to the mine car.

The rake and pan method of loading coal was formerly in common use in Illinois mines.* After the largest lumps of coal had been loaded into the mine car by hand, the remainder was raked onto an iron or wooden plate or pan and transferred to the mine car, while the fine coal and dirt not gathered by the rake were left in the mine. "The pan was in fact a hand scoop made of sheet wrought iron or steel, the front end being flat and open, the sides being usually curved outward and upward with handles affixed for lifting and carrying; they were also made with straight sides. The back of the pan was straight up. The conditions under which the pan was used varied its dimensions. The purpose of the pan was to have only clean lump coal carried to the pit box or car; and whether this coal was loaded on the pan by hand or scraped on by a rake or fork, the end attained was the same. In some parts a penalty was attached to any person found loading coal with a shovel."†

Fig. 4, is a photograph of an old pan, part of a wooden rake, an iron rake, and a sledge recently found in abandoned workings in the Belleville district, and presented to the Mining Museum of the University of Illinois. These implements were practically out of use by 1884. They were, however, used in a few mines until the strike of 1897, after which their use was discontinued.

Another common method of preparation underground was by the use of "grills" or wooden bar screens. These were wooden bars, spaced ½ inch to 2 inches apart, nailed into a wooden framework, and set up at an angle of about 45 degrees. The coal, before being loaded was shoveled against these bars. The finer coal which passed through the bars was left in the mines and the oversize was rehandled and loaded. Similar practice may be seen with the common gravel screen of today.

The forms of underground preparation described made unnecessary the loading and hoisting of a product then considered useless; namely, coal equivalent in size to the screenings of today. Although a limited amount was utilized in the eighties, screenings were not even considered as a coal in the tonnage reports of the State Bureau of Labor Statistics until 1891, and in some districts it was not until 1900 that they became a commercial product. The use of screenings began to assume important proportions about 1895. The total estimated production of lump coal for the period from the beginning of the industry (about 1830) to 1895 is 273,000,000 tons. The first report including screenings in the tonnage (1891) showed that, considering the state as a whole, 17.24 per cent of the coal produced was screenings. If this average be accepted for the preceding years there was probably a total of more than 57,000,000 tons of this small coal, which was either mined and left underground to be buried beneath the falling mine roof and consumed in gob fires or, if separated on the surface, was hauled to dump piles and there destroyed by spontaneous combustion.

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*Report of Ill. Bureau of Labor Statistics, 1897, p. xxxv.

†W. L. Morgan, Ex-State Mine Inspector. Personal communication.

When the mines grew, when steam hoists and improved systems of underground haulage were generally introduced, and when labor cost underground and the value of fine coal or screenings increased, the coal was hoisted as mined and prepared in the mine surface buildings or tipple. The introduction and use of the automatic stoker with fine hole or chain grates and other special grates which made possible the generation of power from the finer sizes of coal were important factors in the increase in use and value of these small sizes of coal. Such devices were introduced into Illinois about 1890 and during the succeeding ten years their use in large power plants became general. Then the consumer with the average steam plant realized the possible saving through their use in connection with cheap coal screenings, and within a few years these stokers became common. Now, there is a constant demand for the once despised waste product, coal mine screenings.

Weighing Practice.—Formerly payment to the miner and operator alike was made almost wholly on the basis of the bushel, there being usually 25 to 28 bushels to the ton. This standard is still used at some of the country "banks" or local mines. In some places an arbitrary standard was based on the volume of the box or contents of the mine car. An early report* records prices paid to the miner based not only on the short ton of 2,000 lb., but also on tons of 2,050 lb., 2,100 lb., 2,200 lb., and 2,250 lb.

By 1880 some of the larger mines had adopted weight by scales as a basis of payment, such weights being taken by company men, while others still adhered to the volume basis. This naturally led to considerable trouble between operator and miner, which, together with the difficulty experienced by the state in gathering adequate statistics concerning tonnage, led to the passage, in 1883, of a law governing the weighing of coal at the mines. (Ill. L. 1883, p. 113.) The act required that all coal companies in the state, shipping coal by rail or by water, should provide standard track scales at the mines and should weigh all coal hoisted before or at the time of loading for shipment. The weights so determined formed the basis upon which the wages of the miners were computed. The miners could employ their own check weighman who should be an employee at the mine. The act seems to have referred to lump coal only, since screenings were not considered salable coal. The act also declared that all contracts for mining coal not based on the stipulated requirements should be null and void. This was the first attempt in Illinois to regulate preparation practice.

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*Report of Ill. Bureau of Labor Statistics, 1885, p. xxvii.

Although most of the shipping mines complied with the act, it was immediately attacked in the courts, and though held constitutional, it was declared to apply only to mines at which weight was accepted as a basis of payment (Reinecke v. People, 15A, 241). In the case of Jones v. People (110 Ill. Rep. p. 590) the law was declared to have no application if the wages of miners were computed on a basis other than that of weighing; namely, on the basis of volume; and it was not held to require miners' wages to be based on the weight of coal mined.

The law was amended (L. 1885, p. 221) by the addition of a provision requiring shipping mines to keep their weights on record for inspection by miners, inspectors, and other interested persons. The law as amended did not require the check weighman of the miners to be an employee at the mine in question, but stipulated that both the company's weighman and miners' check weighman must make affidavit faithfully to weigh and record the coal. In the test case of Millet v. People (Ill. Rep. 117, p. 294) the court decided that if an operator bought and sold coal by weight, the law compelled him to keep reliable scales for that purpose, but it did not oblige him to make contracts for coal on a basis of weight. Moreover, it was declared that the requirement that operators should keep a record of weights for public information was the taking of private property for public use without a provision of just compensation and therefore was unconstitutional.

A bill passed in the state (L. 1887, p. 235) repealed the former laws of 1883 and 1885 and substituted a new law of different wording but having about the same practical effect. It provided, "that at all mines, where miners are paid by weight, a standard scale shall be provided for the weighing of all coal hoisted or delivered." According to this law the check weighman should be an employee of the operators of the mine, and all coal delivered by the miner should be weighed and the records kept open for the inspection of the miner and other interested persons. This act was held unconstitutional (Harding v. People, Ill. Rep. 160, p. 459) because it singled out an especial class of mines, and interfered with the right of the operators and miners to contract among themselves.

A new law, designed to correct the weaknesses of the old one, was passed by the legislature in 1899 (L. 1899, p. 301), and revised in 1911 (Ill. Stat. Ann. Ch. 93, 7501). Its provisions are substantially as follows: Operators at mines where miners are paid by weight of their output shall provide accurate scales for weighing such coal and the record so obtained shall be open to inspection by interested parties; a sworn weighman shall be provided by the company; and the miners may provide a sworn check weighman. This privilege is taken advantage of uniformly throughout the state, and there are today never less than two men in the weighroom of the mine tipple. As a result all contentions as to false weights, so prominent in the past, have disappeared.

At the present time, as a rule, at mines where the pit cars hold 1½ tons or more, the weight is read to the nearest 100 lb.; that is, the weighman and check weighman "give and take" on 50 lb. If the load weighs 4,135 lb., the miner is credited with 4,100 lb., and the company gains the extra 35 lb.; if the weight is 4,165 lb., the miner is credited with 4,200 lb. In mines with cars holding less than 3,000 lb., the weight is generally read to the nearest 50 lb., with "give and take" on 25 lb. The fact that at some mines three cars per minute are weighed, checked, and dumped from the tipple weigh box, makes it evident that considerable care and engineering skill have been devoted to bring tipple weighing to a degree which is nearly perfection. The beam scales formerly used have in many cases been replaced by the self-indicating dial scales, and in some of the newer tipples by automatic self-recording weighing devices. Thus, a just and equitable system of weighing, giving satisfaction to operator and miner alike, has been evolved.

Lump Coal vs. Run of Mine Payment.—During the same period in which the troubles over weighing occurred, a still sharper fight was being waged with reference to the quantity and kind of coal for which the miner should be paid. It had become customary at some of the larger mines to hoist all the coal mined, and to roughly clean and prepare it above ground before loading into railroad cars.

The common method of procedure then followed in Illinois, and still followed in Indiana, Michigan, Western Pennsylvania, and several other coal producing states, was to dump the coal from the mine car over a bar screen in the tipple. This bar screen consisted of a row of iron bars about ⅝ inch wide and 2 inches thick, set on edge with a space of about one inch between the individual bars, the whole rack being possibly 6 feet wide and from 8 to 15 feet long, and set at an angle of from 26 to 45 degrees. The oversize from the bar screen, or coal too coarse to pass between the bars, was weighed and sold as lump coal and the miner was paid, if by weight, according to the lump coal so produced. The finer coal passing through the screen bars, called slack or waste, was usually given away or hauled to a dump in the neighborhood. There was no market for this product since it could not be burned on the type of furnace grates then in use.

In the early eighties attempts were made to recover a part of these screenings. At a number of mines installations were made at the tipple by means of which the screenings were elevated and rescreened in a revolving screen, having holes about ¾ inch in diameter. The material passing through this screen was called slack and was discarded. The oversize, varying from about ¾ inch to 1 inch more or less in diameter, was called nut coal. This coal was in considerable demand, especially as a domestic coal and was used by miners and by the public if the mines were located in the neighborhood of a city. The records show that by 1887 most of the large mines in the central part of the state were equipped to rescreen the small coal, and the value of this rescreened, or nut coal, as noted in several contracts made between miners and operators, was about one-third that of the lump. The proportion of nut coal so recovered to the lump varied with the districts, the method of mining, and the size of screen used, but was estimated as 13 per cent of the lump.* This treatment of coal, making lump as oversize on the bar screen, nut as oversize, and slack as undersize in the revolving screen, probably constituted the fullest dry preparation that coal received in 1887, excepting in one or two cases in which four sizes were made, two from the lump and two from the screenings, to meet the demand of domestic city trade. By 1891 several of the city mines were making three distinct sizes of screenings. The largest, called nut, was above 1 inch or ¾ inch in size; the medium size, called pea, below nut and above ½ inch or ¼ inch, and the slack below this size was thrown away.

T. B. Comstock,† writing in 1887 on coal mining in Illinois, said that Pennsylvania methods were followed blindly; that market rating of coals was based on crude trials in unskilled hands; and that one of the subjects just beginning to attract attention was the sizing of coal for market. He noted that assorted products from one or two mining plants in the state threatened to revolutionize the trade. At these plants small portable crushers and screens were placed at the car door to prepare the coal before loading, but probably the extra cost was not covered by the increased profit. Commenting on the fact that washing and other methods of preparation received little attention, chiefly because consumers did not recognize the enhanced values of prepared coal, he prophesied, "The time will come when these advantages will appear as necessities."

At a few mines in the state miners were paid for gross weight of coal hoisted, an allowance agreed upon being made for slack. For instance, at Oglesby, in the longwall field, where conditions favored a minimum of slack, 36,000 lb. gross of coal were required to obtain payment on 30,000 lb. of lump, an allowance of 16⅔ per cent being made for slack. At Mt. Olive, in the central field, 20 lb. of coal were deducted for slack from each 100 lb. mined, an allowance of 20 per cent. However, these were isolated instances, as four-fifths of the product of the state was screened, the oversize only being weighed and paid for. The thought that they did not share in the revenue from this merchantable nut coal was a source of considerable grievance among the miners, but it was overshadowed by a greater one; namely, the lack of uniformity in width of opening and area among the bar screens at the various mines.

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*Report of Ill. Bureau of Labor Statistics, 1888, p. 331.

†Eng. and Min. Jour., Vol. 44, p. 24.

In 1885 the secretary of the Illinois Miners' Protective Association issued a report, part of which dealt with the miners' grievances against the lack of uniformity of weighing, screening, and preparing coal for the market, and in 1886, the report of the Illinois Bureau of Labor Statistics (p. 549) dealt at length with the question of screens. The report, of which the following is a condensed table, covered screening practice at 218 mines in the state.

TABLE 1.

SCREENING PRACTICE IN ILLINOIS IN 1886.

Number of Mines	Tons Screened Lump Coal Produced	Space Between Main Screen Bars	Tons of Nut Produced by Second Screen (Estimated)	Tons Total Product Lump and Nut	Per Cent of Nut Coal (Not Paid for by Operators)
12	175,425	½"- ¾"	20,579	196,004	10.0
98	4,352,252	⅞"	516,631	4,868,883	10.6
23	437,074	1 "	62,972	500,046	12.6
10	354,305	1⅛"	60,356	414,661	14.6
44	1,198,739	1¼"	224,417	1,423,156	15.6
25	665,533	1½"-1⅞"	167,220	832,753	20.1
6	175,409	2"-2½"	49,705	225,114	22.1
Total 218	7,358,737		1,101,880	8,460,617	13.0

This table represents 80 per cent of the production at that time, based on lump coal. By far the most common width of bar space was ⅞ inch, 62 per cent of the product being so prepared; in fact, this width of bar space was recognized as a standard in several parts of the state. The percentage of coal passing such a bar screen is about equal in amount to that passing the 1¼-inch round hole screen common today, except that the shapes of the larger particles of coal are of course different. This common screen, however, was overshadowed by those with 2-inch and even 2½-inch spaces. Such a screen must have allowed a considerable proportion of the miner's coal to pass through, probably an amount equivalent to that passing a 3-inch or even a 4-inch round hole screen. The average area of screen used in Illinois was about 60 sq. ft., usually 5 ft. wide and 12 ft. long, but certain screens were at least 16 ft. long and some had an area as great as 130 sq. ft.

This condition existed generally throughout the country. At 146 mines,* taken at random from bituminous mines in the United States, the screens used were mostly of ⅞-inch or 1¼-inch bar, though some had spaces as large as 2½ inches. Forty-four of these mines produced and paid on a run of mine basis, while 102 were operated on the screened lump payment basis.

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*Mineral Industry, Vol. 1, 1892, p. 80.

The cleanness of separation of lump and screenings was further affected by the slope of the screens. If the screens were set at a low angle the coal passed slowly over them,—in some cases having to be pulled over, and thus a maximum of coal was sent into the nut or through sizes. One or two strikes were caused because chains or logs were hung loosely close above the bar screens. According to the operators, the lump coal sliding down the bar screen with considerable force was turned over and freed from adhering dust; according to the miners, the lump coal was broken by such devices, and passed through the screen into the nut sizes for which they received no pay.

Such lack of uniformity in the preparation of coal led the miners to demand that screens be made of uniform size and opening, and that payment be made for nut coal produced. They claimed that the average realization to the operator for nut coal was five cents per ton of lump made. Their grievances on these subjects were: (a) The practice of changing screen openings from time to time. (b) Screens of largest size were found where they were least justified by market conditions. (c) The operator using a screen of only sufficient size to clean his coal could not compete with one using a screen which passed enough nut to pay his expenses. (d) The percentage of coal going through the screens was larger than that necessary to clean it. (e) The operators encouraged the use of an excessive amount of powder in order that they might receive the benefit of the large percentage of small sizes made by it, and an extra profit on the excess powder. (f) The operators tried to force the loading of the fine coal for which the miners received no pay.

The operators claimed: (a) Coal had to be screened to get a merchantable grade of lump. (b) Fair miners' wages were based on the percentage of lump and nut sold, even though the miners were nominally paid for lump only. (c) Since different degrees of friability of coals, and different methods of mining, tended to produce a variation in the percentage of finer sizes, a variation in the size of screens was necessary to make a uniform coal for a common market. For instance, coal shot off the solid as in the central part of the state made more fines than that mined longwall and wedged down as in the northern fields. (d) It was necessary to load out the fine coal to prevent gob fires.

Three remedies were proposed: (a) To use throughout the state a uniform bar screen of not more than ⅞-inch space. (b) To pay miners pro rata per ton for whatever proportion of the product was sold. (c) To weigh and credit to the miners all coal before screening.

Although the first of the above met with considerable favor, in 1883 a bill requiring the use at all mines of a screen of uniform dimensions and width between bars failed to pass the State Legislature. In the light of the history of coal preparation revealed during the next few years, it is a question if the passage of this bill might not have prevented the difficulty caused by the increased percentages of fine coal and refuse.

Legislation was then proposed to weigh and credit to the miner, before screening, all coal hoisted. The operators protested against payment on the "mine run" basis, their arguments being: (1) Installation of new scales and tipples would be necessary. (2) Domestic coal must be lump to command a good price. (3) An unskillful or careless miner might make double the amount of slack made by a careful one. (4) A dishonest miner could load dirt and rock in the bottom of his car. (5) The excess of powder used would shatter the coal. (6) The proposal amounted to offering a premium for dishonest work. (7) They would suffer on account of time wasted by weighing in the tipple. (8) If a carpenter is paid for his day's finished work, and not for the chips he makes, why should a miner be paid for the slack he makes?

To this report a reply was issued, signed by a committee of the United Mine Workers of Illinois, as follows:

(1) The miners' organizations during the last few years had forced some reduction in the size of screens used. (2) Where no miners' organization existed, screens were large enough to let through good sized coal. (3) Some of the mines had installed breakers to crush lump coal, proving that fine coal was not bad. (4) There was no law in Illinois limiting the size of coal screens, which varied from ⅞ inch to 2½ inches between bars and had an area of from 50 to 130 sq. ft. (5) Miners were forced to sign contracts stating that screens might be widened without violating such contracts between operator and miner. (6) Wages were based on the price of lump coal, but the operator derived an enormous revenue from screenings which was clear profit to the owner. (7) It was shown that at one mine on the Illinois Central Railroad, of 1,095 tons hoisted, 509 tons or 46 per cent, passed through the screens. (8) The allegation that practical and trained miners would attempt to produce slack was denied.

The same question of run of mine vs. lump coal payment was fought out in Ohio in 1913. A coal mining commission was appointed by the governor to inquire into the merits of the matter. In its report the commission took up exactly the same arguments for and against as those advanced in Illinois. Twenty-five years have thrown very little new light on the subject, the only additional arguments presented by the miners in Ohio being along the line of conservation. They claimed that under the lump system valuable coal was left in the ground, that pillars could not be economically robbed because of the undue crushing of the coal in them, and that they were unable to make fair wages under these conditions. The only new argument of the operators was that a trial of many years had disclosed the detrimental effects of the mine run law in Illinois.

Whichever side had the better of the argument in Illinois, on June 10, 1891, the Illinois Legislature (L. 1891, p. 170), passed a gross weight law, making it unlawful for an operator, whose men are paid on the basis of quantity of coal, to take any portion of the same by any process of screening, or by any other device, without fully crediting the same to the miner. The law required the weighing of all coal in the pit cars before being dumped, and gave the miners freedom to choose their own checkweighman. Nearly all the mines which paid their miners by weight complied with the law, which, "as a whole, has not seriously inconvenienced the business in spite of the claims of the operator that weighing before screening was impracticable without serious reconstruction of the surface plant."*

Generally in the American bituminous coal fields, as long as the coal has been paid for on a screened coal basis, little attempt at detailed preparation has been made. When paying for run of mine coal, however, the operators are forced, through a series of new conditions, such as necessity of disposing of the finer sizes, or by an increase of refuse in the coal, to make greater refinement in its preparation. Moreover, when the operator pays the miner for all his coal regardless of size, he may more easily install whatever system or make whatever sizes best meet his market conditions, since he has no agreement with the miners concerning screens.

Operators in Illinois were now free to screen and prepare the coal as they wished. Consequently, new devices for preparation were introduced among which was the shaker screen (see p. 44), which replaced the bar screen in a number of plants. The first screen of this kind in the state was probably installed in 1890 at the Gillespie Colliery of the Consolidated Coal Co. Since that time the old bar screens have been steadily replaced by shaker screens. As late, however, as 1893, a shaker coal screen exhibited at the World's Columbian Exposition attracted considerable attention. At a number of plants the bar screen was replaced by a revolving screen. It was found, however, that lump or larger sizes of coal were badly shattered during passage through this type of screen, and their use for lump coal was gradually abandoned, until at the present time the writer does not know of a single plant in the state which uses such a screen for the separation of the coarser sizes. For screening the finer sizes of coal, under 3 inches, which are not so much affected by breakage from fall, the revolving screen held its own until a very recent period.

In 1892 the screening or gross weight law of 1891 was attacked in the courts (Ramsey v. People, Ill. Rep. 142, p. 380) and declared unconstitutional, since "it required, regardless of contract, payment on the weight of coal before it was screened, and it thus so far limits the power of employers and employees to contract, as to deprive them of property and rights without due process of law." In spite of this decision, many mining plants which had been equipped for the gross payment basis continued to use it in preference to the old method, partly because consumers had begun to demand a variety of sizes, which could be produced more easily with the new equipment. In some cases as a result of a decided increase in the percentage of screenings and contained dirt, a reversion was made to the older method. As contracts between operators and miners were generally of a local or district nature, such changes were quite easily made. On the whole until 1897 the pendulum swung towards the abandonment of the gross weight system.

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*Report of Ill. Bureau of Labor Statistics, 1891, p. 47.

In 1895 new complaints were made against the enlargement of screen apertures, and it was claimed that the weighing of the coal was inaccurate. The annual reports of the State Board of Arbitration for 1895, '96, '97, and '98 cite numerous instances of coal mine strikes, the principal causes being those just mentioned and the renewed contention by the miners for payment by the gross weight system. The case was aggravated by the general industrial depression during these years and by a decreased demand for Illinois coal, due in part to natural gas displacing coal as a house fuel in the Chicago market,* to smoke ordinances passed in the cities,† and to the increased use of Ohio and West Virginia coal in Chicago and of Missouri coal in St. Louis.‡

On June 3, 1897, the Illinois Legislature passed a new gross weight bill, designed and worded to overcome the legal objections to the law of 1891 and to accomplish all that was intended by the previous law. The law (L. 1897, p. 270) provided "that every person engaged in mining coal . . . . shall be paid in lawful money . . . . for all coal mined and loaded into the mine car by him, including lump, egg, nut, pea, and slack, or such other grades as said coal may be divided into, at such price agreed upon by the respective parties."

As in previous cases the act was attacked in the courts (Whitebreast Fuel Co. v. People, Ill. Rep. 175, p. 51). Since it was held constitutional and is the basis for present laws, not only in this state but elsewhere, it is interesting to note that, "the application of the act does not extend to cases where there is a contract for compensation upon a different basis than that specified in the act, and the employer and miner are free to contract at such price as may be agreed upon by the respective parties. It does not require that the coal shall be weighed, or that the same price shall be paid for different grades."

The efforts of the miners to obtain their demands and to bring their contracts under the provisions of the act helped to bring on the great strike of 1897, which began in Illinois July 4, 1897, and

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*Min. Ind., Vol. 1, 1892, p. 80.
†Min. Ind., Vol. 4, 1895, p. 164.
‡Min. Ind., Vol. 7, 1898, p. 185.

lasted about 100 days. The strike resulted in an almost complete victory for the miners, district after district acceding to their demands for higher wages and payment on the gross weight basis when the same was not already in force. The greatest change perhaps was made in the longwall field. On November 22, 1897, a joint conference of miners and operators of this field was held in Joliet, and the gross weight scale was adopted for the entire field. The system of paying for all coal mined, provided for in terms of settlement, was considered important enough to justify the miners of the northern field accepting a rate for mining a few cents less per ton than agreed upon at the previous State Convention.

The report of the Illinois Bureau of Labor Statistics for 1897 contains a full history of the strike. From the viewpoint of preparation, before the strike at 156 of 284 mines in the state, miners were paid on the gross ton basis and at 128 on the screened ton basis. After the strike 198 mines paid the miners on the gross ton basis and only 86 paid on the screened ton basis. These totals included 86.5 per cent of the total coal mine employees in the state, and probably about the same percentage of the total tonnage. The large mines were now all in the gross tonnage column. Little change resulted in the southern fields, which previously had been gradually changing to a mine run basis, based either on weight or on volume.

Operators' and Miners' Agreements.—The miners of the state had now become firmly organized as District No. 12 of the United Mine Workers of America, and following the national convention of that order at Chicago, January, 1898, District No. 12 met the Illinois coal operators associations at Springfield, February 24-26, 1898. The wage scales adopted at this joint conference by the different state districts were "understood in every case to mean that coal is to be weighed before screening and the system of paying miners by the ton of R. O. M. (run of mine) coal shall obtain in all the mines in the state of Illinois.* Thus the present system was adopted in Illinois, not so much through the strictness of the gross weight law, as by agreement between the operator and the organized miner.

The next year (1899) the first complete state agreement regarding mining prices and general conditions was adopted at a meeting between operators and miners held in Peoria. Concerning cleaning of the coal, no reference was made in the agreement regarding the mining of impurities or the increase in amount of screenings. Testimony given at this meeting, however, showed that at Streator the percentage of screenings increased 15 per cent and at La Salle from 2 to 5 percent through the same screens, as the result of a year's trial under the new gross weight system. There was, for the last time, some talk of accepting a double standard; namely, one price for lump and another for screenings, but no action was taken.†

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*Minutes of Springfield Conference.

†Proceedings Joint Convention, Dist. 12, U. M. W. of A. and Operators, Peoria, 1899.

This first year under the new system has been called "an era of good feeling in the coal mining history of the state." The old grievances and troubles had disappeared and no new ones had arisen as yet. By the time, however, that the 1900-1901 agreement was made the modern refuse question had become so acute that the following clause was inserted: "Where slate, bone, etc., are sent up by the miners, it is the duty of the car trimmer or inspector, who shall be a member of the miners' union* and appointed by the operator, to call the attention of the weighman and check weighman to the same. If they agree, the offender shall be fined fifty cents for the first offence; one dollar for the second; and two dollars for the third or subsequent offence, or discharged by the company; providing that no miner shall be discharged unless he is guilty four times in one month."

This clause was evidently not sufficient to enforce the loading of clean coal since the Peoria agreement for the year ending March 31, 1903, contained more elaborate rules regarding preparation. These rules, except for minor changes, are in force today. The pertinent sections, as given in the 1915-16 ageement, are as follows:

Section 4. "The scale . . . shall be per ton of 2,000 lb. R. O. M. (run of mine) coal, practically free from slate, bone, and other impurities."

Section 5. (b) "The system of paying for coal before screening was intended to obviate the many contentions incident to the use of screens and not to encourage unworkmanlike methods of mining and blasting coal, or to decrease the proportion of screened lump, and the operators are hereby guaranteed the hearty support and cooperation of the United Mine Workers of America in disciplining any miner who from ignorance or carelessness or other cause fails to properly . . . load his coal."

Section 6. (a) "In case slate, bone, clay, sulphur, or other impurities are sent up with the coal by the miner, it shall be the duty of whomever the company shall designate as inspector to report the same, with the estimated weight thereof, and the miner or miners so offending shall have such weight deducted from the established weight of the car and for the first offence in any given calendar month shall be fined fifty cents; for the second offence in the same month he or they shall, at the option of the operator, be fined two dollars and for the third or any subsequent offence in the same calendar month, he or they may be fined two dollars or be suspended for not to exceed six days of mine operation.

(b) "For a malicious or an aggravated case, ... the operator may either indefinitely suspend or discharge."

The term "malicious or aggravated case" is then defined "as a case in which the quantity, character, or appearance of the impurities indicate that they were loaded with intentional carelessness or wrong purpose. In case of discharge for an aggravated case as above, the inspector shall preserve the impurities for 72 hours, Sundays and legal holidays excepted. All impurities subject to being docked shall be preserved for the balance of the working day except at mines where it is impossible to do so without seriously impeding the mine. Where it is claimed by the operator that to so preserve the impurities will serously impede the output of the mine and where it is claimed by the miner that the case is not a malicious or aggravated one, the question shall be taken up jointly for determination."

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*The 1904 and subsequent agreements provide that the company coal inspector "shall not be a member of the union."

(c) "The company weighman shall post in a conspicuous place at the pit head the names of all miners dealt with hereunder."

(d) "The inspector designated by the operator . . . . shall not be a member of the U. M. W. of A., and in the discharge of his duties shall not be subject to the jurisdiction of the union. Any miner . . . . seeking to embarrass the inspector . . . . shall, at the option of the operator, be suspended two days."

(f) "The proceeds from all fines shall be paid to the miners' subdistrict treasurer, and . . . . shall not be remitted."

"The foregoing is designed to secure to the operator the loading of clean coal, while protecting the miner from any abuse of the penal code."

It is evident from the various clauses noted that every possible means is being taken to compel the loading of coal free from impurities. The present demand of the consumer for clean coal has made it necessary that care be taken to load, at least as regards lump coal, only pieces that will pass inspection by the company coal inspector, or as he is often called, the "dock boss" or "rock man."

Regulations concerning fine coal and screenings have not been so thoroughly worked out. At present the only provisions in the operators-miners agreements regarding such coal are:

Section 5. (c) "That all 'bug dust'* or machine coal cuttings when practically free from impurities shall be loaded out with the snubbings† or other coal so as to produce a merchantable R. O. M. coal."

"The above does not contemplate any change in the present method of handling bug dust or machine cuttings in Franklin county, or other mines when it is necessary to load the same out before shooting the coal, as a protection against explosions or fire."

"Where the operator desires the bug dust loaded out separately, this shall be done by the miner working in his place during his regular shift at the regular tonnage price and the company shall furnish cars promptly to load the same."

Much of the roof, "draw slate," floor, and bands which comprise the bulk of the impurities mined with the coal either break into small pieces or tend to soften quickly on exposure to the air. Any piece of such impurity smaller than 1¼ inches in diameter passes into the screenings during the screening process of preparation. Since the impurities are small and pass quickly from sight through the screens, and since they mix with the dust and multitude of small pieces of coal which make up the screenings, they so blend with the prevailing black that close inspection is impossible. It is sometimes easier, while loading, to break a piece of flat shale with the back of a shovel so that the pieces will enter the screenings, than it is to pick this same piece out of the coal and throw it into the gob.

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*See p. 101.

†See p. 106.

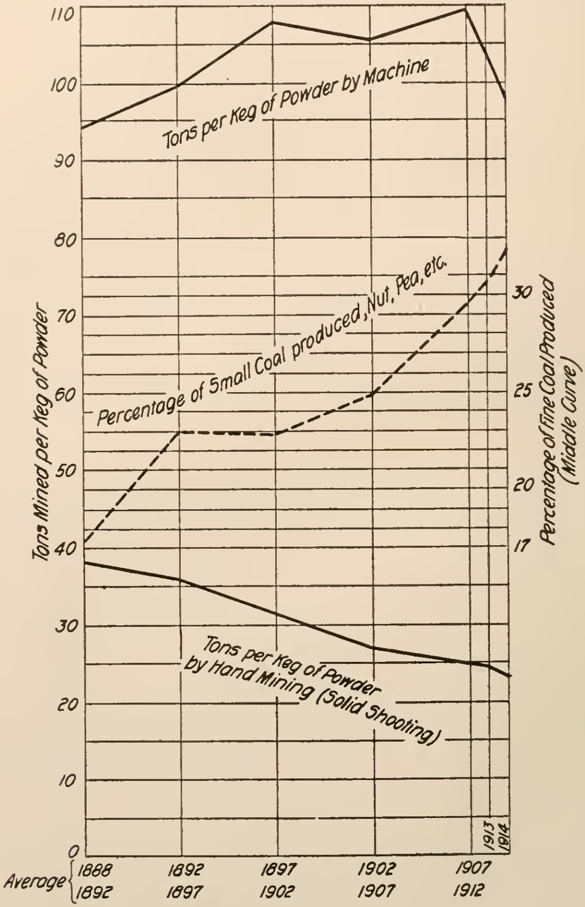

FIG. 5. TONS OF COAL MINED PER KEG OF POWDER (1888-1914).

From a large number of analyses the Department of Applied Chemistry at the University of Illinois determined that, on the average, Illinois screenings contained about twice as much ash as the lump coal from the same mine. The writer has observed a like difference, varying considerably as to daily conditions at the individual mine. Samples of screenings taken on different days from the same mine often vary several per cent in ash content. The mere fact that a coal is of screenings size does not necessarily impair it for use in modern furnaces, but the value of screenings generally is lowered and sale retarded by a high and uncertain refuse content. Impurities in screenings, even though concealed to the eye, appear to their full value upon chemical analysis or in the furnace ashes. Thus, the increasing tendency of consumers to insist upon ash analysis and specifications is based on reason and is certain to continue.

The coal from some of the mines in Illinois is held in disfavor in the general market, and a differential price is made against it. Some of this coal is in every way equal in quality to that from more favored mines. Investigation reveals that carelessness in loading impurities and negligence in enforcing inspection are the main reasons for this condition.

Another clause of Section 5 in the recent joint agreements is of interest, since it relates to the effect of powder used on the sizes of coal produced. It is given as follows: "Where practicable, miners shall shoot coal with two pounds of powder or less."

In general, little or no powder is used in longwall mines, the coal being loosened by roof pressure or by wedges; in mines in which the coal is undercut only a moderate amount of powder per ton of coal is used; while in mines in which the coal is "shot off the solid," without previous preparation of the face, a much greater amount of powder is used since there is only one free face to break to.

Before the general introduction of the run of mine payment system in 1907, it was evidently to the miner's interest to shatter and break his coal as little as possible. This could only be done with a minimum of powder. The accompanying chart (Fig. 5), prepared from data in the annual coal reports, shows the number of tons of coal mined per keg of powder for each year since 1888, for both machine and hand mines (solid shooting). Since longwall mines come under the general heading of hand mines, and since they use little or no powder, they have been omitted from the compilations. The charts are based on five year averages in order to avoid accidental fluctuations.

The percentage of fine coal produced by the mines for the same period is also plotted in Fig. 5. Since the year 1900, the coal production has been reported in tons of the different sizes produced; namely, run of mine, lump, egg, pea, and slack. For general purposes of comparison the percentages since 1900 were obtained by considering run of mine as lump coal, and considering nut, pea, screenings, and slack sizes as fines. Since today these are mostly made through round hole screens, while before 1897 they were mostly made through bar screens, the first part of the curve up to 1902 shows an uncertainty due perhaps to the change from bar to round hole during these years. There has been a regular increase in the percentage of the fine sizes produced since 1902. Thus, from 17.3 per cent in 1892 to 22.9 per cent in the period from 1897 to 1902 and to 32.5 per cent in the year 1914, is a serious increase in fine coal in the state as a whole. The rise of this curve, coincident with the increased use of powder in solid shooting mines, recorded in the lower curve, is more than a coincidence, and shows plainly the close relationship between them.

It has been stated that as the amount of powder per hole increased so increased the percentage of fatal accidents.* This fact, more than the evil of increased coal breakage, led to the passage in 1903 of a law regulating the use of powder in coal mines (L. 1903, p. 252), which limited to about 5 lb. the amount of powder used in a single charge in seams over 5½ feet thick, and to about 4 lb. if the seam were under this thickness. The sections covering the above were repealed in 1913, and substantially inserted as Section 19, Act of 1913. The coal report of 1904 (p. 3) commenting on the law of the previous year states that disregard of its provisions had been general. It did not serve to stop the increase in the percentage of fine coal.

More legislative remedies were proposed: (1) All coal should be undercut. (2) Special shot firers should be employed by the company. Had the first measure passed and been enforced, probably the percentage of coarse coal would have increased. The second measure finally became a law July 1, 1905. It provided, "that where more than 2 lb. of powder are used in one blast . . . a sufficient number of practical miners to be designated as shot firers shall be employed by the company to inspect and fire all blasts in the mine."

Evidently the miners, now freed from firing their own shots, loaded heavier than ever, taking the chance that their holes would be fired without inspection. For this reason, apparently, the law was amended in 1907 (Sections 47, 47a, and 47b) forbidding a miner to alter a hole after inspection by the shot firer and forbidding the shot firer to fire an unlawful hole.

A. Bement† comments on the increased use of powder during the last few years, and in Bulletin No. 16 of the Illinois State Geological Survey he states that its increase per ton of coal produced was over 100 per cent between 1897 and 1908.

Laws and collective agreements in Illinois as regards percentage of fine coal and the contained amount of impurities have not proved wholly successful. The writer believes further action should be taken, especially regarding the impurities and excessive dust in screenings. The close competitive market prevailing today on all sizes of coal, and especially on screenings, makes it appear imperative that in the near future this coal be placed on the market in a cleaner condition without preparation.

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*"Powder Accidents in the Coal Mines of Ill." Issued by The Illinois Coal Operators' Association, 1909.

†Jour. West. Soc. Eng., June, 1909, p. 307.

The effort in Ohio, one of the mining states competing with Illinois, to avoid the difficulties in changing to the run of mine payment basis, if successful, may offer a remedy for many of the difficulties which have hindered effectiveness with the run of mine payment practice.

According to the new mining laws of Ohio, passed in 1914, payment to the miner is changed from the screened coal to the run of mine basis; but, in case the miner and his employer cannot agree, the State Industrial Commission has the power to fix, for any mine, maximum percentages of fine coal and impurities allowable in the miner's coal. Loading of excess impurity is made a misdemeanor and punishable by fine on conviction. Thus, Ohio has attempted to solve by law a matter which in other states has been regulated by collective bargaining between operator and miner.

A suggestion has been made to introduce a bonus system of payment to the miner for his lump coal, as a means of overcoming the evils of the present run of mine system in connection with breakage and impurities. Briefly stated, it is proposed to continue the present practice of paying an agreed price for all coal as mined and hoisted, but since clean lump coal is of extra value to the operator, it is suggested that he pay the miner for any excess of such product over a fixed percentage to be agreed upon for each district. This would not conflict with present state laws and would compensate both the operator and the careful and skillful miner. The possibility of such a method proving impracticable on account of difficulty in determining rapidly in the tipple the respective amounts of lump and slack in each miner's car of coal should be given consideration.

RECENT DEVELOPMENTS IN PREPARATION PRACTICE.

Fig. 3 shows that sizing and close preparation had become important enough by 1900 to justify the separation of the total production into the separate sizes produced. The public was beginning to demand sized coal. Fresh impetus to this demand was a result of the anthracite strike of 1902, when the public, deprived of closely sized anthracite, tried sized Illinois bituminous coal for the first time as a substitute. In the period from 1902-1907 many of the small companies were consolidated, and larger corporations entered the field and erected new and larger modern steel surface plants.

Beginning about 1906, considerable prominence was given to the abatement of the soft coal smoke nuisance in cities, and the burning of more closely sized coals proved to be of some advantage in overcoming this trouble. In 1908 considerable public attention was given to "The Purchase of Coal by the Government under Specifications," by Geo. S. Pope, Bulletin 428, U. S. Geological Survey, and reprinted as Bulletin 11 of the U. S. Bureau of Mines. The author (p. 5) states, "Until recent years coal consumers purchased coal merely on the statement of the dealer as to its quality, relying on his integrity and on the reputation of the mine or district from which the coal was obtained. It is surprising that the important question of whether value was being received for the money expended was not sooner seriously considered." In speaking further (p. 8) on the size of coal as influencing combustion, and without qualifications as to kind of coal or to conditions of burning, he states that, "coal of uniform size forms the most satisfactory fuel." Attention is also called in the bulletin to the limiting values of ash and refuse allowed and to the general advantages of some specification system. Frequent articles in the general press also emphasized the advantages to the consumer of prepared coal.

The period from 1907 to 1915, especially the last five years, is notable for the remodeling of coal tipples to handle larger outputs and to meet the growing demand for cleaner and more evenly sized coal. This has necessitated improvements in mechanical details, such as weighing devices, sizing screens, picking tables, and loading booms, and has caused the introduction of special engineering features in improved rescreening plants and special dry processes for cleaning the coal. A surface plant in Illinois must be capable of handling more than 4,000 tons per eight-hour day in order to be considered one of large capacity. At a mine in Macoupin county recently a record has been made of 5,116 tons hoisted and prepared in an eight-hour day, or an average of 640 tons per hour. Upon comparing this with a record of 1,655 tons hoisted in 9½ hours, or about 174 tons per hour, made at Braidwood, Illinois in 1888, and mentioned in Colliery Engineer* of that year as being remarkable, it is evident that remarkable changes have taken place not only in underground methods, but in weighing, handling, and general preparation at the surface.

The large increase in capacity of individual mines has at times tended towards overproduction with a consequent decreased profit per ton and increased competition. The average shipping mine in Illinois is operated 172 days per year. In other words, without increasing present mine plant capacity, if the mines were worked 300 days per year, Illinois could supply over 100,000,000 tons of coal per year. This factor has led to complication regarding preparation, some plants having introduced devices on account of competition rather than through real need of such treatment for the coal.

It has been stated that Illinois has been obliged to improve preparation, in order to keep ahead of her competitors in the neighboring coal fields. The interstate commerce in Illinois coals has extended to the west, perhaps as far as Omaha, to the southwest into Texas, and to the northwest into Minnesota and the Dakotas until competition with the high grade West Virginia and other eastern coals carried over the Great Lakes has become too severe. Thus, at various places Illinois coals may come into competition with West Virginia, Ohio,

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*Vol. VIII, 1888, p. 222.

Kentucky and Indiana coals from the east, with the coals of Oklahoma and Arkansas in the southwest, and with the generally lower grade coals of Missouri, Kansas, and Iowa in the west. Further west, the eastern shipment of the bituminous coals of the Rocky mountains is the determining factor in the situation.

That Illinois has kept abreast and ahead of her competitors in preparation is apparent by noting the number of new screening and preparation plants erected in these competing states within the last two or three years. All things considered, there is probably no bituminous coal district in America having refinement in preparation equal to the general Illinois field, and especially equal to that at the tipples producing coal for domestic and retail use, in which an even size and freedom of impurities visible to the eye are prerequisites.

CHAPTER II.

STANDARD TYPES OF ILLINOIS COAL MINE TIPPLES.

INTRODUCTION.

Tipple Units.—At Illinois mines today the coal is hoisted to the surface for screening and final cleaning, although a rough preliminary hand picking often takes place at the face underground. This necessitates for preparation a surface structure composed of one or more of the following units:

(1) A headframe or tower over the mine shaft, supporting and taking the reaction from the pulleys or sheaves over which the hoisting rope runs, and which generally must further resist the shock of the mine car being dumped.

(2) A screen structure, sometimes called, if of the common type, a shaker house or shaker, containing the particular type of screen used to separate the mine run coal into various market sizes, and also some weighing device for the coal.

(3) Picking belts, for use in cleaning the screened coal, and loading chutes, etc., and for loading it gently into railroad cars.

(4) At certain mines the mine cars are run off the mine cage before being dumped, necessitating an additional structure in which the mine cars are handled and recaged, and in which disposition is made of the waste rock brought to the surface.

In Illinois these units are grouped into one general structure at the mouth of the shaft which is called the tipple. To avoid vibration and promote stability the screen frame in the tipple is generally built with independent supports and foundation. A frequent auxiliary and separate structure is the rescreening plant or rescreener, into which the smaller coal is re-elevated, and there separated into various small sizes. In the design of these units every precaution should be taken to prevent breakage of the coal, to obtain capacity, to do clean screening, and to avoid vibration, all with a minimum of labor. These are the five requisites of good tipples.

Materials of Construction.—Until about 1900, mine tipples in Illinois were built of wood, excepting the screens, chutes, etc. Since then the use of steel in the construction of tipples has become more and more common. In June, 1911, the state legislature passed an act requiring that all structures thereafter erected on the surface within 100 feet of the mouth of any shaft, slope, or drift should be built of metal, rock, clay, cement, clay or cement products, or a combination of the same. Tipples erected since that time have been made of steel, and although concrete and other fireproof materials have been used in other parts of the country for mine tipples, they have not as yet found favor in this state.

Many of the older wooden tipples have given remarkable service, especially those well built and cared for, and in many districts are still the common form. A number of the earlier steel tipples were constructed of a comparatively large number of small steel members, and following the custom prevalent at the wooden tipples, the sides, especially over the shaft, were tightly enclosed, usually with corrugated iron. Since the hoisting shaft is generally the upcast air shaft, the corrosive action of this moist mine air on the thin steel framing of the tipple was rapid, especially on those members closest to the mouth of the shaft. In the later steel tipples trouble from this source has been eliminated by any or a combination of the following simple precautions:

(1) Leaving the sides of the headframe open.

(2) Spreading the legs of the headframe; thus removing them from the shaft collar.

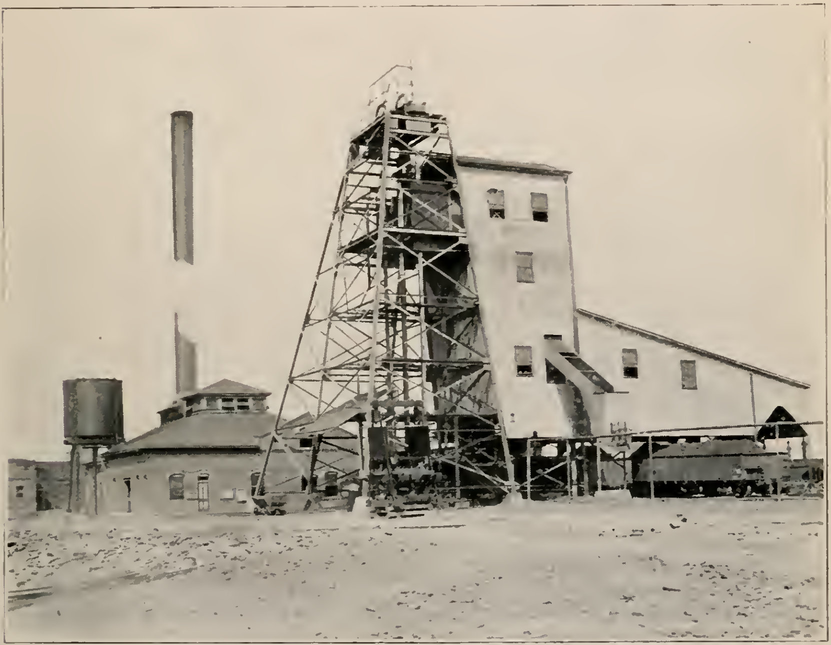

Fig. 6. Class I Tipple.

(3) Using a few large steel members for headframe construction, instead of the more quickly corroded small sections.

(4) Encasing in concrete the legs or posts close to the shaft.

(5) More frequent scraping and painting.

It is to be regretted that so many correctly designed steel headframes in this state have been allowed to deteriorate solely through lack of attention on the part of the operator. No design can withstand for many years the severe conditions imposed without being properly protected by an occasional coat of paint. If this simple expedient is carried out, there is no reason why the steel tipple should not last almost indefinitely.

Types of Tipples.—Mine tipples differ according to (a) their structural material, (b) method of receiving coal (whether from a shaft, slope, or drift), (c) position of hoisting engines in reference to the axis of the structure, (d) type of mine car dump used, (e) method of handling mine cars of coal and waste, (f) whether or not the coal is rescreened, and (g) the type of screen used to prepare the coal.

In Illinois the structural material is uniformly wood or steel, and as shaft mines are most common, mine tipples may be grouped into three general classes or types as follows:

(1) Those equipped with self-dumping cages and with shaker screens for sizing the coal.

(2) Those equipped with self-dumping cages and with gravity bar screens for sizing the coal.

(3) Those equipped with rigid or fixed cages, with cross over dumps in the tipple to empty the mine cars, and with shaker screens for sizing the coal.

To illustrate the general practice in coal preparation, one tipple representing each of the three types just mentioned, has been chosen at random from a number of similar ones in the state. Although constructed of steel they are by no means the largest or most recent installations in each class, but each represents many features found at many of the tipples in the state. The designs illustrated are for steel construction, but they are also representative of wooden construction since there are no especial differences in principle, operation, or appliances. Any of these types may have an auxiliary rescreening plant.

Although the length or long axis of a tipple is usually perpendicular to the long axis of the shaft, in several cases, in order to accommodate a more favorable position of railroad tracks, the screen structure is built at an angle to the long axis of the shaft, and in several instances parallel to it.

Hoisting engines may be placed so that the hoisting ropes will run over the sheaves either in the same plane as the long axis of the shaft or perpendicular to it. In the first case the headframe is said to be "end pull" and the pulleys or headsheaves are tandem and in approximately the same vertical plane. Headframes in the second case are "side pull" and the pulleys are parallel. Neither type is more popular in Illinois. Both types are often used in the same district, the choice depending on the designer, the operator, the permanent position of some part of the old plant necessary to combine with the new structure, the ease of conveying coal to the boiler room, the topography, the position of railroad, etc. Accidently the three plants chosen to illustrate typical surface plants in this chapter are of the end pull type. The frontispiece (Fig. 2) shows a side pull headframe.

CLASS I. TIPPLES EQUIPPED WITH SELF-DUMPING CAGES AND WITH SHAKER SCREENS FOR SIZING THE COAL.

The type of tipple commonly used in Illinois at room and pillar mines and in which the bulk of Illinois coal is prepared for the general market belongs to Class I and is shown in the photograph (Fig. 6) and in sectional elevation (Fig. 7). The operation is best explained by following the course of the coal from mine car to railroad car.

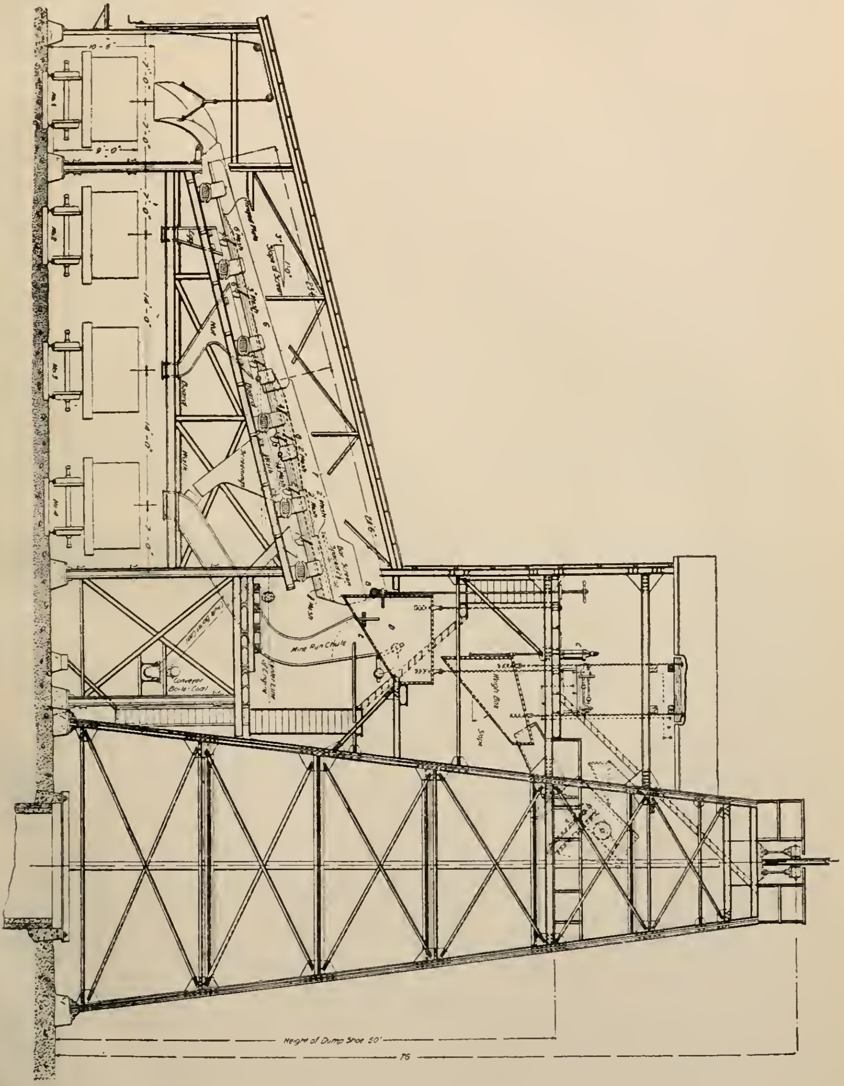

FIG. 7. ELEVATION OF CLASS I TIPPLE.

The self-dumping cage *A,* containing the end dump mine car, dumps at a height of about 50 feet above the ground, this height varying with the design of the tipple, the kind of screen used, the number of railroad tracks to be served, etc. The general scheme is to elevate the coal sufficiently before dumping to enable all necessary preparation to take place and the coal to be loaded into railroad cars without re-elevation. As the cage tips, the coal slides out of the mine car over the dump chute and into the weigh box. The weighman, or in the larger mines a check puller at the scale, takes the miner's check from the mine car in order that proper accounting for the coal contained may be rendered.

After the coal has been weighed, the gate *B,* controlled by hand or by the piston *C,* is opened, thus allowing the coal to fall into the hopper or pocket *D.* In many mines this hopper is absent, the coal being dumped directly from the weigh box on the shaking screen. At other mines an automatic feeder which supplies the coal evenly to the shaking screen is installed instead of this hopper. A valve *E* may be opened to allow rock, refuse, or run of mine coal to enter the mine run chute and thus be loaded into a railroad car on track 4. Above the screen proper is a relief bar screen 6 ft. long and with 2½-inch spaces. This prevents a rush of coarse coal on the upper fine screens and in general prevents their choking; thereby increasing capacity and the completeness of screening. The coal from *D* falls on this relief bar screen and thence to the top section of the upper shaking screen *F,* which in the case illustrated, is 10 feet wide and 24 feet 6 inches long.

The top section *F* of the shaking screen has three decks; the upper one is equipped with a screen plate with 2-inch round perforations, the middle one with a plate with ¾-inch round holes for the upper ten feet and 1¼-inch round holes for the lower end, and the bottom plate is solid, excepting for discharge gates or doors, Nos. 1, 2, 3 and 4.

The lower section *G* of the shaking screen has approximately the same area as the upper one and has two decks only. The top one is fitted with sections of plate having 3-inch round holes and 6-inch round holes as shown, and the bottom deck is solid excepting for the discharge doors.

The driving eccentrics of the shaker screens have a throw of about 6 inches and make about 100 revolutions per minute. To minimize vibration the motion of one screen opposes the other and their total weights when vibrating, including the loads of coal, are approximately the same. The screen supports are independent of the rest of the tipple. The slope of the screens is usually 3 to 4 inches per foot. The chutes leading to the cars are curved and built at a low angle in order to prevent breakage and spilling of the coal during the loading of the railroad cars. The boiler coal chute takes the finest coal, ¾-inch screenings, to the trough conveyor leading to the boiler room of the power plant.

With such an arrangement of screens and chutes, by opening or closing variously the gates, numbered 1 to 8 inclusive, in the bottom plates of the screens, a great number of sizes of coal can be made without changing the screen plates. The possible sizes are given below in Table 2, and Table 3 gives the arrangements of the gates necessary to obtain the different combinations of sizes.

TABLE 2.

SIZES OF COAL PREPARED. CLASS I TIPPLE.

Combination Number	Size of Coal			
	On Track 1	On Track 2	On Track 3	On Track 4
1	¾″ lump			¾″ screenings
2	2″ "		¾″x2″ nut	¾″ "
3	6″ "	2″x6″ egg	¾″x2″ "	¾″ "
4	1¼″ "			1¼″ "
5	2″ "		1¼″x2″ "	1¼″ "
6	6″ "	2″x6″ "	1¼″x2″ "	1¼″ "
7	3″ "		1¼″x3″ "	1¼″ "
8	6″ "	3″x6″ "	1¼″x3″ "	1¼″ "
9	6″ "	1¼″x6″ "		1¼″ "
10	2″ "			2 ″ "
11	3″ "		2 ″x3″ "	2 ″ "
12	6″ "	3″x6″ "	2 ″x3″ "	2 ″ "
13	3″ "	3″x6″ "	3″ screenings	
14	6″ "		3″ "	
15	6″ "	6″ egg run		

TABLE 3.

OPERATION OF SCREENS. CLASS 1 TIPPLE.

To Make Combination Number	Open Gate Number	Close Gate Number
1	2	3, 8, 4, 5, 6, 7
2	2, 4, 5	3, 8, 6, 7
3	2, 4, 5, 7	3, 8, 6
4	2, 3	8, 4, 5, 6, 7
5	2, 3, 4, 5	8, 6, 7
6	2, 3, 4, 5, 7	8, 6
7	2, 3, 4, 5, 6	8, 7
8	2, 3, 4, 5, 6, 7	8
9	2, 3, 4, 7	5, 6, 8
10	2, 3, 8	4, 5, 6, 7
11	2, 3, 8, 6	4, 5, 7
12	2, 3, 8, 6, 7	4, 5
13	4, 5, 6	2, 3, 8, 7
14	4, 5, 6, 7	2, 3, 8
15	4, 7	2, 3, 8, 5, 6

Besides these sizes mine run or unscreened coal can be loaded on track No. 1 by closing all the gates or on track No. 4 by opening the valve *E* and running the coal through the mine run chute. The usual combinations of sizes of coal prepared are 1¼-inch or 2-inch screenings, 2-inch to 3-inch nut, 3-inch to 6-inch egg, and 6-inch lump. At many tipples, having adjustable screens of this type, only the above standard sizes are prepared, since it is found that changing the screens and valves to prepare different sizes lowers the capacity, especially if close screening of the smaller sizes is attempted. If there is a demand for sized nut coal, a separate rescreening plant is used.

A plant of this size will prepare up to 4,000 tons in eight hours, and similar tipples are in operation which will handle 6,000 tons daily if required.

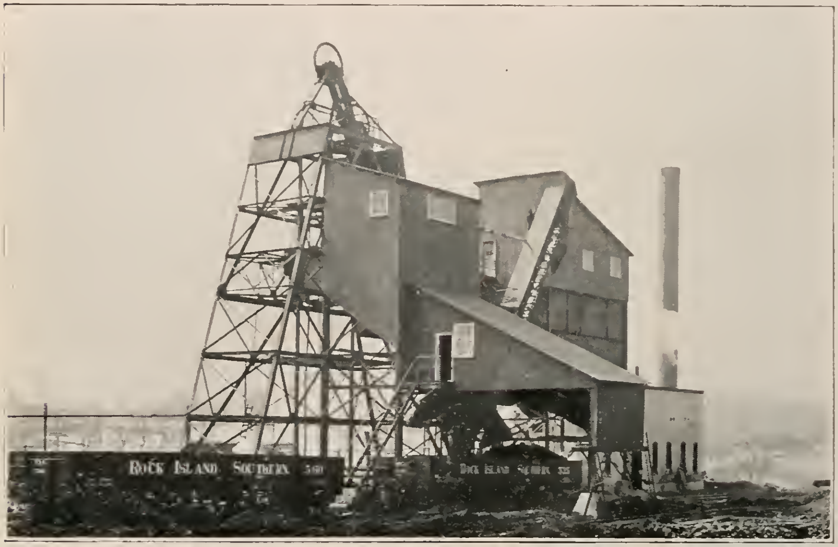

Fig. 8. Class II Tipple.

CLASS II. TIPPLES EQUIPPED WITH SELF-DUMPING CAGES AND WITH GRAVITY BAR SCREENS FOR SIZING THE COAL.

Although the gravity bar screen has been replaced generally by the shaker screen, on account of the better sizing, less breakage, and less headroom and slope required, yet at many mines, especially if the coal is prepared for a special market, the gravity bar screen is still used, because of its low cost of installation and upkeep, simplicity, and freedom of the tipple from vibration, as well as its almost unlimited capacity.

Fig. 8 shows a photograph, Fig. 9 a ground plan and end elevation, and Fig. 10 a side elevation of a bar screen tipple and accompanying surface plant. The self-dumping cage (Fig. 10), dumps the coal from the mine car into the weigh box, and after being weighed, it flows on the top bar screen *AA′*. This bar screen is in two equal sections, 6½ feet wide and 8 feet long, and is set at an angle of 27 degrees, which is just enough to allow the coal to slide over it by gravity. The spaces between the bars are 5 inches in width. Lump coal passes over the bars *AA′* into chute *B*, set at the same angle, and from here into the railroad car on track No. 1. The coal passing the 5-inch spaces of the screen *AA′*, falls on the lower bar screen *DD′*, which is of the same width as *AA′*, is 12 feet long, is set at an angle of 32 degrees, and has only 1-inch spaces between the bars. Thus, the coal between 5 inches and 1 inch in size passes over the lower screen into chute *E*, from which, as egg coal, it is loaded on track No. 2.

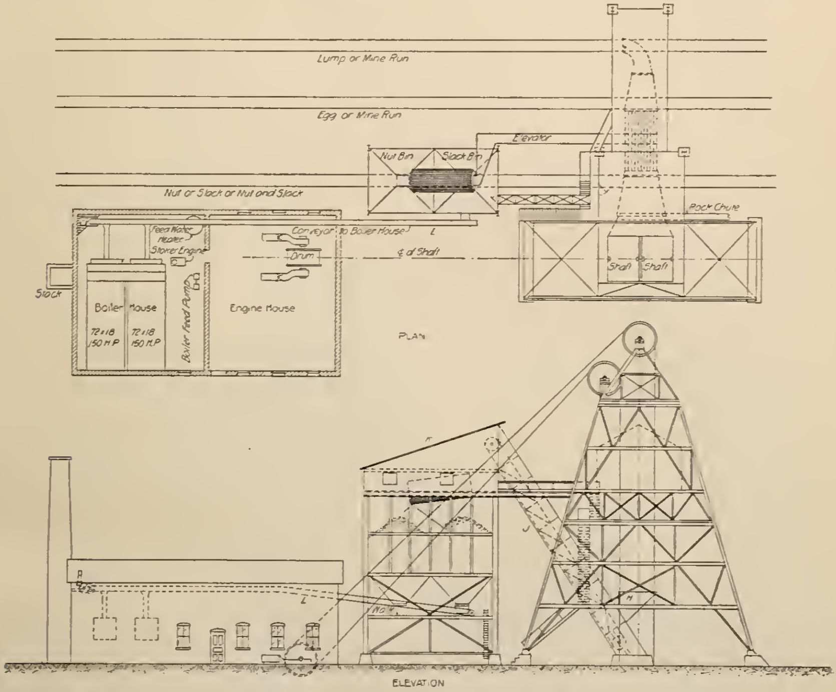

FIG. 9. GROUND PLAN AND END ELEVATION OF CLASS II TIPPLE.

The smallest coal, having passed through screen *DD′*, is collected by chute *F* and may be loaded on track No. 3 as raw or untreated screenings. By means of a hinged gate *G* this coal may also be deflected into chute *H* and from here to the elevator *J* (Fig. 9), leading to the rescreening plant *K*. This particular rescreening plant is provided with a revolving screen 5 feet in diameter and 12 feet long, sloping 5 degrees, and enclosed with wire screen cloth having ¾-inch diameter square meshes. The oversize falling into the steel nut bin is called rescreened nut or pea coal; the undersize ¾ inch to zero in size, called slack, falls into the steel slack bin. The capacity of each of these bins is about 60 tons. Railroad cars can be loaded underneath as desired. A conveyor *L* carries part of the slack to the boiler room.

By lowering veils or blank plates over the screen *AA′*, it is possible to load mine run coal on track No. 1. Also by opening the chute *B′*, the oversize from the 5-inch bar screen may be loaded on track No. 2. Provision is also made for running either mine run coal or refuse from the weigh box directly into the chute *C* and from here into the railroad car on track No. 3. By using veils on the screens as required,

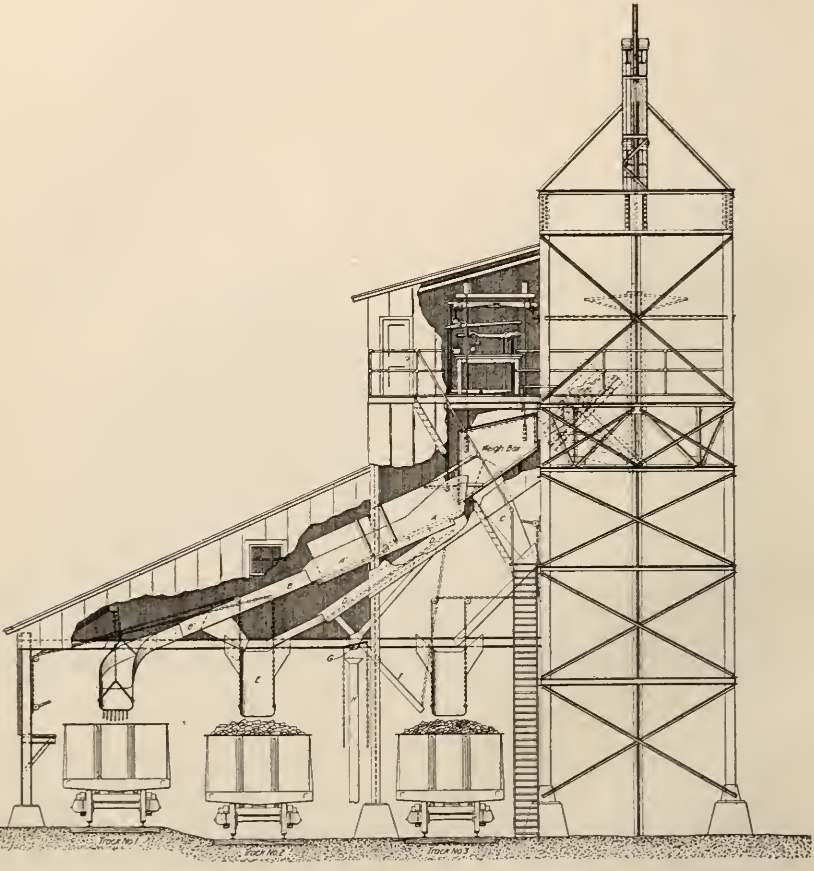

FIG. 10. SIDE ELEVATION OF CLASS II TIPPLE.

together with the rescreener, the combinations of sizes given in Table 4 are possible in the tipple described.

TABLE 4.

SIZES OF COAL PREPARED. CLASS II TIPPLE.

Combination Number	Size of Coal			Rescreener	
	On Track 1	On Track 2	On Track 3	Nut or Pea	Slack
1	mine run				
2	5″ bar screen lump	5″ bar egg run			
3	5″ bar screen lump	1″-5″ bar egg	raw screenings		
4		1″ bar screen lump	" "		
5			mine run		
6	Raw screenings in combinations 3 and 4 may be rescreened into			¾″ sq. to 1″ bar	¾″ sq. to zero

The average daily capacity of the tipple illustrated is 800 tons, but at individual tipples of this class in the state more than 5,000 tons daily are prepared.

CLASS III. TIPPLES EQUIPPED WITH RIGID OR FIXED CAGES, WITH CROSS-OVER DUMPS IN THE TIPPLE TO EMPTY THE MINE CARS, AND WITH SHAKER SCREENS FOR SIZING THE COAL.

The tipple illustrated by the photograph (Fig. 11), and shown in side elevation (Fig. 12), has found favor in and is confined almost wholly to the longwall field in the northern part of Illinois for the following reasons: (a) Many of the mines are from 300 to 600 feet deep and at some two seams at different levels have been worked from the same shaft. (b) The thin seams and method of working usually necessitate small mine cars holding 1 to 1½ tons, and to secure adequate hoisting capacity, two cars are often hoisted end to end on the same cage. (c) One car of waste is also often hoisted to three cars of coal, and large amounts of timber are lowered.

The loaded cars (Fig. 12), are pushed from the cage along a track having a gradient with the load of about 1½ per cent, to some form of crossover or end dump and dumped into the weigh box. The empty car runs down the sharp grade *A* (6 to 12 per cent), through a spring switch at *B* to the kickback at *C*, while returning the empty car runs by gravity down the side track *D* (1½ per cent grade), to the transfer car *E*. When two mine cars have entered the transfer car, it is moved up inclined tracks by means of an air or steam piston, not shown, until the mine cars arrive at the level of the cage. By this time two more full cars have been hoisted and the mine cage is in the position referred to in the beginning. By means of a piston working in the long plunger *F* a ram *G* gently pushes the two empty cars into the cage, at the same time forcing the two loaded cars off.

The tracks, dumps, etc., in the tipple are in duplicate to provide for the cars coming from the cage in the other compartment of the shaft.

Mine cars containing shale or refuse are taken over the cross-over dump without dumping and switched to a track leading to the left end *H* of the tipple where their contents are dumped into a chute, usually by means of a horn dump, after which the cars are returned to the empty tracks *D*. In the same way, coal intended for local or wagon trade is dumped into a chute at *H* and roughly bar screened before being loaded into the wagons.

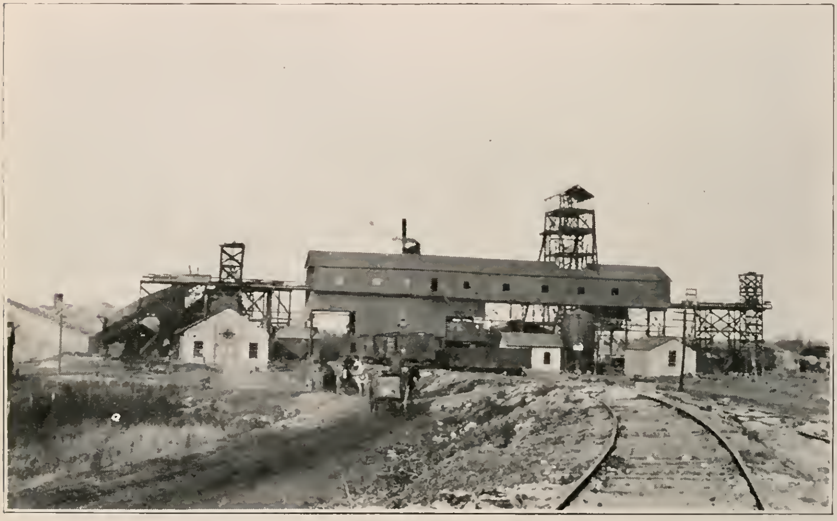

Fig. 11. Class III Tipple.

Cars to be loaded with timber are taken through the end *J* of the tipple to an elevator (shown in Fig. 11), lowered to the ground, loaded, and returned to the system at *E*.

The screening plant in such a tipple may consist of either gravity bar screens or shaker screens. As illustrated in Fig. 12 on the shaker screen *M* one, two, or three sizes of coal can be prepared at once. It is of all steel construction and rests on independent foundations.

The coal dumped into the weigh box is weighed by scales in the weighroom *K*, the check puller at the dump having taken the miner's

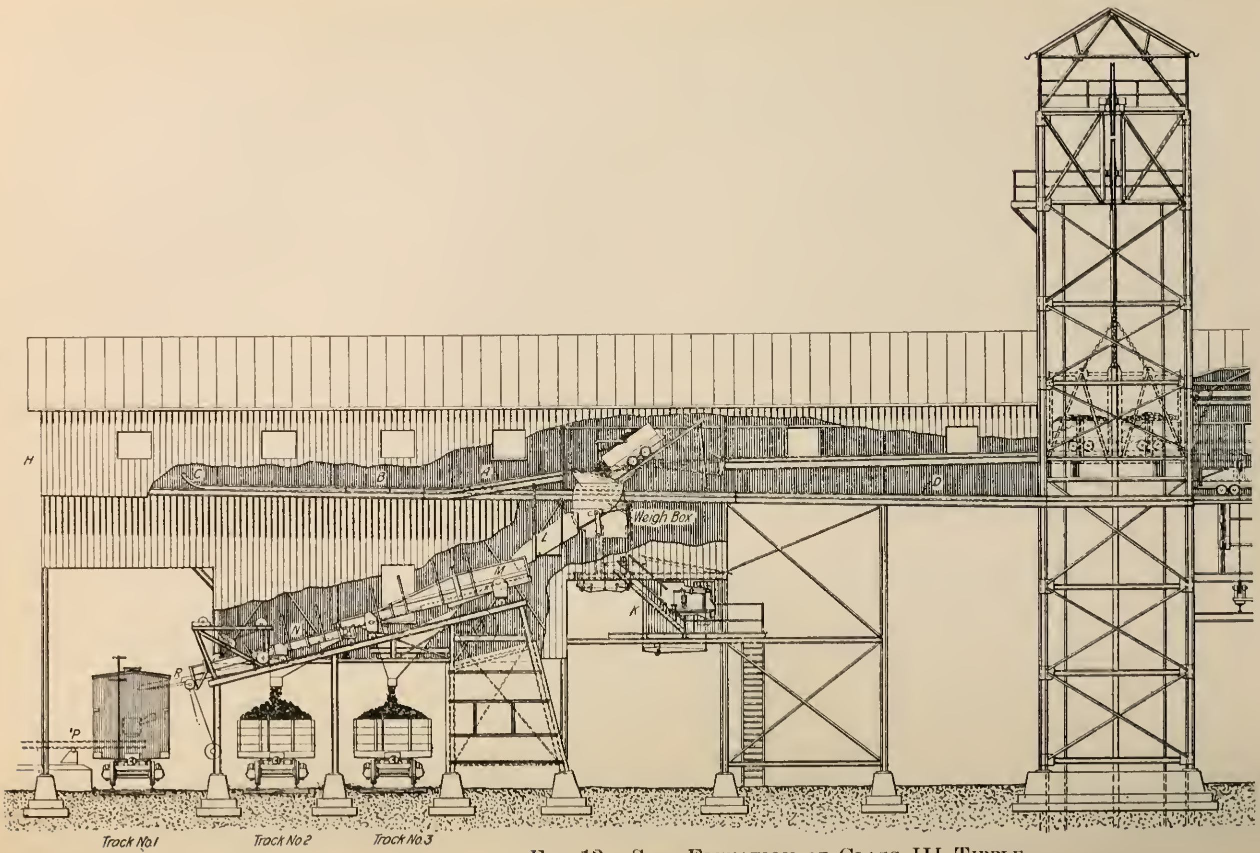

FIG. 12. SIDE ELEVATION OF CLASS III TIPPLE.

not shown, until the mine cars arrive at the level of the cage. By this time two more full cars have been hoisted and the mine cage is in the position referred to in the beginning. By means of a piston working in the long plunger *F* a ram *G* gently pushes the two empty cars into the cage, at the same time forcing the two loaded cars off.

The tracks, dumps, etc., in the tipple are in duplicate to provide for the cars coming from the cage in the other compartment of the shaft.

Mine cars containing shale or refuse are taken over the crossover dump without dumping and switched to a track leading to the left end *H* of the tipple where their contents are dumped into a chute, usually by means of a horn dump, after which the cars are returned to the empty tracks *D*. In the same way, coal intended for local or wagon trade is dumped into a chute at *H* and roughly bar screened before being loaded into the wagons.

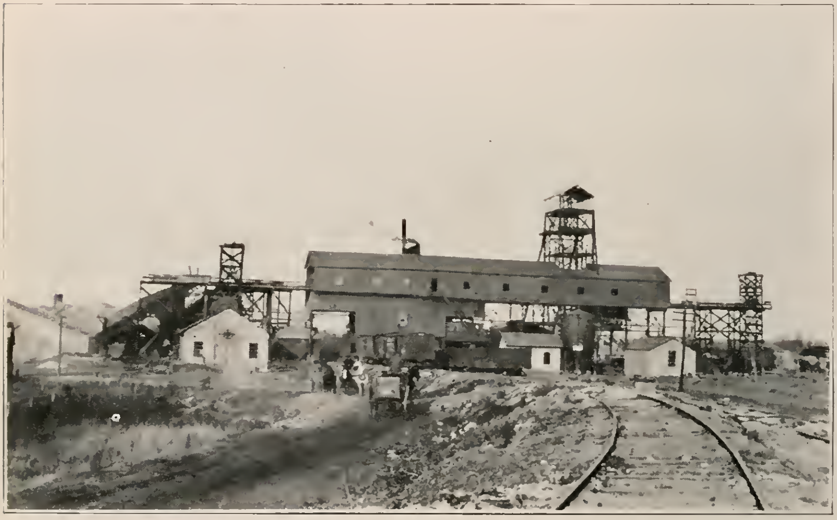

Fig. 11. Class III Tipple.

Cars to be loaded with timber are taken through the end *J* of the tipple to an elevator (shown in Fig. 11), lowered to the ground, loaded, and returned to the system at *E*.

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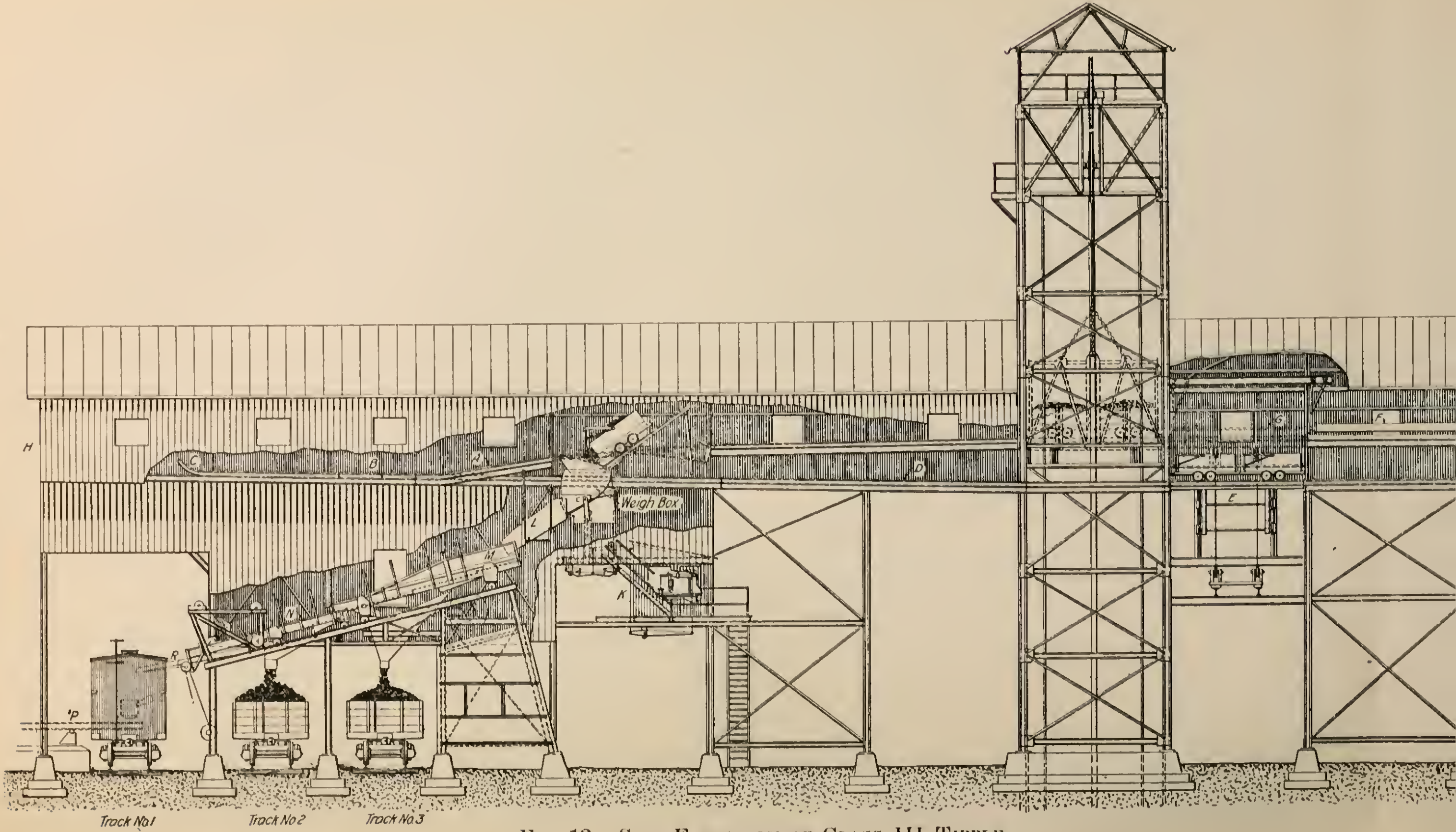

FIG. 12. SIDE ELEVATION OF CLASS III TIPPLE.

check from the car and dropped it into a chute (not shown), which carries it to the weigh room, so that proper credit is given for the coal.

The coal passes over the dead plate or chute *L* to the upper shaking screen *M*, which is 8 feet wide by 16 feet long, and consists of plates with 1⅛-inch round hole perforations. The undersize from here is loaded as screenings on track No. 3. In some cases it is washed before being marketed. The oversize of the screen passes over a short dead plate to the lower shaking screen *N* of which the upper part consists of 5-inch hole plate. Thus 1¼-inch to 5-inch egg may be loaded on track No. 2. The lump or chunk coal passes over the 5-inch screen and through the adjustable chute *S* into the car on track No. 1. If it is necessary to load box cars, the box car loader is run out from *P* into the car as shown, and the chute *R* is adjusted to feed it. It is possible by veiling the screens to load mine run on either track No. 1 or No. 2, or to load 1¼-inch lump on track No. 1.

Owing to facts previously stated, the capacity of these tipples is relatively small in comparison with those at pillar and room mines, a daily production of 1,500 tons being considered large. A large proportion of the coal produced is clean lump, and consequently, most of the screening plants in tipples of this type are small and simple.

CHAPTER III.

IMPURITIES AND BREAKAGE.

PART I. GENERAL.

Removal of impurities and avoidance of breakage are the two standards by which coal preparation is measured. The object of coal mining is not only to produce the greatest possible tonnage at the least cost, but to produce the greatest possible percentage of clean lump coal. Perfect mining would mean, not only the extraction of all the coal, but the winning of it in a perfectly clean condition with 100 per cent lump. Were this possible, subsequent mechanical breakage could easily and cheaply produce finer sizes, as needed, in as pure a condition as the original lump.

"On the whole, Illinois has pure coal compared with many sections and it is possible to clean lump by hand in the railroad car. If the market calls for a steam coal, very little preparation is necessary. If the market requires domestic, we must screen and pick and even rescreen. The biggest problem is to load without breakage."*

No seam of Illinios coal is free from at least small amounts of impurities. Since mines situated in widely separated districts over an area of approximately 36,800 square miles are worked in five different seams, great variations are possible locally in the nature and in the extent of the impurities in an individual seam and in the physical characteristics of the coal itself.

The most persistent impurity in Illinois coal is the famous "blue band" or shale band in seam No. 6, which averages 1 to 2 inches in thickness, and is situated about 20 inches above the bottom of the seam. It extends with considerable regularity over an area of at least 5,000 square miles.† Impurities such as pyrite ("sulphur") and shale bands are of frequent local occurrence in all the seams. On the whole, it cannot be said that any one district in Illinois has an advantage over others in this respect. Considerable variance occurs among mines in the same district, however, not only in the impurities in the coal, but also in the care displayed in removing the same before the coal is shipped to market.

Friability means the tendency to produce fines under like conditions. The Illinois coals are not so friable as the coking bituminous coals of the Appalachian region, and in freedom from the breakage which occurs during mining, preparation, and shipment, they are perhaps not excelled by coal from any other section of the bituminous fields of America. Taken as a whole the different seams in Illinois show considerable uniformity as regards friability, although sometimes the same seam in different districts or even in the same district will vary, depending probably on the thickness of the cover and the regularity of the bed. A seam buried under several hundred feet of cover is likely to be firm; a seam which has been tilted or folded is likely to be somewhat friable. At only two places in the state where mining is undertaken have the beds undergone any extensive regional faulting and tilting, all being practically horizontal. This has tended to keep the beds regular in general physical structure over large areas. Since Illinois coal is relatively firm, the preparation of sized coals intended for competition with the domestic sizes of anthracite has recently become a noticeable feature.

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*W. R. Roberts, Black Diamond, November 30. 1912.

†T. E. Savage, Journal of Geology, Vol. 22, No. 8, 1914.

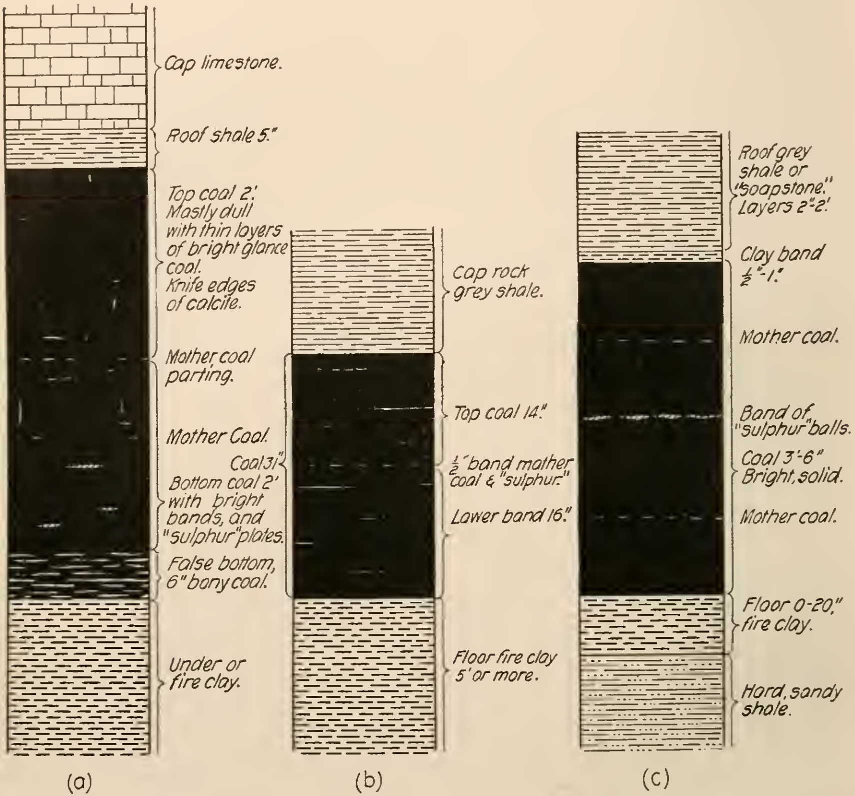
Cap limestone.
Roof shale 5."
Top coal 2.' Mostly dull with thin layers of bright glance coal. Knife edges of calcite.
Mother coal parting.
Mother Coal.
Coal 3½"
Bottom coal 2' with bright bands, and "sulphur" plates.
False bottom, 6" bony coal.
Under or fire clay.
Cap rock grey shale.
Top coal 14."
½" band mother coal & "sulphur."
Lower band 16."
Floor fire clay 5' or more.
Roof grey shale or "soapstone." Layers 2"-2.'
Clay band ½"-1."
Mother coal.
Band of "sulphur" balls.
Coal 3'-6" Bright, solid.
Mother coal.
Floor 0-20," fire clay.
Hard, sandy shale.
(a)
(b)
(c)

FIG. 13. TYPICAL SECTIONS OF SEAMS NOS. 1 AND 2.

The sections shown in Figs. 13 to 16 have been prepared to illustrate the relative occurrence and distribution of impurities in the different coal seams worked in Illinois together with the nature of roof and floor. These in no way reflect on or give prominence to any one seam, since the quality of the merchantable coal is largely affected by

TABLE 5.

AVERAGE ANALYSES* OF COAL IN SEAMS DESCRIBED.

Illustration Fig. No.	Seam	Number of Samples	Proximate analysis of coal: 1st; "as rec'd," with total moisture. 2nd; "Dry," or moisture free				Sulphur	B. t. u.	Unit Coal B. t. u.
			Moisture	Volatile Matter	Fixed Carbon	Ash			
13a	1	11	15.58	39.17	35.80	9.45	4.69	10,673	.....
			Dry	46.40	42.41	11.19	5.55	12,643	14,546
13b	2	3	17.40	33.30	41.48	7.82	2.03	10,811	.....
			Dry	40.32	50.20	9.48	2.45	13,091	14,663
13c	2	33	16.18	38.83	37.89	7.08	2.89	10,981	.....
			Dry	46.34	45.21	8.45	3.45	13,101	14,528
14a, 14b	5	54	15.10	36.79	37.59	10.52	3.52	10,514	.....
			Dry	43.32	44.28	12.40	4.15	12,384	14,447
14c	5	27	6.75	35.49	48.72	9.04	2.92	12,276	.....
			Dry	38.06	52.25	9.69	3.13	13,165	14,812
15a, 15c	6	76	12.56	38.05	39.06	10.33	4.01	9,848	.....
			Dry	43.52	44.67	11.81	4.59	12,406	14,377
15b	6	58	9.21	34.00	48.08	8.71	1.53	11,825	.....
			Dry	37.45	52.96	9.59	1.68	13,025	14,585
16a	2	15	9.28	33.98	51.02	5.72	1.29	12,488	.....
			Dry	37.46	56.24	6.30	1.42	13,765	14,818
16b	6	31	14.45	35.88	40.33	9.34	2.55	10,919	.....
			Dry	41.94	47.14	10.92	2.98	12,764	14,557
16c	7	18	12.99	38.29	38.74	9.98	2.93	11,143	.....
			Dry	44.01	44.52	11.47	3.37	12,807	14,740

*S. O. Andros, "Coal Mining in Illinois," Bulletin 13, Co-operative Agreement, p. 57.

the care and skill exercised in its mining and preparation. These sections and the accompanying analyses of the coal listed in Table 5 are based mainly upon data gathered by the Illinois Coal Mining Co-operative Investigation.

The instructions given the samplers engaged in gathering the face samples for the analyses called for the exclusion of all impurities in the seam more than ⅜ inch thick, or thinner partings or impurities if, in the judgment of the sampler, such are excluded by the miner in loading the coal. The latest directions of the U. S. Bureau of Mines for taking face samples are practically the same, with the addition that "lenses or concretions of 'sulphur' or other impurities more than 2 inches in maximum diameter and one-half inch thick are excluded if, in the judgment of the sampler, they are being excluded by the miner from the coal as loaded out of the mine or as shipped."* It is evidently assumed that these are the maximum sizes the miner would allow to enter the car with the coal. The commercial sizes of prepared coal may be higher or lower in impurities and ash than these face samples, depending largely on the care used in preparation.

Seam No. 1 in Mercer county (Fig. 13a), averages 4 feet in thickness. The coal has weak or incipient cleavage, along which plates and films of pyrite or calcite may be developed, and locally, sulphur bands are interbedded with the coal. Directly over the coal is a shale band 2 to 5 inches thick, which tends to disintegrate and fall on exposure to the air, and which may, therefore, mix with the coal. Below the mineable coal there may be a thin band of bone giving way to a soft fireclay, that swells badly on exposure. This has been likened to bread rising. The output from this seam is scarcely large enough to affect the general market.

Seam No. 2 in McDonough county (Fig. 13b), averages about 30 inches in thickness and consists of a top and a bottom band which are separated as shown. The coal as a whole is built up of fine laminations of bright and dull coal. Irregular bands and lenses of pyrite occur in places. The roof is a grey shale or "soapstone," and the floor fireclay; usually soft.

Seam No. 2 in LaSalle county (Fig. 13c), averages about 3 feet, 6 inches in thickness. The coal is uniformly long grained, hard, bright and firm. It tends to split parallel to the bedding, being aided by the mother coal layers. One or two irregular bands of sulphur balls are found in some places. The roof is a brittle blocky gray shale or soapstone separated from the coal by a clay band ½ inch to 1 inch thick. The floor, in which undercutting is often done, may vary from fireclay to a sandstone. In general, this is a clean seam and little or no picking is necessary to prepare an excellent lump coal for the market. Considerable fire clay, which may be removed by washing, is mixed

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*A. C. Fieldner, "Notes on the Sampling and Analysis of Coal." U. S. Bureau of Mines, Technical Paper 76, p. 8.

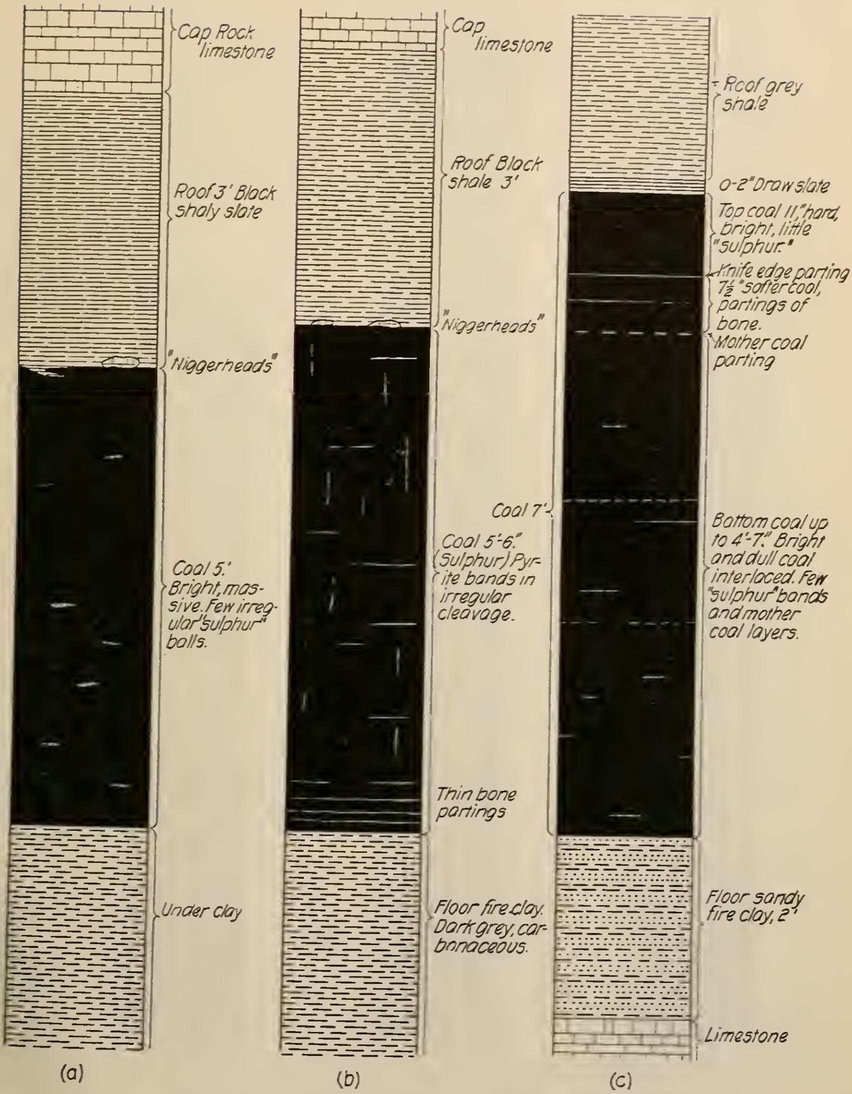
Cap Rock limestone
Roof 3' Black shaly slate
"Niggerheads"
Coal 5.' Bright, massive. Few irregular "sulphur" balls.
Under clay
(a)
Cap limestone
Roof Black shale 3'
"Niggerheads"
Coal 5'-6." (Sulphur) Pyrite bands in irregular cleavage.
Thin bone partings
Floor fire clay. Dark grey, carbonaceous.
(b)
Roof grey shale
0-2" Draw slate
Top coal 11," hard, bright, little "sulphur."
Knife edge parting
7½" softer coal, partings of bone.
Mother coal parting
Coal 7'
Bottom coal up to 4'-7." Bright and dull coal interlaced. Few "sulphur" bands and mother coal layers.
Floor sandy fire clay, 2'
Limestone
(c)

FIG. 14. TYPICAL SECTIONS OF SEAM NO. 5.

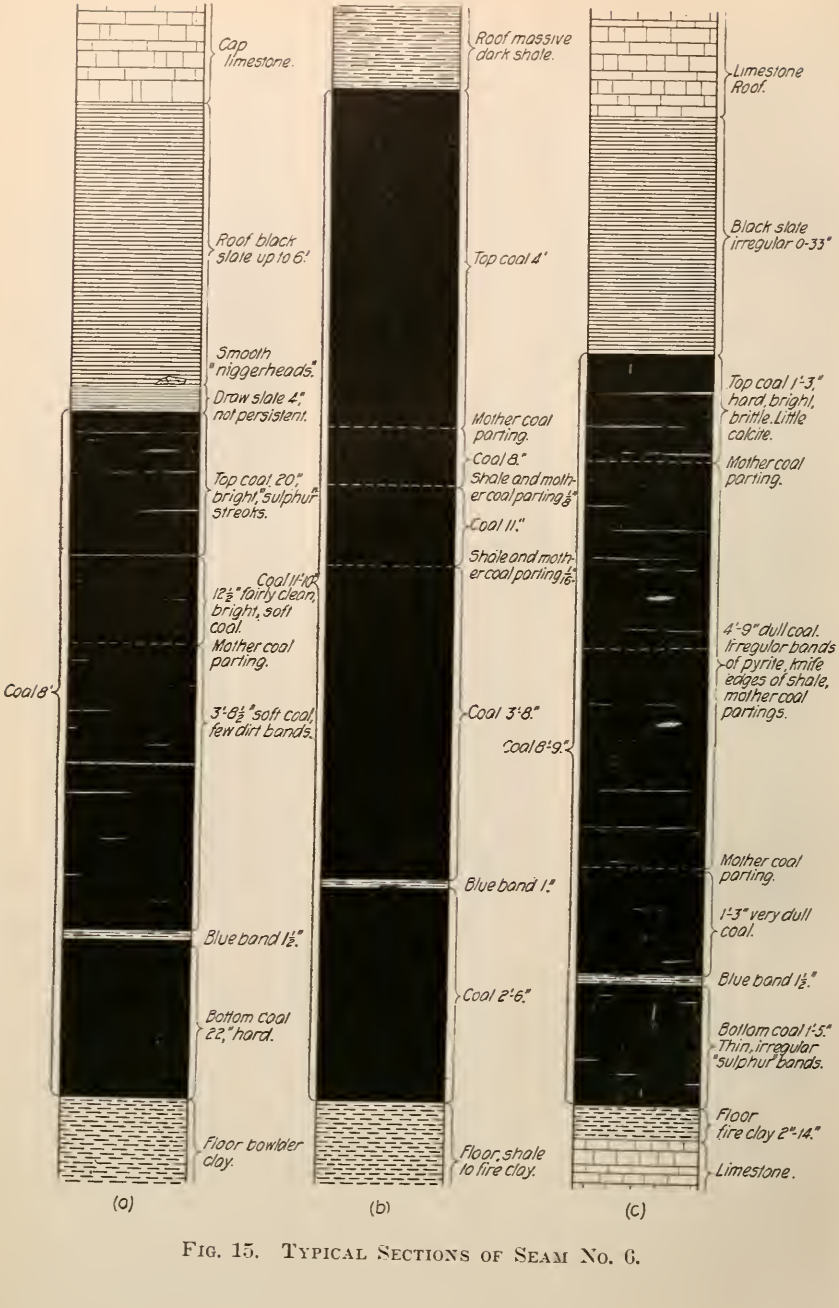
Cap limestone.
Roof black slate up to 6'.
Smooth "niggerheads."
Draw slate 4." not persistent.
Top coal 20," bright, "sulphur" streaks.
Coal 11'-10"
12½" fairly clean, bright, soft coal.
Mother coal parting.
Coal 8'
3'-8½" soft coal, few dirt bands.
Blue band 1½."
Bottom coal 22," hard.
Floor bowlder clay.
(a)
Roof massive dark shale.
Top coal 4'
Mother coal parting.
Coal 8."
Shale and mother coal parting ⅛"
Coal 11."
Shale and mother coal parting 1/16"
Coal 3'-8."
Blue band 1."
Coal 2'-6."
Floor, shale to fire clay.
(b)
Limestone Roof.
Black slate irregular 0-33"
Top coal 1'-3," hard, bright, brittle. Little calcite.
Mother coal parting.
4'-9" dull coal. Irregular bands of pyrite, knife edges of shale, mother coal partings.
Coal 8'-9."
Mother coal parting.
1'-3" very dull coal.
Blue band 1½."
Bottom coal 1'-5." Thin, irregular "sulphur" bands.
Floor fire clay 2"-14."
Limestone.
(c)

FIG. 15. TYPICAL SECTIONS OF SEAM NO. 6.

with some of the screenings.* As an illustration, raw screenings from this field containing 28.87 per cent ash, are reduced by washing to nut coal containing 8.75 per cent ash and to slack coal containing 9.96 per cent ash.† The top coal is generally harder than the bottom.

Seam No. 5 at Lincoln, Logan county (Fig. 14a), consists of about 5 feet of bright massive coal. There are some irregular balls and bands of sulphur and other impurities present. The roof contains frequent concretions or "niggerheads."

Fig. 14b shows the same seam in the Springfield district. The sulphur here often is irregularly distributed in thin plates in the cleavage planes of the coal. The seam in this district is relatively free from impurities. The chief of these is sulphur in vertical faces. Also, in places a rather high percentage of ash is noted in the lump coal, due to a slight tendency of the coal to be bony. This same tendency, however, results in a coal that is unusually tough, hard and firm, and able to withstand breakage as well as any in the state. Rolls, horsebacks, and clay veins, which are of frequent local occurrence, are often avoided by careful mining. The fact that of these clay veins the large ones are soft, and the small ones are hard, has considerable effect on the cleanness of the adjacent coal.

Fig. 14c shows the seam mined in Saline county, and correlated by the State Geological Survey with seam No. 5. In general it is a bright laminated coal, the hard top coal being succeeded by a much softer coal in the middle of the seam, and this by a harder coal on the bottom. The noticeable feature of this seam here is its low moisture content. This coal has been compared favorably with the famous Hocking Valley coal.‡ Locally, in the roof there are bone and stringers of coal up to 3 feet or more above the true coal. The roof is generally a hard calcareous shale, while the fireclay bottom in places is sandy and heaves badly when wet. Only incipient cleavage is developed, and the coal has a good reputation for hardness.

Seam No. 6 (Fig. 15a) in the Standard district east of East St. Louis is usually divided into several benches by partings, sometimes of mother coal, often by sulphur or by shale bands, which include the blue band. In many places the roof consists of a thin layer of dark drawslate, locally called clod, containing frequent niggerheads. In places in which this drawslate is absent, the cap limestone forms a firm roof. According to A. J. Moorshead,¶ the coal is harder where limestone forms the direct roof. The floor, although a clay shale containing frequent boulders, as a rule does not heave, except when wet. In the mines in which the coal is thick (8 feet) the top coal

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*A. Bement, "Illinois Coal Fields." J. W. S. E., June, 1909.

†F. C. Lincoln, "Coal Washing in Illinois." Bulletin No. 69, Engineering Experiment Station. University of Illinois, p. 90.

‡A. Bement, "The Illinois Coal Field." J. W. S. E., Vol. 14, 1909, p. 319.

¶Colliery Engineer, 1914, p. 435.

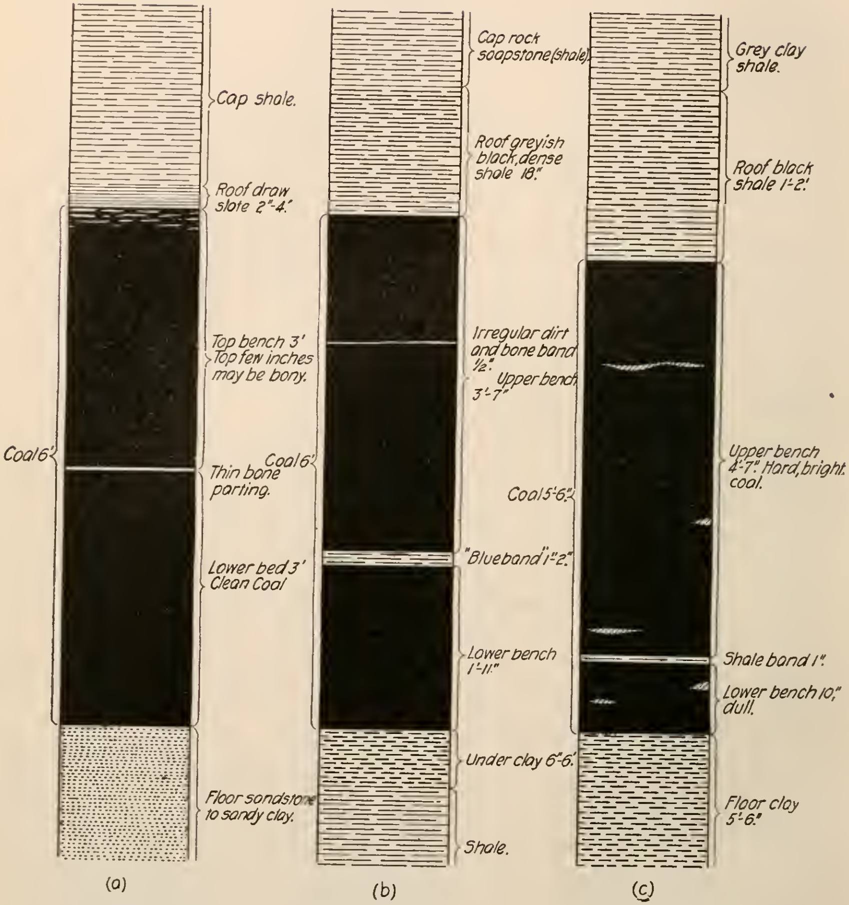

FIG. 16. TYPICAL SECTIONS OF SEAMS NOS. 2, 6, AND 7.

shown is often left in order to hold the roof in place. Since the district covered in the analyses embraces fifteen counties in whole or in part, considerable difference is evident in local physical characteristics. On the whole, the coal contains enough bands of impurities to require care and attention in removing them during its preparation.

Seam No. 6 in Williamson and Franklin counties (Fig. 15b), varies from 7½ to 14 feet in thickness. Excepting for the blue band and locally sulphur in thin, irregular vertical plates, the coal is clean. Although from the standpoint of heating value, it is not the best coal in the state, its toughness, strength and clean glossy appearance make it a favorite, especially for domestic trade and for northwest shipments often requiring several handlings enroute. Also, it withstands storage well. In places a definite east and west vertical cleavage affects to some degree the sharpness of its breakage. The top coal is frequently left in place.

The coal in seam No. 6 in Macoupin county (Fig. 15c), is often more brittle than that in the same seam farther south. This county is included in the average analysis given for seam No. 6, Fig. 15a.

Seam No. 2 in Jackson county (Fig. 16a), is often called Big Muddy or New Kentucky. Its chief characteristics are general freedom from impurities and high fixed carbon, as compared with most other coals of the state. The thin bone parting varies from zero up to 30 feet in thickness, in which case only the lower and better bed is mined. The cleavage is more pronounced than in any other district in the state, the coal breaking freely northeast and southwest, thus making blocky lumps of a bright lustre. As a rule, the floor is more stable than those in the other districts. The former use of this coal for the manufacture of blast furnace coke indicates its generally low sulphur content. A. Bement called this coal the best west of the Appalachian region,* and it is to be regretted that the district is so limited.

Seam No. 6 in the Danville district (Fig. 16b), called the Grape Creek field, consists of two benches, separated by the blue band, the upper bench being usually duller and dirtier than the lower. The roof, varying from a gray to a dark shale with little bedding, breaks away easily and falls, often in conchoidal masses, and thus is mixed more or less with the coal. The underclay swells readily. The bands and nodules of pyrite present are usually thick enough to allow hand separation. Rolls occur in both floor and roof, and frequent horsebacks tend to mix considerable fine impurity with the coal.

Seam No. 7 (Fig. 16c), as it occurs in the Danville district, is the uppermost seam exploited commercially in the state. The feature of the bed is the large lenses and bands of sulphur, which can be easily removed in mining and preparation. In general, the lower bench of the seam is hard and dull, overlaid by a softer, cleaner coal near the center of the seam and again by a harder bright coal at the top. The cap rock, a gray clay shale, is soft, and where open cut mining is practiced is dug and removed by steam shovels. The bottom is soft and rolls are frequent.

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*Black Diamond, June 27, 1914, p. 53.

PART II. IMPURITIES.

Impurities in the Coal Bed.—The diagrams or logs of the various Illinois coal seams (Figs. 13-16), show that they contain various thin bands or streaks of impurities, such as are found in practically all bituminous seams. In order to discuss these impurities, they have been divided by authorities into groups relating either to their origin, position in the seam, or ease of removal. The writer prefers a grouping by origin which divides these coal impurities into three general groups which have been variously named as follows:

Group 1. Innate, Intermixed, Inherent, Normal Ash, or Inseparable Impurities.

Group 2. Sedimentary, Interbedded, Intercalated, or Separable Impurities.

Group 3. Infiltrated, Extraneous, Interstitial, Segregated, Subsequent, or Precipitated Impurities (Sometimes Separable).

Group 1. Innate or Inseparable Impurities.—All the terms listed under this group have been used to describe the true or normal ash of the coal. Since coal is a product of plant remains, it must contain the original mineral matter or ash of such plant. Trees or plants of the present day contain about one per cent ash, but since ancient plant remains have undergone great changes and losses of weight through partial decomposition before arriving at their present stage of coal, and since the mineral matter remains without loss in the decreasing residue, the inherent ash may amount to a much larger percentage in the coal than in the original vegetable matter. Such impurity may be considered a part of the original coal substance. It is present in every piece of coal and cannot be removed or altered by any process of preparation. Certain coals have probably as low as 2½ per cent inherent ash. The author determined approximately by a number of experiments the inseparable or inherent impurities in Illinois coal. Each sample was crushed to pass a ¼-inch mesh screen and then subjected to a "float and sink" test in a solution of 1.35 specific gravity, which allowed the purest coal to float and the separable impurities to sink. The results indicated that the inseparable impurities varied from 3 to 7 per cent, with an average of about 5 per cent.

If the coal bed were formed slowly by accumulations under sheltered and shallow water conditions, and if this water were clear and without sediment, a coal would result containing only inherent ash. If, however, the waters of the marsh contained sediment, such as might arise from the influx of rivers, etc., the slow settlement of this slimy mineral matter simultaneously with the growth of the coal forming bed would produce a coal containing a uniform mixture of this impurity and coal. Such an impurity would increase the ash content, and as it increased in amount would cause the coal to assume more and more a rocklike character, both in appearance and physical characteristics. Since such sediment is, in most cases, a clay-like slime, or clay in a more or less colloidal state, which becomes shale under heat and pressure, its presence in increasing percentages tends to lower rapidly the commercial value of the coal. These impure coals are called bony coal or simply "bone," and by English engineers "bass." Geologically, there may be a range of bone coals containing as much as 50 per cent clayey matter; beyond this point the substance is no longer coal, but a carbonaceous shale, with pure shale as a limit.

If these conditions persisted to a slight degree only during the formation of the coal bed, this admixture of impurity, while increasing slightly the ash content of the coal, would produce through its own cementing tendency a harder and firmer coal than the average. As an illustration, the clean lump from seam No. 5 in the Springfield and Peoria districts is usually slightly higher in ash than the usual clean coal from some other districts, but is of such a recognized hard nature that special provision is made for its mining in the agreement between operators and miners.

If these conditions favoring the formation of bone coal were periodic or infrequent during the growth of the seam, certain benches only of the coal seam would be bony. At many places in this state the bottom bench of the seam is of a higher ash content and is harder than the upper benches, although not bony to a degree which interferes with its commercial value. This is illustrated by seam No. 7 in the Danville district. On the whole, the coal seams of Illinois are unusually free from benches of true high ash bone coal, and trouble is caused by it locally in one or two seams only.

The moisture and oxygen in the coal are also inherent impurities, but since they are removed only by weathering, heating and similar methods, they are not impurities removable in the dry preparation of coal at the mines. S. W. Parr says, "A coal with 14 per cent moisture may reach the consumer with 10 per cent only, therefore the value per ton is greater than when the coal left the mine."* This, however, is probably due to part of the moisture in Illinois coals being held mechanically in the pores and drying out on exposure; therefore, it is not an impurity under control of the operator in the preparation of the coal.

The effect of oxygen in coal is fully discussed by David White, who shows in Bulletin 29, U. S. Bureau of Mines, that oxygen is an inert constituent in coal, and unit for unit of weight takes the place of so much combustible material.

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*Bulletin No. 16, Ill. State Geol. Survey, p. 227.

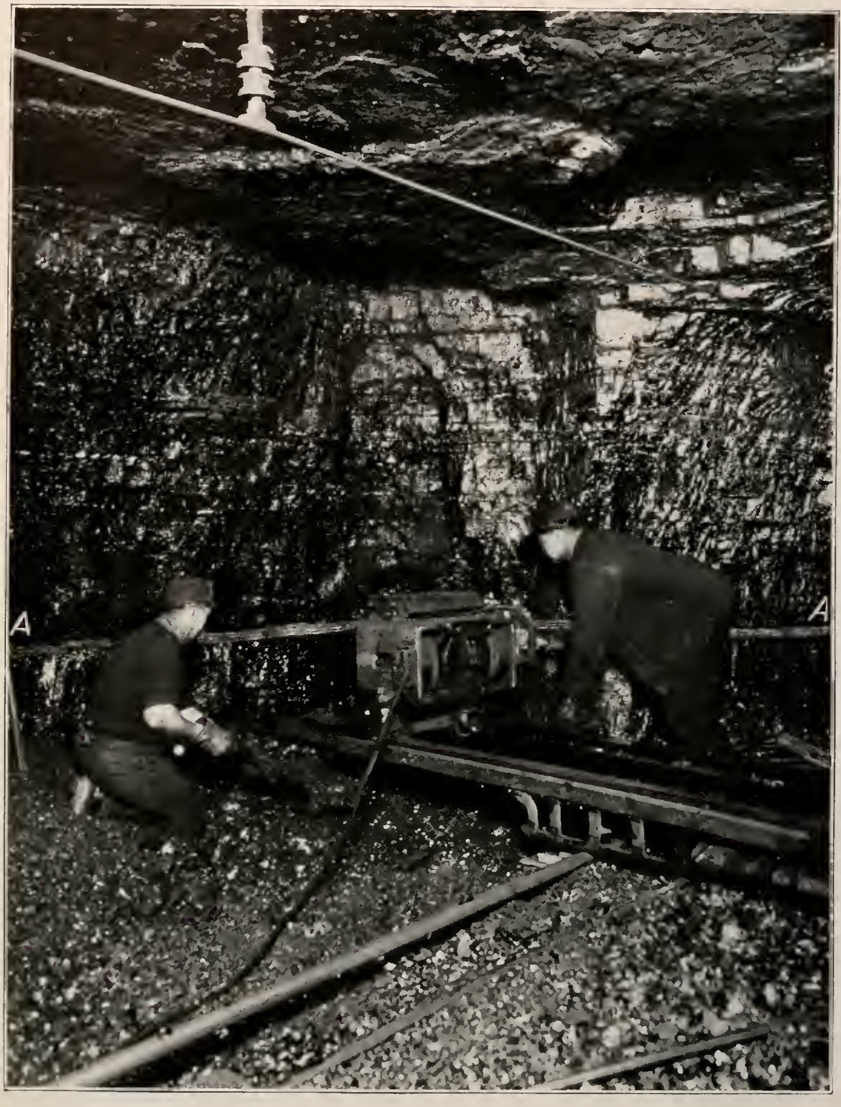

FIG. 17. THE "BLUE BAND."
AA=LINE OF BLUE BAND.

Group 2. Sedimentary or Interbedded Impurities.—Bone coal may be formed in layers or bands intermixed with and separable from the better coal. Such bands may be of any percentage mixture of pure coal and mineral matter and range in thickness from a knife edge to a dimension that separates the coal into two benches. Moreover, such a band may be either flat or lenselike in shape, and either local or of even thickness over a considerable area.

During mining these bands tend to break free from the coal into characteristic flat pieces. As they approach a pure shale in composition ($Al_2O_32H_2O + XSiO_2$) and also containing varying percentages of lime ($CaCO_3$), the color becomes lighter, usually a stony gray, and the specific gravity increases. This makes easy, detection and removal of the coarsest bands by the miner at the face, or by the picker in the better light of the tipple above ground. If these bands are in coal below two or three inches in size and above 5 to 10 per cent in quantity, their removal is accomplished only by some mechanical means, chiefly washing. From the larger sizes of coal, approximately those above three inches, and not exceeding 5 to 10 per cent in quantity, these shale bands can be removed by hand picking, although at the present time the largest percentage of such an impurity hand picked in an Illinois tipple does not exceed 3 per cent of the total coal, or about 4.5 per cent of the sizes above screenings.

Bands of bone coal, containing more than 50 per cent of coal substance, are dark in color and not greatly different from the coal in weight. This renders their removal difficult, whether attempted by hand picking or by washing, which process depends for success upon a considerable difference of specific gravity between coal and impurity.

An examination of the coal sections illustrated on pages 47 to 52 shows that the thinner of these impure bands, those under ¼ or ⅜ inch in thickness, may easily become mixed with the finer coal and not be detected. The thicker ones must be removed at some stage of the preparation, since the appearance and consequently the sale of the coal is injured by their presence, perhaps more than the coal is actually deteriorated chemically. Unfortunately, many of the shale bands in Illinois coal are rather soft and tend to soften, peel, and disintegrate rapidly upon exposure to air and moisture. For this reason, their complete removal from the finer sizes of coal is a problem of some difficulty. In most cases, the purer the shale, the more it tends to soften and disintegrate. The floor shale "fireclay" tends to disintegrate much more rapidly than the interbedded or the roof shale. The miners of the state call these bands of impurities "blue band" (referring to the well known impurity of seam No. 6), "black band," "blackjack," "stone," "grit," "dirt," or "brash." Fig. 17 shows the blue band of Seam No. 6 in Franklin county. Lumps in which coal layers and shale bands are intimately mixed in the same piece are called "intermixed," or "intergrown" coal.

Group 3. Infiltrated or Subsequent Impurities.—This group includes the visible impurities in the coal which were introduced subsequent to the formation of the bed, such as pyrite, calcite, or gypsum.

Underground circulating waters contain considerable amounts of iron, lime, and gypsum salts in solution, which deposit or precipitate under favorable conditions. Such conditions are furnished by the reducing tendencies of the carbonaceous matter of the coal, and by the more porous layers of the seam which furnish easy channels of circulation for the solutions. The firmer bands of the seam tend to define and limit these channels. Having started deposition around some favorable nucleus, further deposition tends to enlarge the particle. The final result will be nodules, bands or lenses of pyrite (iron sulphide, FeS_2), containing if pure 46.6 per cent iron and 53.4 per cent sulphur. By the miners they are usually called "sulphur balls," "sulphur," "cat faces," "kidney sulphur," or "brasses." The nodules have an irregular maximum thickness of several inches. Less resistance usually has been offered to the growth of these masses along the bedding or lamination planes of the coal; for this reason sulphur bands

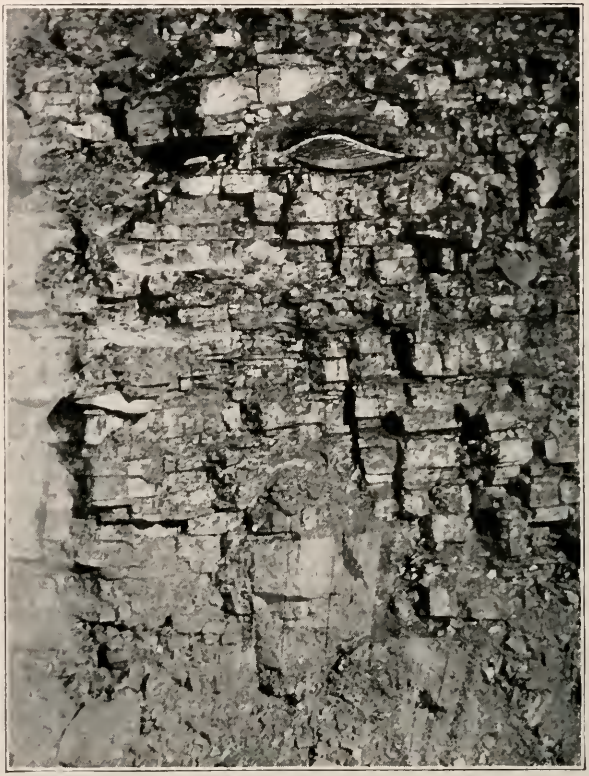

FIG. 18. PYRITE LENSE IN ILLINOIS COAL.

are horizontal in the bed, and may be either flat or slightly lenticular in shape. Often the bands are quite flat, are as much as one or two inches in vertical thickness, and may have a horizontal extent of many square feet. The lenses are sometimes 5 or 6 inches in vertical dimension and considerably greater in length along the bed. Like interbedded shale the pyrite is rarely pure in composition, an analysis of one lump showing 33.2 per cent of volatile matter and fixed carbon present. Occasionally, lumps of pyrite are seen, the forms of which suggest the replacement of bits of branches or other woody tissue. Fig. 18 illustrates the occurrence of pyrite lenses in Illinois coal.

The sulphur balls or bands, being brassy yellow in color and of high specific gravity (if pure from 4.9 to 5.1) are easily distinguishable by the miner, and are usually thrown into the gob. Frequently the pieces are more or less coated with adhering coal, and if missed by the miner are removed by hand picking in the tipple or on the railroad cars.

At one mine in the Standard (Belleville) district at which seam No. 6 is worked, enough of this lump pyrite is picked in a clean condition from the coal during screening and loading to justify saving and shipping the product to various chemical companies for use in the manufacture of sulphuric acid. At certain mines in the Danville district (seam No. 7), numerous large bands of pyrite are hand picked from the coal by the loaders, or in the tipple by pickers during screening and cleaning. Enough impure pyrite is secured from several mines to justify the erection and operation of a washing or jigging plant in which the raw pyrite or sulphur with its adhering bands or bunches of coal is crushed and washed. After the completion of this process the clean pyrite is shipped, and a quantity of fairly clean small coal is recovered as a by-product. In most other places in the state pyrite is justly regarded as a deleterious impurity, not only harming the appearance of the coal if not removed, but aiding materially through its combustion products FeS and FeO in the formation of clinker when the coal is consumed.

Information obtained by correspondence with various chemical companies indicates that the possible market for such a product as pyrite is limited since pyrite is used in quantities for the manufacture of sulphuric acid only when there is a scarcity of the usual and cheaper raw material—the sulphur in zinc blende. Any attempt on the part of Illinois coal operators to generalize the commercial production of pyrite as a by-product of washing or of picking belt, would result at present in a complete demoralization of the market and in a consequent lack of sale.

If all the sulphur in Illinois coals were in this lumpy form its removal would not present any serious difficulties since it can easily be removed from the smaller sizes by mechanical washing processes. Unfortunately, in many of the districts in this state the pyrite is found adhering to the coal in very thin leaves or plates, often several inches square and of almost infinitesimal thickness. These plates have originated by reason of the incipient vertical cleavage in the coal, such planes of weakness having afforded opportunity for the deposition of pyrite in irregular thin plates. When the coal is mined, it breaks into lumps more or less along these cleavage lines, thus exposing to view the thin brassy plate of pyrite. By actual weight or percentage the amount of sulphur in such a coal may be small, even smaller than in coals which show no sulphur to the naked eye. Coal with these glistening films is at a disadvantage on the market since consumers believe that they are an indication of an inferior fuel.

Such pyrite is difficult to remove from the coal underground; in fact, so many lumps of coal may have one or more glistening sulphur faces that a clean separation would involve the waste of a large percentage of the coal. Unfortunately for miner and operator alike, every fresh break in the coal during preparation is likely to expose fresh brassy faces. Again, these thin films hang closely to the coal, and their removal by any process of breaking and picking involves the loss of much lump coal. Another factor is that coal dust easily sticks to and hides these faces, making them indistinguishable until subsequent drying and jarring again bring them to light.

At one mine as many as fifty tons of coal having blotches of this pyrite are picked out in the tipple each day in an effort to ship coal that looks well. This is about 2½ per cent of the daily production of the mine. The coal thus separated is crushed and used as second grade fuel. At another mine which makes a specialty of sized domestic grades, the egg and nut coal are wetted or rinsed by sprays of water before being hand picked, thus removing any adhering dust and bringing to light the pyrite films. This rejected coal is also used for purposes for which appearance is not a requisite.

The author has not observed any regularity by districts in the distribution of this leaf pyrite, every seam in Illinois containing some of it, at least locally. For instance, at one mine no leaf pyrite is found in seam No. 6, while at another mine only a few miles distant it is present in the same seam. This form of pyrite cannot usually be removed by mechanical washing, because it adds little to the weight of the individual piece of coal to which is it attached. In general, operators troubled by it, though at a disadvantage in the open and domestic markets, should have little difficulty meeting competition based on specifications since the trouble often looks worse than it really is.

The total sulphur content of Illinois coals ranges from 1 to 6 per cent, most of it being held in the forms previously noted.* The remainder, probably from ¼ to ¾ per cent of the coal, is in some not well understood chemical organic combination, probably with the hydrogen and carbon of the coal. Such organic sulphur supposedly is completely burned in the process of combustion. This form of sulphur is present in all coals, and is so intimately combined with the coal substances that it is not apparent to the eye and cannot be mechanically separated from it. For this reason, it may be classed with the ash of Group 1. The minute percentages of phosphorus present in coal may be classed with this sulphur. Such organic sulphur does not lessen the value of a coal for combustion.

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*S. W. Parr, Bulletin 16, Illinois State Geological Survey, p. 226.

Only those forms of sulphur, such as pyrite, which occur as a mineral in the coal and which after combustion leave an incombustible basic residue such as iron oxide (FeO), promote the formation of the ash slag known as clinker. However, a complete solution of the clinker problem involves a study of combustion temperature and conditions* and of the proportion in the ash of the basic compounds (iron or lime) to acid compounds† (silica and alumina), and of their relative sizes and admixture. Pyrite alone need not cause trouble.‡

Calcite ($CaCO_3$), and in smaller quantities, gypsum or calcium sulphate ($CaSO_4 . 2H_2O$), and still smaller amounts of other salts occur in much the same way as flake pyrite. Many Illinois coal beds contain these thin flakes of calcium sulphate and of gypsum in the small joints and cleavage planes of the freshly mined coal. If these minerals are present in considerable quantity their whitish color offers an easy means of detection, and removal may be made by hand picking. Seams Nos. 1 and 7 especially contain in places considerable amounts of these impurities. The presence of gypsum adds a percentage of sulphur to the coal. Whether or not these impurities may be removed by washing depends upon their amount, thickness, and tenacity to adhere to the coal. At only a few mines is attention paid to them during preparation, their presence usually being of chemical interest only.

Another probably subsequent impurity is the clay or shale which often fills small vertical fissures through, or in the top or bottom of the coal bed. These are locally called slips, horsebacks, or mud seams. In some districts, for instance around Springfield, these impurities occur frequently in seam No. 5, and are called "clay slips." They are probably caused by the soft floor or roof material having worked, under pressure, into local slips or faults in the coal bed. Local rolls in roof or floor, causing thinning of the seam, together with more or less intermixture of coal and shale often occur. The method of separation of any of these from the coal is similar to that of removing the shales as noted in Group 2. Many such disturbances are avoided in mining and they are not necessarily an impurity.

Niggerheads are sometimes present in or just above the coal bed. In the same way that pyrite through deposition forms sulphur balls, calcium carbonate, or iron carbonate, upon deposition around a favorable nucleus, may form oval concretions or niggerheads often several inches or even feet in diameter. These impurities, readily detected on account of their size and shape, can be easily removed by the miner when shot down with the coal in the mine.

Even the coal part of a seam is not a solid homogenous mass of

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*S. W. Parr, Bulletin 16, Illinois State Geological Survey, p. 226.
†F. R. Wadleigh, Coal Age, June 22, 1912, p. 1206.
‡W. B. Phillips, Coal Age, July 27, 1912, p. 111.

pure shining coal. A casual inspection shows that the seam is built up of alternate bands or laminae of bright, shiny coal, and dull, lustreless coal. According to T. E. Savage,* a close examination of much of the Illinois coal shows that these alternating laminae are generally between 1/32 and ½ inch and often more in thickness, and that in places the dull laminae make up nearly one-half of the coal bed. Coal in which the bright bands predominate has in the small sizes a bright appearance approaching anthracite, popularly supposed to be necessary with a good coal.

A powdery dull thin band generally of paper thickness, called mother of coal, mineral charcoal, or carbonized wood, forms a distinct parting between many of these layers or laminae in Illinois coals. On exposure such a band leaves a slight smut when touched with the finger. If of any thickness, it dusts badly when the coal is mined and must in such cases be a definite factor in the formation of fine dust. Although somewhat unlike the other dull coal in appearance, this mother of coal is usually associated with and constitutes a part of such a band.

David White and R. Thiessen,† and James Lomax,‡ have studied microscopically the bright and dull bands in coal and their conclusions are that dull bands may be as truly coal substance as the bright bands with which they are associated. E. C. Jeffrey¶ came to the same conclusions concerning mother of coal or mineral charcoal. These statements do not apply to the definite bone or shale bands which, although dull in color, are entirely different in appearance. An analysis of the dull mother of coal layers from seam No. 6 in Williamson county§ showed them to contain the same amount of ash as the average coal from the seam. Several analyses made recently by M. L. Nebel,** however, showed in every case higher ash values in the dull bands than in the bright bands in the same lump of coal. A sample of mother of coal taken from a band ½ inch in thickness in seam No. 6, Williamson county, and analyzed under the direction of the writer, gave the following proximate analysis: Moisture (as received) 0.16 per cent; volatile matter 9.75 per cent; fixed carbon 87.47 per cent; ash 1.72 per cent; and sulphur 0.90 per cent. This indicates that mother of coal is a high grade coal of different composition than the rest of the seam. Since the presence of the dull laminae and the mother coal are specially prominent in Illinois coals and since coal of a dull appearance is at a disadvantage in the open market, further analytical work concerning the relative purity and composition of these bands is desirable.

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*Journal of Geology, Vol. 22, No. 8, 1914.

†"The Origin of Coal," Bulletin 38, U. S. Bureau of Mines, pp. 29 and 64.

‡"Microscopic Examination of Coal," T. I. M. E., Vol. 42, p. 2.

¶Economic Geology, Vol. 9, No. 8, p. 734.

§T. E. Savage, Journal of Geology, Vol. 22, No. 8, 1914.

**Results of these analyses will appear in Bulletin 89 of the Engineering Experiment Station, University of Illinois.

Impurities Entering the Coal from Roof and Floor.—In a majority of the mines in Illinois the roof directly above the mineable coal is of a soft crumbly nature. In some of the thicker coals, as in seam No. 6 in the southern districts, this roof is protected by leaving the bench of top coal in place. In other districts the shale roof is only a few inches thick, and when it spalls off or is taken down, is found to be overlaid by a firm limestone or other resistant roof. In mines in which the coal is under about six feet in thickness and in which it is necessary to take out the whole seam, excessive use of powder weakens the roof, which frequently breaks off, sometimes in large slabs, but oftener in small scales. When this "drawslate" is removed some of it unavoidably becomes mixed with the coal. Seam No. 6 in the Danville district has a notably crumbly roof. This impurity is more likely to occur in solid shooting mines than in machine worked or in longwall mines. In the longwall field, however, the soapstone or slippy shale roof is weak and brittle, and falls unless closely propped. In the state as a whole there are conditions of roof and customs in the use of powder which tend to introduce portions of this roof into the coal as an impurity. As a rule, this material keeps its structure well enough to make possible its removal at the proper place by picking or by mechanical means unless extra breakage allows it to pass into the screenings.

The different seams in the state generally have a soft weak clay shale or fireclay bottom. Often this floor is extremely hard when first exposed and makes a suitable bottom from which to shovel the coal; in other places it is softer and mixes with the coal. Usually, after exposure for a few weeks to the dry atmospheric conditions in the mine, the top layers of such a floor crumble and become loose and dusty. Sometimes the floor expands, swells, and rises, thus increasing the probability of the presence of impurities in the coal should shoveling be necessary at such places. In most of the cases which come under the writer's observation, the lumps of bottom fireclay, although they may be hard and firm, disintegrate if placed in water even for five minutes. Under like conditions in the mine the bulk of this fireclay passes into the screenings during preparation. In parts of the longwall field hand pick undercutting is practiced in this under shale band, producing considerable amounts of fine shale, which become mixed with the coal and pass into the screenings. Under this condition a clean high grade lump coal and dirty screenings are produced. This practice is not uniform.

In the room and pillar mines floor impurities are generally introduced through three causes:

(1) Careless shoveling.

(2) Excessive disturbance of the floor by heavy shooting.

(3) Undercutting in the bottom clay instead of in the lower bench of coal.

Careless shoveling explains itself. Concerning the second cause, W. R. Coleman* says that long 6 to 7 feet holes in solid shooting mines in Illinois may pick up from 4 to 10 inches of bottom mud, which becomes mixed with the coal during shoveling into the mine car. Although these are extreme cases, they illustrate the possibility of the addition of considerable impurity if such conditions are not kept under control. Little difficulty should be experienced in undercutting if the bottom or floor is fairly level. In some places uneven floors formed by rolls and horsebacks may be cut into.

PART III. REMOVAL OF IMPURITIES.

Removal of Impurities Underground.—Most of the larger pieces of separable impurities which have been mined with the coal may be removed underground by hand sorting during loading, but the time spent in doing this reduces a miner's daily output of loaded cars; consequently, there is an inclination to minimize this work. The regulations pertaining to this subject, as a part of the evolution of present preparation practice, have been outlined in Chapter I, pp. 23 to 29.

The claim has often been made that the most efficient place for inspection of coal for impurities is underground during loading. However, as the miners usually work in pairs only and in separate rooms, continual underground inspection in the dim light is impossible. At several mines in which persistent bands of impurities occur, the assistant mine manager makes regular trips through the mine and by noting the size of the waste or rejected pile, or the character of the coal in a partly loaded car, he is able to estimate rather closely the percentage of rejected impurities. If he detects carelessness, word is sent to the regular coal inspector at the surface plant to watch closely for the cars of the miner in question. It is probable, however, owing to the multiple duties of the assistant mine manager and to the limited number of these officials allowable under the agreement between the operators and miners, that underground inspection is casual rather than systematic.

Once in the mine car further inspection is impossible until the coal is dumped or spread out on the screen in the tipple. One mine only was visited in which inspection was attempted in the loaded cars at the shaft bottom before hoisting. Since the natural tendency is to put clean coal on the top of the car, it is difficult to see how such inspection could be effective. At the particular mine noted, condemned cars were not hoisted into the tipple, but were set off at the surface landing and kept there for the inspection of the interested parties, and in full view of all men entering or leaving the cage. It was explained that the effect of this was noteworthy.

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*Proc. 1st Annual Convention, International Railway Fuel Association, 1909, p. 22.

It is necessary to emphasize how closely all the impurities loaded with the coal are within the control of the miner at the face. If he takes pride in his trade, is careful, undercuts with skill, uses powder with judgment, and loads with care, a considerable part of preparation has been accomplished, and he has done his part towards making a good name in the markets for the mine in which he works.

Removal of Impurities in the Tipple.—At practically all mines in Illinois an attempt is made to remove the bulk of impurities loaded underground at one or more of the following places in the surface plant:

(a) In the tipple during screening.

(b) On picking bands or belts after screening.

(c) During the loading of the railroad cars underneath the tipple.

At one of these places a coal inspector, dock boss, or rock man is stationed, who not only watches for impurities and penalizes the miner according to the joint agreement (see page 24), but often has general charge over the men picking and loading the coal. Some companies place an inspector at each tipple, others employ a single inspector who covers two or more of their neighboring surface plants. Large companies frequently have one chief inspector who superintends dock bosses at the individual tipples. In all cases close inspection was reported beneficial in lowering the percentage of ash in the coal. Frequently, the buyer is notified if a railroad car which has been loaded contains defective coal. An economy of 6/10 of one per cent will pay for inspection at the mines.*

At a few tipples in the state the dock boss "rides" the screen; that is, he is seated on a stationary support just above the shaking screen, from which with perhaps one or two assistants, he can bend down and remove the impurities from time to time. If the screen is narrow (7 feet or less), the dock boss and his assistants may stand at the sides and remove impurities; in either case throwing the rejected material into chutes which lead to a refuse car or pile below. Such inspection is not very efficient in the dusty surroundings if a considerable amount of impurities is present, if a large tonnage (perhaps 2,000 tons or more) passes over the screens, or if wide screens (7 to 10 feet) are used. Another factor hindering the detection of impurities is the mixture of sizes from nut to the largest lumps, which must be examined at the same time. In such a mixture of sizes, lumps are bound to hide the impurities of nut size. This difficulty is overcome at several tipples by the use of a patented combined screen and picking table. The screen is horizontal and of sufficient length and movement to separate efficiently the sizes and allow picking on the screen under favorable conditions. This screen will be described later.

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*Eugene McAuliffe, Proc. 4th Annual Convention, Int. Ry. Fuel Assoc., p. 280.

The Black Diamond, under date of Feb. 1, 1913, stated that picking tables after screens were new in Illinois. The demand for cleaner coal forced Illinois operators, especially at the large mines, in which from 2,500 to 5,000 tons per day were handled in the tipple, to adopt these picking tables or belts as a method of removing the refuse. The picking tables (Fig. 19), are from 3 to 5 feet wide, often 30 feet long, 30 inches above the floor, and are endless belts either of rubber or of steel links covered with small over-lapping sections of steel plate. The tonnage handled is up to 1,000 tons per day per belt. These belts catch the coal as it falls through or leaves the screen and convey it horizontally at right angles to the screen. Each size of coal may have an individual belt or picking table. Thus the coal, spread out on the belt and traveling at a speed of from 30 to 60 feet per minute, passes before men or boy pickers who can easily pick out the refuse. Dirty coals, of course, require low speed and additional pickers. Speed depends also on the size, on the color, and on the shape of the impurities; that is, the ease with which they can be distinguished.

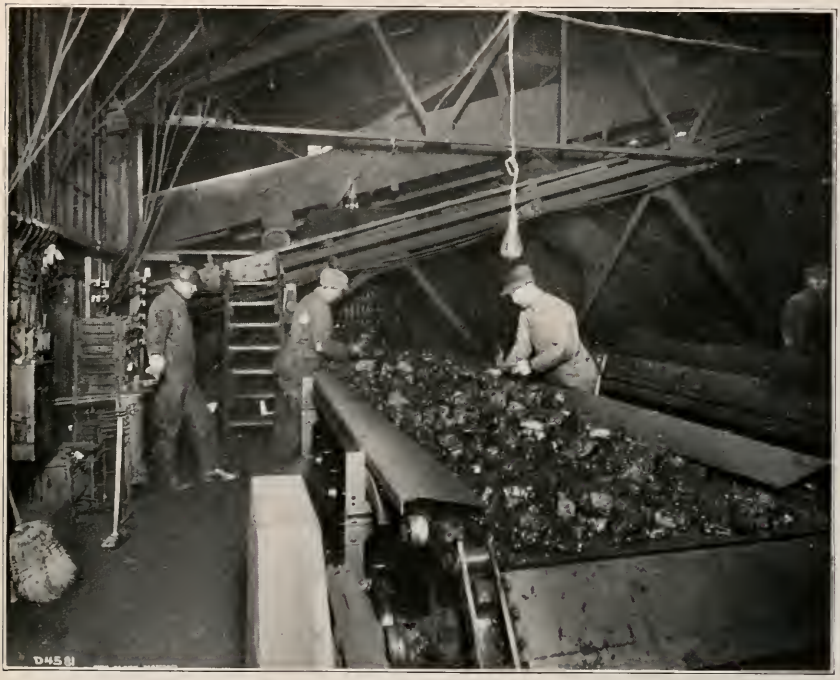

FIG. 19. OPERATION OF A PICKING TABLE.

In general, picking tables, bands, or belts have the following advantages:

(1) Thorough inspection; all coal being spread out in fixed sizes and moving at a fixed rate.

(2) Decreased length of chutes and height necessary to feed certain tracks.

(3) Loading of coal in a constant stream with minimum velocity and breakage.

(4) Safety and effectiveness of pickers; they are no longer exposed to the dust on the screens or to danger in the loading cars.

(5) Combined with a movable loading boom, they reduce further the breakage and allow easy loading on cars of different heights. The only disadvantages are first cost and maintenance.

At one mine in which pyrite refuse is saved from the tables, six rock pickers, three on lump and three on egg, pick out 50 tons of pyrite per day, or about 8 tons per man per day. On account of the high percentage of refuse, stationary plows are fixed on the tables close above the belt, so that the moving coal may be turned over and thus expose to view any hidden impurity.

At other mines, including some of the largest, inspection and picking are carried on only when the railroad cars are being loaded with the screened coal. The stream of coal filling the car is closely watched by one or two men, who throw overboard from time to time the noticeable pieces of impurities. They often use large rakes to pull the impurities from under the falling stream of coal. In such cases picking is for appearance only, the actual amount of impurities removed being usually under 1 per cent of the size loaded. At some mines only the lump sizes are picked. At other mines the egg and nut sizes as well as lump sizes are picked. At these mines the dock boss usually watches the railroad cars and not the screens for impurities. Fig. 20 illustrates car picking on egg coal.

Car picking is sufficient if the amount of refuse is small or consists of only occasional and accidental pieces, possibly one or two tons per thousand tons loaded. In these cases it is possible to remove all noticeable impurities from the top of the car before shipping. Car picking is not sufficient to clean systematically a dirty coal. A piece of impurity even if detected cannot always be removed from a stream of lump coal falling and rolling into a car without exposing the workman to danger of injury from the large lumps. Consequently such impurities are buried rather than removed. The danger of being injured by the small egg and nut sizes is not so great. In one case as much as 1 per cent of the egg size is thrown out of the car during loading; however, the percentage is usually much smaller. The removal in this way, as refuse, of a noticeable percentage of the coal would probably be commercially impossible. Table 6 shows detail of picking practice at several mines chosen at random.

The chief difficulty with hand picking in any form is that of securing conscientious labor to do the work. At several of the tipples visited the laborers made little pretense at picking unless the dock boss was in constant attendance. Another feature is the amount of good coal going to waste with the shale and sulphur. For instance, if an impure band ½ inch thick is noted in perhaps an 8-inch lump of coal, the whole is likely to be discarded. An examination of some of the refuse heaps to which the rejected lumps are hauled, revealed many tons of good coal. It is unfortunate that this waste may be

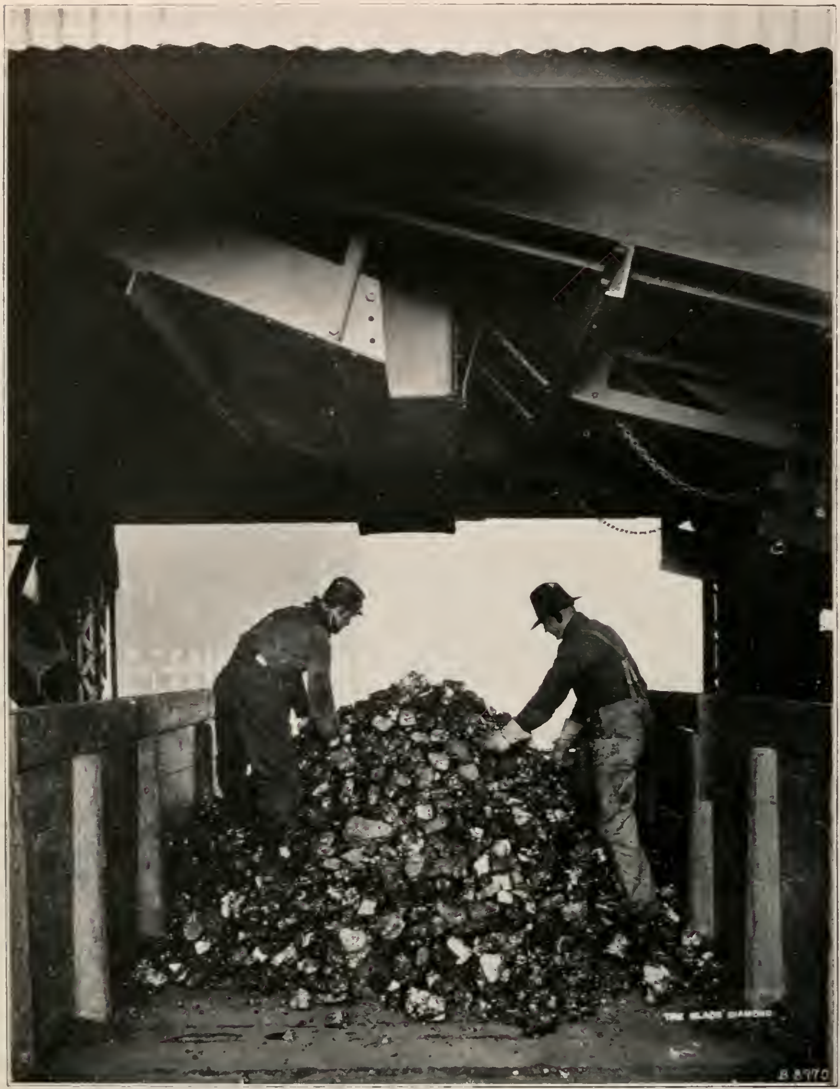

Fig. 20. Hand Picking an Egg Coal During Loading.

TABLE 6.

DETAILS OF TIPPLE PICKING PRACTICE IN ILLINOIS MINES.*

Mine No.	Commercial Designation District	Picking Lump Coal				Picking Egg Coal				Picking Nut Coal				Coal Inspector	Remarks
		Tons P'k'd per Day	How Picked	Tons Wasted per Day	No. of Men	Tons P'k'd per Day	How Picked	Tons Wasted per Day	No. of Men	Tons P'k'd per Day	How Picked	Tons Wasted per Day	No. of Men		
1	Saline county...	375	In R. R. cars only	2	1	...	Not picked.......	–	–	...	Not picked.......	–	–	Part time	
5	Franklin county.	800	In R. R. cars only	1–2	1	800	On picking belt...	8–12	2–3	...	Rescreened	–	–	Yes	
7	Franklin county.	600	In R. R. cars only	–	–	660	R. R. car........	–	–	480	R. R. Car.......	–	–	Yes	
9	Williamson " .	900	In chute and car	7	2	420	On picking table..	3	3	...	Washed	–	–	Yes	Waste: 10 tons per day. 8–10 trimmers and pickers.
11	Franklin county.	600	On picking table and loading boom	–	–	600	On picking table..	–	–	400	Rescreened	–	–	No	Total waste: 3 tons per day.
17	Sangamon " .		No picking except occasionally if coal is dirty.........			...	....	–	–	...	....	–	–	Yes	
24	Montgomery " .	450	On picking table and loading boom	11	2–3 boys	400	On picking table..	10	2 boys	...	Not picked.......	–	–	Yes	
32	Longwall field...	360	In R. R. cars....	2 3	1	360	Not picked.......	–	–	...	Not made........	–	–	Yes	Occasional pieces of sulphur.
34	Standard district	650	On screens.......	5	2	650	On screens.......	5	1	390	On screens included in other sizes.	–	–	Yes	3 men pick
35	Standard district	1300	On screens.......	5	–	1300	On screens.......	5	–	...	Not made........	–	–	Yes	2 men and Inspector pick on screens.
40	Standard district	1000	In R. R. cars....	2–3	1	...	Not picked.......	–	–	165	Not picked.......	–	–	Yes	
42	Marion county..	550	On screen and table	–	–	440	On screen and table	–	–	440	On screen and table	–	–	Yes	12 pickers; 60 tons waste per day.
43	Marion county..	720	In R. R. cars....	1	1	...	Not picked.......	–	–	...	Not made........	–	–	Yes	Inspector picks.
45	Perry county....	700	On screen and table	–	–	360	On picking table..	–	–	340	On screen and table	–	–	Yes	4 pickers; 6 tons waste per day.
47	Perry county....	700	In R. R. cars....	1	1	420	On picking table..	4	2	420	On picking table..	4	2	Yes	50 tons impurities per day.
48	Danville district	150	On picking tables.	–	3	600	On picking table.	–	3	...	No nut made.....	–	–	No	

*Mines were chosen at random.

a commercial necessity at the present time. At a few mines these lumps are pulled from the picking belt, the impure band split off, and the good coal returned to the belt. This operation is called "skinning" the coal.

Sizing should take place before the picking is begun. If picking is attempted on 1¼-inch lump coal the eye of the picker cannot readily detect the different sizes of impurities mixed with the different sizes of coal. The more uniform the size of the coal that passes the picker, the better the impurities can be removed. A recently built dock cleaning plant sizes the coal for picking purposes, and then reunites the sizes to meet the demands of the market.

Other important questions regarding picking are: How small sizes can be hand picked? How shall the screenings and small sizes be cleaned? Should they be hand picked or washed? In general, what are the limits of refuse today in a commercial Illinois coal?

In European bituminous coal fields hand picking has been practiced for many years. In comparing their practice with our own, it must be kept in mind that cheaper labor and higher priced coal allow a closer and a greater range of work than is possible here. Various authorities treat this problem as follows:

"The process of hand picking can be successfully applied to pieces of coal and shale of 1½ inches to 2 inches in diameter, but not to smaller pieces. Washing can be used from 3 inches to 1/20 inch. There is no known process of separating shale dust from coal dust."*

"Hand picking cannot economically be applied to coal less than 2 inches in diameter, except in exceptional cases as where coal is fairly clean or wages low, in which case picking takes place even at 1 inch. It is usually done by boys, as it requires alertness of hand and eye."†

"On the continent sizes over 4 to 5 centimeters (1.6 to 2 inches) are generally picked by women or boys, sizes smaller than these are frequently washed."‡

In many places in Illinois hand picking is practiced on the lump size only, in others the 3-inch egg is the lower limit. It is probable that unless the refuse in such sizes had some special value it would not pay to mine and hand pick coal in Illinois containing more than from 3 to 5 per cent of refuse in the large sizes. Fortunately, in most districts the bulk of refuse passes into the finer sizes.

At present in Illinois washing is practiced on 3 or 3½-inch coal as a maximum size.¶ This is partly on account of the mechanical difficulties encountered in building jigs for washing larger sizes. The average amount of impurities removed from the washed coals in 35 plants examined was 11 per cent, with a maximum of 36 per cent and a minimum of 5 per cent. It is difficult to estimate an exact limit for commercially profitable washing because of the varying character of the refuse and its degree of freedom from contained coal. On the whole 3½ inches is the maximum size, and about 7 per cent refuse, containing at least 60 per cent of ash or its equivalent, represents a possible minimum of removable impurities. Under European conditions this limit has been put at from 4 to 6 per cent impurities.* As a lower limit, coal under ¼-inch in size is benefited little by washing and under from 20 to 50 mesh probably not at all. Considerable doubt exists as to the proper method of handling the small sizes of Illinois coals on account of the frequent occurrence of a high percentage of ash. These sizes are discussed further under Sizing in Chapter IV.

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*W. Galloway, "Lectures on Mining." Subject 8, p. 2.

†W. S. Boulton, "Practical Coal Mining." Vol. 3. p. 315.

‡J. Callon, "Cours d'Exploitation des Mines." Texte 3, p. 152.

¶F. C. Lincoln, "Coal Washing in Illinois," Bul. No. 69, Engineering Experiment Station, University of Illinois.

At a considerable number of mines, especially in the southern part of the state where rescreening plants are used for the separation of nut coal and screenings, all sizes below from 2 to 3 inches are divided into as many as five distinct sizes. Most of the impurities generally stay with the finest of these sizes, leaving two or three of the largest sizes of nut coal practically as clean as the lump. At several mines one or two men were noted in these rescreeners picking the largest sizes of nut, perhaps from 3 to 1¾ inches in size. Although no figures of amounts picked were obtainable, it is evident that the large number of pieces of this size that must be picked out of the coal to produce a ton of refuse makes the problem of doubtful economic value. If two men are picking 100 tons per day of such nut, the extra cost is roughly five cents per ton. From the ash standpoint one man must pick one ton per day of this fine material to reduce the ash content one per cent. Does the comparatively small amount picked pay for the better appearance? This question must be solved for each coal and each market condition.

Impurity and Inspection Standards.—Coal inspection is becoming more rigid, and large users, such as railroads, frequently have their own inspectors, even at the mines of the producer. Much coal is inspected by the buyer by a hasty examination of the top of the railroad car; but to secure the best results, more strict examination, such as inspecting the inside of the load while it is being loaded is necessary. If hopper bottom railroad cars are used and delivery is made through these to a bin or to a stock pile, individual car inspection is difficult.

A set of impurity and inspection standards has been proposed for railroads using bituminous coal as follows:

"The seller further agrees that all coal delivered under this contract shall not contain more than....per cent removable noncombustible or nearly noncombustible impurities. The quality of coal furnished....shall be subject to the inspection of the buyer, and the buyer's inspector....has the right to reject any of said coal which.... does not conform to specifications, at whatever point the same may be

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*Soc. Min. Ind., Vol. 17, p. 384.

TABLE 7.

MAXIMUM PERCENTAGE OR LIMITS OF REMOVABLE IMPURITIES PERMISSIBLE IN GRADES OF COAL FROM THE DISTRICTS AS LISTED.

Mining District	Lump		Egg		Mine Run	Egg Run†
	Size	Percentage Impurities Allowed	Size	Percentage Impurities Allowed	Percentage Impurities Allowed	Percentage Impurities Allowed
Canton, Ill.......	1¼" R*	2.0	1¼" x 8" R	3.0	2.0	...
Virden, Ill.......	1½" R	1.5	1½" x 6" R	2.5	1.5	...
Centralia, Ill.....	1½" R	1.5	1½" x 6" R	2.5	1.5	...
Herrin, Ill.......	2 " R	1.0	2 " x 6" R	1.5	1.0	...
Oskaloosa and Albia, Iowa....	1¼" bar	4.0	....	...	...	...
Centerville, Iowa	1¼" bar	1.0	....	...	1.5	...
Bevier, Mo.......	¾" bar	2.0	....	...	3.0	...
Novinger, Mo....	⅞" bar	3.0	....	...	3.0	...
Walsenburg, Colo.	....	...	....	...	2.0	...
LaFayette, Colo..	....	...	....	...	0.5	...
Cambria, Wy.....	¼" Sq.	4.0	....	...	4.0	...
Sheridan, Wy....	4 " bar	0.2	2½" x 4" bar	0.2	0.5	0.7
Kirby, Wy.......	....	...	....	...	0.8	1.0

*R = Railroad.
†See p. 104.

found. The buyer's inspectors shall have access to the seller's tipple screens, scales, washer, and yards while the coal is being hoisted and prepared.

"All coal delivered....may be inspected at the mines by the.... (representative) of the buyer....such inspection and refusal to be final and conclusive."*

A further paragraph covers the discovery of inferior coal after shipment when the same was not inspected at the mines.

Besides individual contracts, the most specific standardization of Illinois and other middle west bituminous coals in regard to the percentage of noncombustible or nearly noncombustible impurities allowable has been used by several railroads drawing their supply from mines in the territory mentioned. Through the kindness of C. G. Hall, Secretary-Treasurer, Int. Rwy. Fuel Assoc., figures of the limits or removable impurities as allowed by these railroads have been secured (see Table 7).

Table 8 shows the amounts of different impurities, etc., which are allowed to remain in the coal in the standard preparation of Pennsylvania anthracite,† and is given for comparison with Table 7.

TABLE 8.

IMPURITIES ALLOWED WITH STANDARD ANTHRACITE PRACTICE.

Size of Coal	Allowable Percentage			
	Slate	Bone*	Of Next Size Larger	Of Next Size Smaller
Broken	1.0	2	—	20
Egg	2.0	2	5	50
Stove	2.5	4	5	50
Nut	4.0	5	10	15
Pea	8.0	5	5	15 Buckwheat 15 Rice
Buckwheat	10.0	—	8	15
Rice	15.0	—	8	15
Barley	15.0	—	8	25

†Bone equals product of between 40 to 55 per cent of carbon.

PART IV. BREAKAGE.

Bituminous coals vary from 0.5 to 2.0 on Mohr's scale of hardness.‡ They are, as a rule, decidedly brittle and friable, tending to break into more or less cubical blocks, depending on physical structure, such as frequency of bedding laminae or planes of stratification, and development of cleat or vertical cleavage. Cleat may be defined as the tendency for the coal to break into more or less cubical blocks along vertical or highly inclined planes either parallel or normal to the face of the coal.

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*Inspection Specifications, Proc. 5th Annual Convention, Int. Rwy. Fuel Assoc., 1913, p. 28.

†Paul Sterling, "Preparation of Anthracite." T. A. I. M. E., 1911, p. 757.

‡Henry Lewis, "The Dressing of Minerals." p. 8.

Natural Physical Factors Causing Breakage in Illinois Coal.—Illinois coal may be considered comparatively hard, rather porous, more or less brittle, and friable; tending to break or split easily along definite major bedding laminae 5 to 12 inches apart and somewhat less easily along small or minor bedding planes. Vertical cleavage, prominent in many bituminous coals, is generally lacking or is developed only incipiently. This incipient cleavage is more marked in one direction than in the other, the lump coal usually being rather smooth and regular on the breakage faces in two directions, but tending to be irregular on the third face. This incipient cleavage determines the lines on which the coal frequently breaks into small blocks on first being heated. This cracking, which is probably due to shrinkage caused by the heat expelling the water, is also often more strongly developed in one vertical plane than in the other.

Exceptions to this rule occur in the case of east and west cleavage in parts of seam No. 6 in Franklin county, and in that of a cleavage in seam No. 2 in Jackson county (Big Muddy district). Here the cleavage is developed sufficiently to influence the direction of mining, as the ease of mining when working along the face cleavage here, tends to produce a larger percentage of lump coal with less powder than in the majority of mines. In most districts, however, mining takes place with no thought of cleavage.

There is considerable difference in the natural tendency towards breakage of coal in the different seams and in the different districts in Illinois. The coals of seams No. 1 and No. 2 in the northern field are of average hardness (firmness or cohesion); an example appears in the coal from seam No. 2 in the La Salle district, which breaks into rectangular lumps capable of withstanding much handling before breakage.* The coal from seam No. 5 in the Springfield and Peoria districts is harder and firmer than the average. Seam No. 6 west of the DuQuoin anticline, although covering a large area, is generally regarded as somewhat friable, especially in the northeast and southwest portions and also in those mines just west of the anticline, in which it is close to the surface. Seam No. 6, east of the DuQuoin anticline, and in general wherever it is situated under several hundred feet of cover, consists of a hard firm coal, especially in the smaller sizes. This coal withstands without breakage handling and shipment, and therefore forms a good domestic fuel in markets in which clean coal and close sizing are prerequisites. The coal of seam No. 5 in Saline county is also of a firm texture and well adapted for rescreening. Seam No. 6 in the Danville district contains rather friable coal. Seam No. 2 (Big Muddy), partly by reason of the cleavage noted, produces blocky lumps of a bright lustre. The coal of seam No. 7, Danville district, on the whole is softer and more brittle than the others, perhaps on account of its nearness to the surface. It cannot undergo transportation as well as the average.* It is interesting to note that in the deepest mine in the state, at Assumption, a coal is produced from seams Nos. 1 and 2, which commands a premium, partly on account of its blocky firm character.

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*T. A. I. M. E., Vol. 29, p. 187.

Importance of the Breakage Problem.—Aside from the inherent friability or tendency of the coals to break, as just discussed, the mining and preparation, or in general, any moving, dropping, or handling of a brittle substance like Illinois coal results in more or less breakage with each operation. Breakage, therefore, commences with the first operations of mining, continues with each succeeding operation, and ceases only when the coal is consumed. This gradual breaking of such a material into smaller sizes due to handling, exposure, etc., is called degradation.

None of the problems confronting the Illinois mining industry today is more important commercially than that of breakage. Although this problem, as far as surface preparation is concerned, could be solved so that a considerable increase in value per ton of the whole product would result, yet it has been neglected in all except a few plants recently erected. Breakage constantly reduces the larger sizes to form the smaller. The effect of this sizing on the prices received for the coal by the operator is shown in Fig. 21, prepared from the figures appearing weekly in "Black Diamond" during the years 1913 and 1914 for the average circular price f. o. b. cars at the mine in Franklin county. Although the curves do not represent the true price received in individual contracts, yet taken over such a long period, they represent the average difference in value of the sizes produced. The egg and nut sizes, although ranging with the lump in circular price, are frequently unsalable as such, and must be thrown into the cheaper steam sizes. The average price of lump coal during this period was $1.50 per ton, and the average price of screenings $0.73 per ton. The mere question of size reduces value over 50 per cent. Thus it is advantageous to produce the free burning Illinois coal in as coarse lumps as possible.

If these figures represent the average difference in price between lump coal and screenings, each per cent of lump gained (or each per cent of breakage reduced) is worth 0.77 cents per ton to the operator. At a mine at which three thousand tons per day are produced this means $23.10 per day or nearly $600.00 per month saved.

In the tipple a long, high angle dump chute, a poorly designed weigh box, or a drop of several feet onto the screens may cause the breaking of several per cent of lump coal into the finer sizes. At some mines the sound of the coal when dumped from the car is audible above the rest of the surface noises at a distance of several hundred yards from the tipple. The louder the noise the greater the breakage. In some cases the dumping takes place with such force that the coal actually seems to be thrown from the car through a distance of from twelve to fifteen feet into the bottom of the weigh box, in which it is more or less broken by the rest of the load falling upon it. While the great speed of such apparatus, in some cases three dumps per minute, is a notable achievement, good engineering demands a closer study of the problem of decreasing the breakage during preparation.

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*Eng. and Min. Jour., Vol. 63, p. 165.

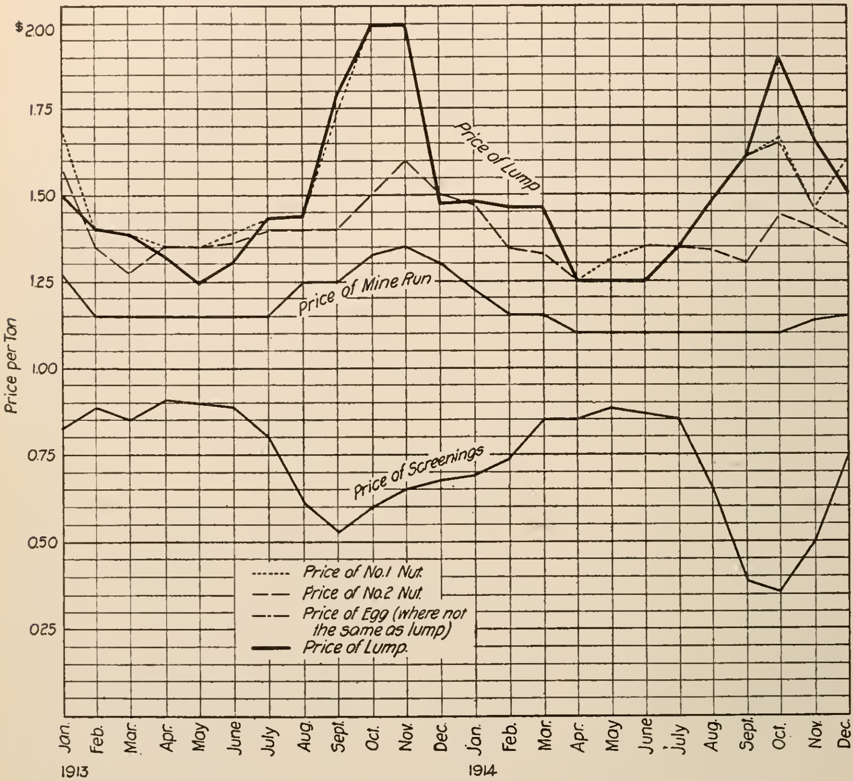

FIG. 21. AVERAGE CIRCULAR PRICE OF THE DIFFERENT SIZES OF COAL. FRANKLIN COUNTY (1913-14).

Breakage or Degradation Standards.—The adoption of some standard of breakage for Illinois coal is desirable. Especially in making specifications in which close clean sizing is required it would be of advantage to have a standard allowable amount of breakage under fixed conditions with which each coal could be compared, in order that it might be designated as harder or softer (more friable), or as containing more or less fines than the standard. If Illinois coal withstands transportation and handling better than competitive coal from other states, such a standard inserted into specifications, would emphasize a fact that is of great importance to retail dealers, especially to those to whom fine coal represents almost a dead loss. To show the difference in the friability of different bituminous coals, a certain bituminous coal in dock handling on lake shipments degraded from 30 to 35 per cent in two cases, while another bituminous coal under the same treatment degraded only from 12 to 14 per cent.*

While there has been much discussion concerning degradation on various coals no standard of degradation has been introduced into general practice. Attempts have been made by the U. S. Geological Survey† to fix such standard for the purpose of comparing the friability of coal briquets. For illustration, 50 lb. of briquets made from an Illinois coal were dropped five times through a distance of 6½ feet, and then screened on a one-inch wire screen; 35.5 per cent of the material passed through the screen. The briquets were then compared with others in this respect.

The U. S. Bureau of Mines‡ has adopted a similar standard for comparison of cokes. A fixed weight of coke is dropped 6 feet onto a cast iron plate. This operation is repeated four times. Then the sample is screened over a standard screen, and the percentage of fines is determined.

J. B. Porter¶ determines the comparative friability of fine bituminous coal and dust, by taking coal under ¼-inch screen in size and screening it through the following sieves: ¼-inch; ⅛-inch; 14-mesh; 24-mesh; 50-mesh; and 100-mesh, a screen ratio of practically 2. The percentage of each size is then plotted, and the assumption made that the several coals range in friability in the order of their percentage of fine material.

It is evident that in any set of breakage test on bituminous coal the large lumps of such a friable and non-homogenous material may break easily and the smaller sizes not so easily, or vice versa. The position of the coal in the seam, whether in the top, middle, or bottom bench, and the percentage of ash or other impurity in a particular piece have a decided effect on breakage. For these reasons any set of tests must be relative rather than absolute. Tests along the above lines are now being conducted in the Mining Laboratory of the University of Illinois to ascertain if such breakage standards are of any practical value.

Theoretical Considerations of Breakage.—Considered from a theoretical standpoint, breakage of coal takes place by splitting along planes of minimum strength rather than by actual shearing or crushing of the coal substance itself. Thus the actual measure of the force in pounds per square inch required to crush or break coal is extremely variable. The application of Rittinger's theory* that the work necessary to break or crush minerals is proportional to the reduction in diameter, shows, for example, that three times as much power must be expended in breaking 6-inch cubes into 1½-inch cubes as in breaking 6-inch cubes into 3-inch cubes.

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*Records of Interstate Joint Conference, Philadelphia, Pa., Feb. 10, 1914, p. 914.

†Bulletin No. 332, p. 44.

‡Technical Paper No. 50.

¶"Coals of Canada." Vol. 1, p. 195.

The kinetic energy developed in any lump of coal being moved; in other words, the force with which one piece strikes another and causes breakage, is directly proportional to its weight and to the square of its velocity, or if dropping, directly proportional to the distance through which it is dropped. In simpler language a piece thrown or moved with twice the speed of another, has four times as much power to break or to shatter. The practical effects of these laws will become evident when breakage in bins, in chutes, and in other surface plant devices is discussed. Since force is proportional to weight, the general effect of size or weight is that large pieces striking on their own ragged edges sustain breakage from a drop that will not affect the smaller sizes.

If coal is dropped piece by piece from a height of from 10 to 15 feet upon iron or steel it will show from three to four times as much breakage as if dropped onto wood.† It is claimed, in unloading a bin or a car, that the first drop only causes breakage and that after the pile below has started there is little breakage. Probably the breakage is greater in the dumping of large volumes than in the dumping of small volumes of coal. Still another point is the effect of squeezing and rubbing of the particles against one another in a bin from which coal is being drawn from the bottom. If other conditions are equal, the actual breakage is greater in handling or transporting sized coals than in handling or transporting run of mine coal because the fines in run of mine coal form a bed or nest upon which the large sharp pieces of lump may ride without breakage.

Data on Degradation of Bituminous Coals.—There is a practical limit to the freedom of a particular size of coal from smaller sizes, because reduction in the amount of smaller sizes beyond a certain point by rescreening means the making of an additional amount of the large coal into fines. At one mine it was reported that each rescreening of the coal meant 5 per cent was broken into small sizes and passed the screens. Concerning this point, "Experiments show that there is a limit beyond which there is no advantage in attempting to rescreen and further prepare anthracite and bituminous coal. In one instance in which a ton of hand picked coal was used the process of screening furnished 3 per cent undersize which went through the screen, while a larger per cent of undersize formed in the handling still remained in the coarse coal. In repeating the operation, another 3 per cent of undersize was caught, while the (coarse) coal again held a proportion of undersize similar to that previously found. It is believed that the process might be repeated indefinitely without materially benefiting the grade of the coal."*

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*R. H. Richards, "Text Book of Ore Dressing. p. 167; also, H. von Rittenger, Lehrbuch der Aufbereitungskunde, p. 19.

†Black Diamond, August 29, 1914, p. 165.

In one test made by a consumer in Chicago clean Illinois nut coal over ¾-inch screen was shoveled into a wheelbarrow, weighed, dumped, and then rescreened over the same screen. In one lot 4 per cent and in another lot 2 per cent degradation resulted from the shoveling and dumping.

At a mine in the central part of the state, the degradation of clean screened coal of from 3 to 6 inch size, which was rescreened after passing through a bin 20 feet in height was 5 per cent. In a similar case in the southern part of the state 2 per cent degradation was reported.

Another test, made on a car of carefully loaded domestic lump coal by removing clean lumps from the car with a fork, showed 5 per cent of smaller sizes left in the car. J. D. Rogers† states that a breakage of 5 tons per car, or 10 per cent, is common and is even greater if loading is carelessly done.

In elevating and passing an Arkansas bituminous coal through a 50-foot railroad coaling station the increase of slack was 25 per cent.‡ This coal is somewhat softer than Illinois coal.

An English test on bituminous coal falling 10 ft. showed a loss of 1s. 3d. per ton or 15.6 per cent on coal valued at 8s. per ton.¶

At a large Chicago retail yard a degradation of 7 cents per ton is allowed on all sizes of bituminous coal passing through the yard; on an estimated average value of $2.50 per ton this would be a loss of about 3 per cent. J. W. Hardy** records the breakage on bituminous coal as 4 per cent for every 10-foot vertical drop.

Transferring Coal in Railroad Cars.—Considerable discussion has taken place regarding the amount of degradation caused by the transferring of coal from one railroad car to another en route. H. C. McKinney in The American Coal Journal for Dec. 5, 1914, states that the loss of 25 cents per ton (at the point in question this amount represents possibly 10 per cent of the value) conceded by the railroads is insufficient. Since the practice of transferring coal from one car to another en route is common, more data on this point would be valuable.

The random references given show the present day practice in estimating the general breakage on Illinois and other bituminous coals. In every case dropping coal even once causes serious breakage. Considering the treatment that Illinois coal receives in the tipple in preparation, it is doubtful, especially on the prepared sizes, if it pays to strive for tonnage records at the expense of extra breakage of the coal.

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*Eng. and Min. Jour., Feb. 16, 1907, p. 339.

†"Preparation of a Domestic Coal." Kentucky Min. Inst., December, 1912.

‡Proc. 5th Annual Convention, Int. Rwy. Fuel Assoc., p. 257.

¶Gillott on Kirkby Colliery. Inst. of C. E. of London, Vol. 127, 1897, p. 177.

**Black Diamond, Aug. 29, 1914, p. 165.

As anthracite is essentially a domestic coal prepared under standard practice, the results of tests made to determine its breakage are given for the purpose of comparison.

(1) Test on Dropping.* Sized anthracite dropped 7 feet onto iron plate gave 0.77 per cent breakage into finer sizes, and dropped 7 feet onto wood plate gave 0.34 per cent breakage into finer sizes. In other words, the breakage of anthracite upon striking iron is more than twice as great as upon striking a softer material like wood.

(2) Tests on Handling and Shipping.† The loss by breakage in storage and picking up averages 2 per cent, but varies from ½ per cent with the coal and with the amount of handling. At one plant in which coal is dropped into a hopper beneath the car, though the fall is small, the breakage measures ½ per cent. The breakage in shipping east without rescreening is from 2 to 3 per cent. In shipping west, in which case the anthracite is transferred and rescreened, breakage is from 8½ to 9 per cent. By breakage is meant the actual amount of all sizes smaller than the size specified.

(3) Breakage by Dropping.‡ A summary of tests made both by dropping carefully sized anthracite through measured distances and by dropping carload lots into pockets is given in Table 9.

TABLE 9.
BREAKAGE OF ANTHRACITE BY DROPPING.

Size	Amount of Breakage into Smaller Prepared* Sizes	Amount of Breakage into Sizes Smaller than Nut	Total Amount of Breakage
Broken	3% plus 43/100D†	2% plus 17/100D	5% plus 6/10D
Egg	4% " 43/100D	2% " 17/100D	6% " 6/10D
Stove	2% " 33/100D	2% " 27/100D	4% " 6/10D
Nut	....	4% " 40/100D	4% " 4/10D
Pea	....	2% " 50/100D	2% " 5/10D
Buckwheat	....	1% " 25/100D	1% " 25/100D

*Prepared sizes are broken, egg, stove, and nut.
†D equals drop of coal in feet.

Table 9 shows that not only is the percentage of degradation through fall for a hard coal like anthracite surprisingly large, but also that breakage increases with an increase in size and in height of drop. The breakage in an Illinois coal must be considerably greater.

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*Black Diamond, July 12, 1913, p. 16.
†Mines and Minerals, Vol. 25, p. 23.
‡R. V. Norris, "The Storage of Anthracite." T. A. I. M. E., Vol. 42, p. 316.

General Analysis of the Breakage Problem.—In general during mining and preparation of a coal, breakage may occur in any of the following necessary operations:

- A. Breakage in the mine incident to......
  - a. Undercutting.
  - b. Snubbing and drilling.
  - c. Breaking the coal from the face.
    - 1. In longwall mines.
    - 2. Shooting after undercutting and solid shooting.
    - 3. Permissible explosives.
  - d. Handling ..
    - 1. Loading.
    - 2. Haulage.
    - 3. Dumping into a skip at shaft bottom.
- B. Breakage in surface plants incident to...
  - e. Use of self-dumping cage.
  - f. Dumping into weigh box and onto the screen.
  - g. Screening.
  - h. Loading ....
    - 1. Passage over loading chutes, aprons, or booms.
    - 2. Loading and trimming railroad cars.
  - i. Rescreeners and bins.
  - j. Washing (not included in this bulletin).
- C. Breakage in transportation and rehandling (not discussed fully in this bulletin).

Breakage in the Mine.—Undercutting.—About 52 per cent of the tonnage in Illinois is produced in mines in which undercutting is practiced either by hand picks, by machines of the puncher type, or by electric chain machines. The last named type is the most common, and its use is increasing. Contrary to the practice in many bituminous fields, undercutting in Illinois takes place almost entirely in the coal; the exceptions being at several mines in the longwall field, in which hand picks are used to undercut in the clay bottom. At other scattered mines, the line of undercutting follows some thin dirt band in the lower part of the seam, and at others in which the floor rolls or is irregular, accidental undercutting of the floors takes place, in all of which cases the cuttings are high in ash or impurities. In general, however, the miners avoid working in the tough bottom clay. This leads to the reduction into fine sizes of a certain portion of any particular seam, the percentage depending on the height of the undercut; that is, on the particular machine or method used.

The cuttings obtained, called "bug dust" by the miner, are generally assumed to consist only of the finest sizes of coal, and on account of a high percentage of dangerous dust contained, are often loaded out before breaking down the face of the seam undercut. Therefore, cars of bug dust are generally at the foot of the shaft when hoisting begins in the morning; consequently, the first railroad cars of screen-

TABLE 10.

AMOUNTS AND SIZES OF CUTTINGS MADE BY VARIOUS MINING MACHINES.

1	2	3	4	5						6		7	8	9	10	11	12	13
Mine	District	Seam Number	Kind of Undercutting	Screen Analysis Percentage						Chemical Analysis of Cuttings		Total Undercut During Test. Sq. ft.	Total Weight of Cuttings, lb.	Pounds of Cuttings per sq. ft. of Seam Undercut.	Total Per Cent of Cuttings if Seam Were 6 ft. Thick.	Per Cent of Cuttings Under 1¼″ r'nd hole; i.e., 1¼″ Screenings.	Weight of Cuttings per sq. ft. passing 1¼ in. Round Hole Screen.	Per Cent of 6 ft. Seam Made into Screenings by Hand or Machine Mining.
				a	b	c	d	e	f	a	b							
				On 1″	On ½″	On ¼″	On 10 Mesh	On 100 Mesh	Throu 100 Mesh	Per Cent Ash	Per Cent Sulphur							
1	Northern Longwall	2	Hand	33.2	20.2	18.7	17.3	10.3	0.3	17.5	4.7	14	1180	84.3	17.3	66.8	56.3	11.6
2	Southern Room and Pillar	6	Electric	11.2	12.3	13.9	21.4	39.4	1.8	14.0	2.7	129	4075	31.6	6.5	88.8	28.0	5.8
3	Southern Room and Pillar	6	Ch. Mach.	3.7	15.2	23.7	34.5	22.1	0.8	10.2	0.9	166	5290	31.8	6.5	96.3	30.6	6.3
4	Southern Room and Pillar	6	Ch. Mach.	5.9	16.4	20.4	31.0	25.2	1.1	9.2	1.5	130	4393	33.8	6.9	94.1	31.8	6.5
5	Southern Room and Pillar	6	Ch. Mach.	6.6	19.4	22.8	25.6	25.2	0.4	13.0	1.2	94	2935	31.2	6.4	93.4	28.1	5.8
6	South-western Room and Pillar	6	Ch. Mach.	12.8	15.6	19.8	22.4	28.2	1.2	17.1	6.8	64	2150	33.6	6.9	87.2	29.3	6.0

7	South-western Room and Pillar	6	Air Puncher	37.2	13.5	13.8	17.8	17.7	0.0	26.5	5.1	55	3780	68.7	14.1	62.8	43.1	8.9
8	Danville Room and Pillar	7	Air Puncher	35.4	16.4	15.4	17.3	14.8	0.7	17.5	4.1	26.8	2750	103.0	21.2	64.6	66.5	13.7
9	South-western Room and Pillar	6	Air Puncher	45.1	13.2	13.5	15.6	12.0	0.6	27.6	6.7	87	11550	133.0	27.3	54.9	73.0	15.0
10	Southern Room and Pillar	6	Air Puncher	28.5	17.8	16.6	19.9	16.2	1.0	15.8	1.2	32.6	2250	69.0	14.2	71.5	49.3	10.1

ings loaded may contain more than the usual percentage of this fine coal, which is often of lower grade than the regular screenings.

There has been considerable discussion as to which type of machine makes the least cuttings, coarsest cuttings, lowest percentage of dangerous dust, etc. To throw light on these and other pertinent questions in connection with the use of machines in Illinois mines, H. H. Lauer of the Department of Mining Engineering of the University of Illinois, in connection with the Co-operative Investigation of Illinois mining conditions, made a series of ten tests at mines in which undercutting was variously practiced with hand picks, punchers, and electric chain machines of different types. In each test a room or a definite length of face in a mine was carefully cleaned, and then undercut by one of the above methods as in regular practice. The total amount of cuttings produced was carefully collected, weighed, and sampled. One sample was saved for analysis while another was passed through various standard screens, the object being to ascertain the percentages of the various sizes made, and in this way to compare the cuttings produced by the various methods of undercutting. These operations were all carried out underground at the face. The full results have not yet been published, but through the courtesy of the Co-operative Investigation the results regarding amounts and sizes of cuttings, summarized in Table 10, were secured.

The screens used in these tests were of square mesh; the openings, therefore, were slightly greater in area than the corresponding sizes of round holes, which are more convenient as a standard. A screen ratio or sieve scale of ½ inch was used, excepting with the smallest size of screen—100-mesh. The percentage of coarse coal on the 1-inch screen is high in mines in which pick mining or puncher machines were used. The very small percentage of cuttings through a 100-mesh screen is surprising. In other words, machine cuttings are more granular than they are commonly supposed to be, and the percentage of dangerous dust (at least through 100-mesh) is smaller than was anticipated.

Column 6, a and b, shows that, as a general rule, cuttings are of lower grade than the average face sample from the same seam (see Table 5). The presence of a much smaller amount of ash in the top benches of the seam at many mines in the state than in the lower benches influenced unfavorably the results given in Column 6. Column 7 gives the total square feet of the seam undercut during the test, and Column 8 the total weight of cuttings; consequently, dividing Column 8 by Column 7 gives Column 9, which shows the pounds of cuttings made per square foot of the seam undercut.

To standardize the test still further, the seams are assumed to be 6 feet thick, and Column 10 shows the total percentage of a seam made into machine cuttings on the basis of a six-foot seam. The proper percentage to allow for a seam of any other thickness may be easily calculated.

If machine cuttings were large enough to pass into the lump sizes their quantity would be of minor importance. If a 1¼-inch round hole is taken as the standard size over which lump may be prepared (this size is nearly equivalent to a 1-inch square hole opening); Column 11 shows that considerable difference exists in the percentage of the cuttings which are fine enough to pass a 1¼-inch round hole; namely, to be screenings. The percentages in Column 11, recalculated on the basis of weight in Column 12, are reduced to the basis of a standard six-foot seam, and are given in Column 13. Percentages for thicker or thinner seams may be easily figured from this calculation. If there were no other breakage in coal mining and preparation, this percentage of the seam would represent the amount of 1¼-inch screenings made by undercutting.

The tonnage won after undercutting has in each case been assumed to be that part or block of the seam directly over the undercut. The frequent practice of boring holes deeper than the undercut increases the tonnage, and so decreases the percentage of the seam made into screenings by undercutting. The puncher machine and hand pick show more favorably in Column 13 than in Column 10; that is, while these machines make more cuttings than the electric chain machines, the sizes of their cuttings are coarser. An average of the electric chain machines shows 6.08 per cent screenings, and of punchers 11.92 per cent. The writer visited two mines of the same company within four miles of each other at which ½-inch bar screens of the same size were used. At one of these mines 11 per cent screenings was produced with electric mining machines; at the other, 17 per cent screenings was produced with punchers. These facts tend to confirm the reliability of the figures given in Table 10.

The agreement of the electric chain machines in Column 13 is remarkable, five machines in five different mines differing only 7/10 of 1 per cent in total cuttings when reduced to a common basis of measurement. Such a difference or even a greater one might depend upon the nature of the coal cut, upon the number of positions of bits used in the chain, upon the kind of bits; that is, chisel or pick point, and also upon whether the bits were sharp or dull. The height of the cutting opening made, or "kerf," also varies somewhat with different machines.

Excepting special manufacturers' tests, this is believed to represent the first work on this special problem carried out in the central bituminous field. In Mines and Minerals for March, 1908, p. 397, a table is given for similar work carried on in the Westmoreland mine, Pittsburg Seam, Pennsylvania. For purposes of comparison the results are given in Table 11.*

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*See also G. S. Rice, "The Explosibility of Coal Dust." U. S. Bureau of Mines, Bul. No. 20, p. 35.

TABLE 11.

AMOUNTS AND SIZES OF CUTTINGS MADE BY VARIOUS MINING MACHINES IN THE PENNSYLVANIA BITUMINOUS DISTRICT.

Method	Total Cuttings		Through 40 Mesh	
	Pounds	Per Cent	Pounds	Per Cent
Puncher	3436	10.95	394	1.250
Chain Machine..	1836	5.86	155	0.494
Hand Pick......	4533	14.45	128	0.408

The percentage of total cuttings here is also reduced to a six-foot seam as a basis of common measurement.

Considering the physical differences between the coals of the two fields and other possible conditions, as noted previously, the general agreement of these results with those of Table 9 is significant.

Snubbing and Drilling.—By snubbing is meant the practice common in some districts before shooting, of cutting a triangular section from 18 inches to 3 feet high from the lower face of the coal above the undercutting. If this is not done the shooting is likely to loosen the coal only enough to fill the space undercut and, unless excessive powder is used, does not break it sufficiently to allow easy loading. Proper snubbing causes the coal to roll and spread out in the room with a minimum of powder.

The amount of snubbings made is less than 3 per cent of the average seam, and since the work is often done with a hand pick and there are two free faces to break to, the relative sizes of the snubbings are large. No determinations of the sizes have been made. Since from only two to five 2-inch drill holes are bored to a depth of from 5 to 8 feet in a single face, the percentage of fine coal made in this way is too small to be considered. These cuttings, however, contain a considerable percentage of fine dust. One per cent or less screenings probably represents the fine coal made by snubbing and drilling.

Breaking the Coal from the Face.—By the method of applying roof pressure or wedging as in longwall mines, the coal breaks slowly and into blocks, governed by the resultant direction of pressure and by the larger incipient cleavage planes, and consequently loosens ready for loading with a minimum of breakage. At certain longwall mines as small an amount as 15 per cent screenings (1¼-inch round hole or ⅞-inch bar) has been reported after loading, hauling, hoisting, and dumping over the screen. "On the whole, the longwall field shows 15 to 20 per cent more lump coal over 1¼-inch screen as compared with the rest of the state."* These figures agree with those gathered by the author at individual mines. Although the question is somewhat complicated by partial hand pick undercutting at some of the longwall mines, the conclusion seems logical that the amount of screenings in the coal at the face need not be greater than from 5 to 10 per cent.

The amount of fines or breakage produced in shooting is largely under the control of the miner (Chapter I). Heavy charges of powder shatter the coal. No conclusive data on this subject are available, and it will be discussed by making a comparison of general results obtained from the practice of two forms of mining; namely, shooting after undercutting vs. solid shooting.

In the bituminous field of Arkansas the change at one mine from undercutting to solid shooting increased the slack coal 14 per cent and the amount of slate from 11 to 23 per cent in three years; in the state as a whole, the change to solid shooting increased the amount of slack coal 50 per cent.† It is also claimed that overshooting weakens the lump coal so that it readily slacks off on standing and is more easily broken by handling. A retail dealer stated that when a car of clean lump made with undercutting was unloaded at destination, from 3 to 5 per cent of degradation was left in the car, while under solid shooting the amount was as great as 15 per cent.‡ A. A. Steel states that Oklahoma has had like experience,¶ and other writers complain that heavy solid shooting not only increases the amount of fines but also jars the coal and breaks the grain, producing incipient shattering, and even though it may hold together until it passes over the screen, it disintegrates more easily afterwards.

A direct comparison of these methods has been made in several cases of Illinois. A. J. Moorshead, in a paper read before the 1913 meeting of the Illinois Mining Institute,** states that from long experience he believes Illinois coal mined by machine has from 3 to 10 per cent less screenings on the average than the same coal shot off the solid, but that some of the harder coal gives as few screenings when shot off the solid as when undercut. Also he states that the percentage of screenings made from the coal from seam No. 6 in Williamson county when shooting off the solid is larger than that made when undercutting is practiced, the difference being probably from 7 to 10 per cent. The difference varies with the manner of shooting. At a mine†† in one district in the state, where shooting off the solid is practiced 55 per cent lump over 1¼ inches is made, while at three mines where undercutting is practiced 70 per cent lump is made over the same size screen. A study was made of 100 typical mines throughout the state;* at 33 mines where shooting off the solid is practiced an average of 65 per cent of the coal produced is larger than 1¼ inches, while at 43 mines where undercutting is practiced 67 per cent of the coal is larger than this size.

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*S. O. Andros, Co-op. Bul. No. 5, p. 40.

†A. H. Perdue, Proc. American Mining Congress, 1909 to 1911, p. 227.

‡J. E. Turney, Ibid., p. 233.

¶Proc. 3rd Annual Convention, Int. Rwy. Fuel Assoc., p. 56.

**From reprint in Colliery Engineer, 1914, p. 435.

††S. O. Andros, Co-op. Bul. No. 6, p. 23.

A personal communication to H. H. Stoek gives the following test made at an unnamed Illinois mine:

TABLE 12.

COMPARISON OF SIZES PRODUCED, SOLID SHOOTING VS. UNDERCUTTING.

Test	Per Cent of Lump	Per Cent of Nut	Per Cent of Slack
Solid Shooting.......	35	30	35
Undercut by Electric Machine	60	20	20

Generally, the amount of screenings made by either process of shooting is measured by screening through the tipple screens. Since the amount of breakage in handling, in weighing, and in screening the coal, varies for different mines, a direct comparison of breakage by shooting in sometimes impossible. Figures gathered by the writer concerning these two classes of mines in the same districts, show in every case as great or a greater percentage of screenings at solid shooting mines than at undercutting mines. Of two neighboring mines in the southern part of the state, one at which solid shooting is used, 35 per cent of screenings through an 1¼-inch round hole are produced. At the other, where electric machines are used, the same percentage of 1½-inch round hole screenings are produced. At a third mine in this district, where electric machines are used, 33 per cent 1½-inch round hole screenings are produced.

Permissible Explosives.—Since the amount of permissible explosives used in the state in 1915 amounted to 1,342,334 pounds, their influence on the coal is of considerable importance. Concerning permissible explosives vs. black powder, S. O. Andros† doubts that permissibles make more slack if properly used.

J. J. Rutledge and Clarence Hall state, "Permissible explosives have come into use so recently that it is not easy to get reliable figures showing the increased proportion of fine coal they make as compared with black blasting powder. The estimates of this increase given by mine superintendents run from no increase to 10 per cent. Some superintendents maintain that although smaller lumps of coal may be made by using permissible explosives, yet changing from black blasting powder to these explosives does not increase the proportion of fine coal. Some persons state that the lumps of coal produced by using permissible explosives are not so easily broken up during transportation or exposure to the air as are those made by using black blasting powder, whereas other persons maintain the reverse. However, if the coal is undercut or sheared and the blasting is done with judgment, the permissible explosives make as good coal as black blasting powder and at approximately the same cost."*

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*S. O. Andros, "Coal Mining in Illinois," Co-op. Bul. No. 13.

†Co-operative Bul. No. 8, p. 34.

J. R. Fleming, Assistant Engineer of the U. S. Bureau of Mines, has gathered data for a bulletin on "Use of Explosives in Illinois with Special Reference to Permissibles." These data show that the percentages of fines made by the use of permissibles at several mines are less than when black blasting powder is used; in other cases little difference is noted, while in still others an increased amount of fines in recorded when permissibles are used. Following are summaries from several mines:

Mine No. 1. The extreme case reported in which permissibles made more fines than black blasting powder.

Size of Coal	Year 1910 Black Blasting Powder	Year 1912 Permissibles
6 in. lump	24.19 per cent	20.41 per cent
6 in. x 3 in. egg	19.28 per cent	21.29 per cent
3 in. x 2 in. nut	16.50 per cent	12.88 per cent
2 in. screenings	40.03 per cent	45.72 per cent
Total	100.00 per cent	100.00 per cent

Mine No. 2. A more favorable case.

Size of Coal	Black Blasting Powder	Permissibles
6 in. lump	15.2 per cent	13.5 per cent
1¼ in. x 6 in. egg	49.5 per cent	51.0 per cent
1¼ in. screenings	35.3 per cent	35.5 per cent
Total	100.0 per cent	100.0 per cent

Mine No. 3. A very favorable case.

Size of Coal	Black Blasting Powder	Permissibles
2 in. lump	47.4 per cent	50.0 per cent
2 in. screenings	52.6 per cent	50.0 per cent
Total	100.0 per cent	100.0 per cent

The conclusion is that if permissibles are used with judgment no material increase of fines results.

Handling the Coal.—Loading.—Loading the broken coal into the mine car in Illinois is done with a hand shovel. The only exception to this rule occurs in the case of two stripping mines at which the broken coal is loaded with small steam shovels. At only one mine, near La

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*"The Use of Permissible Explosives." U. S. Bureau of Mines, Bul. No. 10, 1912.

Salle, is the seam inclined enough to necessitate the use of chutes or other rehandling devices. Breakage from this cause, which is often a serious question in other states in which the seams are highly inclined, is therefore at a minimum. Whether or not the miner loads his coal in the largest possible lumps or rebreaks it, either by more powder, by pick, or by the back of a shovel into sizes convenient for shoveling, is a somewhat disputed point. It is not uncommon, however, to see mine cars containing lumps possibly as large as 12 inches by 18 inches, 2½ feet long, and weighing 300 pounds; as large in fact as the two men in the room can lift into the car.

Haulage.—The coal, having been loaded into mine cars, is moved to the foot of the shaft by mule or electric locomotive, or both. At times the writer has heard complaint about breakage caused by such transit, especially if in making up a trip the locomotive bumps the loaded cars, jarring the contents considerably and even breaking the top load.

Probably the custom of excessive topping, or loading the coal to too great a height above the top of the car, is responsible for most of the breakage in haulage. Bumping easily knocks off lumps from a poorly topped car which fall on the roadway and become broken. At comparatively few mines in the state is the load on the car limited; these few, however, have a maximum limit. As an illustration, at one mine the car is supposed to hold 4,000 pounds and if any carload weighs over 4,500 pounds, the excess weight goes to the check weighman fund. Such a rule is intended to prevent loss and breakage caused by excessive topping of cars. It is probable that breakage from actual car transportation, whether below ground or in railroad cars, has been rather overestimated.

Breakage Through Dumping into a Skip at the Shaft Bottom.—At nearly all the mines in the state coal is hoisted to the surface and into the tipple in the mine cars. Attempts have been made to dump the coal from the mine car at the bottom of the shaft into skips which are hoisted into the tipple and there dumped automatically. A similar arrangement is common at ore mines and is used at some coal mines in the Appalachian districts in which the coal is used for coking, and in which extra breakage is a benefit.

When a mine in the southern part of this state abandoned this system for the ordinary self-dumping cage, an estimated gain of 10 per cent in the coarser sizes resulted. A mine in the longwall field using skips produces 15 per cent more screenings over the same size of screen than the average of six other mines in the district using cages. In Franklin county a mine operating 7-ton skips reports 6 per cent more 1¼-inch screenings than the average of thirteen other mines in the district, and only 12 per cent of 6-inch lump against an average of 22 per cent for the other mines mentioned which hoist the mine car in self-dumping cages.*

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*S. O. Andros, Co-op. Bul. No. 8, p. 48.

If, however, as in one or two cases in the state, the output of a mine is used for special purposes not affected by the general commercial market requirements for large lumps, skips may be the best solution of the engineering problem of a small shaft and high tonnage requirement.

Breakage in Surface Plants.—Final preparation and sizing in the tipple are carried on under the advantages of daylight and easy access to all parts of the plant. Underground the coal is subjected to a series of movements causing breakage, some necessary and some unnecessary, but all more or less under the control of the operator and miner. In the surface plant the success attained depends largely upon the design and erection of the tipple building, chutes, screens, conveyors, loading devices, etc. The operator usually feels that this part of his mine plant is distinct from the rest, and its design and erection is usually entrusted to some engineering firm which makes a specialty of distinctive and often patented designs of headframes, weigh boxes, screens, chutes, and other equipment.

The general types of Illinois coal mine tipples or surface plants have been described in Chapter II. While special engineering features of tipples are reserved for future discussion, several features causing breakage should be mentioned at this point.

Breakage from Use of the Self-Dumping Cage.—In older designs, the cage, after being brought to a quick stop is tipped suddenly with considerable throw and jerk, throwing the coal out of the tipped car. At the required angle for dumping, the coal at the top end of the car is frequently from 6 to 8 feet above the dump shoe, thus giving a considerable momentum to the outgoing load. If the shape of the dump shoe and connecting chutes is carefully designed, the coal will slide out with a minimum of breakage and danger.

In the interests of safety as well as breakage more care should be taken to prevent the coal being dumped from missing its proper chute and falling either down the shaft or to the surface around the headframe. It is not unusual to see considerable coal falling when dumping takes place.

Dumping into Weigh Box and onto the Screen.—The coal on being dumped from the car slides down a short dump chute into the weigh box. In one of the older tipples a dump chute, 6 feet long and set at an angle of 75 degrees, gives an additional falling height to the coal of about 5 feet. At this mine 4 per cent more screenings are produced than at a neighboring one with a modern tipple. The increase in screenings can be attributed to such defects. The tipple breakage is even apparent to the eye.

The slope of the bottom of the type of weigh box, pan, hopper, or basket illustrated in Fig. 22 is usually 35 to 40 degrees, and since the upper end may be from 10 to 12 feet above the lower end, the distance moved and the speed acquired by the coal may cause considerable breakage before it comes to rest and is ready to be weighed.

This is evident from the flying bits of coal noticeable in the usual type. The weigh box or basket is usually made large enough to hold two or even three loads from the mine car, and although this means waste space, lost headroom, and chance for extra breakage if used for single weighings only, the large capacity allows the hoisting to proceed continuously even though the screens below may be stopped.

Weigh boxes with a more gentle slope or at least a changing and decreasing slope, which would bring the coal to rest without a shock and then would deliver it on the screens below in a more gradual manner, would be an improvement. Weigh boxes of this type have been designed, but complaint is made that they tend to decrease the speed of operation. A weigh box having a bottom sloping at a low enough angle to allow a car of dry lump to slide gently may not be steep enough to handle a car of moist or of fine coal, and if the angle is great enough to allow wet coal or slack to run, lump coal usually acquires considerable velocity in passing down the incline.

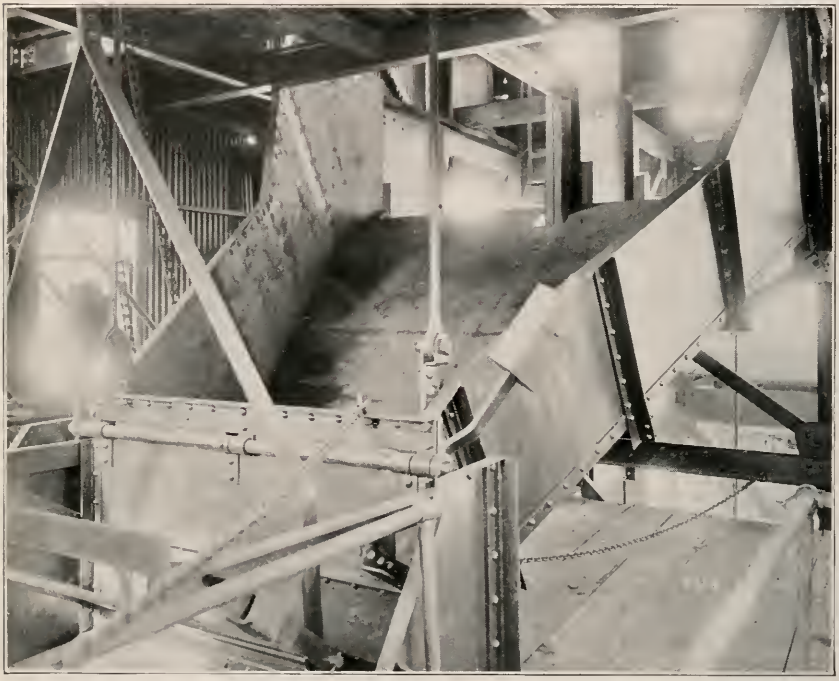

Fig. 22. Weigh Box.

Use of Screen Feed Hoppers.—The custom of allowing coal from the weigh box to discharge, either at the end or bottom, direct to the shaking screen, is meeting with disfavor for the following reasons:

(1) A drop between the weigh box and screen is a factor in the amount of breakage; in certain tipples a drop of 3½ feet has been noted.

(2) Coal is weighed in from 2 to 5-ton lots, and when this amount is dumped suddenly on the upper screen, it throws an extra weight upon it, which seriously affects the balance of weight that should be maintained between the upper and lower screens, and gives additional vibration to the screen structure.

(3) The sudden rush of material from the weigh box frequently chokes the fine screens (which usually come first), and allows considerable fine coal to be carried into the coarser sizes, damaging their appearance.

(4) The sudden load thrown on the screen puts an undue strain on the driving belt, causing slipping unless it is laced to a considerable and often damaging tension. At one mine two engines are used to drive the screens, thus avoiding the dead center troubles and lessening the strains just mentioned. This scheme has not been adopted generally.

(5) Any stoppage of the screens for the purposes of repair or to shift railroad cars causes the cessation of hoisting after one or two dumps have filled the weigh box; that is, there is no storage capacity between weigh box and screen.

For these reasons, steel feed hoppers (Fig. 23), with sloping sides and holding from two to four mine cars of coal are, in many of the newer installations, placed under the weigh box and at the head of the screen. The bottoms of these hoppers have a reciprocating motion, due to an adjustable crank arm, the full stroke being from 8 to 12 inches, and the speed from 40 to 60 revolutions per minute. The lower part of the end of the box next to the screen is cut away, thus allowing coal to fall on the screen gently and regularly with each back stroke of the bottom. The partial load of coal usually in these hoppers reduces the breakage since a new load from the weigh basket falls gently onto coal instead of striking steel. If such a feed hopper cannot be used on account of a lack of sufficient headroom some form of conveyor with sides may accomplish the same purpose.*

The most serious objection to the use of a feed hopper is that when two or three mine cars of coal are in it at the same time, and the coal is being mixed and fed continuously onto the screen, it is impossible for a coal inspector to properly single out individual cars of dirty coal. Some companies claim that by good engineering design of the various tipple appliances these feed hoppers may be dispensed with.

Screen Breakage.—Gravity Bar Screens.—Usually there is a dead or blank plate at the top and bottom of the bar screen, the former to spread out the coal before screening, the latter to collect it into the loading chute to the railroad car. Since at certain times the coal may be damp or extra fine, or weather conditions unfavorable, a considerably greater slope is given the screens than is necessary to allow dry lump to screen by gravity. The result is that a lump attains considerable velocity on the screens and suffers breakage when allowed to shoot directly into a railroad car. It is difficult to see how a car of commercial lump of good size and appearance can be made under such conditions, unless a simple steel adjustable chute is installed between bar screen and railroad car to slow up the coal and allow it to fall more gently into the car. The breakage is frequently increased by the chains, car wheels, or short logs of wood which are hung above and about halfway down the screen for the purpose of checking the flight of the coal or of turning the lumps in order to free them from adhering fines. Also lumps frequently become wedged in the bars, and upon being struck by succeeding lumps are broken and drop into the screenings. At many tipples at which bar screens are used to prepare coal for special purposes not in commercial competition with lump coal from adjoining mines, breakage is of secondary importance; the

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*Coal Age, Vol. 1, p. 1143.

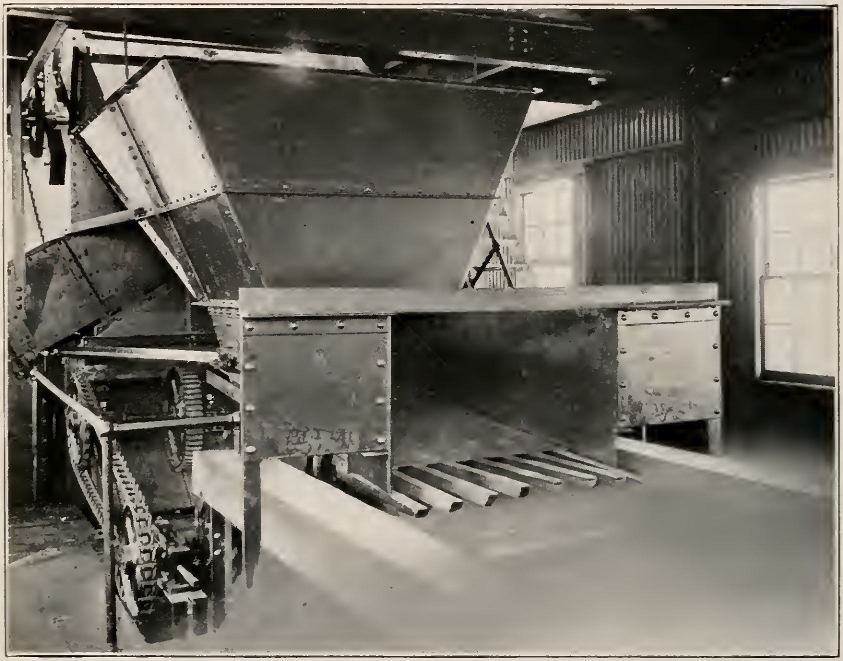

FIG. 23. A STEEL FEED HOPPER.

bar screen being used to free the lump from the impurities in the slack rather than from the slack coal itself.

Shaking Screens.—The great variance in speed, in slope, and in size of holes, found in the different designs of shaker screens, cause differences in breakage and freedom of one size from another. In most designs the small holes come first, and often become clogged by sudden rushes of coal. The best results are obtained at the mines at which this fine screen is protected by a false one of larger opening placed about a foot above it, for the purpose of providing against overloading and crushing by the bulk of the lump coal. Where drops of more than one foot are allowed in passing from upper to lower screen some splintering of large lumps can generally be detected.

On a number of screens, lumps of coal fall into the larger screen holes and are not dislodged until worn or broken enough to pass into the undersize. This fault may be remedied by increasing the slope, speed, or length of throw of the screen, or by a combination of these points.

Revolving, Trommel, or Roller Screen Breakage.—On account of the breakage caused by roller screens they are not used in Illinois tipples for the screening of lump coal. They are, however, commonly used, especially in the southern part of the state, for producing the smaller sizes of coal, and indeed they were the only kind of screens used in rescreening plants till about 1910. Now, they are being replaced in the newest rescreeners by the Parrish screen or other approved shakers designed especially for small coal. Roller screened coal can often be distinguished by the rounded corners of the individual pieces, showing that attrition has taken place in the screen, with a consequent production of the dust sizes.

The severest criticism of the roller screen is that in shape and in general principle of action it is similar to the Bradford disintegrator, an efficient machine built for breaking coal. At the Joliet plant of the Illinois Steel Company a Bradford disintegrator 11 feet in diameter and 25 feet long with screen openings of 1⅝-inch diameter breaks Illinois lump coal so completely that less than 3 per cent oversize is discharged, and this is mostly hard refuse. Practically all the coal is broken during its passage through the screen. Although the diameter of the disintegrator is 5 feet greater than that of the ordinary roller screen, and although ribs are placed inside to lift the coal and secure extra height of drop, the general parallelism of action holds.

Breakage in Loading—Passage Over Loading Chutes, Aprons, or Booms.—In England the loading of clean lump coal is assured often by loading from the screen or belt by hand. Two men and a boy can load 60 tons per 8 hours in this way. Because of the price of labor and low value of the coal at Illinois mines coal must be loaded mechanically or automatically, even at the expense of slight breakage. Since most tipple screens have considerable slope, inclined chutes are constructed to lower the various screened products into the railroad cars. Such chutes may be fixed, hinged, shaking, or movable, or may consist of a moving or travelling adjustable steel loading boom, belt or apron (Fig. 24), with which is often combined the picking belt (Fig. 19). Most of the tipples built within the last five years have been equipped for loading the lump and egg sizes of coal with these travelling adjustable loading booms, or with some other form of movable loading chute which can be lowered into the empty car and raised again when that end of the car fills.

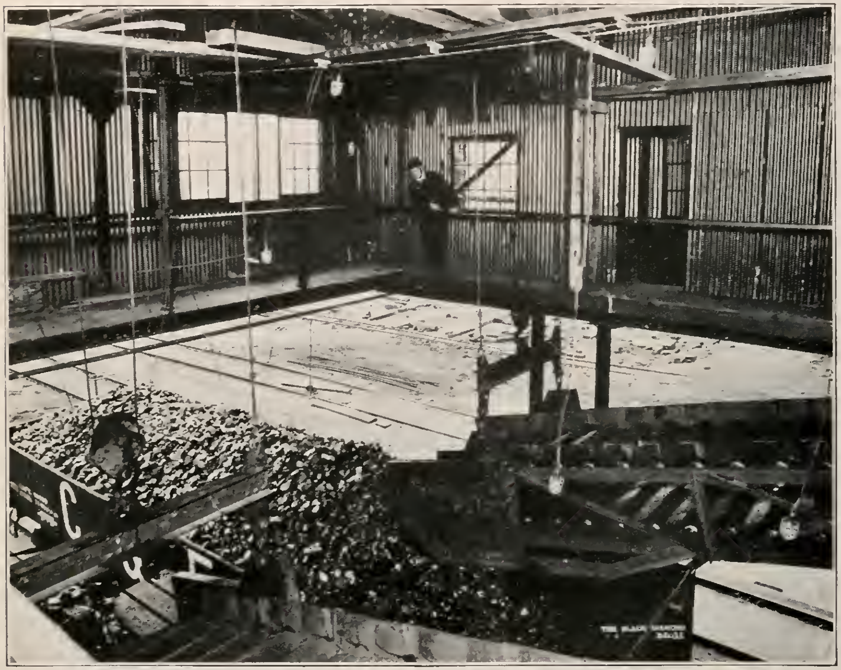

Fig. 24. Loading Boom.

Loading chutes may lead directly into the car at right angles to the track or if they have bends or right angle turns or drops they may lead into the car in a direction parallel with its length. Chutes which discharge straight into the car at right angles to the track, unless flat and shaking, shoot the coal into the car. At one or two of the older mines in which these chutes are still in use, a sheet steel buffer is hung over the opposite side of the car in order to prevent spilling, and since the coal comes with considerable force, even trimming and hand picking are rendered difficult. In a chute with curved bottom the lumps slide down only in the lowest part and one at a time, so that a swiftly moving piece may overtake and break the piece ahead. Otherwise, these chutes give good results, especially if bends are necessary, because they turn the direction of the coal gradually.

At most new installations the loading chutes are placed parallel with the track since from this position the coal can be loaded with less breakage, the chutes can be lowered into the cars, spilling of the coal over the sides of the car can be eliminated to a great extent, hand picking can be conducted efficiently and with greater safety, and the coal can be trimmed more uniformly. Whether or not the flow of coal in such a chute should be with or against the direction in which the loading car is moving is open to argument. Most chutes in Illinois are placed so that the coal moves in the same direction as the car. Advocates of this system claim:

(a) If movable loading booms are used, it is possible to get them well down into the empty car.

(b) Lumps can be examined and picked with safety.

Those in favor of the other system claim:

(a) More regular trimming is possible, especially if the cars are moved by hand or by gravity.

(b) Less breakage occurs, since the coal is always moving in the direction of the slope of the coal in the car. The only reference noted on this subject favors this design.*

Breakage in the loading of the smaller prepared sizes—egg and nut—is not so severe on account of their smaller size, lighter weight, and greater uniformity. In order to meet the demands of domestic trade, these sizes must be unusually free from degradation products. A number of mines have recognized this fact and have installed small degradation or secondary screens in the bottom of the loading chutes for these sizes. These screens, bar, round hole, wire screen hole, or lip, are usually stationary and form the bottom of the chute for their length. They usually have less than 1-inch openings, but they remove the last trace of accidental fines and allow only the clean, sized coal to enter the car. The undersize usually passes into the screenings car or to the boiler room.

The perforations in the lip screens (Fig. 25) are long in comparison with their width, and they become slightly wider at the lower or discharge end. A slight drop in the screen at the end of each series of slots allows wedged pieces to be released at these points. This screen also aids in removing small flat impurities which have been sized on the round hole shaker with the more cubical pieces of coal.

Loading and Trimming Railroad Cars.—If all railroad cars were of the same dimensions as regards height, it would be a simple matter to regulate a chute so that loading might take place with a minimum of breakage. Coal cars differ greatly in design, in height, and in capacity. They consist of three classes: (a) Gondola cars, with or without dumping devices. (b) Hopper bottom cars. (c) Box cars. The special loaders of the box cars will be treated later. Open gondolas and hoppers are the common classes, and at present range in capacity from 25 to 50 tons, and in height or clearance of sides above the rails they vary from about 7 to more than 11 feet in the newer "high sides" or "battleships," as they are often called at the mines. Such a range of clearance seriously affects the successful loading and trimming of the cars, especially if the chutes are not easily and quickly adjustable, since in "spotting" the cars one with steel sides is likely to be followed by a low wooden gondola. At some of the old mines, screens and chutes are so low that the highest cars cannot be loaded on the lump track. In such a case, they are usually switched under the screenings chute, where more headroom is available and breakage not so important. Also, by this practice, excessive shifting of the loading devices is avoided.

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*A. J. Reef, Coal Age, Dec. 14, 1912.

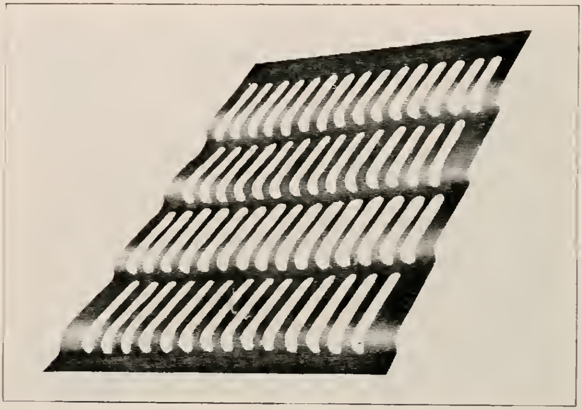

FIG. 25. LIP SCREEN (CUT LOANED BY CROSS ENGINEERING CO., CARBONDALE, PA.)

There is a wide difference in the inside height of coal cars. They range from 3 feet 6 inches in the smaller flat bottom gondolas to 9 feet in the deepest part of a modern steel hopper car. The difference in breakage in loading two such cars from a chute of the same height is evident. At one mine the loading chute had been raised to a height of about 14 feet above the tracks in order to load properly and trim a high side car. The next car was only 8 feet in height and, therefore, the coal dropped about 10 feet before striking the bottom of the car. Another measurement showed that the end of the lump coal chute had a clearance of 8 feet above the car being loaded at the time.

In general, car loading without breakage is an engineering problem which has been practically solved. Frequently the loaders do not take advantage of the adjustable features of the chutes by following the loading closely, and often no attempt whatever is made to use them. In some cases bad balancing makes the shifting of the automatic loading devices laborious. The newer inclined loading belts or booms are generally controlled by small electric hoists which make adjustment easy.

When loading a car under the tipple, first the end should be filled to the full height of the car, and then the car gradually and slowly moved along, preferably only a few inches at a time. In this way the coal coming from the loading chute strikes near the top of the loaded pile, and after dropping a minimum distance, rolls gently down the slope of the pile into the car. Sometimes the cars are pinched by hand while being loaded, along a grade which has become somewhat irregular by reason of long service, while on other tracks the grade is sufficient to start the car on releasing the brakes; in either case, there is danger of excessive breakage from lumps dropping the full distance to the floor if the car is allowed to move too far. At the more modern and larger mines the cars are retarded and moved either by means of a car puller, which is simply a wire rope attached to a power driven drum, or by a patented car retarder, which consists of a rope attached to a small friction drum under control of an operator. This allows the car to be moved forward as desired and insures a uniform unbroken lump.

While it may be argued that car breakage as discussed has no effect on the percentage of sizes made, the following illustration gives an idea of its effect on the buyer. A large retail dealer gives an average figure of 5 per cent degradation on Illinois lump coal after careful loading, transporting to Chicago, and then unloading with a fork. Other cars, which he thinks must be carelessly loaded, frequently contain up to 15 per cent fines. A saving of 10 per cent breakage in carefully loaded coal is shown by these figures.

Box cars are loaded at many Illinois mines for the following reasons:

(1) General shortage of regular coal cars.

(2) To avoid wetting and possible freezing of the coal in winter.

(3) Prevention of theft in distant shipments.

(4) To prevent box cars returning empty to the states west of Illinois.

(5) Less shrinkage in weight of the coal en route.

(6) Railroads sometimes require box cars to be used for shipments to Texas in order that they may be available there for reshipping cotton.

According to J. J. Rutledge,* twenty years ago in Illinois it was common to see from six to twenty laborers shoveling and carrying lump coal from the center doors of the car and piling it in the ends. Since that time a number of mechanical devices have been perfected, called box car loaders, which load the coal evenly in the car, or where desired.

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*Min. Mag., March, 1906.

A modern box car loader in the tipple should fulfill the following requirements: (a) A capacity equal to that of the shaker screen. (b) Ready adaption to the regular coal chutes. (c) Mobility.—Can be moved in and out of the cars quickly. (d) Freedom of injury to the car ends. (e) Freedom from breakage in conveying and depositing the lump coal in the ends of the cars. The first four requirements will come properly in the discussion of the engineering features of the tipple. The fifth feature, breakage, is being overcome in the newer designs.

At a number of the tipples the box car loaders, which had been installed several years previously and were of older designs, were rusty and unused. Inquiry revealed that the main reason for this was "too much breakage of coal," although it is fair to say that other reasons were, "we don't get the box cars we used to," and "the trade don't call for it loaded that way." Another objection to their use is that no inspection or picking is possible in the car. At many mines, however, the loaders are used constantly, and the fact that some of the newest plants have installed them is proof that they fill a need.

Much of the breakage in connection with a box car loader is due to sliding the coal from the shaker screen down the steep chute necessary to bring the lump coal to the box car loader. Mr. A. J. Reef describes the difficulty as follows: "It has been the universal practice to feed box car loaders over stationary chutes, and even when shaking screens are used, the oscillating portion is at such height as to load open cars, and for the loading of box cars, a long and rather steep chute is always employed. The coal necessarily attains such speed in passing through this latter that it is certain to splinter more or less on striking the loader."* Recent installations in Illinois have improved this obvious defect.

Railroad cars are usually loaded to the limit of 110 per cent of their rated capacity, and the railroads require loading at least to their rated capacity. Several devices have been put on the market to make the top or exposed portion of the carload of coal appear free from small coal. A well appearing and nicely trimmed car is attractive and calls the purchaser's attention to the fact that preparation is receiving due attention.†

The St. Louis and San Francisco Railroad Company has published a bulletin on the proper way to load a railroad car with coal.‡ This states that steel cars should not be loaded more than 4 inches above or below the top of the car against the sides and ends. The top load should not be over 26 inches high. The same rules apply to wooden cars, excepting that the top load should not be over 20 inches high. Cars are properly loaded according to the rules if they contain 10 per cent in excess of the marked capacity. This saves one car in ten, which is important when difficulty is experienced in getting cars spotted. Further, they note that the above advantage should more than pay for careful trimming. These rules also reduce loss and breakage to a minimum.

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*Coal Age, Dec. 14, 1912, p. 830.

†B. S. Thym, Coal Age, Vol. 2, p. 248.

‡For abstract see Black Diamond, Dec. 6, 1914, p. 21.

Rescreener and Bin Breakage.—These have been considered briefly under Date on Degradation of Bituminous Coal, p. 76. Further data concerning this subject are not available for this bulletin.

Attention has been called to every possibility of breakage because it is believed that the great care taken at the face to avoid the production of fines can be rendered useless by carelessness in handling below and especially above ground, from the time the coal is hoisted until it is shipped. A single illustration, chosen from a reliable source, shows the importance of modern engineering devices in the tipple in preventing breakage.

At a mine in southern Illinois a new tipple was recently installed in which particular attention was given to the question of breakage. Comparative data given in Table 13 show the percentages of sizes made in the old and new tipples, together with a probable realizable value per ton of coal produced. The prices per ton are an average of the circular prices for the Franklin county field for the years 1913 and 1914.

TABLE 13.

Relative Percentages and Prices Received for Coal with Old and New Tipple Equipment.

Sizes	Old Tipple Equipment			New Tipple Equipment		
	Percentage of Output	Value per Ton	Percentage times Value or Value of Each Size in a Ton	Percentage of Output	Value per Ton	Percentage times Value or Value of Each Size in a Ton
Screenings	37	$0.73	$0.2701	33	$0.73	$0.2409
Small Nut	6	1.38	0.0828	7	1.38	0.0966
2x3-in. Nut	12	1.50	0.1800	11	1.50	0.1650
3x6-in. Egg	24	1.47	0.3528	23	1.47	0.3381
6-in. Lump	21	1.50	0.3150	26	1.50	0.3900
Total	100	....	$1.2007	100	....	$1.2306

Without changing underground methods and by tipple improvements alone the value of the coal was increased 3 cents per ton. At a 2,000-ton mine this is an added income of sixty dollars per day; enough to yield considerable profit after covering the extra fixed charges. Another benefit not apparent from an inspection of the table is that with the new tipple the percentage of domestic sizes is somewhat less than with the old. Although about the same prices are quoted for these sizes as for lump, it is well known that they are at times a drug on the market and must be sold at a lower price.

CHAPTER IV.

SIZING AND SIZES OF ILLINOIS COAL.

STATEMENT OF PROBLEM.

No feature of coal preparation in Illinois has caused so much discussion as the question of sizing coal in the mine tipple for the various markets. The tendency during the past few years undoubtedly has been towards a multiplication in the number of sizes with a consequent increase of mechanical equipment and preparation cost for each ton produced. The introduction of a new size for a particular purpose at one mine has forced producers at competing mines to introduce a like or even greater refinement in order to hold their trade. Such competition has been severe especially among the smaller mines at which the limited tonnage produced makes difficult the disposal of small amounts of special sizes made as a by-product during the production of the more salable special sizes. The domestic trade today demands the larger coal;* thus the considerable amount of fines produced in its preparation must be absorbed by the steam trade. At times this causes demoralization of the steam coal market.

The coal mining industry is seriously asking the question, "Are too many, enough, or too few sizes of coal being made in Illinois today?" A majority of the operators have expressed themselves in favor of a simplification of sizing and of sizes made, contending that (a) formerly the trade was satisfied with fewer sizes, (b) present sizing tends to cause undue loss through the production of considerable amounts of fines during screening and preparation, (c) at certain periods of the year such carefully prepared sizes are not in demand and must be sacrificed with the smaller and cheaper coal, and (d) consumers do not receive sufficient benefit from sized coals to warrant their paying the increased price per ton which careful preparation demands. They also claim that retail dealers do not like to handle so many sizes in their yards. Some even favor shipping run of mine coal only, since the lumps in such coal, having a cushion of fines to ride on, ship with minimum breakage. If sizes are needed for domestic trade, the retail yard could prepare them as needed in a perfect condition. This plan has been adopted at one Chicago coal yard.†

Others contend that (a) the present sizing practice is the legitimate evolution of an increased knowledge and consequent demand on the part of the consumer, (b) much capital expense has been incurred by mining companies in the erection of special screening and rescreening plants to meet this demand, and (c) having won satisfied customers in this way, a change would be folly. Still others contend that the benefits resulting from very closely sized coals are great, and that even closer sizing is in demand. They favor the multiplication of sizes as fast as the market will absorb them.

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*Black Diamond, Oct. 10, 1914, p. 281.

†Black Diamond, March 8, 1913.

Many of the arguments over the above questions fail to consider the basic question: namely, "Have the correct sizes been made?" Is not the present practice in sizing, although complex, based on custom and convenience, rather than on accurate knowledge of what sizes or range of sizes are most efficient for the consumer and consequently of greatest potential value to the producer?

Necessity for Close Sizing.—In discussing the question of sizing Illinois bituminous coal, care should be taken to differentiate it from Pennsylvania anthracite and from the Appalachian bituminous coal with which it competes.

No doubt, close sizing is necessary for the successful burning of anthracite coal. One of the best summaries on this question is given by J. Callon,* who says that Pennsylvania anthracite will not decrepitate enough in the fire to burn without sizing; that it cannot be in too large sizes because the cold mass would be in too great proportion to the incandescent surface; that all the fragments must be of nearly the same size so that all the particles will be under the action of the flame at the same time; that little pieces must not be so numerous as to fill up the space between the large; and that these little particles are useless because they do not coke. Moreover, he states that the hard nature of anthracite permits close sizing without excessive breakage.

In burning the friable Appalachian bituminous coal, a new set of conditions arises. The coal, whether large or small, when introduced into the fire, heats, gives off its volatile combustible matter, swells, becomes pasty, and fuses or cokes into a more or less coherent mass. In hand fired furnaces this coke must sometimes be broken with a slice bar in order to give the air free access to the fixed carbon or coke remaining, although the coke is in itself porous, and usually admits air, causing combustion to take place throughout the whole mass. Much of the value of close sizing is lost through this coking of the pieces, whether large or small.

Illinois coals are essentially free burning or non-coking in an open fire, are higher in volatile matter than either of the two coals just discussed, and in point of hardness lie between them. On account of its lack of coking properties it might be well to imitate the close sizing used with anthracite; at least to size enough to prevent fines from falling through the grate bars, although in some cases wetting of the coal is said to cause this fine material to adhere to the larger lumps long enough for it to be burned with success. On the other hand, its property of decrepitating readily in the fire into more or less rectangular blocks, allowing free admission of air would seem to refute the general claim that increased furnace efficiency is a result of close preliminary sizing.

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*Cours d'Exploitation des Mines, Texte 3, p. 153.

The following reasons have been advanced for closely sizing Illinois coal:*

(1) Increased furnace and boiler efficiency.
(2) Less loss of fuel in ash.
(3) Less smoke.
(4) Uniformity in quality better assured.
(5) Uniform combustion insuring great capacity.
(6) Less clinker and cinders.†
(7) Less draft needed.
(8) Longer life of grates.
(9) Less cleaning of fires and tubes.
(10) Greater efficiency of fireman, through handling evenly sized lumps.
(11) Furnace under better control; steam can be increased more rapidly.
(12) Sized coal may be stored with little danger of spontaneous combustion.
(13) Increased capacity of furnace through more rapid combustion.
(14) Even and regular air supply encircling each piece being burned.
(15) Sizing allows depth of fire to be adjusted so as to give each size its proper draft.
(16) Quicker fire on account of great number of surfaces in proportion to the weight immediately exposed to the fire.
(17) Less dirt and dust.

It is not province of this bulletin to discuss all of these reasons. Some which are true in the case of anthracite, coking bituminous, or even briquetted coal, are not necessarily true in the case of Illinois coal, or may be true when the coal is used for a particular purpose only. In general, the size of coal which can evaporate the most water for a dollar under the conditions imposed will ultimately determine which sizing practice will be adopted. Until tests have settled this point, a universally acceptable standardization, at least of steam coal, is not feasible.

Another view point is the following: "From an engineering standpoint, exact sizing is right for effective combustion, but consumers for whom sizing is done do not want to pay for it, therefore in a commercial sense sizing is wrong."‡

Sizes Prepared in Illinois.—In order to bring out present sizing practices in Illinois a list of the various sizes of coal prepared has been compiled and is given below, together with trade names and general use or significance of these sizes. The screen in each case is understood to be a standard round hole plate, unless otherwise stated.

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*Partly after C. T. Malcolmson. "Briquetted Coal." Proc. 1st Annual Convent., Int. Rwy. Fuel Assoc., 1909.

†Bul. 325, U. S. G. S., p. 40, gives a table showing that size has no effect on clinkering.

‡Black Diamond, December 5, 1914, p. 455.

(1) Run of Mine.—Coal sold as mined without preparation, and including all sizes mixed together.

(2) Lump Coal.—The larger sizes, from which the finer have been removed. Lump may be made by screening over any one of a variety of sizes of holes. In different parts of the state the size of perforation varies from ½ inch to 8 inches. A few sizes, however, are so common that special names have been given to the lump coal made over them. They are:

(3) Big Lump or Fancy Lump.—Made over a screen with 8-inch holes.

(4) Standard Lump or Chunk.—Made over a screen with 5-inch, 6-inch, or even 8-inch holes. This is the common large size made for the retail trade, and is a favorite, especially if coal must stand considerable handling before using, as in the northwestern trade. In one district a mixture of egg and lump mixed is called standard lump.

(5) Domestic Lump.—Usually a lump made over a screen having 3-inch or 4-inch holes; or it may be identical with (4).

(6) Steam Lump or Ordinary Lump.—Usually made over a screen with 1¼-inch or frequently 1½-inch or 2-inch holes; at some mines the holes may be 1⅛ inches or smaller. Bar screens are also used to prepare this product.

(7) Three Quarters Coal.—Lump and nut coal mixed. (A term adopted from the Pittsburgh district in which bar screens are used.)

(8) Modified Lump.—Made by taking only part of the screenings from run of mine coal. (More frequently used in the Kansas districts, and is made by taking 25 per cent screenings from run of mine containing about 40 per cent screenings.)

(9) Railroad Lump or Standard Railroad Lump.—About the same size as No. 6. In places this is dumped over a bar screen with 8 to 10-inch spaces for the purpose of reducing the largest lumps in order that they may be fired without the necessity of breaking. In this case it may be called Engine Coal or Locomotive Coal.

(10) Engine Coal or Locomotive Coal.—(See (9).) These terms may also refer to run of mine coal broken to pass bars from 8 to 10 inches apart, placed over the receiving hopper or pockets of the coaling station. Locomotives burn other sizes of coal and at least locally, the terms defined may refer to almost any size.

(11) Chute Coal.—A coal containing all sizes under about 6 inches, including screenings, which is frequently sold for local uses. Railroad Chute Coal may be any size used in locomotives.

(12) Local Trade or Wagon Coal.—At many of the mines situated in or near towns there is an auxiliary chute leading from below the weigh box to the ground at a place convenient for loading wagons. This chute usually contains a bar screen with as small as ¾-inch or as large as 2-inch spaces, over which this size is prepared.

(13) Egg Coal.—If screens with from 4 to 8-inch holes are used in preparing a lump, the largest of the sizes below this limit is called egg. Thus egg coal may be as small as 1¼-inch or as large as 8 inches in diameter. If a wide range is included in the egg size, as from 1¼ to 6 inches, it is frequently called Railroad Egg or Railroad Coal. The most common egg coal, however, is that made through a 6-inch and over a 3-inch hole. In retail trade this is sometimes called Furnace Coal.

(14) Egg Run.—Made by leaving all the smaller sizes in the egg coal. May be all coal passing a screen with 5, 6, or even 8-inch holes. Used mostly by locomotives.

(15) Nut Coal.—The sizes smaller than egg and from which the smallest coal has been removed. Nut is made as large as 3½ inches and as small as ¼ inch in diameter. The usual size, however, is through a 3-inch and over a 1¼-inch screen.

(16) Nut Run.—Includes nut coal and all smaller sizes. If nut run is rescreened, it may be divided into any combinations of five sizes, the usual dimensions of which are as follows:

(17) No. 1 Nut.—Through 3-inch and over 2-inch. (May be called Small Egg.)

(18) No. 2 Nut.—Through 2-inch and over 1¼-inch. (May be called Stove.)

(19) No. 3 Nut.—Through 1¼-inch and over ¾-inch. (May be called Chestnut.)

(20) No. 4 Nut.—Through ¾-inch and over ¼-inch. (May be called Pea, Buckshot, or the smaller sizes, Buckwheat.)

(21) No. 5 Nut.—Through ¼-inch and over 0-inch. (May be called Dust or Slack.)

Nut coal is frequently washed, in which case sizes may be prepared, ranging as follows:*

(22) No. 1 Extra Washed.—Always under 3¾-inches and always over 2½-inches.

(23) No. 1 Washed.—Always under 3½-inches and always over 1¾-inches.

(24) No. 2 Extra Washed.—Always under 2¼-inches and always over 1⅜-inches.

(25) No. 2 Washed.—Always under 2¼-inches and always over ⅞-inches.

(26) No. 3 Washed.—Always under 1½-inches and always over ⅝-inches.

(27) No. 4 Washed.—Always under ⅞-inch and always over 3/16-inch.

(28) No. 5 Washed.—Always under 7/16-inch and always over 0-inch.

If only two sizes are made at a washery, the larger is designated "washed nut" and the smaller "washed slack."

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*F. C. Lincoln, "Coal Washing in Illinois." Bul. No. 69, Engineering Experiment Station, University of Illinois, p. 44.

(29) Pea Coal.—In Illinois this term refers to the intermediate sizes of nut. (See (20).) Coal ranging in any of the sizes between 2 inches and ¼ inch has been called pea. Generally this term refers to some size of coal made with less than 1¼ inch as a maximum dimension and with greater than ¼ inch as a minimum.

(30) Screenings or Raw Screenings.—The general term for the fines made in the tipple, if they are not rescreened, includes all sizes passing 2-inch holes or more generally 1¼-inch holes, although screenings are made with larger or smaller maximum limits than these. It is the standard chain grate stoker coal.

(31) Slack.—Refers to screenings from which the larger sizes have been taken. It may be from ¾ inch (plus or minus) to zero in size. The term is frequently used interchangeably with screenings.

(32) Coarse Slack or Stoker Coal.—A combination of Slack, Pea, and Nut.

(33) Fancy Screenings.—Refers to Pea and Slack sizes not separated, and is sometimes called Pea Run.

(34) Duff.—The smaller sizes of rescreened coal. It may be ½ inch (plus or minus) to zero, or may refer to the same sizes as Slack or as Dust Coal.

(35) Dust Coal.—Usually refers to the smallest coal. Thus No. 5 Nut (See (28)) of a size ¼ inch to zero may be called Dust Coal. This coal is also referred to as Culm.

(31), (34), and (35) are sometimes called Waste Coal, especially if they are so dirty that they have little market value.

(36) Pulverized or Powdered Coal.—Coal used in metallurgical furnaces or cement kilns is frequently ground before burning so that 92 per cent or more passes a 100-mesh screen. This is usually done at the plants in which it is used.*

Common terms used in connection with prepared Illinois coal, although not restricted to any special sizes are the following:

Secondary Coal.—Coal in general not of the first grade. It may be a product from the picking belts in the tipple or from the second compartment of a washing jig.

Bug Dust.—Machine cuttings.

Clean Coal.—A coal properly prepared without visible impurities; and not one free from small sizes, as sometimes thought.†

Dirty Coal.—A term ordinarily used when fines or degradation products are visible in the coal; should be used only to designate the presence of visible foreign matter or impurities.†

Conditions Affecting the Sizes Produced.—Considerable differences exist in the actual sizes of the coals just defined, due to differences in the screening surface and to the conditions under which screening is carried on. The screening surface may be composed of bars, of screen plate with round or oblong holes, or of wire screens with square or rectangular openings. The latter are used only on roller sceens in rescreening plants. Also, the steeper the angle at which the screen is placed, the smaller is the maximum piece which will just pass the holes.

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*For a symposium on powdered fuel see: Journal American Society of Mechanical Engineers, October, 1914.

†Black Diamond, May 11, 1912.

Round and Square Hole Screens.—The round hole screen is by far the most common in Illinois, and is generally taken as a standard. No tests are available showing the width of bar screen or the diameter of square opening which will pass the same percentage of an Illinois coal as will a round hole of given diameter.

Tests on other bituminous coals show that a ⅝-inch bar screen will pass the same percentage of run of mine coal as a 1-inch round hole.* Another test shows that a square hole screen will pass as much material as a round hole screen with a diameter 1.23 times as large.† Thus, a 1-inch square hole may be equivalent to about a 1¼-inch round hole, and a 1¼-inch bar screen equivalent to a 2-inch round hole screen. The largest pieces made on a round hole screen are more nearly of uniform dimensions in two directions. For these reasons, if exact sizes are desired, specifications should state not only the sizes of coal desired, but also the kind of screen over which they are to be made, since an operator using a 1¼-inch bar screen produces a considerably coarser screenings than one using a 1¼-inch round hole screen.

Sizes in Competing Districts.—The market for Illinois coal covers at least parts of eighteen states. In different portions of the territory this coal comes into competition chiefly with the bituminous coal from West Virginia, Kentucky, Ohio, and Indiana on the east, and from Iowa, Missouri, Kansas, Oklahoma, etc., on the west. Multiplication of sizes in all of these districts has been rapid during the past few years, and many mines formerly producing only lump and screenings, now ship two or more prepared sizes.

Until recently the initial preparation of coal in much of the eastern territory mentioned was governed by an agreement between the coal operators and the United Mine Workers of America, which specified as follows:

"Screens hereby adopted for the State of Ohio, Western Pennsylvania, and the bituminous district of Indiana shall be uniform in size, six feet wide by twelve feet long, built of flat or Akron‡ shaped bar of not less than ⅝ of an inch surface with 1¼ inches between bars, free from obstructions, and that such screen will rest upon a sufficient number of bearings to hold the bars in proper position."

Frequently, the slack or screenings made through this standard screen are rescreened over a bar screen having ½ or ¾-inch spaces, making pea and slack. In other cases this second screening is over a rotary screen with square or round holes. As has been stated, Ohio has recently passed to the mine run basis which will lead to a change in preparation and possibly to a multiplication of sizes to meet competing districts.

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*Henry Louis, "The Dressing of Mineral," p. 12.

†Eng. and Min. Jour., March 13, 1915, p. 493.

‡The name Akron bar refers to a bar of particular cross section, taking its name from the city of Akron, Ohio, where it originated, and not from the resemblance of the cross section of the bar to an acorn, as has been suggested.

In districts of West Virginia in which coking coal is mined not so much attention is paid to sizing as in districts in which splint and gas coals are mined. Thus, in the New River field 3-inch bar lump, 1 or 1½-inch bar to 3-inch bar egg, and 1 or ½-inch bar screenings are prepared. From the Fairmont region of northern West Virginia the following sizes are shipped:*

(a) Lump.—Four inches, 3 inches, 2½ inches, 2 inches, 1½ inches, 1¼ inches, and ¾ inch.

(b) Egg and Nut.—The sizes depend upon the size of lump.

(c) Pea.—Through ¾-inch and over ½-inch.

(d) Slack.—Through ½-inch. This district has been sizing its coal closely since 1900. The recent tendency is towards a simplification in the number of sizes.

In Indiana 1¼-inch bar screenings are prepared which may be rescreened into pea or slack. The 1¼-inch bar lump is frequently made into nut, egg, and lump which competes with Illinois sizing.

In Kentucky the practice varies greatly, but in many of the newer tipples, especially in the eastern part of the state, and in several cases in the western part, shaker and other screens have been installed to prepare the domestic and other sizes common in Illinois.

In Iowa, Missouri, and Kansas, although payment to the miner is on the run of mine basis, preparation is generally more simple than in Illinois. One, two, or three sets of bar screens are often used, making at the most four sizes; through 1¼-inch, through 2½-inch, then through 4, 5, or 6-inch, and a large lump over this size. Round hole shaker screens, however, are not uncommon in these fields. For example, Kansas coal for general domestic trade is prepared over round hole shaker screens into 3-inch lump, 2-inch to 3-inch egg, ¾-inch to 2-inch nut, and a ¾-inch slack.

Oklahoma. (For Oklahoma sizes see p. 109.)

Standardization of Sizes.—Such a multitude of sizes and names of bituminous coal must be a cause of uncertainty, trouble, and annoyance to producer and consumer alike; to the producer in endeavoring to adjust his own screens to meet the extremely varying market, and to the consumer from lack of information regarding the nature and real worth of the many sizes. Aside from the question as to whether more or fewer sizes are advantageous, Illinois and the rest of the bituminous field are badly in need of some standardization of sizes.

In November, 1914, a radical change, proposing to do away with the many sizes now made and to return to the old methods of producing 1¼-inch screenings and 1¼-inch lump coal only, was advocated in the present method of preparing bituminous coal at the mines in Illinois and in the eastern competing region. This plan contained a provision for producing a domestic nut size when necessary.* The plan was not to be adopted until accepted by enough operators to represent 90 per cent of the tonnage in the field.

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*H. H. Stoek, "Mechanical Preparation of Coal." Proc. 2nd Annual Convention, Int. Rwy. Fuel Assoc., 1910.

Mr. H. C. Adams, the originator of the plan, states that Ohio, Indiana, West Virginia, and western Kentucky appeared to be favorable to the change, but that Illinois and the southeastern Kentucky did not give sufficient support to make it successful.† Figures given for Illinois show that operators representing about two-thirds of the tonnage were willing to standardize if competing districts would do likewise, and that over 14 per cent of the remainder represented mines not entering the general market. It is remarkable that such a percentage of the operators were willing to return to the simplest of all preparation,—two sizes, and of course run of mine. Further opposition to the plan was made by the railroads which in some cases were afraid the change would mean forced sizing in their own chutes, especially if they had been accustomed to burn lump.‡

Standard Practice in Various Districts.—Standardization has been accomplished in several districts, as is shown by the following:

TABLE 14.

COMMERCIAL SIZES OF ANTHRACITE.

Name of Size	Diameter of Ring		Uses
	Over inches	Through inches	
Lump	6½	. . .	Locomotive Steam Coal. (Very little now made.)
Steamboat	4½	6½	Blast Furnace Coal. Smith's Forge Coal. (Very little now made.)
Broken	3¼	4½	Domestic Furnace Coal.
Egg	2 5/16	3¼	" " "
Stove	1⅝	2 5/16	Domestic Range Coal.
Nut	15/16	1⅝	" " "
Pea	⅝	15/16	Domestic Furnace Coal.
Buckwheat	7/16	⅝	Steam Boiler Coal.
Rice	¼	7/16	" " "
Barley	3/32	¼	" " "

In 1916 the standard sizes of anthracite were changed and a reduction made in the number.¶

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*The Coal Trade Journal, November 14, 1914.
†Black Diamond, February 20, 1915.
‡Coal Age, March 6, 1915.
¶Coal Age, p. 839, May 13, 1916.

TABLE 15.
STANDARD SIZES OF BITUMINOUS COAL. (OKLAHOMA.)

Name of Size	Diameter of Round Perforations in the Shaker Screen Used	
	Through, inches	Over, inches
Domestic Lump	...	2½
Screened Nut	2½	1¼
Chestnut or Pea	1¼	¾
Slack	¾	...

In the anthracite districts of Pennsylvania the commercial sizes given in Table 14 are produced,* and the standardized sizes adopted by the producers in the Oklahoma bituminous fields are given in Table 15. This standardization has been in force since about 1912, and is used at practically all mines in Oklahoma, irrespective of district or seam. Five sizes cover the entire range of these bituminous coals. They are probably somewhat softer than Illinois coals and, with the exception of one seam, are non-coking. It has proved satisfactory to both producers and consumers.

A committee on standard form of contract covering purchase of railroad fuel reported in favor of a standard for bituminous coal as follows:† (See Table 16.)

TABLE 16.
STANDARD SIZES OF BITUMINOUS COAL FOR RAILROADS.

Kind of Coal	Screen Made Over			Screen Made Through		
	Round Inches	Square Inches	Bar Inches	Round Inches	Square Inches	Bar Inches
Pea	...	⅜	...	...	1¼	... or
Pea	½	...	...	...	1¼	... or
Pea	...	¾	...	1¼	...	... or
Pea	½	...	...	2	...	...
Nut	1¼	...	...	2½	...	...
Egg	2½	...	...	7	...	... or
Egg	3½	...	...	6	...	... or
Lump	7	...	...	...	...	... or
Lump	...	...	4	...	...	...
Pea Run	...	...	...	1¼	...	...
Nut Run	...	...	...	2½	...	...
Egg Run	...	...	...	...	...	5 (not less than)
7″ Round Egg Run	...	...	...	7	...	...

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*Paul Sterling, "The Preparation of Anthracite." T. A. I. M. E., Vol. 42, 1911, p. 264.

†Proc. 5th Annual Convent., Int. Rwy. Fuel Assoc., 1913, p. 31.

Mine Run Coal shall not contain more than 30 per cent screenings through 1-inch diamond bar screen, but shall contain more than 30 per cent of 5-inch bar screen lump or more than 40 per cent of 7-inch round hole screen lump.

The recommendations also included the use of relief screens to prevent overcrowding of the fine screens, and specified a minimum area for screens making the different sizes. Although favored by the convention in question the provisions have not been adopted by the railroads.

"In 1903 the operators in Williamson county, Illinois, adopted the following standard of sizes for washed coal. The numbers used refer to screens with round perforations."* This standardization has not been maintained in all cases.

Designation	Through	Over
No. 1	3 in.	1¾ in.
No. 2	1¾ in.	1 in.
No. 3	1 in.	¾ in.
No. 4	¾ in.	¼ in.
No. 5	¼ in.	......

Standardization means little to the small consumer, whose aggregate consumption, however, forms a considerable percentage of the coal produced. The general ignorance on this subject is well brought out in a report of a committee of the Boston Chamber of Commerce on Buying and General Handling of Steam Coal (November, 1909), wherein it is stated that of 225 manufacturing firms using steam coal, 25 per cent did not even know the particular kind of coal they were using.

The recent trend of public opinion towards some form of standardization for coal is significant. In 1913 the United Improvement Association of Boston voted to present three bills to the legislature:

(1) To establish a standard of quality for coal sold for domestic purposes.

(2) To establish a standard for sizes of meshes in sieves used in screening coal sold for domestic purposes.

(3) To require that a list of prices and notices of change of the same be filed with the state police.

Requirements for Standardization.—In considering the standardization of Illinois coal several viewpoints may be taken: (1) That of the producer, who desires to make only the largest percentage of the sizes of greatest market value. (2) That of the dealer who must furnish the coal which customers demand and are accustomed to.

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*C. S. McGovney, "Tests of Washed Grades of Illinois Coal." Bul. No. 39, Engineering Experiment Station, University of Illinois.

(3) That of the fuel agent, or purchaser for the larger consumers, who generally wants the most heat units for a dollar. (4) That of the consumer, each class of whom, for various reasons, demands a different fuel. It would appear that his viewpoint must be the final judgment as to what coal may be produced.

Consumers of Illinois coal may be subdivided into seven groups as follows:

(1) Railroads using coal for locomotives.

(2) Stationary power plants in which coal is used to produce steam.

(3) Domestic, retail, or household users.

(4) Large heating plants (as brick kilns and metallurgical plants).

(5) Those using pulverized coal. (Generally as a fuel for cement kilns and for certain metallurgical purposes.)

(6) Coke oven operators. (Ovens operated primarily for manufacture of metallurgical coke.)

(7) Gas Manufacturers using (a) gas producers for power purposes, and (b) retorts.

Many consumers in the last four classes obtain their coal from mines operated for their particular demand, or if buying in the general market they consume only a small fraction of the total production; therefore, they will not be given detailed consideration. Pulverization (5) has been referred to (p. 105). It is noteworthy that recently considerable Illinois coal has been used to produce metallurgical coke in the by-product ovens in northern Illinois and Indiana, 25 per cent and even more of the high volatile coal of Illinois having been mixed with low volatile eastern coals with success. Low ash and sulphur contents are the prerequisites for the coal used for this purpose, and since the coal is crushed to ¼ inch and smaller before using, size is of minor importance.

The first three classes noted—railroads, power plants, and domestic users—enter most strongly into the general market, the railroads alone buying 18 per cent of the total production of Illinois coal in 1914.* There is doubt that these three users, with their widely varying demands, can be satisfied with the same coal.

Railroad Fuel for Locomotive Use.—The railroads have and can use almost any size of fuel produced in Illinois today. The type of locomotive, class of service, grate and fire box design, draft and load (whether light or heavy) may render one size of fuel more efficient than another. Ease of firing, kind of fuel to which the fireman is accustomed, desire to assist a mine being served by the railroad, and relative price converted into ton miles under the conditions imposed, also influence the sizing or lack of sizing favored.

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*Illinois Coal Report, 1914.

It has been stated that Illinois run of mine coal under present conditions is the most economical coal per ton mile obtainable.*

The sizes listed on p. 103 as railroad fuel are the ones frequently used, although a number of companies, considering the difference in price, prefer run of mine; and recently the automatic stoker, burning the lower priced screenings, has been successfully introduced for this work. Most authorities agree that the ideal fuel for hand fired locomotives would be one with lumps of a maximum size of from 3 to 5 inches,† which would allow the fireman to shovel without taking his time to break lumps; and of a minimum size of about ½ inch, which would lessen losses through the grates and prevent the draft drawing the fine fuel out of the stack. Large lumps generally necessitate breaking in the railroad chutes or coaling stations.

From the standpoint of absolute efficiency, a number of railroad tests made with run of mine vs. variously sized coals, do not show conclusively that it is cheaper to use a closely sized fuel for this work.

Domestic, Retail, or Household Fuel.—Individual fuel users burn only a few tons per year and usually buy the size of fuel recommended by their dealer as best suited to their needs. The standard domestic fuel, anthracite, is sized closely, and until recently most stoves and heating furnaces were designed for this fuel; consequently a domestic bituminous coal, to sell well, should resemble anthracite in appearance. Absence of dirt and dust are important factors which make closely sized fuel a favorite. In discussing this subject, J. D. Rogers states that in grading a bituminous coal for domestic use three points must be noted: (a) absence of slack in the prepared grades, (b) absence of impurities in all grades, and (c) uniformity of size of the smaller grades.‡ For interstate shipments designed for house heating furnaces, or for threshing engine boilers and similar uses, where the coal must be handled several times, often with excessive breakage, it seems essential that a large lump should be shipped. Many householders favor a sized coal because it is said to sustain better a mild, steady combustion.

Tests by J. M. Snodgrass¶ conducted for the Engineering Experiment Station of the University of Illinois, have shown that with Illinois coal as a fuel, water can be evaporated in house heating boilers at about 50 per cent of the fuel cost of anthracite, and at about 75 per cent of the fuel cost of Pocahontas coal or coke. The relative worth of the different sizes in use for this purpose is not discussed.

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*Proc. International Rwy. Fuel Assoc., 2nd Annual Convention, p. 88.

†Proc. International Rwy. Fuel Assoc., 2nd Annual Convention, pp. 19 and 21.

‡"Preparation of a Domestic Coal." Kentucky Mining Institute, December, 1912.

¶Proc. Illinois Fuel Conference, Urbana, Ill., 1909.

Whatever the comparative effective heating power of unsized vs. sized coal for domestic trade, it is outweighed by appearance, freedom from dust and cleanliness in handling of the prepared sizes. These demands must be met by the producers furnishing this trade.

Stationary Power Plants Using Coal to Produce Steam.—The selection of coal for steam boilers depends on five things; namely, (a) relative price per ton, (b) total heating value, (c) relative percentage of the heating value that can be utilized in the boiler (d) maximum capacity which may be developed in the boiler, and (e) cost of handling different coals and the ashes produced by them.*

The results obtained by this selection depend in turn on: (a) the nature and condition of the coal, (b) character of the furnace, and (c) conditions of firing and furnace control.†

Effect of Size of Fuel.—The great number of variables thus introduced into any boiler test have tended somewhat to obscure the effects of sizing or of the range of sizes to be used under any particular condition. Regarding sizes of Illinois coal for a steam boiler fuel the following data are available.

"Tests made with different sizes were negative in results, and emphasize the statement—that the study of the effects of the various elements of size of coal has not been made in sufficient detail."‡

"The size of coal influences the capacity of any given equipment owing to its effect on the draft. . . . When dust and fine coal are fed into the furnace they either check the flow of air or are taken up by the draft and after being only partly burned are deposited back of the bridge walls. . . . Coal of uniform size forms the most satisfactory fuel, as it does not pack so closely as coal of different sizes mixed. The furnace design may be changed to suit the coal in view."¶ In the same bulletin (p. 6) it is stated that almost any fuel may be burned with reasonable efficiency in a properly designed apparatus.

L. P. Breckenridge states, "When coal fed into a furnace is fairly uniform in size it is much easier to burn it without smoke than when it is of different sizes. . . . Just to what extent it will pay to size coal for regular use is not yet clear."** Still another investigator states, "The size of the interstices between the coal particles increases with the size of the coal particles. Consequently with lower grades (meaning smaller sizes) of uniform coal a large proportion of the total air passes through the fuel bed. . . . For each size a depth of fuel bed will be found for which a minimum excess (of air) occurs."††

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*Colliery Engineer, Vol. 19, p. 64.

†Colliery Engineer, Vol. 10, p. 114.

‡U. S. G. S., Bul. No. 325, p. 49.

¶U. S. G. S., Bul. No. 428, p. 8.

**"How to Burn Illinois Coal Without Smoke." University of Illinois, Engineering Experiment Station, Bul. No. 15, p. 43.

††C. S. McGovney, "Tests of Washed Grades of Illinois Coal." University of Illinois, Engineering Experiment Station, Bul. No. 39, p. 55.

In general none of the references quoted give information specific enough to aid the operator of a hand fired stationary grate power plant in choosing among a variety of roughly or more closely sized coal. Such a consumer demands a coal large enough to avoid losses through the grate and to burn freely under the weak draft conditions existing in these plants. The tendency of fines to produce smoke may be an added factor, although "any fuel may be burned economically and without smoke if it is mixed with the proper amount of air and at the proper temperature."*

The ordinary coal sold for this purpose is steam lump with more or less screenings removed as demanded. Certain types of automatic stokers also are run with such lump coal.

Use of Screenings.—Most of the automatic stokers in power plants using Illinois coal today are designed for burning the sizes of coal under two inches, usually raw screenings. A favorite type is the chain grate stoker. In 1914 Illinois mines produced 19,740,000 tons of screenings size, or 32.5 per cent of their coal output, and although considerable of this was rescreened, a larger part was burned in automatic stoker boiler plants for the generation of steam. At the power houses of one public service corporation in Chicago, over 1,500,000 tons of coal were burned in 1914. This was mostly Illinois screenings, the cost of which represented about three-fourths of the total cost of each kilowatt of electricity generated.

"During the past twenty years the price of screenings has risen from practically nothing to a figure that in 1913 threatened for a time to make possible the crushing of larger prepared sizes for use in this work."† Normally, however, screenings are of lower value. Considerable success has been attained by rescreening this raw product, the average circular price of the different sizes so produced being higher than that of the raw screenings. A market for these sizes must necessarily be limited and unable to absorb the major tonnage of the raw screenings, unless some such separation is proved to be of added benefit in those sizes used in automatic stoker steam plants.

Because of this great production and market any proposed standardization of coal sizes should take into account the following questions: What size of screenings, if any, is best adapted for chain grate work? Will 1¼-inch or 2-inch or even some larger or smaller sizes or range of sizes give the coal for the most economic combustion? Other factors being equal, 2-inch screenings bring a higher price than 1¼-inch. Is this justified? Another point concerns the occurrence, distribution, and effect of the ash and impurities in the various sizes of screenings. Unlike many eastern bituminous coals, Illinois screenings are uniformly higher in ash than the larger sizes of coal and are not available for coking purposes, which throws practically the entire output into the steam trade.

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*University of Illinois, Engineering Experiment Station, Bul. No. 15, p. 16.
p. 16.

†H. W. Weeks, Black Diamond, March 20, 1915, p. 233.

Mr. H. W. Weeks, in discussing the general subject, says of the standard 1¼-inch screenings, "Screenings of this size are not satisfactory when used on many different kinds of equipment, the nature and amount of the load having an important bearing on their efficient use."* On the whole he favors larger screenings than these, especially if heavy loads are frequent.

To show the general effect of size on the efficiency with which Illinois screenings can be burned, the following are quoted: "Coals ranging in size from ¼ inch to 1½ inch burn much more rapidly than either very small or very large sizes."† (They must therefore give a greater capacity for any given equipment.) "Many Illinois screenings contain 40 per cent or over of material under ¼ inch."‡ "A coal with evaporative efficiency of 66 per cent when 10 per cent of it is under ¼ inch diameter, will give an efficiency of only 62 per cent when 60 per cent of it is under ¼ inch in diameter."¶ "Small sizes of coal burned with less smoke than large sizes, but developed lower capacities;"§ however, "31 tests on coal with an average diameter of 0.39 inches give an average efficiency of 66.88 per cent and 53 tests on coal with an average diameter of 1.46 inches give an average of 65.97 per cent."**

A. P. Kratz made a number of boiler tests with 1¼-inch round hole Illinois screenings from which the following is abstracted.†† In test No. 6 using 1¼-inch screenings of which 19.6 per cent were smaller than ¼-inch round hole opening, the overall efficiency of the boiler was 68.09 per cent; while in test No. 8, with 50.5 per cent passing the ¼-inch screen, the overall efficiency was 61.08 per cent. He states (personal interview) that draft and thickness of fuel bed are factors in high efficiency as well as any particular sizing of the coal.

Recent specifications frequently limit the amount of the screenings under ¼ inch in diameter, one examined limiting the amount to 40 per cent.

Tests made by the Commonwealth Edison Company of Chicago in 1906 ‡‡ show that for their chain gate stoker work with a constant thickness of fire, maximum efficiency was obtained when using sized coal of an average diameter of ¾ inch, and that especially below this size efficiency decreases rapidly and is very low for screenings under ¼ inch in size. This view has been somewhat modified since that time. Recently the above company have conducted a great number of boiler tests with Illinois screenings to ascertain among other points the size or combination of sizes best adapted for their work. Through the courtesy of W. L. Abbott, Chief Operating Engineer of the company, the writer was able to examine the results of many of these tests, which throw considerable new light on the whole subject of screenings. They point towards the following conclusions:

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*"Influence of Size on the Cost of Making Steam." Black Diamond, March 20, 1915, p. 233.

†U. S. G. S., Bul. 325, p. 176.

‡U. S. G. S., Bul. 290 (Tables).

¶University of Illinois, Engineering Experiment Station, Circular No. 3, p. 37.

§U. S. G. S., Bul. 373, p. 10.

**U. S. G. S., Bul. 373, p. 148, Table 37.

††"A Study of Boiler Losses," University of Illinois, Engineering Experiment Station, Bul. No. 78.

‡‡W. L. Abbott, "Some Characteristics of Coal as Affecting Performance with Steam Boilers." J. W. S. E., Vol. 11, 1906.

(1) There is a great difference in ash content among the various raw screenings produced in Illinois today, car samples received showing a minimum of 10.1 per cent and a maximum of 29.7 per cent, with an average of 17.4 per cent for thirty-two samples examined (dry basis).

(2) Screenings made through the same size of screen and coming from different mines or even from the same mine at different times show great differences in the percentages of relatively coarse and fine coal contained. Certain cars showed as much as 50 per cent and others as little as 17.8 per cent of coal under ¼-inch round hole in size. A large percentage of this fine coal in screenings greatly lowers the efficiency of the boiler under test.

(3) The company has placed a maximum limit to the amount of screenings they will buy of 55 per cent under ¼ inch in size. With such material, combustion is slow because the air will not go through it. Most of the trouble comes from the part of this fine material which is smaller than ⅛ inch in size. There is no objection to pieces of ⅛ inch size.

(4) Three per cent moisture added to screenings containing high duff greatly aids combustion.

(5) By removing all material below ⅛ inch from an Illinois screenings in one test the engineers were able to raise the furnace temperature from 2,500° F. to 2,800° F.

(6) On the other hand, coal sized with an entire absence of the sizes under ¼ inch does not show increased efficiency, probably due to difficulty in securing uniform air admission. The engineers report crushing an egg coal to stoker sizes and obtaining indifferent results because of a lack of a certain necessary amount of fine material. Ten per cent of the material under ¼ inch probably represents roughly a minimum desirable.

(7) Screenings with a maximum size of 1 inch are as efficient for these conditions as larger sizes.

Briefly, the best screenings for these and similar conditions are those with a maximum size of 1 inch and containing a limited amount only (15 or 20 per cent) of sizes under ¼ inch. (It is evident that a sized coal would produce this amount of fines as a degradation product if it received two or three handlings before being burned.)

The effect of a variation of ash in the screenings is to reduce the capacity and efficiency of the particular size of coal slowly until 15 to 20 per cent is reached and then more rapidly, until with about 40 per cent ash the efficiency of that size of coal is zero.*

Other references state, "Ash in dry coal has little effect on efficiency up to 15 per cent, above this point efficiency drops rapidly;"† and "overall efficiency of coal decreases only 2 to 3 per cent until ash content rises above 20 per cent when efficiency falls rapidly. In general, ash increase of 1 per cent causes a drop in efficiency of about 2 per cent."‡ In other cases this was less. U. S. government specifications for coal also recognize the greater decrease in efficiency with an increase of ash than would be expected by the actual decrease in combustible matter shown by such added percentage of ash.¶

Tests Made with Illinois Screenings at the Mining Laboratory of the University of Illinois.—Believing that ash and possibly sulphur were unevenly distributed among the various small sizes of coal which compose Illinois screenings, and that the location of certain sizes containing uniformly a high percentage of ash might be of assistance in solving the problem of how to make screenings more valuable, tests were undertaken in the Mining Laboratory of the University of Illinois. It was thought that screening tests, followed by analysis of the separate sizes, might reveal sizes the removal of which might pay both operator and consumer if cost, freight, and comparative boiler efficiency were considered. An outline of these tests follows:

(1) Requests were sent to ten mines in the state, asking that a barrel of screenings be collected from under the tipple at random during loading and be shipped to the laboratory. The mines were chosen with a view of representing all the seams mined in the state and widely separated districts.

Such a sample in no way represents an average of the screenings being made at any of the mines, and does not reflect on or bring credit to any mine or district. It was believed, however, that such samples would give a preliminary guide as to the relative percentages present in the different sizes, how impurities occurred in the screenings, and a hint as to their possible removal.

(2) In any set of sizing tests a standard set of screens must be chosen. Unfortunately many sizing tests are made with no idea of uniformity in the succession of screens used, either regarding shape or relative sizes of the holes. The round hole screens were adopted in each case as conforming to the usual commercial conditions. A 1-inch diameter hole was taken as a standard, and with screens below this size, a ratio of ½ was used; thus the diameter of each hole is ½ the diameter of the hole of next greater size. The screens used were 1 inch, ½ inch, ¼ inch, and ⅛ inch in diameters. For sizes above an inch a 2-inch screen would be the next in the ratio; this was used when the size of the screenings tested warranted it, but

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*W. L. Abbott, Jour. West. Soc. Eng., Vol. 11, 1906.

†U. S. G. S. Bul. 325, p. 176.

‡U. S. G. S., p. 38.

¶U. S. G. S. Bul. 339, p. 13.

since some of the screenings were of smaller size, as 1¼ inch, this size of intermediate screen was also used.

(3) Upon receipt, the whole barrel of screenings was carefully weighed, sampled, and a portion saved for analysis.

(4) Another portion was subjected to a float and sink test to determine the relative amounts of high grade and low grade coal present. In this test a solution was brought up to a specific gravity of 1.35 by the additions of zinc chloride. A representative lot of the screenings was stirred into the solution, the light weight purer coal floating, and the heavier bone coal and refuse sinking. The two products were carefully separated and analysed. The results appear in the tests under the heading "Float and Sink Tests." These results are relative only, as another standard of specific gravity of the test solution would produce different results.

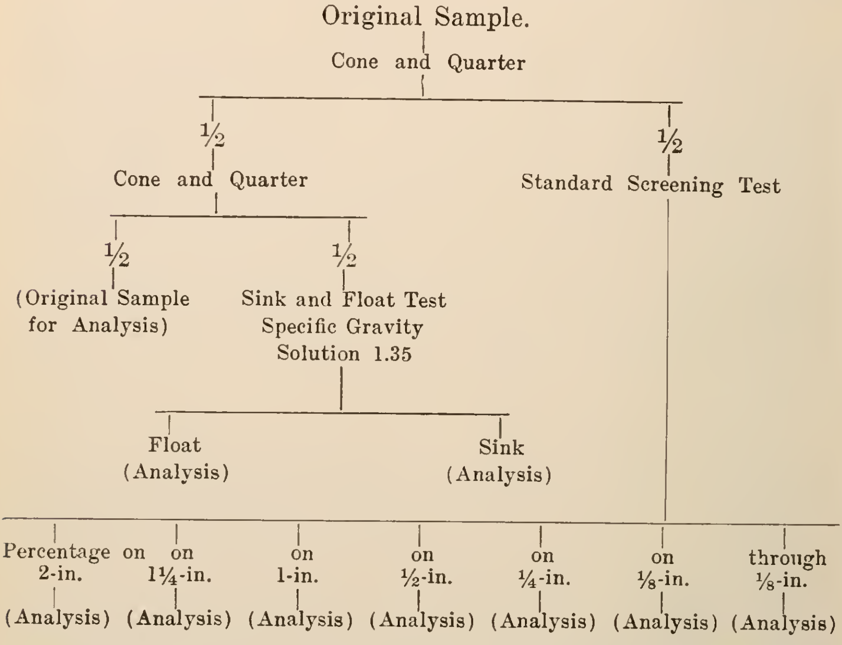

FIGURE 26. OUTLINE OF WORK. SCREENINGS INVESTIGATION.

(5) Half of the original sample was carefully screened in succession through the standard set of screens mentioned under (2), effort being made to prevent breakage.

Fig. 26 shows an outline of the work just described.

As received, the eleven samples varied in ash from 6.72 to 24.88 per cent and in sulphur from 0.90 per cent to 7.16 per cent. The float and sink tests show that the greatest part of the screenings is a rather pure coal of from 4.22 to 8.74 per cent ash and of low sulphur, and that on the average 23.2 per cent of the screenings is represented by a separable coal having an average ash content of 36 per cent and correspondingly high sulphur.

Test (1) was made with a 1-inch bar screenings, the other tests were upon screenings through round holes of the sizes indicated. When test (1) was subjected to screening on round hole screens, 23.5 per cent of the material was over 1 inch, showing the relatively large coal produced by bar screens. The presence of 1.47 per cent of coal on a 2-inch round hole in this test, and an ash content of 49.2 per cent and sulphur of 10.78 per cent show how these impurities, by breaking into flat pieces, pass a bar screen when they would not pass a round hole screen.

The largest amount of coal under ¼ inch was in test (4), the total being 30.1 per cent. The largest amount of dust under ⅛ inch occurred in test (5), 21 per cent, and this test had an average ash content of 17.81 per cent or 40 per cent more than the coarsest sizes in the same screenings.

In every test excepting (1) the largest sizes of the screenings show the least ash; this ash increases slightly until the dust coal under ⅛ inch is reached, when it shows in every case a decided increase, tending to confirm the opinion that the universal high ash content of this finest coal may be one cause of its low efficiency. In test (1) this fine material had an ash content of 40.9 per cent, enough to render it practically useless as an efficient fuel. Several others show striking increases measured on a percentage basis.

Contrary to the usual opinion, sulphur showed no increase in the fine sizes; a decrease even being noted in several cases. This tends towards the opinion that a large percentage of the separable sulphur is contained in the lump or ball form rather than in the brittle leaf form as is the case in many coals.

Regarding moisture, the results were irregular, and for this reason they are not included in the table. This may have been partly due to working with considerable quantities of the samples in a steam heated laboratory. It is significant, however, that in no case did the moisture vary greatly among the different sizes or show any tendency to increase in the finer sizes.

The percentage of the total ash was obtained by multiplying per cent weight by per cent ash. It shows the percentage of the total ash in the screenings in any particular size. As an illustration, in test (1) 18.7 per cent of the screenings which passed ⅛ inch, contained over 32 per cent of the total ash.

Calculated on an efficiency basis, if the screenings in tests (1), (2), and (11) were to stand any considerable freight rate, there is no doubt that the removal of the material under ⅛ inch would prove of benefit to producer and consumer alike.

Résumé.—This chapter leads to the conclusion that Illinois coal is badly in need of some standardization. As a possible guide, the essential differences of Illinois coal compared to certain others have been mentioned, and relative sizing has been discussed. It seems possible that the different markets, with their own needs, customs, and prejudices may always require particular coals. Furnace efficiency and relative price of any particular size will influence the final decision. It is to be regretted that in most boiler tests so many other variables have obscured the effect of sizing the various coals.

In regard to screenings, it has been shown that in the few tests available this important and often neglected product has a surprising individuality among its own sizes. So far as its standardization is concerned, further investigation is necessary before fixing a definite size or range of sizes. Its efficiency when sized may have a close relationship to the ash content of any sizes removed.

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BY

MERLE L. NEBEL

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ENGINEERING EXPERIMENT STATION

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SPECIFIC GRAVITY STUDIES OF ILLINOIS COAL.

BY MERLE L. NEBEL.
RESEARCH FELLOW IN ENGINEERING EXPERIMENT STATION.

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SPECIFIC GRAVITY STUDIES OF ILLINOIS COAL

I. INTRODUCTION.

The specific gravity of coal, especially of high moisture bituminous coal, such as is found in Illinois, is not a fixed value. Coals of different types may have characteristically different specific gravities, and it is equally true that coals of the same type vary greatly in specific gravity.

The object of this bulletin is to present the results of a study of (1) the effect of moisture upon the specific gravity of coal, and (2) the methods of determining the specific gravity of coal. The commercial aspects of the problem are discussed, and values are given for the specific gravity of coal from many of the mining districts of the State of Illinois. The specific gravity of bright and dull coal is also considered.

The investigation was carried on under the direction of Professor H. H. Stoek of the University of Illinois, who suggested the problem and whose advice and criticism were of great value. Experiments were conducted in the laboratories of the Department of Mining Engineering of the University of Illinois, and field trips were made to the Vermilion county coal field, where samples of coal were collected from a number of mines. Other samples were obtained from various sources. The author is indebted to Professor E. A. Holbrook of the Department of Mining Engineering of the University of Illinois for assistance in securing samples of coal and for valuable suggestions and criticisms during the progress of the work; and also to Professor S. W. Parr of the Department of Chemistry of the University of Illinois, and Mr. Fred H. Kay, of the Illinois State Geological Survey, for advice.

II. SPECIFIC GRAVITY OF COAL AS AFFECTED BY ITS ASH AND MOISTURE CONTENTS.

The chief factors which affect the specific gravity of coal, aside from difference in type, are (a) the amount of impurities, and (b) the amount of moisture present in the coal. A high-ash coal has a greater specific gravity than a low-ash coal of the same type. In general, a wet coal has been found to have a greater specific gravity than a dry coal of the same type. However, the effect of moisture in varying proportions upon the specific gravity of coal is more or less complicated, and it is chiefly with this phase of the subject that the present bulletin deals. Though the effect of ash content has long been recognized, apparently very little study has been made of specific gravity under varying conditions of moisture content. This is probably because most of the coals of Europe and of the eastern United States which have been studied have a low moisture content, seldom over 3 or 4 per cent; whereas the coals of the Middle West may contain moisture as high as 15 or 20 per cent.

The ordinary method of determining the specific gravity of a solid, such as coal, is to weigh it in air, then immerse it in water and weigh it again, the specific gravity being equal to the ratio of the weight in air to the loss of weight in water. However, since coal is porous, probably most of the contained moisture is held mechanically in its pores. When coal is dried the moisture leaves the pores, which become filled with air; if the dry coal is placed in water, the air in the pores is displaced by the water, and the coal becomes saturated. Under such circumstances the length of time coal is immersed in water before it is weighed affects materially the value obtained for the specific gravity.

As early as 1892 Eckley B. Coxe recognized the effect of ash upon the specific gravity of coal. In his laboratory at Drifton, Pennsylvania, in which analyses of coal were made and boiler tests run, he required specific gravity determinations to be made of all coal samples. In his Presidential Address before the New England Cotton Manufacturers' Association in 1893, he said, "There seems to be no doubt that there is a close relation between the specific gravity of coal and its percentage of ash. . . . A careful study of a great number of analyses of coal and determinations of specific gravity has led us to believe that, although our experiments are not as yet absolutely conclusive, there is a strong probability that, for a given size of coal from the same colliery under ordinary circumstances, the determination of the specific gravity of an average sample will give very nearly the same percentage of ash as will be determined by analysis, although the relation may not be exactly the same for different mines or for different sizes of coal.

"If the specific gravity and percentage of ash, in any sample of coal below egg size, are known, the percentage of ash in any other sample of the same coal, and from the same colliery, can be satisfactorily determined (we are inclined to think) from the specific gravity of that sample by the following formula:

$y' = y + (x' - x)a$, in which
x = the standard specific gravity,
y = the standard percentage of ash,
x' = the specific gravity of coal determined by our apparatus,
y' = the percentage of ash to be determined,
a' = a constant for coal from same mine.

"In the Lehigh region, for any size of coal, we find that, within what may be called the commercial limit of purity, an increase of 0.01 in the specific gravity corresponds to about the increase of 1½ per cent of ash; that is to say, that if coal of the specific gravity of 1.54 contained 8 per cent of ash, the same size of coal from the same mine when its specific gravity was 1.56 would contain twice 1½ per cent more ash, or 11 per cent."

Mr. Coxe evidently neglected the effect of moisture upon the specific gravity. This effect would not be great, however, in the case of the low-moisture anthracite with which he was chiefly concerned. M. S. Hachita* has suggested that if ordinary coal be considered a mixture of impurities with a pure coal substance, then the specific gravity of the mixture is equal to the sum of the specific gravities of each component multiplied by its percentage of the total mixture: that is,

$100\ gm = gpx + gi\ (100 - x)$, in which
gm = specific gravity of the mixture,
gp = specific gravity of the pure coal,
gi = specific gravity of the impurities,
x = per cent of pure coal in mixture,
$100 - x$ = per cent of impurities.

"Float and sink" tests of coal are very commonly made to determine the relation between the coal and the shale and other impurities which are mixed with it, and to study the possibilities of clean separation. Among the earliest of such tests in this country were those of Eckley B. Coxe, made in connection with the work on specific gravity previously referred to. The method in general use today is essentially the same as that used by him twenty-five years ago.

About 1910 the Department of Mines of Canada conducted an investigation of Canadian coals, including float and sink tests. Dr. J. B.

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*Eng. and Min. Jour., Vol. 83, pp. 670-73.

Porter, who was in charge of the work, states that the specific gravity of pure bituminous coal is from 1.28 to 1.37.*

The Technologic Branch of the United States Geological Survey (now the United States Bureau of Mines), at the St. Louis Fuel Testing Plant in 1904, carried on float and sink tests, and with a Nicholson hydrometer determined the specific gravity of eighty-two samples of coal. The average value obtained for clean coal was 1.29 and for "average coal" was 1.34.†

III. Terminology of Specific Gravity.

The United States Bureau of Mines, recognizing that coal is a porous substance and that the pores of the coal may contain air or moisture or both, most of which can be easily removed, takes this factor into account by determining two values for the specific gravity; (a) the "apparent" and (b) the "true" or "real." These terms are defined as follows:‡

The "apparent" specific gravity is the specific gravity of the coal including the moisture or air contained in its pores. In determining this value no correction is made for moisture content, and care is taken that no air escapes from the pores during the weighing of the coal in water.

The "true" or "real" specific gravity is the specific gravity of the actual coal substance corrected for air and moisture content. In determining this value the weight of the moisture present is deducted from the weight of the coal and all air is removed from the pores of the coal before it is weighed in water. The methods employed by the Bureau in determining this value are described on page 38.

The true specific gravity of coal, as thus defined, may be compared to a coal analysis calculated to the "moisture-free" or "dry" basis. The true specific gravity is then merely the apparent specific gravity calculated to the dry basis.

Coal analyses are calculated to a dry basis because the moisture content of a given coal may vary greatly with differences in the atmospheric conditions to which it has been exposed. It is often desirable to compare analyses with respect to the thermal or other qualities of the coal, and in order to make such comparison accurate, the error introduced by the variable moisture content must be excluded by calculating to the dry basis.

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*J. B. Porter and R. J. Durley, "An Investigation of the Coals of Canada," Vol. 1, p. 165, 1912.

†Bul. 323, U. S. Geological Survey.

‡Private communication from Dr. A. C. Fieldner, Chemist, U. S. Bureau of Mines.

In order to obtain the true specific gravity it is necessary to know the moisture content of the coal; the specific gravity may then be calculated by the formula:

$$\text{Sp. Gr.} = \frac{W-W_m,}{(W-W_m)-W'}, \text{ in which}$$

W = weight in air,
W' = weight in water,
m = percent of moisture.

Since W_m is equal to the weight of moisture present in a lump of coal of weight W, $(W-W_m)$ equals the weight of the actual coal substance excluding the moisture, and $(W-W_m)-W'$ equals the weight of the actual coal substance in water.

In Table I values of the true and apparent specific gravity of the same samples are compared for different contents of moisture. The true specific gravity is greater, a difference of from 0.10 to 0.21 being recorded.

TABLE 1.

TRUE AND APPARENT SPECIFIC GRAVITY FOR DIFFERENT MOISTURE CONTENT.

Sample No.	Moisture Per Cent.	Specific Gravity		Moisture Per Cent	Specific Gravity	
		Apparent	True		Apparent	True
138	4.04	1.23	1.35	4.91	1.26	1.38
141	3.30	1.25	1.35	5.06	1.23	1.37
144	3.07	1.24	1.34	5.64	1.24	1.34
147	3.26	1.24	1.40	5.55	1.22	1.40
150	3.79	1.22	1.38	5.01	1.15	1.36
154	4.85	1.21	1.34	5.48	1.24	1.36
156	4.51	1.24	1.37	4.94	1.22	1.36
159	4.31	1.21	1.35	4.62	1.26	1.38

The term "true" or "real" specific gravity is a rather unfortunate one, since its use implies that any other value for the specific gravity would be unreal or untrue. Such is not the case. Other values are as distinctly true or real as the value so termed, and each has a practical use, which may be even more important than that of the so-called true specific gravity. For this reason it is proposed that in this bulletin the term be replaced by the more logical term "dry specific gravity," which is directly analogous to the "dry" values of fixed carbon, ash, or B. t. u., given in proximate analyses of coal.

In order to arrive at a more accurate basis for the comparison of the thermal quality of coals of the Illinois type, the idea of recalculation of analyses has been carried still further, and the "ash and moisture-free" or "unit-coal" basis has been developed by Parr.* To this end the effect of all of the impurities—the ash and sulphur, as well as the moisture—has been taken into account, and Parr has shown that the unit B. t. u., determined by such a calculation, is remarkably constant for the coal from a given bed over a considerable area.

Since the "unit-coal" basis of comparing analyses has been found so useful, and since the impurities in the coal affect its specific gravity as well as its other properties, the analogy might be carried still further and a "moisture-free" and "ash-free" or a "unit-coal" basis might be used for specific gravities. The specific gravity on such a basis would be called the "unit-coal" or simply "unit" specific gravity. Such a value might prove of considerable practical utility in comparing coals.

Since there is no term referring to proximate analyses of coal which corresponds to the term "apparent" specific gravity, and since it fills a very definite place, the term will be retained to designate the specific gravity of coal without any correction being made for variation in moisture or ash content.

The term "fresh" specific gravity is proposed to correspond to the "fresh" or "as-received" values of coal analyses. It designates the apparent specific gravity of the coal in its fresh condition; that is, in the condition in which it exists in the ground—saturated with moisture. It is a form of the apparent specific gravity, but it has a very distinct usage and is worthy of a distinct name.

Values of the true and fresh specific gravities of the same samples are compared in Table 2. In this case the true specific gravity is the greater by an amount varying from 0.02 to 0.07. The amount of variation in either case depends upon the moisture content of the coal.

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*Bul. No. 16, Ill. State Geol. Survey, 1909, p. 212.
Bul. No. 29, Ill. State Geol. Survey, 1914, p. 40.

TABLE 2.
COMPARISON OF TRUE AND FRESH SPECIFIC GRAVITY.

Sample No.	Moisture Per Cent	Specific Gravity	
		Fresh	True
125	8.37	1.24	1.27
126	10.24	1.29	1.34
127	9.10	1.29	1.33
128	7.93	1.30	1.32
129	9.04	1.26	1.29
130	7.75	1.28	1.30
131	8.27	1.28	1.32
132	6.79	1.32	1.34
133	7.58	1.26	1.29
134	9.00	1.29	1.32
135	9.13	1.32	1.35
136	8.61	1.26	1.29
137	7.51	1.30	1.33
138	13.58	1.31	1.38
139	15.07	1.27	1.33
140	15.08	1.29	1.36
141	13.64	1.30	1.37
142	14.62	1.28	1.35
144	12.65	1.28	1.31
145	14.57	1.28	1.35
146	13.85	1.28	1.35
147	14.95	1.27	1.34
148	15.02	1.32	1.39
149	14.10	1.29	1.36
150	13.67	1.32	1.39
151	13.40	1.27	1.33
152	13.91	1.30	1.36
153	13.40	1.29	1.35
154	14.72	1.30	1.34
155	14.13	1.31	1.38
156	13.30	1.31	1.37
157	14.30	1.31	1.37
158	13.21	1.29	1.35
159	14.21	1.31	1.38
160	14.46	1.30	1.37
161	13.83	1.30	1.37

IV. LABORATORY STUDY OF SPECIFIC GRAVITY OF COAL.

After trying out a number of the well known methods of determining specific gravity, such as the hydrometer, pyncnometer, and Jolly Balance, it was agreed that the Jolly Balance was the form of apparatus best adapted for carrying on the investigations contemplated in the present research. A discussion of the experiments carried out and a description of the various forms of apparatus used for determining specific gravity will be found in Appendix 1.

1. *Specific Gravity of Air-Dry Coal.*—In studying the specific gravity of air-dry coal it is necessary that all air be removed from the pores of the coal. The chemists of the United States Bureau of Mines accomplish this by boiling the coal about three hours under a partial vacuum obtained by the use of an aspirator.

When determining the specific gravity of air-dry coal by the Jolly Balance method, it was noted that very soon after the lumps were immersed in water, usually within a minute, bubbles of air began to rise through the water and that air continued to be given off for several hours. Two series of experiments were conducted to study (a) the effect of this expulsion of air on the specific gravity, and (b) the length of time required for the removal of all the air.

Experiment (a)—In the first experiment to determine the effect on specific gravity of expulsion of air the apparent specific gravity was determined for seventeen lumps from the same sample of coal, lumps which represented the average of the sample being selected. The lumps were immersed in water at room temperature for three days, and their weights in water were determined at the end of 1, 2, 3, 4, 5, 15, 20, 24, 48 and 72 hours. The specific gravity was determined in each case from the formula:

$$\text{Sp. Gr.} = \frac{Wa}{Wa - W}, \text{ in which}$$

Wa = weight in air,

W = weight in water.

The results are shown graphically by the curve in Fig. 1, which represents the average of the seventeen samples immersed for a period of 24 hours. The specific gravity increased in value as the air was expelled.

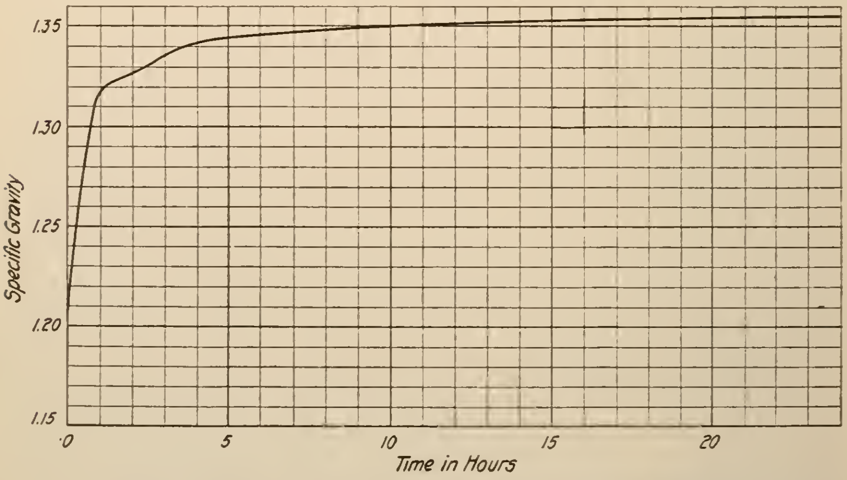

FIG. 1. GRAPH SHOWING EFFECT ON SPECIFIC GRAVITY OF IMMERSING COAL IN WATER TWENTY-FOUR HOURS, AVERAGE OF SEVENTEEN TESTS.

No appreciable change was noted after 15 hours, but by the end of 24 hours the action seemed to have been completed and all of the air to have been removed from the pores since no further increase was noted at the end of 48 and 72 hours.

Experiment (b)—As the greatest change took place during the first two hours, other experiments were conducted to determine the period of change. Three lumps were treated as before, but these were weighed at the end of 15, 30, 45, 60 and 120 minutes. The results are indicated graphically by the curve in Fig. 2, which represents the average values for the three lumps. These results show that the greater part of the change took place within one hour after immersion.

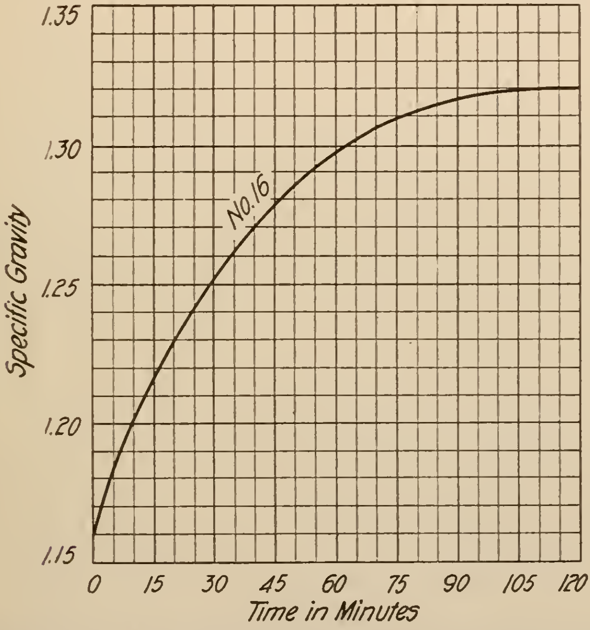

FIG. 2. GRAPH SHOWING CHANGE IN SPECIFIC GRAVITY OF COAL IMMERSED IN WATER AT ROOM TEMPERATURE, AVERAGE OF THREE TESTS.

Other similar experiments were conducted in which two lumps of coal were used. Each lump was weighed in water at five-minute intervals for an hour, then at the end of two hours, and finally after twenty-four hours. The average results for the two lumps for the two-hour period are shown graphically by the curve in Fig. 3. There was no material change after two hours.

These two series of experiments, (a) and (b), indicate:

(1) That air-dry coal contains a considerable amount of air in its pores.

(2) That this air may be removed by immersing the coal in water, and that the specific gravity increases as the amount of included air decreases.

(3) That most of this air is displaced by water within about an hour after immersion.

(4) That this air is practically all removed at the end of twenty-four hours' immersion.

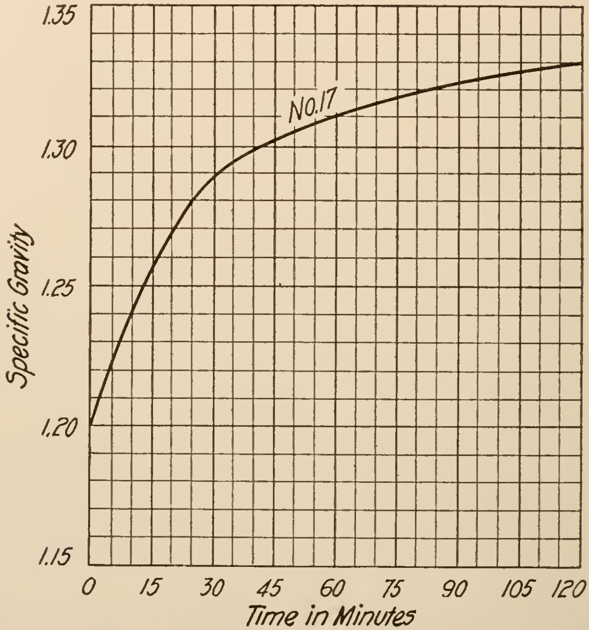

FIG. 3. GRAPH SHOWING CHANGE IN SPECIFIC GRAVITY OF COAL IMMERSED IN WATER AT ROOM TEMPERATURE, AVERAGE OF TWO TESTS.

In order to avoid delay in making specific gravity determinations it is desirable to remove the contained air as rapidly as possible. An increase in the rate at which the air is displaced may be accomplished in several ways:

(1) By increasing the temperature of the water in which the coal is immersed the density of the air in the pores of the coal is decreased, and consequently its tendency to rise is increased so that it will be displaced more rapidly by the water.

(2) By decreasing the pressure on the water and consequently on the air in the pores of the coal by placing it under a partial vacuum, the air expands and its density decreases, hence it is displaced more rapidly by the water.

(3) A combination of decrease of pressure and increase of temperature would have a double effect in increasing the rate at which the air is displaced by water. If this combination is effected by boiling under a vacuum, however, another factor must be considered; namely, the lowering of the boiling point of water with decrease in pressure. If the pressure is reduced to about 20 mm., water will boil at about room temperature, and the only effect accomplished is that due to decrease in pressure. It is probably that simple boiling under atmospheric pressure would have more effect.

Experiment (c)—To test the effect of boiling in hastening the displacement of the air, the specific gravity of each of a number of samples was determined, the weighed lumps of coal being boiled for about an hour and then weighed in water. The values are compared in Table 3 with the values obtained for samples of the same coal by immersing in water for twenty-four hours, each value representing the average of from six to eight determinations upon the same coal.

TABLE 3.

EFFECT OF BOILING UPON REMOVAL OF AIR.

Sample No.	Initial Sp. Gr.	After Boiling 1 Hour	Immersion for 24 Hours	Per Cent Air Removed by Boiling 1 Hour
9	1.26	1.34	1.34	100
15	1.19	1.31	1.32	92
42	1.28	1.32	1.32	100

Experiment (d)—In order to compare the relative effects of boiling and evacuating in hastening the displacement of the air the specific gravities of three samples were determined in duplicate, the weighed lumps of coal being evacuated in water for about an hour and then weighed in water. The evacuation was accomplished by means of an aspirator, and at the end of an hour the pressure was reduced to about 18 mm. so that the water was boiling at room temperature. The results are shown in Table 4, determinations being made in duplicate.

A comparison of the results given in Tables 3 and 4 shows that

TABLE 4.

EFFECT OF EVACUATION AND IMMERSION UPON REMOVAL OF AIR.

Sample No.	Initial Sp. Gr.	After Evacuating 1 Hour	Immersion for 24 Hours	Per Cent Air Removed by Evacuating
9	1.23	1.31	1.34	73
15	1.21	1.30	1.35	64
42	1.28	1.30	1.32	50

boiling the coal for one hour is much more effective than evacuating it for the same length of time.

2. *Specific Gravity of Fresh Coal.—Experiment (e)*—In order to study the specific gravity of fresh coal, samples were obtained from fresh faces in several mines in Vermilion county from Coal No. 7, and in Montgomery county, from Coal No. 6. Each sample contained about four pounds of coal, and was obtained by quartering down a large sample from a cut across the entire bed. The samples were sealed in air-tight cans and shipped to the laboratory in which the specific gravity of selected lumps was determined. Care was taken to select only those lumps which represented as nearly as possible the average of the entire sample. Lumps which contained pyrite or bands of shale were excluded, but both bright and dull coal were used in order to get a fair average.

The specific gravity of the fresh coal was determined with the Jolly Balance; the lumps were then boiled for an hour and the specific gravity again similarly determined. Instead of showing an increase in weight after boiling, as was characteristic of the air-dry coal previously tested, the lumps of fresh coal, with few exceptions, showed no increase (see Table 5). This indicates that the pores of fresh coal contain no air and are filled with moisture. Since this held true for coals which differed geologically and chemically, and since as-received analyses of coal always show a considerably higher content of moisture than air-dry or dry analyses, the evidence seems strong that Illinois coal in the fresh condition is saturated with moisture. It is probable that this moisture is held mechanically in the pores of the coal, since it is given off readily by drying upon exposure to the air, and since it can be taken up again in approximately the same amount if the coal is boiled in water for about one hour or simply immersed in water for a greater length of time, not exceeding twenty-four hours. If further investigation confirms these results a method will have been established for comparing the specific gravities of different coals under standard conditions, in which the samples are boiled in water for an hour before determining the specific gravity. Although experiments (e) and (f) show the specific gravity thus determined to be slightly above the specific gravity of the fresh coal, it is much closer than the apparent specific gravity as usually obtained and near enough for all practical purposes.

TABLE 5.

SPECIFIC GRAVITY OF FRESH COAL.

Danville Field No. 7 Coal			Central Illinois Field No. 6 Coal		
Sample No.	Specific Gravity		Sample No.	Specific Gravity	
	Before Boiling	After Boiling		Before Boiling	After Boiling
125	1.24	1.24	138	1.31	1.31
126	1.29	1.29	139	1.27	1.27
127	1.29	1.29	140	1.29	1.29
128	1.29	1.30*	141	1.30	1.30
129	1.26	1.26	142	1.28	1.28
130	1.27	1.28*	144	1.28	1.28
131	1.28	1.28	145	1.28	1.28
132	1.32	1.32	146	1.28	1.28
133	1.26	1.26	147	1.27	1.27
134	1.29	1.29	148	1.32	1.32
135	1.32	1.32	149	1.28	1.29*
136	1.26	1.26	150	1.32	1.32
137	1.30	1.30	151	1.27	1.27
	—	—	152	1.29	1.30*
Average	1.28	1.28	153	1.29	1.29
			154	1.30	1.30
			155	1.31	1.31
			156	1.31	1.31
			157	1.31	1.31
			158	1.29	1.29
			159	1.31	1.31
			160	1.30	1.30
			161	1.30	1.30
				—	—
			Average	1.29	1.29

*In one or two instances a slight increase in weight was noted, which may have been due to one or two causes; either the sample was taken from a portion of the bed in which the coal was not perfectly fresh, or the lumps had been accidentally permitted to dry a trifle before being used.

3. *Comparison of Specific Gravity of Fresh and Dry Coal.*—In order to study the effect of drying upon the specific gravity, portions of a number of the samples of fresh coal were placed in shallow containers and allowed to dry exposed to the air of the laboratory. The pieces of coal ranged in size from ½ in. to 2 in. in diameter. The air of the laboratory was at a fairly constant temperature of about 24°C.

Experiment (f)—The samples of Vermilion county coal were allowed to dry for sixty days, and at the end of that time lumps were selected and their specific gravities determined. In Table 6 the values are compared with those obtained for the fresh coal. They show that upon drying, the apparent specific gravity of the coal decreased considerably, but that the specific gravity obtained after boiling the air-dried coal showed an increase over the fresh coal. This variation is probably due to loss of moisture during the drying.

TABLE 6.

COMPARISON OF THE SPECIFIC GRAVITY OF FRESH AND AIR-DRY COAL FROM VERMILION COUNTY.

Sample No.	Fresh		Air-Dry for 60 Days	
	Before Boiling*	After Boiling	Before Boiling*	After Boiling
125	1.24	1.24	1.16	1.26
126	1.29	1.29	1.18	1.33
127	1.29	1.29	1.22	1.32
128	1.29	1.30	1.20	1.32
129	1.26	1.26	1.19	1.29
130	1.27	1.28	1.18	1.29
131	1.28	1.28	1.18	1.29
132	1.32	1.32	1.22	1.36
133	1.26	1.26	1.16	1.29
134	1.29	1.29	1.19	1.31
135	1.32	1.32	1.18	1.37
136	1.26	1.26	1.16	1.28
137	1.30	1.30	1.19	1.34
Average	1.28	1.28	1.19	1.31

*These values represent the apparent specific gravity of the coal.

Experiment (g)—The samples of coal from Montgomery county were dried in the same manner as those from Vermilion county, but lumps were selected and their specific gravity determined at the end of four weeks, and again at the end of five weeks. Moisture determinations were made each time in order that the relation between the loss of moisture and the variations in specific gravity might be studied quantitatively. In Table 7 the values obtained are compared with those obtained for fresh coal.

These values show that, as in the case of the Vermilion county coal, the specific gravity of the air-dry coal before boiling (the apparent specific gravity) is less than that of the fresh coal; also, that the specific gravity after boiling is more than that of the fresh coal. In addition they show that the coal lost from 9 to 10 per cent of moisture during the drying.

This loss of moisture explains the high values obtained for the specific gravity after boiling because, although the loss of moisture causes a decrease in the weight in air, the weight in water is approximately the same, whether the coal is fresh or dry. To determine the latter point the following experiment was carried out:

Experiment (*h*)—A number of samples of fresh coal were selected and their specific gravities determined. These samples were put aside and allowed to dry in the air for sixty days, and at the end of that time their specific gravities were again determined, each lump being weighed in water before and after boiling. These weights, together with the corresponding specific gravities, are given in Table 8, which shows that the weight of the lumps in air decreased from 8 to 10 per cent after drying, while the weight of the lumps in water after boiling was approximately the same for the dry as for the fresh coal, the average difference for the thirteen samples being only 0.09. Thus the higher values for the specific gravity after boiling, given in Tables 6, 7, and 8, seem to be the natural consequence of the loss in moisture during the drying.

TABLE 7.

COMPARISON OF THE SPECIFIC GRAVITY AND MOISTURE CONTENT OF FRESH AND AIR-DRY COAL FROM MONTGOMERY COUNTY, ILLINOIS.

Sample No.	Fresh Coal			Air-Dry Coal					
				Four Weeks			Five Weeks		
	Moisture Per Cent	Specific Gravity		Moisture Per Cent	Specific Gravity		Moisture Per Cent	Specific Gravity	
		Before Boiling	After Boiling		Before Boiling	After Boiling		Before Boiling	After Boiling
138	13.58	1.31	1.31	4.04	1.23	1.33	4.91	1.26	1.37
141	13.64	1.30	1.30	3.30	1.25	1.33	5.06	1.23	1.34
144	12.65	1.38	1.28	3.07	1.24	1.33	5.64	1.24	1.31
147	14.95	1.27	1.27	3.26	1.24	1.37	5.55	1.22	1.36
150	13.67	1.32	1.32	3.79	1.22	1.36	5.01	1.21	1.34
154	14.72	1.30	1.30	4.85	1.21	1.32	5.48	1.24	1.33
156	13.30	1.31	1.31	4.51	1.24	1.34	4.94	1.22	1.34
159	14.21	1.31	1.31	4.31	1.21	1.33	4.62	1.26	1.36
Average	13.84	1.30	1.30	3.89	1.23	1.34	4.90	1.24	1.34

The amount of moisture lost when fresh coal is exposed to the air probably depends upon (a) the original moisture content, (b) the degree of fineness of the coal, (c) the humidity of the air in which the coal is exposed, and (d) the length of time the coal is exposed to the air. Table

7 shows that the moisture content was higher after five weeks' exposure than after four weeks. This increase may have been due to the fact that the air was more humid when the latter determinations were made, for the initial moisture and degree of fineness were about the same for all samples and the moisture determinations were made in the same manner, but the moisture in the air was not determined.

It was shown in Tables 6 and 7 that the apparent specific gravity of the coal decreased as the coal dried out. This might be explained by the loss in weight of the lumps due to the loss of moisture. If a 25-gram lump of fresh coal weighs 5 grams in water, it displaces 25—5—20 grams of water, and its specific gravity is equal to

$$\frac{25}{25-5} = \frac{25}{20} = 1.25.$$

If the same lump loses 10 per cent of its moisture when it dries out, its weight in air is decreased by 10 per cent and is then equal to 25 (1—0.10) = 22.5 grams. In determining the apparent specific gravity the lump is weighed before any water can enter its pores, so that it displaces the

Table 8.

The Effect of Drying Upon Weight of Lumps of Coal in Air and in Water.

Sample No.	Fresh Coal			Dry Coal				
					Before Boiling		After Boiling	
	Weight in Air	Weight in Water	Specific Gravity	Weight in Air	Weight in Water	Specific Gravity	Weight in Water	Specific Gravity
125	16.53	3.18	1.24	15.00	1.97	1.15	3.08	1.26
126	20.23	4.57	1.29	18.51	2.95	1.18	4.53	1.33
127	19.12	4.32	1.29	17.76	3.16	1.22	4.35	1.32
128	21.92	5.03	1.30	20.08	3.32	1.20	4.87	1.32
129	15.13	3.09	1.26	13.80	2.10	1.18	2.95	1.28
130	20.27	4.40	1.28	18.55	2.78	1.18	4.15	1.29
131	25.20	5.59	1.28	23.06	3.62	1.18	5.20	1.29
132	29.61	7.04	1.32	28.56	4.85	1.20	7.08	1.33
133	20.51	4.22	1.26	18.72	2.58	1.16	4.25	1.29
134	25.91	5.76	1.29	23.61	3.69	1.19	5.57	1.31
135	14.98	3.60	1.32	13.32	2.08	1.18	3.60	1.37
136	25.92	5.38	1.26	23.64	3.26	1.16	5.24	1.28
137	27.87	6.39	1.30	24.92	4.10	1.19	6.42	1.34
Average	21.78	4.81	1.28	19.96	3.11	1.18	4.72	1.31

same amount of water as before, or 20 grams. Then, its apparent specific gravity is equal to

$$\frac{22.5}{20} = 1.125.\quad \text{Of, if}$$

W = weight of fresh coal in air,
w = weight of fresh coal in water,
S_f = specific gravity of fresh coal,
L = per cent loss of moisture during drying, and
S_a = apparent specific gravity of air-dry coal,

$$\text{then } S_f = \frac{W}{W-w}, \text{ and } S_a = \frac{W-WL}{(W-L)-(w-L)} = \frac{W\,(1-L)}{W-w}$$

$$= S_f\ (1-L)\ \left(\text{since } S_f = \frac{W}{W-w}\right).$$

Therefore, if the loss in moisture alone causes the decrease in the apparent specific gravity, the value of the latter for air-dry coal can be easily calculated, knowing the loss of moisture and the specific gravity of the fresh coal. This is a simple relation and would prove of considerable practical value if it could be shown to accord with experimental facts. It does not seem to hold true, however, for the type of coal used in the present investigation. Values were calculated for a number of samples, according to the above formula, and compared by experiment; these are given in Table 9.

In every case except one (that of Sample 150), the calculated values are too low, or in other words the apparent specific gravity did not decrease with loss of moisture as much as it should if loss of moisture were the only factor affecting this decrease in specific gravity. Evidently, for this type of coal (Illinois bituminous) some other factor, or factors, partially counteracted the effect of drying on the apparent specific gravity.

One factor which might have an appreciable effect in this way is that of oxidation. A number of investigators have shown that if coal is exposed to the air for some time it absorbs oxygen, that oxidation takes place between the absorbed oxygen and some of the light hydrocarbons which are present in the coal, and that the oxidation is attended by an appreciable increase in weight. Sommermeier noted an increase in weight from about 1 per cent to 2½ per cent for Illinois coals after from eight to thirteen months' exposure.* Parr states that one of the principal products of this oxidation is water, and that all coal samples prepared for laboratory work, even though kept in rubber-stoppered bottles, change in moisture content, so that after only a few weeks the moisture in the coal under these conditions will increase, sometimes to an extent of 1 per cent or 1½ per cent.† Hundreds of determinations have been made in his laboratory showing such increase in moisture.

In whatever form the light hydrocarbons existed in the coal, it is safe to conclude that if they were replaced by an oxidation product, such as water, an increase in the specific gravity, as determined by the boiling or soaking process, would result.

TABLE 9.

COMPARISON OF CALCULATED AND EXPERIMENTAL VALUES OF THE APPARENT SPECIFIC GRAVITY.

Sample No.	Fresh Coal		Coal Dried 4 Weeks			Coal Dried 5 Weeks		
	Per Cent Moisture	Specific Gravity	Per Cent Moisture	Apparent Sp. Gr.		Per Cent Moisture	Apparent Sp. Gr.	
				Calculated	Experiment		Calculated	Experiment
138	13.58	1.31	4.04	1.18	1.23	4.91	1.20	1.26
141	13.64	1.30	3.30	1.16	1.25	5.06	1.19	1.23
144	12.65	1.28	3.07	1.16	1.24	5.64	1.18	1.24
147	14.95	1.27	3.26	1.12	1.24	5.55	1.15	1.22
150	13.67	1.32	3.79	1.19	1.22	5.01	1.20	1.15
154	14.72	1.30	4.85	1.16	1.21	5.48	1.17	1.24
156	13.30	1.31	4.51	1.19	1.24	4.94	1.20	1.22
159	14.21	1.31	4.31	1.18	1.21	4.62	1.19	1.26
	Average	1.30	Average	1.17	1.23	Average	1.19	1.23

4. *Special Method of Determining Fresh Specific Gravity.*—It has been shown that, in the case of the coal under consideration, the calculation of the specific gravity of coal in the fresh condition, based upon tests in the dry condition, or vice versa, is not feasible. It is not always possible to obtain fresh coal for specific gravity determinations, since such coal must be taken directly from unaltered portions of the coal bed. It would, however, be of considerable advantage to have some method of finding a value which would represent the specific gravity of coal in the

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*Bul. 323, U. S. G. S., p. 22.

†Oral communication.

fresh condition, when only the air-dry coal or commercially-dry coal is available.

The experiments described previously suggested a simple method of determining the specific gravity of the fresh coal from samples of air-dry or commercial coal, such as can be easily obtained. It has been shown that if lumps of coal are soaked in water for about twenty-four hours they reabsorb about the same amount of water as they lose in drying, and their weight in water is nearly the same as when the coal was fresh. If the weight in water is the same as that of the fresh coal, the weight in air ought to be the same, if care is taken not to weigh any extra water which may cling to the surface of the coal. If both the weight in air and in water are the same for saturated dry coal as for fresh coal, then the specific gravities must be the same.

Experiment (i)—In order to ascertain whether the assumption made in the preceding paragraph is true, eight or ten lumps of each of eight samples of dry coal were soaked in water until they were saturated (about twenty-four hours). At the end of that time they were taken out, the water adhering to the surface of each lump was removed with a cloth or with filter paper, and the lumps were exposed to the air for about five minutes. They were then weighed in air and in water and their specific gravity calculated. The values obtained in this way, as given in Table 10, checked closely with the values of the specific gravity previously determined for the same samples when the coal was fresh. Hence, if only air-dry coal is available, by this method the specific gravity of fresh coal can be determined to a degree of accuracy sufficient for all practical pur-

TABLE 10.

COMPARISON OF FRESH SPECIFIC GRAVITY DETERMINED ON BOTH FRESH AND ON AIR-DRY COAL SATURATED.

Sample No.	Specific Gravity	
	Fresh Coal	Air-Dry Coal Saturated
140	1.29	1.30
141	1.30	1.30
144	1.28	1.29
147	1.27	1.29
150	1.32	1.32
154	1.30	1.31
156	1.31	1.31
161	1.30	1.32

poses, and the extent to which the coal has dried out will probably not affect the accuracy of the results.

To determine whether or not the amount of water evaporated from the coal by leaving it exposed to the air before weighing would change the weight of the lump enough to affect its specific gravity appreciably, the following experiment was carried out:

Experiment (j)—Eight lumps of air-dry coal were soaked in water until they were saturated, and then were weighed in water. Each lump was dried only superficially, and was weighed before there was any chance for the moisture to evaporate to any extent. Then, each lump was allowed to dry for fifteen minutes, which gave ample opportunity for some of the moisture to evaporate, and was again weighed. Another weighing was made after the coal had dried for an hour. As shown by Table 11 there was no change in the specific gravity after drying for fifteen minutes, and in only one instance was there a change after an hour, and that was very small. Since in the general method outlined above the lumps were allowed to dry for only five minutes, it is evident that no error is introduced by water adhering to the surface of the lumps.

TABLE 11.

EFFECT UPON SPECIFIC GRAVITY OF DRYING MOIST LUMPS BEFORE WEIGHING.

Sample No.	Specific Gravity		
	At Initial Weighing	At End of 15 Minutes	At End of 1 Hour
1	1.33	1.33	1.33
2	1.35	1.35	1.36
3	1.30	1.30	1.30
4	1.30	1.30	1.30
5	1.35	1.35	1.35
6	1.29	1.29	1.29
7	1.30	1.30	1.30
8	1.29	1.29	1.29

5. *Specific Gravity of Bright and Dull Coal.*—Illinois coal is usually banded in appearance, consisting of alternate bright and dull layers. The coal material consists of organic and mineral matter, and the banded appearance is due to the alternation of layers that are an admixture of the organic and mineral matter in different proportions and in different ways.

The organic matter of coal may be divided into three substances:[¶] (1) lignitoid or glance coal, (2) canneloid or matt coal, and (3) mother of coal or mineral charcoal. Jeffrey calls the bright layers "lignitoid coal," and considers that it is derived from woody material. He calls the dull layers "canneloid coal," and considers that it is derived largely from plant spores, together with fragments of leaves, stems, etc. Thiessen[§] speaks of the dull material as "debris," but his conclusions are similar to those of Jeffrey. These conclusions are based upon the results of refined microscopic study of thin sections of coal from which all mineral matter had been removed.

TABLE 12.

SPECIFIC GRAVITY OF BRIGHT AND DULL COAL.

Sample No.		Apparent Specific Gravity		Fresh Specific Gravity	
		Bright Coal	Dull Coal	Bright Coal	Dull Coal
401	a	1.26	1.31‡	1.29	1.35‡
	b	1.27	1.31‡	1.29	1.35‡
	c	....	1.56†	....	1.57†
402	a	1.30	1.42‡	1.33	1.45‡
	b	1.26	1.42‡	1.29	1.45‡
	c	1.25	.....	1.29	.....
403	a	1.15	1.26*	1.26	1.37*
	b	1.19	1.22*	1.26	1.35*
	c	1.21	1.25*	1.26	1.35*
	d	....	1.26*	....	1.37*
	e	....	1.19*	....	1.32*
404	a	1.22	1.39*	1.31	1.48*
	b	1.19	1.24*	1.26	1.31*
405	a	1.26	1.65*	1.28	1.67*
	b	1.26	1.57*	1.29	1.59*
	c	1.26	.....	1.28	.....
	d	1.26	.....	1.28	.....

* = Banded.
† = Shaly.
‡ = Homogeneous.

Coal which appears dull may, then, be dull because it is composed of the material referred to as canneloid coal, or as mother of coal. It may, however, appear dull because of the admixture of mineral matter, such as mud or shaly material, present either in intimate mixture or as thin layers or partings between layers of organic material. It is probable that the dull appearance of many Illinois coals is due in part to this admixture of shaly material, as well as to the presence of the canneloid type of coal. The difference in specific gravity between bright and dull coal seems to indicate that this is true.

[¶] E. C. Jeffrey, Economic Geology, Vol. 9, No. 8, p. 741.

[§] R. Thiessen, Bul. No. 38, U. S. Bureau of Mines.

Experiment (k)—Specimens of bright and dull coal were selected at random from the same sample and their specific gravity determined. The bright specimens in each case appeared to be homogeneous with little or no admixture of dull or shaly material. The dull specimens in some cases appeared homogeneous, without distinct bright or shaly material; in other cases they were distinctly shaly; and in still other cases they appeared banded bright and dull, with a predominance of dull bands. In every case the specific gravity of the dull coal was greater than that of the bright coal from the same sample. The results are shown in Table 12.

TABLE 13.

RELATION OF SPECIFIC GRAVITY OF BRIGHT AND DULL COAL TO ASH AND MOISTURE CONTENT.

Sample No.		Fresh Sp. Gr.	Moisture (Air-dry)	Ash	Dry Ash
401	BC	1.29	5.83	2.77	2.95
	DC	1.35	3.82	37.15	38.62
	DS	1.57	2.06	36.65	37.42
402	BC	1.30	6.92	2.94	3.16
	DC	1.45	2.96	42.72	44.02
403	BC	1.26	5.79	2.70	2.86
	DB	1.35	2.99	11.32	11.67
404	BB	1.28	6.58	4.21	4.51
	DB	1.40	3.71	34.36	35.69
405	BC	1.28	5.96	2.65	2.82
	DB	1.63	2.78	27.51	28.29

BC = Bright, very clean coal.
DC = Dull coal, bony, but not distinctly shaly.
BB = Bright and banded in appearance with a few dull streaks.
DB = Dull and banded, with a few bright streaks.
DS = Dull and distinctly shaly.

In order to study the effect of admixture of impurities in dull coal and, if possible, the relation between specific gravity and ash and moisture contents, determinations were made of the ash and moisture contents of the bright and dull lumps, of which the specific gravity was determined. The results of this study are shown in Table 13. The most important facts brought out by these data are:

(1) In every case the specific gravity of the dull coal was decidedly greater than that of the bright.

(2) The ash content of the dull coal was much greater than that of the bright coal.

(3) The moisture content of the bright coal was in every case greater than that of the dull coal, although all lumps had been subjected to about the same conditions.

These facts lead to the following conclusions:

(1) Admixture of impurities, especially of shaly material, is a common feature in the dull portions of Illinois coal.

(2) Bright coal is probably considerably more porous than dull coal, since in the experiments made it showed a uniformly higher moisture content, in spite of the fact that it had been exposed to the air for the same length of time and under the same conditions as the dull coal. This is probably due to the admixture of shale in the dull coal, since shale is known to have a very low porosity.

V. FIELD STUDY OF SPECIFIC GRAVITY.

6. *Fresh Coal from Vermilion County.*—In order to determine the fresh specific gravity of Illinois coal on a more extensive scale, field trips were made to the Vermilion county coal field, near Danville, Illinois, and with a complete improved Jolly Balance equipment determinations of the fresh specific gravity were made at the mines, while the coal collected was perfectly fresh lumps ½ to 1 inch in diameter, being used in making the determinations.

Two coal beds of workable thickness, No. 6 and No. 7, of the Illinois State Geological Survey correlation, are mined in the district, and samples were collected from both. The coal is composed of alternate bright and dull bands of variable thickness, with numerous thin partings of shale, mineral charcoal or mother of coal, and pyrite. Some of these partings are quite persistent and divide the coal bed into benches which are known locally as the "top coal," "blacksmith coal," "bottom coal," etc.

Experiment (l)—In order to obtain a value for the specific gravity that would represent the whole coal bed with reasonable accuracy, it was found advisable to sample each bench, instead of sampling the bed as a whole; that is, lumps were collected from each bench appearing persistent over a considerable area. A value was thus obtained which was representative of the particular bench from which the sample was taken, but not necessarily of the whole bed. The average value for the whole bed was found by multiplying the value for each bench by the thickness of that bench in inches, and dividing by the total thickness of the bed.

TABLE 14.

SPECIFIC GRAVITY OF COAL NO. 7 BY BENCHES.

Sample No.	Bench No. 1	Bench No. 2	Bench No. 3	Bench No. 4
230	1.24	1.33	1.25	1.29
231	1.24	1.29	1.26	1.29
232	1.24	1.28	1.27	1.26
233	1.24	1.32	1.26	1.28
234	1.24	1.30	1.26	1.35
235	1.24	1.27	1.27	1.28
236	1.24	....	1.27	1.31
237	1.24	....	1.27	1.31
238	1.24	1.31	1.28	1.32
239	1.25	1.34	1.26	1.30
240	1.25	1.31	1.25	1.28
241	1.25	1.27	1.25	1.34
242	1.24	1.26	1.26	1.30
243	1.25	1.30	1.25	1.36
244	1.23	1.30	1.26	....
245	1.25	....	1.28	....
246	1.24	....	1.31	1.29
247	1.26	1.26	1.26	1.34
248	1.27	1.25	1.28	1.27
249	1.26	1.32	1.25	1.26
250	1.26	1.25	1.26	1.31
251	1.28	1.31	1.33	1.27
252	1.25	1.28	1.26	1.26
253	1.28	1.30	1.29	1.27
254	1.26	1.31	1.30	....
255	1.27	1.32	1.27	1.30
256	1.29	1.28	1.26	1.29
257	1.25	1.27	1.28	....
258	1.22	1.29	1.27	....
259	1.29	1.29	1.27	....
260	1.26	1.27	1.26	....
261	1.25	1.26	1.28	....
262	1.27	1.29	1.27	....
263	1.27	1.27	1.27	....
264	1.27	1.31	1.27	....
265	1.30	1.26	....	....
Average	1.26	1.29	1.27	1.30

Bench No. 1 = Top Coal.
Bench No. 2 = Middle Bench.
Bench No. 3 = "Blacksmith Coal" (local name).
Bench No. 4 = Bottom Coal.

A large number of lumps were selected from different points along each bench and at a number of localities in the district, and the specific gravity of each of these lumps was determined (see Table 14). The coal from different benches showed an appreciable difference in specific gravity, although the value from each bench was fairly constant.

Samples from different benches were obtained from thirteen different mines in the district, and the specific gravities determined immediately by means of the portable Jolly Balance. The average value of each bench and for the entire bed was calculated for each sample by the method described, and the results for the bed are given in Table 15.

TABLE 15.

FRESH SPECIFIC GRAVITY OF DANVILLE COAL.

Coal No. 7		Coal No. 6	
Sample No.	Specific Gravity	Sample No.	Specific Gravity
201	1.27	207	1.29
202	1.29	208	1.28
203	1.27	209	1.29
204	1.29	210	1.27
205	1.29	211	1.27
206	1.27	212	1.28
213	1.27		
214	1.28	Average	1.28
215	1.29		
216	1.28		
217	1.28		
218	1.28		
219	2.26		
220	1.27		
221	1.27		
222	1.27		
223	1.28		
225	1.28		
226	1.29		
227	1.29		
228	1.28		
229	1.29		
230	1.29		
231	1.28		
232	1.28		
233	1.27		
Average	1.27		

7. *Coal From Other Districts.*—It was not found possible to collect samples of fresh coal from various parts of the state; therefore, in order to cover the ground thoroughly, samples of coal in varying stages of dryness were obtained from practically every district in the state.

The apparent specific gravity of each of these samples was determined by the Jolly Balance method. The values obtained for the different samples vary considerably, probably due to the difference in the length of time which elapsed between the mining of the coal and determination of its specific gravity. Also, the fresh specific gravity of each of these samples was determined by the approximate method described on pages 20 and 21, and the values obtained are given in Table 16.

TABLE 16.
SPECIFIC GRAVITY OF VARIOUS ILLINOIS COALS.

Sample No.	County	Coal Bed No.	Specific Gravity		County Average Fresh Spec. Grav.*
			Apparent	Fresh*	
301	Christian	6	1.24	1.30	
302	“	6	1.21	1.29	
303	“	6	1.24	1.34	
304	“	6	1.20	1.29	1.31—Christian
305	Franklin	6	1.25	1.30	
306	“	6	1.22	1.30	
307	“	6	1.26	1.30	
308	“	6	1.28	1.31	1.30—Franklin
309	Grundy	2	1.23	1.34	
310	“	2	1.24	1.34	
311	“	2	1.19	1.35	
312	“	2	1.18	1.29	1.33—Grundy
313	Jackson	2	1.26	1.31	
314	“	2	1.28	1.32	
315	“	2	1.27	1.34	
316	“	2	1.25	1.29	1.32—Jackson
317	La Salle	2	1.14	1.26	
318	“	2	1.21	1.32	
319	“	2	1.12	1.26	
320	“	2	1.17	1.28	
321	“	2	1.12	1.29	
322	“	2	1.13	1.28	
323	“	2	1.17	1.30	
324	“	2	1.16	1.29	1.28—La Salle
325	Logan	5	1.21	1.33	
326	“	5	1.18	1.30	
327	“	5	1.20	1.32	
328	“	5	1.17	1.28	1.31—Logan
329	Madison	6	1.22	1.28	
330	“	6	1.25	1.32	
331	“	6	1.22	1.29	1.30—Madison
332	Marshall	2	1.23	1.29	
333	“	2	1.18	1.27	
334	“	2	1.16	1.29	
335	“	2	1.17	1.28	1.28—Marshall
336	Mercer	1	1.23	1.29	
337	“	1	1.21	1.30	
338	“	1	1.22	1.31	
339	“	1	1.22	1.30	1.30—Mercer
340	Montgomery	6	1.25	1.31	
341	“	6	1.28	1.33	
342	“	6	1.29	1.29	
343	“	6	1.25	1.30	
344	“	6	1.24	1.29	
345	“	6	1.15	1.29	
346	“	6	1.19	1.30	
347	“	6	1.17	1.28	
348	“	6	1.15	1.30	
349	“	6	1.20	1.29	
350	“	6	1.19	1.31	
351	“	6	1.20	1.28	
352	“	6	1.22	1.29	
353	“	6	1.24	1.30	1.30—Montgomery
354	Perry	6	1.29	1.34	
355	“	6	1.26	1.31	
356	“	6	1.27	1.32	
357	“	6	1.24	1.35	1.33—Perry
358	Sangamon	5	1.08	1.27	
359	“	5	1.20	1.30	
360	“	5	1.14	1.28	
361	“	5	1.18	1.26	
362	“	5	1.17	1.30	
363	“	5	1.21	1.32	
364	“	5	1.20	1.30	
365	“	5	1.20	1.29	
366	“	5	1.22	1.31	
367	“	5	1.23	1.34	
368	“	5	1.21	1.30	1.30—Sangamon
369	Williamson	6	1.25	1.29	
370	“	6	1.29	1.33	
371	“	6	1.27	1.30	
372	“	6	1.29	1.32	1.31—Williamson

*Fresh specific gravity as determined on dry coal by the approximate method described on page 20.

VI. PRACTICAL USES OF THE SPECIFIC GRAVITY OF COAL.

There are several practical uses to which a knowledge of the specific gravity of coal may be applied, chief among which are the following:

(1) The calculation of the tonnage of coal in the ground over a given area.

(2) The calculation of the tonnage of quantities of broken coal, such as the coal stored in large bins or in stock piles.

(3) The comparison of coals by the rapid estimation of the ash and moisture content.

(4) The determination of the adaptability of coal to the removal of its impurities by washing processes.

8. *Tonnage of Coal in the Ground.*—It is often desirable to estimate the number of tons of coal in a given area underlaid by coal beds of a known or to be determined thickness. The volume of solid coal can be easily calculated, knowing the thickness of the beds and the area they cover; and the tonnage may then be found by multiplying the volume in cubic feet by the weight per cubic foot, and dividing by 2,000.

$$\text{Tonnage} = \frac{V \times W}{2{,}000}, \text{ in which}$$

$V =$ volume in cubic feet, and

$W =$ weight per cubic foot.

Since the weight per cubic foot of coal is equal to the specific gravity of the coal multiplied by the weight of a cubic foot of water (62.5 pounds), the formula becomes

$$\text{Tonnage} = \frac{62.5\, V S}{2{,}000} \quad (S = \text{specific gravity}).$$

It is necessary to know the specific gravity of the coal with reasonable accuracy if the estimate of tonnage is to have any value. An error of only a small amount in the specific gravity might cause the amount of coal recovered from a given area to fall millions of tons short of the estimated recovery. This fact was recognized by the Commission appointed to investigate the waste in coal mining in Pennsylvania in 1890. This Commission in its report stated that " . . . a variation of 1 per cent in the specific gravity would reduce the total number of tons of coal in the ground 195,000,000."*

The importance of selecting the proper value for the specific gravity is evident when it is remembered that the specific gravity of any coal may be represented by a number of widely differing values, each perfectly correct for the condition which it represents. For example, the apparent specific gravity of a fresh coal may be 1.33, while after the coal has dried out it may be only 1.20. This difference of 0.13 in the specific gravity makes a difference of about 1,020 tons per acre for a bed of coal six feet thick, or over 650,000 tons for an area of only one square mile. This difference is far too great to be neglected, and it is therefore necessary to select the proper value for the specific gravity.

In selecting this value it should be remembered that the fresh specific gravity represents the condition in which the coal exists in the ground; the apparent specific gravity represents any condition of the coal at any stage in the drying process; while the dry and unit values represent only theoretical conditions that are never attained practically, and therefore, can be neglected in this connection.

The most practical value for use in the calculation of tonnage in the ground is one that represents the condition in which the coal exists when it is marketed, since the calculated weight will correspond to the weight upon which the price received for the coal is based. In the state of Illinois, coal that is marketed is usually settled for on the basis of weights at the mine. The coal is weighed as soon as it is placed in the railroad car ready for shipment. Only a short time elapses between the mining of the coal and weighing it, and therefore, the coal when weighed is practically fresh. It is not exposed to the air long enough after being mined to lose sufficient moisture to affect its weight appreciably. Since the coal is sold on a practically fresh basis, all calculations of tonnage in the ground should be based upon the fresh specific gravity of the coal.

9. *Tonnage of Broken Coal.*—In calculating the tonnage of a known volume of broken coal it is necessary to know not only the specific gravity of the coal, but also the amount of void spaces, or the amount of increased volume occupied by the coal when it is broken. For bituminous coal, such as that found in Illinois, this increase in volume probably varies from 60 per cent for large sizes to about 80 per cent for small sizes. Assuming

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*Report of Penn. Coal Waste Commission, p. 11, 1893.

that the coal in question is in small sizes, the tonnage can be calculated by the formula:

$$\text{Tonnage} = \frac{62.5\ V S}{1.80}, \text{ in which}$$

V = volume in cubic feet, and
S = specific gravity.

Again, it is necessary to know which value of the specific gravity to use. In such a case, the manner in which the coal has been stored will affect its moisture content and consequently its specific gravity. Three cases should be considered; (a) when the coal has been stored under water, (b) when it has been stored in a tight bin, and (c) when it has been stored in an exposed stock pile.

(a) *Under Water Storage.*—Coal that is stored under water undergoes practically no deterioration, and its condition is practically the same after a long period of time as when freshly mined. Its specific gravity should not change, and consequently in calculating the tonnage of a given volume of coal stored under water the fresh value of the specific gravity should be used.

(b) *Closed Bin Storage.*—Coal that is stored in a closed bin should lose its moisture very slowly, but after a considerable length of time, for instance, several months or years, a large proportion of its contained moisture would probably be lost, especially if the coal were handled much during the process of transferring it to the bin. Under such conditions it would probably be necessary to obtain samples of the coal and actually determine its apparent specific gravity in order to obtain a reliable value upon which to base any calculations of tonnage.

(c) *Storage in Exposed Piles.*—Coal that is stored in exposed piles will lose moisture in dry weather and gain moisture in wet weather, and its specific gravity will vary accordingly. It is probable also that the specific gravity will not be the same on the surface of the pile as in the center or at the base, because evaporation of the moisture takes place more rapidly at the exterior. Under such conditions it would be almost impossible to estimate with any degree of precision the average specific gravity of the coal from any known values. Samples should be selected from different depths in the pile and the apparent specific gravity determined. On such an average value the estimate of tonnage should be based.

10. *Comparison of Coals.*—If the specific gravity of two coals of the same type and of the same moisture content, such as two bituminous coals from different parts of the same bed or even from different beds are compared, it is probable that the one having the higher specific gravity has also the higher ash content. They can be more conveniently compared if the moisture is entirely eliminated by calculating the dry specific gravity. By comparing the dry specific gravities of different coals of the same type a great deal might be learned about the other properties of the coal, especially the ash content. Since the heat value varies inversely as the ash content for a given coal, some idea might also be obtained as to the relative heat values of the two coals. Such a comparison could not be made between two types of coal such as a bituminous and an anthracite, or a bituminous and a lignite, but for the same types of coal valuable comparisons could be made. By eliminating all the impurities and using unit value of the specific gravity it might even be possible to draw valuable conclusions as to the relative heat values of different coals. Such a possibility is only suggested, but it may be worth further consideration.

11. *Specific Gravity and Coal Washing.*—Before an attempt is made to remove the impurities from coal by washing, the properties of both the coal and its impurities should be investigated to determine whether the coal is adaptable to washing, and which sizes give the best results. The common method of testing is the "float and sink" test, by means of which the impurities are removed from the crushed coal by immersing it in a heavy solution, such as a solution of calcium chloride or of zinc chloride, in which the impurities sink and the clean coal floats and can be skimmed away. The object of washing the coal is twofold; first, to reduce the ash below an arbitrary maximum, which depends upon the purposes for which the product is to be used, and secondly, to reduce the sulphur content to a minimum, especially if the product is to be utilized in manufacturing metallurgical coke. By testing with solutions of different specific gravity, one can be selected which will separate float coal with a little less than the maximum allowable ash. If the sulphur in the float coal has at the same time been reduced to a minimum, the coal is suitable for the manufacture of metallurgical coke, otherwise it is not. However clean the float coal may be, the sink material must contain only a small amount of coal if the separation is to be a commercial success. If the sink contains much coal the loss in the tailings will reduce the profits realized by the increased price of the clean coal until the washing is no longer economical. The lighter the coal and the heavier

the impurities, the cleaner will be the separation. If float and sink tests are carefully made by a standard method of procedure they give valuable comparative data.

Specific gravity tests may be used to advantage in determining the adaptability of coal to washing. They may be made either on average samples from the float coal and the impurities derived from float and sink tests, or they may be made directly upon the coal and upon the shale, which constitutes its chief impurity. All washing methods for the separation of coal from its impurities depend upon differences in specific gravity. The trough, washers, and the grading boxes depend upon the free settling ratio,* and the tub washers, jigs, and bumping tables upon the hindered settling ratio between the coal and its impurities.

Whether the washing takes place under free or hindered settling conditions, the greater the settling ratio the greater will be the range of sizes allowable in the coal and the cleaner will be the separation for a given size.

It has been shown that the apparent specific gravity of the coal itself decreases as the coal dries out. Values as low as 1.12 were obtained for very dry coal by the author, and values of 1.18 or 1.20 are quite common; whereas the apparent specific gravity of the same coal when fresh is about 1.30 or even more. As the specific gravity of the coal decreases its hindered settling ratio increases and the coal becomes more suited to washing, provided the specific gravity of the impurities remains unchanged. Pyrite and shale, the chief impurities, with the exception of fire-clay, are practically nonporous, and their saturation or nonsaturation with water should not affect their specific gravity. Fire-clay breaks up upon coming into contact with water, and in washing operations it is carried away with the water and does not enter into the process, except perhaps to slightly increase the specific gravity of the water. One example will suffice to illustrate the effect upon the specific gravity of drying the coal. Assuming hindered conditions with a quicksand having a specific gravity of 1.10, with impurities having a specific gravity of 2.40 and with fresh coal having a specific gravity of 1.30, the settling ratio becomes:

$$\frac{D}{D'} = \frac{2.40 - 1.10}{1.30 - 1.10} = \frac{1.3}{0.2} = 6.5.$$

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*For a discussion of free and hindered settling ratios. see Bul. No. 69, University of Illinois Engineering Experiment Station, "Coal Washing in Illinois," by F. C. Lincoln.

If the coal is air-dry, with a specific gravity of 1.20,

$$\frac{D}{D'} = \frac{2.40 - 1.10}{1.20 - 1.10} = \frac{1.3}{0.1} = 13.$$

The hindered settling ratio $\frac{D}{D'}$ for the air-dry coal is just twice that for the same coal when fresh, and theoretically the coal when air-dry can be washed with a range of sizes twice as great as when fresh; or with a much cleaner separation for the same range of sizes. Consequently from this viewpoint it would seem advisable:

(1) Not to wash fresh coal, in other words, not to wash coal at the mine; the time required to ship coal to a distant point permits a certain amount of drying to take place. If the question should come up as to whether one large washery should be established near the market and all coal shipped from a number of mines to it, or a number of washeries should be built at the different mines and the product shipped to the market, a consideration of the relation between moisture content and the cleanness of separation would favor the central washery plan.

(2) Not to feed wet coal to the jigs or to other washing machines. The coal should be as dry as possible when it reaches the medium in which the separation of clean coal and impurities takes place. When washing fine sizes of coal it may be necessary to feed it wet in order to keep the coal from matting and floating over the jigs without being wetted. In such cases the wetting should be delayed as long as possible so that the coal will not remain wet long before being fed to the jigs.

These conclusions are based upon theoretical considerations of the results of study of the specific gravity of coal, and have little practical bearing.

VII. SUMMARY.

(1) In general, no fixed value, such as can be given for mineral substances of definite chemical composition, can be given for the specific gravity of coal. Different types of coal, such as anthracite and bituminous, have different specific gravities, and even coals of the same general type may have quite different specific gravities, due to differences in content of ash and moisture.

(2) Fresh coal of the Illinois type is probably saturated with moisture, most of which is held mechanically in the pores of the coal.

(3) The specific gravity of fresh coals of the same general types should be about constant if the content of ash is about the same. The specific gravity of coal in the fresh condition has been called the "fresh" specific gravity.

(4) If coal has dried to any extent its specific gravity will vary, depending upon the amount of moisture lost. As the moisture content decreases the specific gravity should decrease (the ash content remaining the same), but not necessarily in a direct ratio. The specific gravity uncorrected for moisture or ash content has been called the "apparent" specific gravity.

(5) By calculating values of specific gravity to exclude the effect of moisture, values may be obtained which have been called the "real" or "true" specific gravity, and which might well be called the "dry" specific gravity to correspond with analyses of coal which have been calculated to the "dry" or "moisture-free" basis. By excluding both ash and moisture, values might be obtained which could be referred to as the "unit" specific gravity, to correspond with the unit coal B. t. u of Parr.

(6) By saturating air-dry coal with moisture before determining its specific gravity, a value can be obtained which represents quite closely the "fresh" value of the specific gravity, or the specific gravity of the coal in its fresh condition.

(7) The fresh specific gravity of coal from different counties in Illinois was found to vary from 1.28 to 1.33, with an average of between 1.30 and 1.31. These values, excepting those for Vermilion and Montgomery counties, were obtained from air-dry coal by the approximate method described on page 20 of this bulletin.

(8) In computing tonnages it is suggested that (a) for coal in the ground, the fresh specific gravity is the proper value to use, (b) for broken coal stored under water, the fresh specific gravity should be used, and (c) for broken coal stored in bins or in open piles the apparent specific gravity should be found for the average condition of the coal at the time the computations are made, and that value used.

(9) In coal washing operations the coal theoretically should be as dry as feasible when fed to the washing machines, since the settling ratios are considerably greater for dry than for wet coal, and dry coal can consequently be washed more efficiently.

VIII. APPENDIX A.—METHODS OF DETERMINING SPECIFIC GRAVITY OF COAL.

12. *General Methods for Solids.*—A number of methods have been devised for determining the specific gravity of solids, involving various methods of procedure and requiring various kinds of apparatus. Most of these methods are too well known to need description. They are as follows:

(a) Hydrostatic Balance Method.
(b) Jolly Balance Method.
(c) Pycnometer Method.
(d) Heavy Solution Method.

A few modifications of the Hydrostatic and Jolly Balances have been devised which permit especially rapid manipulation, but since these are not particularly applicable to the determination of the specific gravity of coal, a mere reference to them will suffice.*

13. *Special Methods for Coal and Coke Determinations.*—A number of modifications of the general methods have been used for the determination of the specific gravity of coal.

(a) *Ordinary Pycnometer Method of the U. S. Bureau of Mines.*†—"To determine the true specific gravity of coal and coke substance, the procedure is as follows: Approximately 3.5 grams of the 60-mesh coal or coke is weighed and introduced into a 50-c.c. pycnometer with about 30 c.c. of distilled water. In order to avoid loss of particles of the sample during boiling, a one-bulb 6-inch drying tube, *a* (Fig. 4), is connected with the pycnometer by means of a small piece of pure gum tubing or a rubber cork. *c.* The other end of the drying tube is connected with the aspirator. Suction is applied and the contents of the flask are gently boiled on the water bath *d* under partial vacuum for three hours in order to expel all air from the sample. The pycnometer is then detached, almost filled with boiled and cooled water, allowed to cool to the temperature of the balance room, stoppered, and weighed. The temperature of the contents of the pycnometer is taken immediately after weighing. Each pycnometer is accurately calibrated and a table is constructed giving its capacity in grams of water at different temperatures.

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*M. von Schwartz, "Two New Types of Balance for Density Determinations," Munich Centr. Min. Geol., 1913, 565-570.

*Franz Toula, "A Quick Working Hydrostatic Balance," Min. Pet. Mitth., 26, 233-237.

*A. H. Sabin, "A Specific Gravity Balance for Solids," Orig. Com. 8th Inter. Cong. Appl. Chem., I, 441-443.

*W. Bahrdt, "Measurement of the Density of Solid Bodies," Z. Physik Chem., Unterricht, 26, 6-7; Chem. Zentr., 1913, I, 1390.

†Technical Paper No. 8, U. S. Bureau of Mines, 1914, "Methods of Analyzing Coal and Coke," by F. M. Stanton and A. C. Fieldner.

"The true specific gravity is determined by use of the following formula:

$$\text{True specific gravity} = \frac{W}{W-(W'-P)}, \text{ in which}$$

W = weight in grams of dry coke = weight in grams of sample minus its moisture content.

W' = weight in grams of pycnometer plus dry coke, plus water to fill.

P = weight in grams of pycnometer plus water to fill."

(b) *Special Method of the U. S. Bureau of Mines.**—"The Hogarth flask recommended by Blair† for the determination of the specific gravity of iron ores is more convenient and accurate for routine determinations of the specific gravity of coal or coke substances than is the ordinary pycnometer described in the preceding method. With the ordinary pycnometer it is difficult to insert the stopper without catching some floating particles between the stopper and neck.

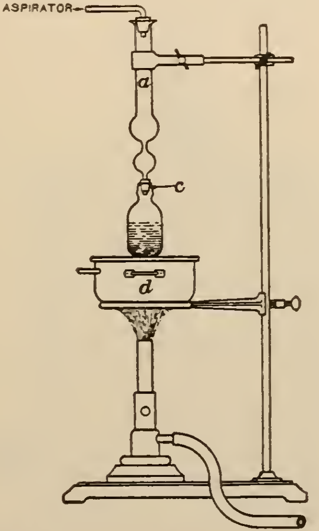

FIG. 4. ARRANGEMENT OF PYCNOMETER, U. S. BUREAU OF MINES.

"With the Hogarth flask is no such difficulty. The method of determination with the Hogarth flask is as follows:

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*Technical Paper 8, U. S. Bureau of Mines, 1914.

†A. A. Blair, "The Chemical Analysis of Iron," 7th ed., 1908, p. 278.

"A 10-gram portion of the 60-mesh coal or coke is weighed and carefully introduced into the weighed flask (Fig. 5) with enough distilled water to fill the flask half full. The capacity of the Hogarth flasks obtained on the market varies from 100 to 125 c.c. The flask is then placed on a small electric hot plate in a 10-inch vacuum desiccator. The desiccator is evacuated by means of an aspirator or air pump. A current sufficient to keep the water boiling is passed through the hot plate. With an efficient vacuum pump all the air is expelled in 30 minutes. The flask is then removed from the desiccator, filled to the tubulure with recently boiled and cooled distilled water, and the stopper inserted. It is advisable to apply a thin film of vaseline to the stopper to prevent leakage.

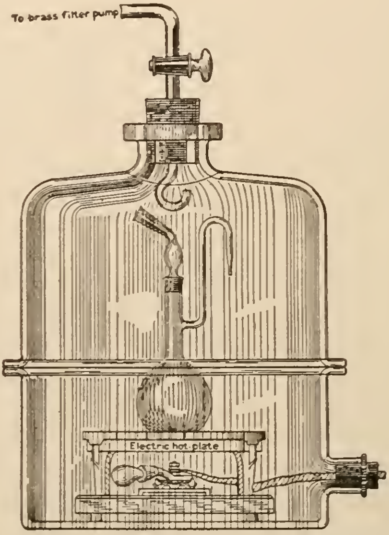

FIG. 5. HOGARTH FLASK METHOD.

"After the flask has been cooled to about 25° C. in a water thermostat, distilled water that has been cooled in the same thermostat is drawn through the tubulure until the water level is slightly above the mark on the capillary of the stopper. This may be done without removing the flask from the thermostat by inserting the end of the tubulure in a small beaker of water and applying a slight suction on the stopper. The flask should remain in the thermostat until the temperature of contents is exactly 25° C. The water level is adjusted to the mark on the capillary by touching a piece of filter paper to the end of the tubulure or by drawing in a little water. The flask is then removed from the thermostat, wiped dry, and weighed. The true specific gravity is calculated as in the preceding method. (See formula, p. 39.) The value for P is obtained by filling the flask with boiled water, cooling, and weighing, as described above.

"By this method no difficulty is experienced in duplicating the figures for specific gravity to two decimal places."

(c) *Blakely and Chance Method.**—Messrs. Blakely and Chance, formerly chemists for the Philadelphia & Reading Coal & Iron Co., have described a method which they used for specific gravity determinations of coal. A representative sample of coal is reduced to 100 grams and placed in a 250 c.c. volumetric flask which has been previously calibrated. Then 100 c.c. of water is run in, suction applied, and the flask shaken. When the air-bubbles cease to rise, the suction is stopped and water run in from a burette up to the 250 c.c. mark.

$$\text{Specific gravity} = \frac{100}{V_0 - V_1}, \text{ in which}$$

V_0 = volume of the flask in cubic centimeters,

V_1 = number of cubic centimeters of water added after the evacuation.

By using a table of reciprocals calculation is facilitated, since the specific gravity equals 100 times the reciprocal of $V_0 - V_1$.

(d) *Brinsmaid's Method.*—Mr. William Brinsmaid developed in the laboratory of Professor S. W. Parr of the University of Illinois a volumetric method for specific gravity determinations. The apparatus consists of a pint fruit jar and an inverted glass funnel of the same diameter. (Fig. 6.) The rim of the funnel and the top of the jar are ground to insure a close joint. The center of the cap of the jar is cut out and the cap is placed over the funnel and screwed down to hold it firmly in place.

The volume of the two-part flask, as it may be considered, is determined by running in water from a burette up to a mark on the stem of the funnel. To make room for the lump of coal of which specific gravity is to be determined, enough water is removed through a pipette inserted in the funnel. The water is placed in a beaker and saved. Then the funnel is taken away and the lump of coal, previously weighed, is placed in the jar. The funnel is replaced, and water run in from the supply previously removed until the flask is again full up to the mark, the air-bubbles adhering to the coal being removed by shaking. Then the remaining water of the supply taken out at first is the water displaced by the coal and has the same volume as the lump of coal. Its weight is determined from the following equation:

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*Mines and Minerals, Vol. 31, p. 499, March, 1911.

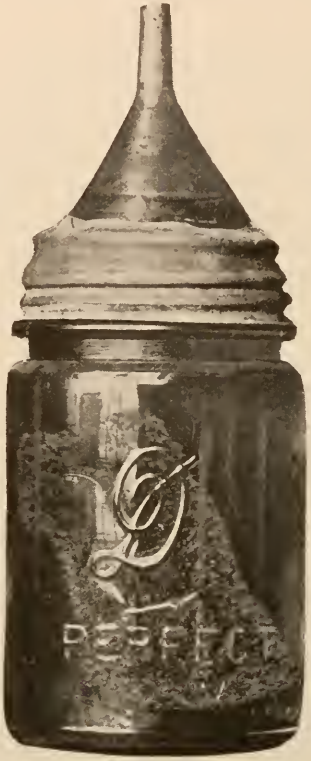

Fig. 6. Parr-Brinsmaid Apparatus.

$$\text{Sp. Gr.} = \frac{Wc}{W}, \text{ in which}$$

Wc = weight of the coal,
W = weight of the water displaced.

(e) *Nicholson Hydrometer Method of the U. S. Bureau of Mines.**— "The apparatus used for determination of the apparent specific gravity (of both coal and coke) consists of a galvanized-iron cylinder (Fig. 7), which is filled with water to the water line, as indicated in the figure. In the cylinder is immersed a hydrometer made of brass. On the top of the hydrometer are two pans. The upper one is used for weights and the lower for the sample. Below the air buoy is a brass cage perforated with many holes to allow the air to escape when the instrument is immersed. The cage carries the sample when it is weighed under water. The method of determining the apparent specific gravity is as follows: Brass weights are placed on the upper pan until the hydrometer sinks to a mark on the stem between the copper pan and the buoy. The total weight required is recorded. The weights are removed, and about 500 grams of the sample in lump form (about 1½ to 2-inch cubes) are placed in the copper dish. Brass weights are then added until the hydrometer sinks to the mark on the stem. The difference in the weights

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*Technical Paper No. 8, U. S. Bureau of Mines, 1914, p. 39.

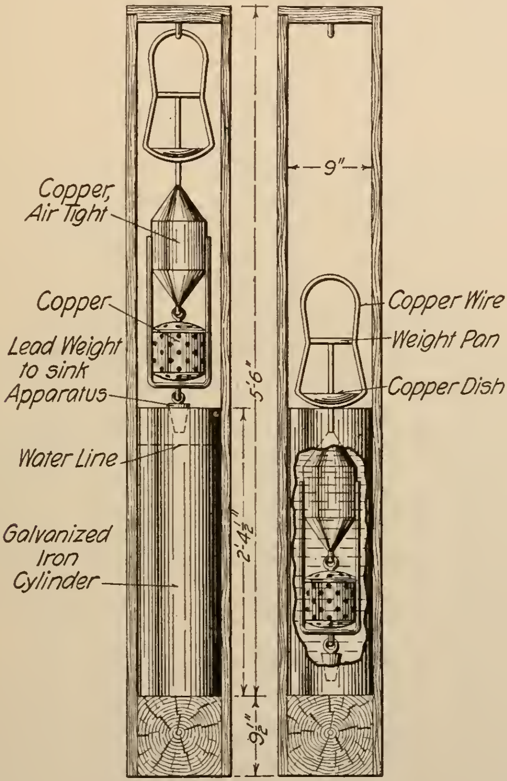

Fig. 7. Nicholson Hydrometer for Coal and Coke.

used gives the weight of the sample in air. The sample is then carefully transferred to the brass cage below the buoy. The weights on the upper pan are now adjusted until the instrument again sinks to the mark on the stem. The weight required to sink the hydrometer to the mark with no sample on the upper pan nor in the brass cage minus the weight required to sink it to the mark with the sample immersed in the cage equals the weight of the coke in water. Then:

If the weight of the sample in air $= x$
and the weight of the sample in water $= y$,

$$\text{the apparent specific gravity} = \frac{x}{x - y}$$

and

$$100 \times \frac{\text{apparent specific gravity}}{\text{True specific gravity}} = \text{percentage of volume of coke substance.}$$

Also,

100 — percentage by volume of coke substance = percentage by volume of cell space.

"In making apparent specific gravity determinations of coke the sample should preferably be in lumps of nearly the same size and shape. When the sample is immersed, the hydrometer should be moved rapidly up and down in the water a number of times in order to remove air bubbles. Since coke samples are porous, they take up water rapidly and should not be allowed to remain in contact with water more than five minutes during a determination. By observing the above-mentioned precautions satisfactory results can be obtained. All samples should be thoroughly dried before specific determinations are made."

(f) *Coxe's Beam Balance Method.*—In 1893 Eckley B. Coxe* described a method for the determination of the specific gravity of broken coal in comparatively large quantities, as used in his laboratory at Drifton, Pennsylvania. The apparatus (Fig. 8) consisted of a Fairbanks market beam scale and ordinary sheet-iron bucket, a cylindrical tin pan about 14 inches in diameter and 7 inches deep, and an ordinary washtub. The weighing beam was supported by a crane attached to a heavy post, and the apparatus was hung on the hook provided for suspending material to be weighed. A yoke was also attached to this hook, and the pan was suspended from it by means of wires and immersed in water in the tub. The coal of which the specific gravity was to be determined

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*Presidential Address, New England Cotton Manufacturers' Association, 1893.

(about 20 lb.), after having been freed from dust and air-dried, was placed in the bucket, and its weight obtained by means of the poise and a suitable rider on the scale beam. It was then poured into the pan under water, and after it had been stirred in order to remove all air, the weight was obtained as before, and the specific gravity calculated in the usual manner.

(g) *Spring Balance Method.*—Mr. M. S. Hachita* has suggested the use of a large spring balance for obtaining the specific gravity of large sizes of coal. It could be used as well for large quantities of small sizes, and exactly in the manner described by Mr. Coxe for his beam balance.

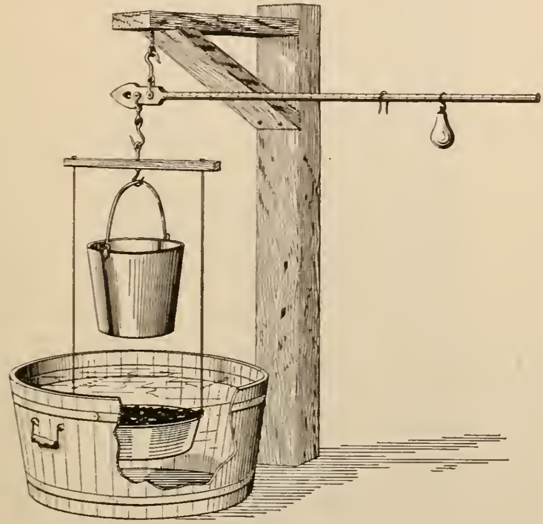

FIG. 8. COXE BEAM BALANCE.

(h) *Jolly Balance Method.*—The Jolly balance (Fig. 9) consists of a long delicate spiral spring suspended from an arm attached to a support *S* which can be moved vertically. This support is in the form of a tube graduated in centimeters and millimeters, and it moves inside of another tube to which a vernier is attached. Two pans are hung from the lower end of the spring, the lower one when in use being immersed in water up to a mark on the wire which holds it. The vertical support is placed so that its zero mark coincides with the zero of its vernier, then a movable mark on the stationary support is made to bisect the marks between the spring and the pans. These adjustments being made, the balance is ready for use.

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*Mines and Minerals, Vol. 31, p. 499.

A lump of coal is placed upon the upper pan, and the spring is stretched by raising the vertical support until a balance is reached. The amount the spring is stretched is read off by means of the vernier from the graduated scale of the vertical support. The lump of coal is then transferred to the lower pan and is immersed in water. The stretching of the spring is again recorded, and the specific gravity calculated by the formula:

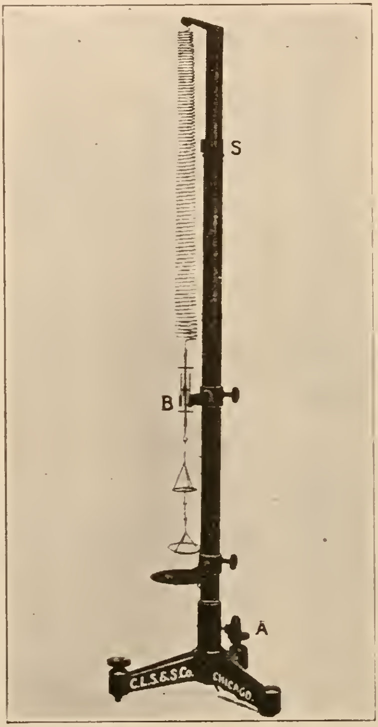

Fig. 9. Jolly Balance.

$$\text{Sp. Gr.} = \frac{Wa}{Wa - A}, \text{ in which}$$

Wa = wt. in air, and

W = wt. in water.

Since the specific gravity is merely a ratio, the stretch of the spring may be used directly for the values Wa and W.

This method of determination may be used directly to obtain the apparent specific gravity. The manipulation of the balance is quite simple, and its operation is very rapid. If the actual weight in grams is desired, the balance may be calibrated with a set of weights. The calibration of the balance used is indicated in Table No. 17.

TABLE 17.
CALIBRATION OF SPRING BALANCE

Grams	Balance Reading	Value of One Space in Grams	Value of One Gram in Spaces
1	3.03	0.330	3.03
2	5.97	0.335	2.99
3	8.90	0.337	2.97
4	11.82	0.338	2.95
5	14.74	0.339	2.95
6	17.67	0.340	2.94
7	20.60	0.340	2.94
8	23.50	0.340	2.94
9	26.40	0.341	2.93
10	29.28	0.342	2.93
11	32.19	0.342	2.93
12	35.10	0.342	2.92
13	37.98	0.342	2.92
14	40.87	0.342	2.92
15	43.75	0.343	2.92

14. *Selection of a Method.*—A method for the determination of the specific gravity of coal should combine as far as practicable accuracy, rapidity of operation, and simplicity of apparatus, and any such method should be adapted to the particular type of work for which it is to be used. If especially accurate results are desired and if time is no object, a method similar to one of those in use at the U. S. Bureau of Mines for true specific gravity determinations might be employed. When rapidity is desired and approximate results are sufficient, as in most commercial work, a more rapid method is better. If the determinations must be made in the field and the apparatus moved about from place to place, a method should be employed which requires only a simple instrument, and one that is easily portable, such as a Nicholson hydrometer or one of the simpler balances. If large lumps or large quantities of material are handled, a method like Coxe's Beam Balance method, or the Spring Balance method would be suitable. If the lumps are of moderate sizes the Nicholson hydrometer, or a method such as that suggested by Prof. Parr would serve the purpose. If only small sizes of coal are available, a pycnometer method should be used for accurate determinations, and a method such as the Jolly Balance method for the less accurate work.

Since the pycnometer methods of the Bureau of Mines are somewhat complicated, the author tried a more simple pycnometer method for purposes of comparison with the Jolly Balance. The specific gravity of a number of samples of dry coal was determined in the following way:

Pycnometers having a volume of 10 c.c. were weighed full of water at room temperature, a portion of the water was removed, and a one-gram sample of 20-mesh coal was placed in each. They were again filled with water, stoppered, and boiled in a beaker of water for about an hour, in order to remove all air from the coal. After they had cooled to room temperature they were filled to the mark with water and weighed. The specific gravity was computed by the formula:

$$\text{Sp. Gr.} = \frac{Wc}{(Wc + W) - W'}, \text{ in which}$$

Wc = weight of coal in air,
W = weight of pycnometer when full of water,
W' = weight of pycnometer when containing coal and filled with water, and
$(Wc + W) - W'$ = weight of water displayed by the coal.

The calculations were simple, as $Wc = 1$, W is a constant, and the specific gravity is the reciprocal of a constant minus W'.

This method was found unsatisfactory. Fine material was easily lost during the process, and it was difficult to remove all the air from the coal.

For purposes of comparison with the pycnometer method just described, the specific gravity of the same samples of coal was determined by the standard Jolly Balance method, the lumps being boiled in water for one hour and then cooled to room temperature before being weighed in water. The values obtained by the two methods are given in Table 18, and an examination of these data shows that the pycnometer method gave lower values than the Jolly Balance method. Evidently, in the latter case all the air was not removed from the pores of the coal before it was weighed in water.

15. *Method Adopted.*—The Jolly Balance method was found to be much more satisfactory, and it was adopted as the method for obtaining the specific gravity determinations made in connection with the preparation of this bulletin. The apparatus is quite simple, it requires lumps of a convenient size and only a small sample of coal, is rapid in operation, only a few seconds being required to make a weighing, and is sufficiently accurate for ordinary work. The chief objection to it is

that the lumps used for the determination are selected arbitrarily, which makes possible the admission of personal errors, since the operator may reject portions of the sample which contain pyrite or shale partings, and select only lumps which represent the average of the sample. In the investigation here described lumps weighing from 5 to 20 grams were used.

16. *Selection of Coal Sample.*—The effect of ash upon the specific gravity has long been recognized. Dull coal may have a higher ash content than bright, glossy coal, and consequently a higher specific gravity. This condition should be taken into consideration, and lumps should be selected so that the average result will represent dull and bright coal in their proper proportions in the sample. Results which show abnormally high values are probably due to the fact that certain lumps contain an abnormal amount of shale or pyrite, and such results should be rejected in the general average, unless a large number of lumps are averaged.

TABLE 18.

COMPARISON OF JOLLY BALANCE AND PYCNOMETER METHODS.

Sample No.	Specific Gravity	
	Jolly Balance	Pycnometer
1	1.28	1.26
2	1.26	1.27
3	1.28	1.27
4	1.33	1.27
5	1.31	1.32
6	1.33	1.26
7	1.33	1.27
8	1.35	1.29
9	1.33	1.29
10	1.32	1.26
11	1.29	1.26
12	1.32	1.27
13	1.29	1.25
14	1.28	1.28
15	1.31	1.25
16	1.31	1.31
17	1.31	1.31
18	1.31	1.28
19	1.32	1.29
20	1.39	1.29
21	1.35	1.28
22	1.36	1.31
23	1.37	1.32
Average	1.32	1.28

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

Circular No. 1. High Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*

Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

Circular No. 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

Circular No. 3. Fuel Tests with Illinois Coal (Compiled from tests made by the Technologic Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907), by L. P. Breckenridge and Paul Diserens. 1909. *None available.*

Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

Bulletin No. 4. Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*

Bulletin No. 5. Resistance of Tubes to Collapse, by Albert P. Carman and M. L. Carr. 1906. *None available.*

Bulletin No. 6. Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *None available.*

Bulletin No. 7. Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr, and Henry B. Dirks. 1906. *None available.*

Bulletin No. 8. Tests of Concrete: I, Shear; II, Bond, by Arthur N. Talbot. 1906. *None available.*

Bulletin No. 9. An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition 1912. *Fifty cents.*

Bulletin No. 10. Tests of Concrete and Reinforced Concrete Columns. Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 11. The Effect of Scale on the Transmission of Heat Through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*

Bulletin No. 12. Tests of Reinforced Concrete T-Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 13. An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *None available.*

Bulletin No. 14. Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 15. How to Burn Illinois Coal Without Smoke, by L. P. Breckenridge. 1908. *Twenty-five cents.*

Bulletin No. 16. A Study of Roof Trusses, by N. Clifford Ricker. 1908. *Fifteen cents.*

Bulletin No. 17. The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1908. *None available.*

Bulletin No. 18. The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1908. *Forty cents.*

Bulletin No. 19. Comparative Tests of Carbon, Metallized Carbon and Tantalum Filament Lamps, by T. H. Amrine. 1908. *None available.*

Bulletin No. 20. Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1908. *None available.*

Bulletin No. 21. Tests of a Liquid Air Plant, by C. S. Hudson and C. M. Garland. 1908. *Fifteen cents.*

Bulletin No. 22. Tests of Cast-Iron and Reinforced Concrete Culvert Pipe, by Arthur N. Talbot. 1908. *None available.*

Bulletin No. 23. Voids, Settlement, and Weight of Crushed Stone, by Ira O. Baker. 1908. *Fifteen cents.*

**Bulletin No. 24.* The Modification of Illinois Coal by Low Temperature Distillation, by S. W. Parr and C. K. Francis. 1908. *Thirty cents.*

Bulletin No. 25. Lighting Country Homes by Private Electric Plants, by T. H. Amrine. 1908. *Twenty cents.*

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SOME GRAPHICAL SOLUTIONS OF ELECTRIC RAILWAY PROBLEMS

BY
A. M. BUCK

CHARTERED 1867

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 90 JULY, 1916

SOME GRAPHICAL SOLUTIONS OF ELECTRIC RAILWAY PROBLEMS

By
A. M. BUCK
ASSISTANT PROFESSOR OF RAILWAY ENGINEERING

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SOME GRAPHICAL SOLUTIONS OF ELECTRIC RAILWAY PROBLEMS

I. INTRODUCTION

In the solution of railway problems involving the characteristics of the motive power it is difficult to use analytical methods, principally because it is impossible to obtain a satisfactory general equation for the curves of an engine or motor of any specified type. The relation between speed and tractive effort, for instance, is so involved that any attempt to obtain a formula leads to assumptions which cannot be made without seriously affecting the accuracy of the final result.* This is true not only of the steam locomotive, but also of the various types of electric motors ordinarily used for train propulsion.

The graphical methods, in contrast with the analytical, form an accurate and at the same time an easy means of attack applicable to any possible combination of characteristics and any range of conditions which may be met in practice. It is the purpose of this bulletin to develop a number of new graphical methods which, in connection with other well-known ones, aid materially in the solution of such problems. While most of these were developed in connection with problems of electric train performance, a number of them are equally applicable to any type of motive power, a fact which is set forth in the paragraphs which follow.

The majority of these methods were developed by the writer in connection with classroom instruction. One of the ways of obtaining motor performance with varying potential and one for finding the "effective" value of the motor current are due to Mr. S. Sekine, a graduate student in Railway Engineering in the University of Illinois, who is also responsible for a portion of the method of plotting speed-time and distance-time curves.

II. MOTOR PERFORMANCE WITH VARYING POTENTIAL†

The performance characteristics of a railway motor are ordinarily furnished by the manufacturer for the normal potential and are usually assumed to be accurate under such conditions. Often it is desirable to find the motor performance when abnormal potential is impressed on the terminals, since in practice the line pressure is subject to wide fluctuations, and the motors are always operating at subnormal potential while the controller is being turned to the full-speed position.

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*See C. O. Mailloux, Discussion on paper by F. W. Carter, Transactions A. I. E. E., Vol. XXII, p. 165 (1903).

†For a brief discussion of this topic see Electric Railway Journal, Sept. 18, 1915.

The torque produced by a given current in a series motor is practically independent of the line pressure,* so that recalculation of this quantity is unnecessary for any ordinary conditions of operation met with in practice, unless, of course, the field strength is purposely reduced. The only other important variable to be considered is the motor speed.

In an electric motor the applied pressure is used up in two ways; a portion overcomes the drop due to the resistance of the windings, and the remainder opposes the counter e.m.f. generated in the armature. If the field flux remains constant, the speed will vary in direct proportion to the counter e.m.f. which is developed. This may be expressed by the equation

$$\frac{V_2}{V_1} = \frac{E_2 - Ir}{E_1 - Ir} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which V_1 and V_2 are the speeds when E_1 volts and E_2 volts are applied at the terminals, respectively, I is the current flowing through the armature, and r is the motor resistance, or that portion in the armature and the circuits in series therewith.

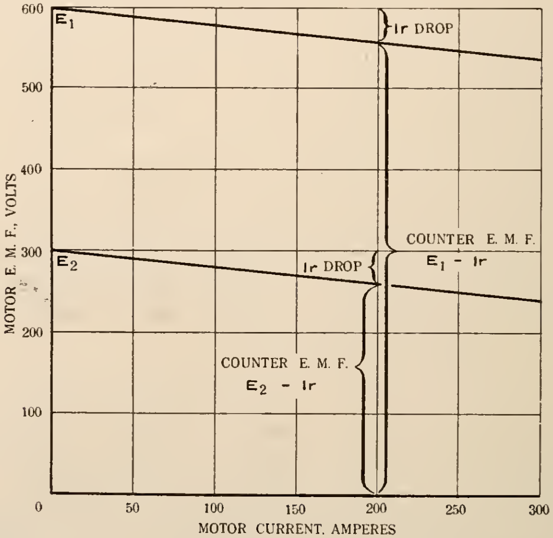

FIG. 1. VOLT-AMPERE DIAGRAM FOR ELECTRIC MOTOR.

In order to make the calculation graphically it is only necessary to determine the relative values of $E_1 - Ir$ and $E_2 - Ir$, from which the ratio of speeds may be found directly. A simple method of showing the relations between these values is to construct a diagram with motor volts as ordinates and armature amperes as abscissae, as shown in Fig. 1. Since the *Ir* drop is a direct function of the armature current, it can be represented for all values of current by the intercepts on a straight line with the proper slope. This may be drawn through the origin, but, since we are principally concerned with the difference between the terminal pressure and the *Ir* drop, it is better to draw it from the line of full pressure at the motor terminals, E_1. If the terminal pressure is then changed to E_2 volts, it will not affect the slope of the *Ir* line, but will change its position so that it begins at the point E_2. In each case the counter e.m.f. is the residue after subtracting the *Ir* drop, as shown in the diagram. All that remains is to obtain a graphical relation between V_1 and V_2, which is proportional to these values of counter e.m.f. Two methods of doing this have been developed.

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*A. M. Buck, The Electric Railway, p. 53.

The first method of calculation is shown in Fig. 2. Here the volt-ampere diagram of Fig. 1 is reproduced, along with the speed-current curve of the motor, as determined by test or from design calculations, the axes of current being in the same straight line. The current scales and their positions along the axis may be chosen as desired, their relation to each other being immaterial. The speed of the motor at the terminal pressure E_1 is represented by the ordinate V_1. It is desired to find the corresponding value of speed V_2 at E_2 volts and the same current *I*. Draw a line through *A* at the value of current *I* on the volt-ampere diagram and also through V_1. This

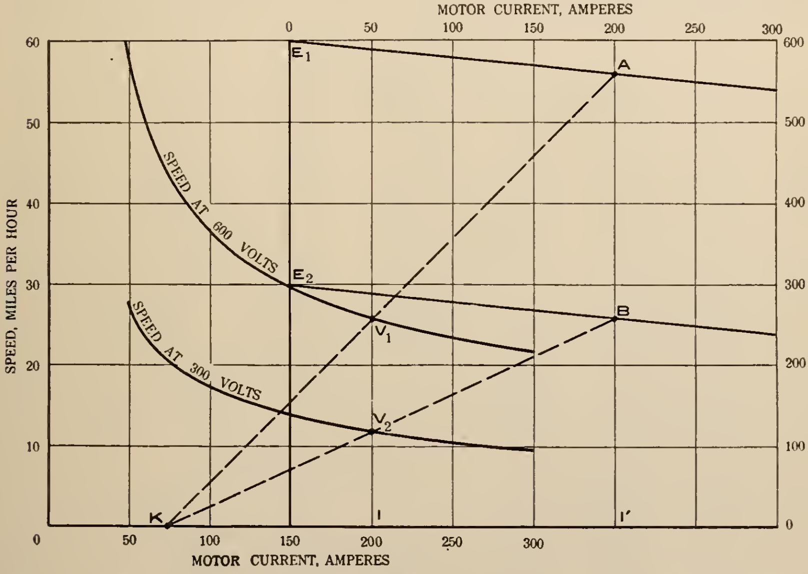

FIG. 2. CONSTRUCTION FOR OBTAINING MOTOR SPEEDS AT DIFFERENT POTENTIALS.

will intersect the axis of abscissae at some point K. From K draw the line KB, through the corresponding point B on the volt-ampere diagram for the same current and the new pressure E_2. This locates V_2, the speed at E_2 volts, at the intersection of KB with the current ordinate. It must be correct since, by similar triangles,

$$\frac{IV_1}{IV_2} = \frac{I'A}{I'B} \cdots\cdots\cdots\cdots\cdots\cdots (2)$$

It may be seen from Fig. 1 that $I'A$ and $I'B$ are the values of counter e.m.f. corresponding to the pressures E_1 and E_2 at the current I.

It should be noted that a different position of the point K will be located for each value of current, and in some cases it may be at too great a distance from the body of the diagram. To obviate this the relative positions of the speed-current and the volt-ampere diagrams may be changed, always keeping their current axes together.

In some cases it is preferable to make the entire construction on the speed current diagram. The arrangement for this method is shown in Fig. 3. Here the base of the volt-ampere diagram is taken the same as that for the speed-current curve, and the proportional division is made by swinging one set of values of counter e.m.f. through an angle of 90 degrees, so that E_1N is equal to OE_2. The two projections of the values of counter e.m.f. will meet at some point, such as P, and a line drawn connecting P with the origin will

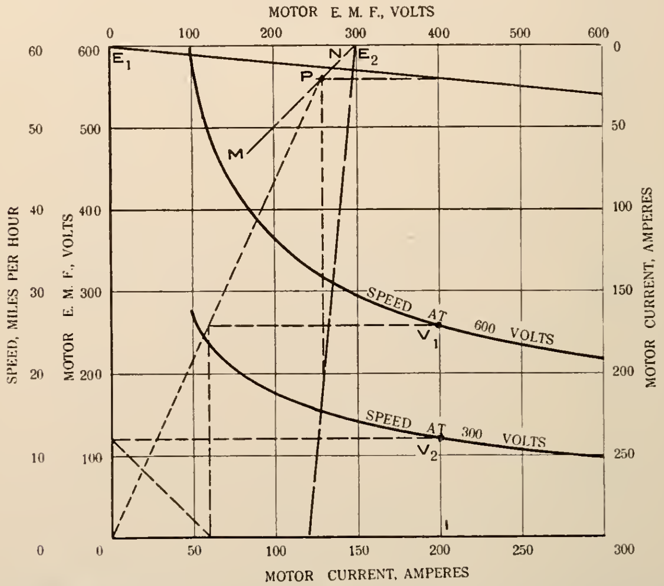

Fig. 3. Second Method for Obtaining Motor Speeds at Different Potentials.

divide the ordinate and abscissa of any point along it proportionally to these two values. Then, by projecting the speed at E_1 volts onto this line, the speed at E_2 volts and the same current are given by the corresponding abscissa, and may be carried back through 90 degrees and plotted on the original current ordinate, as shown.

A further inspection of Fig. 3 shows that the locus of the point P will be a line MN, which passes through N, corresponding to zero Ir drop, and makes an angle of 45 degrees with the axes. The proof of this construction is that the Ir drop is the same for a given current irrespective of the terminal pressure. For this reason it is unnecessary to swing mechanically the counter e.m.f. line through 90 degrees to locate P. Draw MN from the intersection N of the projections of E_1 and E_2 (taken at right angles, as explained above). Any point on the counter e.m.f. line will then give a projection on MN, as at P, thus saving the preliminary construction.

III. Motor Performance with Resistance

To determine the performance of a motor when a resistance is inserted in series with the armature, the constructions given in Figs. 2 and 3 may be used with a slight modification. Fig. 4 is the same as Fig. 2, except that the Ir drop at a different pressure has been replaced by a line E_1B representing the drop $I(R + r)$, in which R is the external resistance in the circuit. The procedure is the

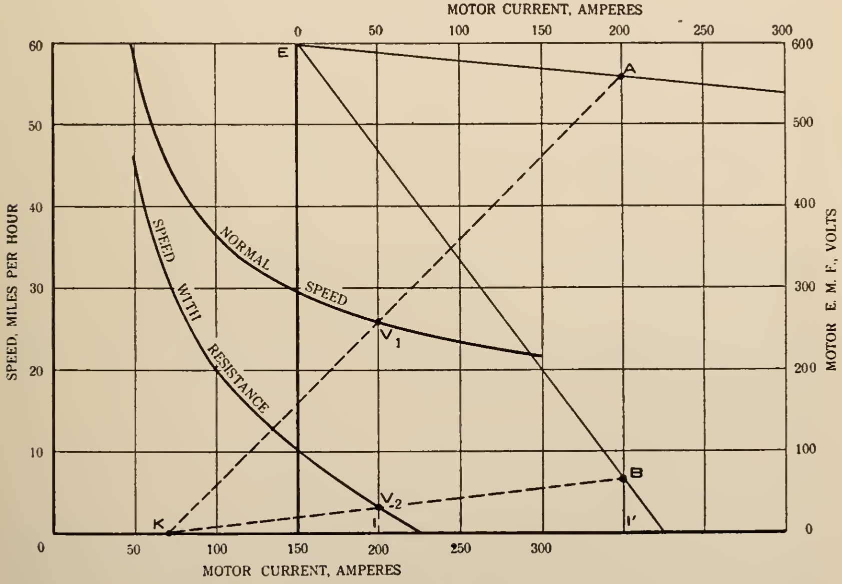

Fig. 4. First Method for Obtaining Motor Speeds with Resistance.

same as that explained in the determination of motor performance with varying potential, and the proof of the construction is identical.

The method of Fig. 3 can equally well be used for determining motor speeds with resistance, as shown in Fig. 5. Since the *IR* drop is not the same, the line *MN* has a different angle, which is determined by the relative values of resistance in the two cases; that is, if the line *MN* of Fig. 5 makes an angle θ with the axis of abscissae,

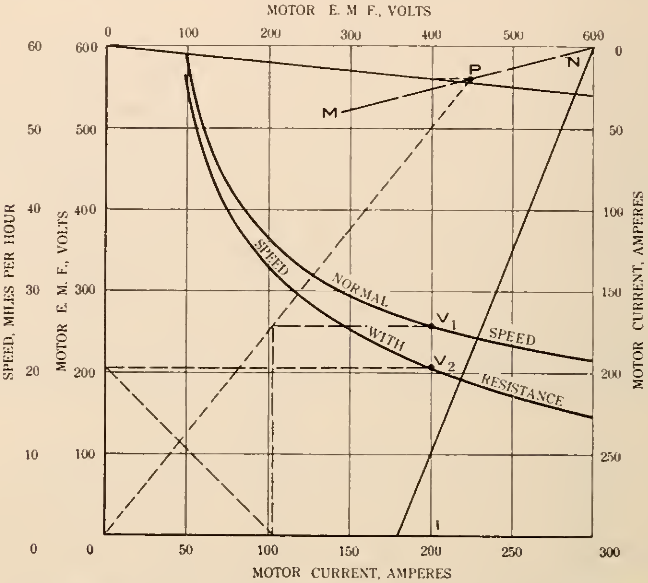

Fig. 5. Second Method for Determining Motor Speeds with Resistance.

$$\tan \theta = \frac{r}{R + r} \tag{3}$$

With this modification the method is precisely the same as that described above.

IV. Starting Resistance for Series Motors with Rheostatic Control

In starting direct-current series motors it is usually not sufficient to reduce the potential at the motor terminals by making different combinations of motors on the supply circuit. When this can be done, as may be possible with very small motors, the performance may be predicted by calculating the performance curves at the lower potentials, as described previously in this bulletin, or by any other ordinary method. In general, however, it is necessary to place a certain external resistance in the circuit, whether or not the potential at the terminals is reduced by any other means. The added resistance should be just sufficient to give the desired values of starting current and torque, the one being dependent on the other. As the motor gains speed, the resistance must be reduced. or the current and the torque will fall off too much. Of course, unless the resistance can be cut out in infinitesimal steps, there will be some variation in these quantities, the range being determined by the allowable difference between the maximum and minimum values of torque and current.

The simplest method of control consists merely in connecting the motor or motors to the line with an external resistance in series, the latter being reduced in steps until finally it is all out of the circuit and the motors are directly across the line. It is essential to determine correctly the exact values of resistance to be placed in circuit on each point of the controller in order that the conditions of current and torque limits may be met. This can be done quickly and accurately by a graphical method based on those given above.

When the motor is stationary the current which will flow is determined entirely by the resistances in the circuit, since the effect of inductance enters only at the instant of connecting to the line, and there is no counter e.m.f. being developed at the time. Since the internal resistance of a well-designed machine is quite small, it is necessary to add a considerable external resistance to keep the initial current down to a proper amount. The exact value of current desired depends on the torque needed and on the capacity of the motor and the connecting wiring. Having determined the required current, it is a simple matter to find the necessary resistance. This may be done directly by the application of Ohm's law. Let I_m be the maximum allowable motor current, E the line e.m.f., r the motor resistance, and R_1 the external resistance to be inserted at starting. Then

$$I_m = \frac{E}{R_1 + r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

from which R_1 may be found at once if the other quantities are known. As soon as current flows through the motor, a torque is developed, and the armature will commence to rotate. This will cause the generation of a counter e.m.f. tending to oppose the e.m.f. of the circuit, so that the current will be reduced. The torque falls off correspondingly, and if the action is allowed to continue the performance will be as shown in Fig. 4 or Fig. 5, the acceleration dropping until the motor operates at some constant speed. Since it is usually desirable to bring the motor up to full speed as soon as practicable, it is customary to reduce the amount of resistance in the circuit so that the accelerating current will remain near the maximum value. The amount of resistance which should be removed from the circuit at one time is a function of the total number of steps in which it is to be cut out or the allowable variation from the mean value of the starting torque. The latter is the simpler case and will be considered first.

Assume that the allowable variation from the mean value of starting torque to give smooth acceleration is 10 per cent. The minimum torque will then be approximately 20 per cent less than the maximum, which latter corresponds to the current at standstill, as determined by equation (4). As previously explained, the current will decrease from the instant of starting until it has fallen to the minimum desired value determined from the proper acceleration. At this point the counter e.m.f. developed by the armature will have risen to some value which can be determined readily, since the sum of the resistance drop, $I(R_1 + r)$, and the counter e.m.f., E_c, must equal the line pressure; that is,

$$E = E_c + I(R_1 + r) \qquad (5)$$

Since the value of resistance has already been found by equation (4), the value of E_c can be obtained.

When the current has fallen to its minimum value I_n the resistance of the circuit should be reduced enough to bring the current up to the maximum value I_m. In order to find this new value of resistance, it is necessary to determine the counter e.m.f. which will exist after the change in connections has been made. If the field flux of the motor remained constant, then, disregarding small variations due to changes in armature reaction and other causes, the counter e.m.f. would be the same for any value of armature current. But in the series motor the field flux is a function of the armature current*, since the latter also flows through the field. The flux will therefore become greater when the current is increased by the removal of some of the resistance. The exact amount of this change depends on the proportions of the magnetic circuits of the motor, and can be determined from the saturation curve of the machine. For practical purposes of calculating starting resistance, this method is not available, since it requires making a special test of the motor. There are, however, methods which may be used for getting approximate proportional values of flux which will serve the purpose equally well.

The speed of an electric motor varies directly with the counter e.m.f. developed and inversely with the field flux. From this it may be seen that the flux is directly proportional to the counter e.m.f. and inversely proportional to the speed; that is,

$$\Phi = \frac{E_c}{kn} \qquad (6)$$

in which Φ is the field flux, n the speed of rotation, and k a constant depending on the winding, etc.

Since

$$E_c = E - Ir \qquad (7)$$

equation (6) may be rewritten

---

*In case the field of a series motor is shunted, the current through it is directly proportional to that through the armature, although not equal to it.

$$\Phi = \frac{E - Ir}{kn} \quad \ldots\ldots\ldots\ldots (8)$$

In the ordinary motor it is not possible to determine with any accuracy the constant k unless access may be had to the design data. But, since in the calculation of starting resistances only *proportional* values of flux are required, the knowledge of this constant is entirely unnecessary. Therefore, the following equation may be used with equal accuracy:

$$k\Phi = \frac{E - Ir}{n} \quad \ldots\ldots\ldots\ldots (9)$$

If the motor resistance is known, the relation of $k\Phi$ to the armature current I may be calculated for any current and a curve plotted if desired.

Another method of getting proportional values of flux depends on the relation of this quantity to the torque developed by the motor. In any electric motor the torque is proportional to the armature current and the field flux; that is,

$$D = K\Phi I \quad \ldots\ldots\ldots\ldots (10)$$

where D is the torque of the motor at a current I, and K is a proportionality constant depending on the winding, but not the same as k in the preceding equation. As before, a curve may be plotted, giving proportional values of flux for any armature current.

When the current is increased from I_n to I_m by reducing the resistance in the circuit, the flux increases from Φ_n to Φ_m. During the infinitesimal time required for changing the current, it is evident that the speed cannot change. It must follow, therefore, that the counter e.m.f. will increase, due to the greater flux. By equation (5), the new value will be the counter e.m.f., E_{cn}, at the minimum current, I_n, multiplied by the ratio of fluxes. The new counter e.m.f., E_{cm}, can then be found as follows:

$$E_{cm} = E_{cn}\left(\frac{k\Phi_m}{k\Phi_n}\right) \quad \ldots\ldots\ldots\ldots (11)$$

or,

$$E_{cm} = E_{cn}\left(\frac{K\Phi_m}{K\Phi_n}\right) \quad \ldots\ldots\ldots\ldots (12)$$

depending on which method was used for getting the proportional values of flux. For brevity, call this ratio of field fluxes Q; that is,

$$Q = \frac{k\Phi_m}{k\Phi_n} = \frac{K\Phi_m}{K\Phi_n} \quad \ldots\ldots\ldots\ldots (13)$$

Then,

$$E_{cm} = QE_{cn} \quad \ldots\ldots\ldots\ldots (14)$$

If the maximum and minimum values of current are to be reached each time the resistance is changed, then the ratio Q becomes constant for the particular conditions assumed, and the calculation of resistances is simplified considerably. On the other hand, it may be advisable to allow different values of current on the various steps of the controller, in which case the ratio of fluxes must be determined separately for each point. When the controller is equipped with a current-limiting device the former condition holds. By the application of the above equations the values of resistance for a rheostatic controller may be calculated.

It is more convenient for the engineer to calculate the resistances by a graphical process, since the use of the equations is somewhat tedious. For this purpose the volt-ampere diagram may be employed conveniently. In Fig. 6 the volt-ampere diagram of Fig. 1 has been repeated, and on it is also plotted the curve of relative values of flux ($k\Phi$ or $K\Phi$) against current. The limits I_m and I_n being chosen, it is evident that the ratio Q will be constant. If, then, a line is drawn through the points Φ_m and Φ_n, cutting the axis of abscissae at X, the latter will be the intercept of all lines cutting the verticals through I_m and I_n at points proportional to these values of flux; that is, in the figure,

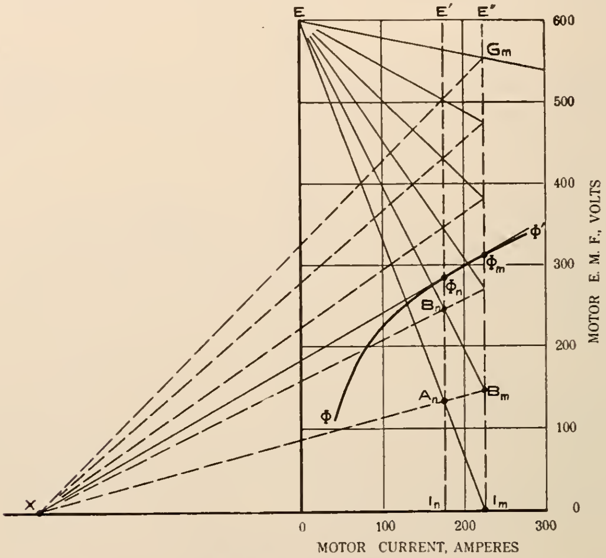

Fig. 6. Diagram for Determining Resistances for Series Motor with Rheostatic Control.

$$\frac{I_m\Phi_m}{I_n\Phi_n} = \frac{I_mB_m}{I_nA_n}\ etc., = Q \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

since all of the triangles whose apexes pass through the point X divide parallel lines into proportional parts.

Starting with the maximum current I_m, the entire external potential E is used up in overcoming resistances. That is, the line I_mE'' represents the IR drop, I_mG_m being that in the external resistor and G_mE'' that in the motor itself. As soon as the armature begins to rotate a counter e.m.f is developed. When the motor current has fallen to I_n this e.m.f. is represented by the ordinate I_nA_n, the line I_mE being drawn through E, for evidently there will be no IR drop with zero current. It is evident that when the current has reached I_n resistance must be cut out in one step until the current rises to the maximum, I_m. Since the counter e.m.f. has a value of I_nA_n when the current is a minimum, it follows that it must increase by the ratio Q when the current is increased to I_m so rapidly that the motor does not have time to change its speed. The new counter e.m.f. may be determined by projecting a line from X through A_n, intersecting the line of maximum current at B_m. The counter e.m.f. at this point is represented by I_mB_m, the drop in the external resistor by B_mG_m, and that in the motor by G_m E''. The external resistance to be employed is found by dividing B_mG_m by the current I_m. The process may now be continued until all the external resistance has been removed and the motor is running on the line. This condition is shown by the line EG_m, and from this point on the normal curves of motor performance apply.

If it is desired to change the current limits at any stage of the controller operation, the proper resistance can be determined in the same manner, the location of the point X being varied to correspond to the proper values of current. For small changes, the location of X may be assumed constant without introducing an appreciable error. If a definite number of steps is called for, as by the adoption of a standard controller, the values of I_m and I_n must be changed until the exact number of steps is obtained on the diagram. This must be done by trial, but the adjustment can be made quickly after a few cases have been solved.

As given above, the diagram has been worked out for a single series motor. If two motors are to be run in parallel, it is only necessary to modify the diagram to give the proper values of current, remembering that the combined resistance of the machines is but one-half that of a single motor. For operation with machines in series the same precautions must be observed, but in this case the motor resistance is twice that of a single machine. With these variations, the diagram can be modified to meet any combinations of rheostatic control of series motors.

V. Series-Parallel Control*

In electric railway practice it is customary to operate series motors in pairs or in groups of motors in pairs. They are ordinarily controlled by the *series-parallel* method, which involves placing the two units in series with resistance which is cut out in steps, changing to parallel with the resistance again inserted, and finally cutting it out again in steps. Generally the current limits are the same for both connections, although sometimes they are different in the series and in the parallel arrangements.

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*See Electric Railway Journal, Dec. 26, 1914, and Feb. 13, 1915.

The calculation of the counter e.m.f. and the resistance for series-parallel control is made in the same manner as for the rheostatic, except that the precautions mentioned under the former topic on p. 15 must be observed very carefully. It is usually convenient to combine the series and the parallel diagrams into one. This is shown in Fig. 7. The method of construction is the same as for

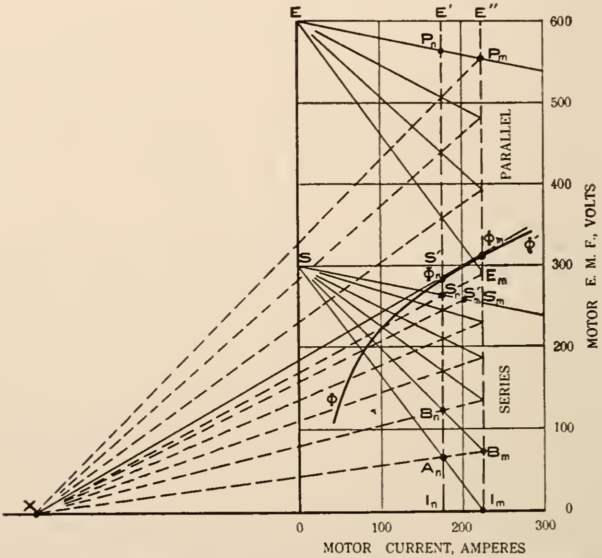

Fig. 7. Diagram for Determining Resistances for Series Motors with Series-Parallel Control.

rheostatic control, the difference being that the point *S*, representing half potential, is taken as the point for drawing the *IR* lines while the motors are in series, and the point *E* for the same purpose after the parallel connection is made. It is necessary to interpret correctly the values of *IR* drop to determine the resistances. When the motors are in series the current flowing through the circuit is that through a single machine, while after they are thrown in parallel the line current is that for two motors. To determine the series resistances, therefore, the external *IR* drop, for instance that on the first point of the controller, is equal to I_mS_m per motor, so that this must be doubled to get the total drop in the external circuit. The correct value of resistance to put in series with the motors on the first point is then

$$R_1 = \frac{2I_m S_m}{I_m} \ldots\ldots\ldots\ldots\ldots (16)$$

and similarly for any other value of series resistance.

When the connections are changed from series to parallel, the counter e.m.f. of each motor is $I_n S_n$ just before breaking the circuit, and $I_m E_m$ after the reconnection is complete. In series, the counter e.m.f.'s of the two motors add, while in parallel they do not. The residue, $E_m P_m$, must therefore be consumed in external resistance. On the first parallel point the resistance must then be

$$R_p = \frac{E_m P_m}{2I_m} \ldots\ldots\ldots\ldots\ldots (17)$$

and so on until the motors are directly on the line. In all other respects the series-parallel diagram is precisely the same as the rheostatic diagram previously described.

VI. Starting Resistance for Shunt Motors

The calculation of starting resistances for shunt motors is made in the same manner as for series machines, the principal difference being that, since the field is supplied by a circuit in parallel with the armature, the field flux is practically constant at a given potential for all values of armature current. It is, therefore, unnecessary to determine any change of flux when the resistance is reduced.

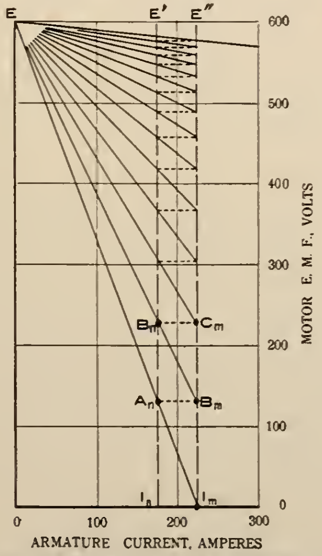

Fig. 8. Diagram for Determining Resistances for Shunt Motor with Rheostatic Control.

The diagram for calculating graphically the values of armature resistance is given in Fig. 8 for a single motor. This diagram is somewhat similar to Fig. 6, except that the lines representing the change from one point to the next are not drawn through a single point *X*, but are all parallel to the base. The method of getting the resistances from measurements on the diagram is the same as previously described. For series-parallel control a similar scheme may be followed. It is not illustrated here on account of the infrequency of the use of series-parallel control with shunt motors.

VII. Plotting Speed-Time Curves

A number of methods have been proposed from time to time to reduce the labor incident to the plotting of speed-time curves for railway trains. The analytical solutions all depend on producing equations representing the characteristic curves of the motive power; and, on account of the difficulty of determining separately the equation of the curve for each separate motor or locomotive, general solutions giving the average of a large number of machines have been used. Although this is satisfactory for approximate calculations in which extreme accuracy is not required, as in preliminary estimates, it is not suitable for problems involving a particular machine. For such cases graphical or semi-graphical methods are usually resorted to if a solution more rapid and less laborious than that obtained by the point-by-point construction is desired.

Of the graphical methods, the first one which was satisfactory was that developed by Mr. C. O. Mailloux.* The construction there described is of a high degree of accuracy, and is so simple that it may be readily applied. It has the disadvantage of requiring a number of charts on which the graphical solution is based, and which take considerable time for preparation. Although the method saves labor when a large number of determinations must be made for the same equipment, the time taken for construction of the charts is a serious disadvantage when but a few runs are to be calculated. A scheme intended to obviate the latter difficulty was devised by Professor E. C. Woodruff,† in which the separate charts are replaced by diagrams drawn directly on the motor curve-sheet. Although the work of plotting is somewhat less than in the Mailloux method, and the intermediate calculations are all on the single motor curve-sheet, considerable time is still required for plotting the diagrams needed in the determination.

From time to time constructions have been developed for accomplishing portions of the desired result. and these may be considered useful for modifications of the original methods just described. They simplify and in some cases reduce the labor incident to the graphical calculation.

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*Notes on the Plotting of Speed-Time Curves, Transactions A. I. E. E., Vol. XIX, p. 901 (1902).

†Graphic Method for Speed-Time and Distance-Time Curves, Transactions A. I. E. E., Vol. XXXIII, p. 1673 (1914).

The plan herein proposed is a graphical solution which possesses the accuracy of the original ones, while at the same time it eliminates nearly all of the intermediate steps. The calculations are all based on fundamentally correct principles, and the results may be determined as closely as desired within the limits of accuracy of the ordinary methods of plotting.

The acceleration produced by a known tractive effort is given in the following equation:

$$A = \frac{F}{91.1(1+r)T} \dots\dots\dots\dots\dots\dots (18)$$

in which A is the acceleration in miles per hour per second, F the *net* tractive effort of the motor in pounds at the wheel treads, T the weight of the train *in tons per motor,* 91.1 the force needed for unit acceleration of translation alone, and r the ratio of force required for the acceleration of rotating parts to that for translation. When extreme accuracy is not necessary, equation (18) can be replaced by the simpler statement

$$A = \frac{F}{100T} \dots\dots\dots\dots\dots\dots (19)$$

in which the rotating parts are assumed to take approximately one-tenth the force necessary for acceleration of translation. It is evident from these equations that for a given weight of train per motor the acceleration produced is directly proportional to the net tractive effort.

The force available for acceleration, or net tractive effort, is the residue of the total torque of the motors, after reducing to the speed at the wheel treads, subtracting the force for overcoming train resistance and curve resistance, and subtracting or adding the force for going up or down grades. The size and type of the cars making up the train being known, and the profile given, it is a comparatively simple matter to determine these quantities. Train resistance may be calculated from tests or by any one of a number of well-known formulæ, as, for example, that developed by Mr. A. H. Armstrong:

$$R = \frac{50}{\sqrt{W}} + 0.03\,V + \frac{0.002\,aV^2}{W}\left(1 + \frac{n-1}{10}\right) \dots\dots (20)$$

in which R is the train resistance in pounds per ton, W, the weight *of the train* in tons, V, the train speed in miles per hour, a, the projected cross-section of the train, and n, the number of cars in the train. This is probably as accurate as any general equation developed for passenger cars. For freight trains, other equations should be used.*

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*See Bulletin 43, Engineering Experiment Station, University of Illinois. "Freight Train Resistance," by Edward C. Schmidt.

Grades require an additional tractive effort of 20 pounds per ton for each per cent of up grade, and correspondingly less for down grade. Curve resistance is quite difficult to determine, but may be assumed from 0.5 pound to 2.0 pounds per ton per degree of curvature. After making the proper subtractions and additions to the gross tractive effort given by the motive power, the force available for producing acceleration, or net tractive effort, is obtained.

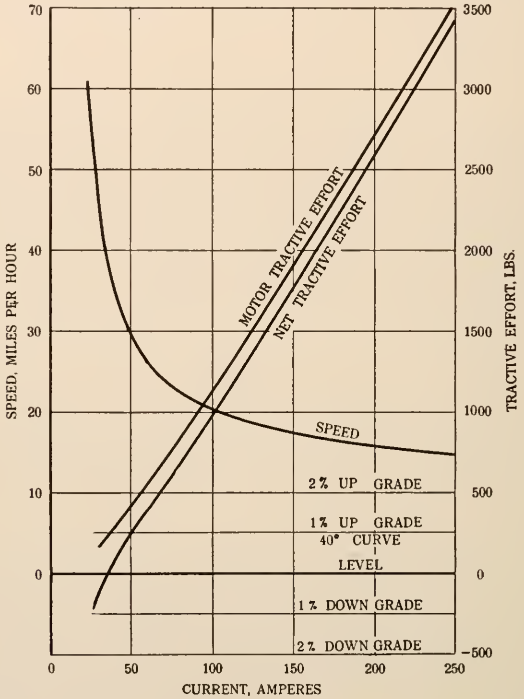

FIG. 9. RAILWAY MOTOR CHARACTERISTIC CURVES.

Since the mathematical operations for getting the net tractive effort are addition and subtraction, the calculation may be made by a graphical process. This has been explained by many previous writers, so that it is not necessary to repeat it here.* It is worth noting, however, that if the train resistance is subtracted directly on the diagram, the residue represents at once the net tractive effort for level track, while if plotted separately the process of subtraction is rendered more difficult, requiring the use of a scale or a pair of dividers, in addition to the coordinate scales of the chart. Grade and curve resistances being single-value functions (i.e., not changing with speed), they may be represented by horizontal lines on the chart, either increasing or decreasing the ordinates of tractive effort.

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*C. O. Mailloux, Notes on the Plotting of Speed-Time Curves, Transactions A. I. E. E., Vol. XIX, p. 901 (1902).

E. C. Woodruff, Graphic Method for Speed-Time and Distance-Time Curves, Transactions A. I. E. E., Vol. XXXIII, p. 1673 (1914).

A. M. Buck, The Electric Railway, p. 39.

The manufacturer's performance curves for a certain electric railway motor are given in Fig. 9, with the addition of the train resistance and grade and curve resistances for use with a particular train or car. The net tractive effort curve gives directly the accelerating force for level track; and for other conditions the base may be moved up or down as required. It is to be noted that the values of resistance are plotted in terms of force per motor, so that if, for example, the equipment consists of four motors, the values on the chart will be one-fourth of the total.

The net tractive effort having been determined, the acceleration produced may be found from equations (18) or (19). These equations show that if the tractive effort is plotted as an ordinate and the quantity $100T$ from equation (19) as an abscissa, the slope of the line connecting the origin with the point thus determined is a measure of the acceleration to the same scale. The actual value of the slope is not important; it depends on the units chosen for the coordinates of the speed-time curve.

In plotting the speed-time curve, the most satisfactory way is to take an increment of speed, ΔV and, knowing the value of acceleration, A, to determine the corresponding increment of time, Δt. It is this method which has been elaborated by all writers and which is the basis of the present article. Since

$$A = \frac{\Delta V}{\Delta t} \qquad \text{(21)}$$

then

$$\Delta t = \Delta V \frac{1}{A} \qquad \text{(22)}$$

This equation is the basis of the former methods of graphical determination of speed-time functions. In Mailloux' method, a chart of inverse values of A and of integral multiples of these values is plotted. An inspection of equation (22) shows that if an increment ΔV equal to unity is taken, Δt is the reciprocal of A, so that it may be taken directly from the chart. A somewhat similar method is followed by Woodruff, who, however, combines the reciprocal curve and the chart of accelerations on one sheet.

A comparison of equations (18) or (19) and (21) shows them to be of precisely the same form, so that they may be equated as follows:

$$\frac{\Delta V}{\Delta t} = \frac{F}{100T} \qquad \text{(23)}$$

using the simpler form of the expression given in equation (19).

Equation (23) makes it evident that if the time is taken to the same scale as $100T$, then the speed must be to the same scale as F, the net tractive effort. If a particular scale for time values has been decided on, the scale for T, which is immaterial to the construction since only one point need be found, is determined as in the following paragraph.

Let V_0 be the scale of ordinates; namely, the number of miles per hour per unit of ordinates, and t_0 the corresponding scale of abscissæ, as required for the speed-time curve. From equation (21) the slope of the acceleration line produced with these unit values may be determined. The scale for $100T$ being arbitrary, then if it is chosen so that the slope of the line is the same as that for unit acceleration, it makes possible the direct construction of the speed-time curve. The diagram, Fig. 10, shows the arrangement. With ordinate OB and abscissa OA on the speed-time curve, each equal to the unit selected, the corresponding acceleration is

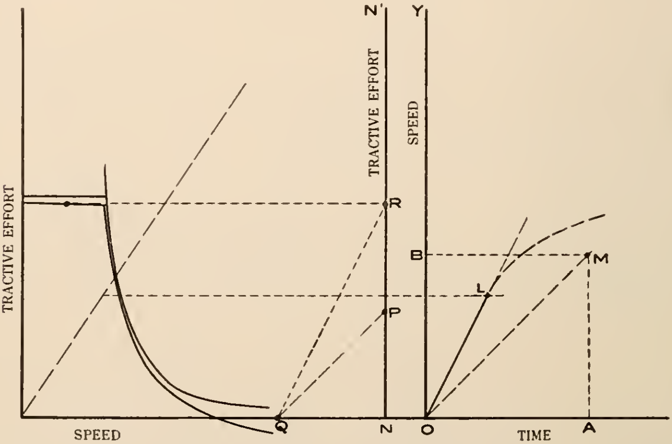

Fig. 10. Construction for Plotting Speed-Time Curve from Tractive Effort-Speed Curve.

$$\frac{OB\ (\text{miles per hour})}{OA\ (\text{seconds})} = A\ (\text{miles per hour per second}) \quad \ldots (24)$$

The discussion shows that a definite amount of force, F_0, is required to produce this acceleration in a given weight of train. Selecting any suitable scale of tractive efforts, as NN', this force may be represented by the ordinate NP. It is evident that if a straight line is drawn through P parallel to the acceleration line OM, cutting the horizontal axis at Q, the length NQ will represent the quantity $100T$ to the proper scale. This is proved by equation (23) and the similarity of the triangles OMA and QPN. The same equation shows that any other value of tractive effort, as NR, will produce an acceleration represented by the slope of the line QR, the corresponding location of the speed-time curve being OL, drawn with the same slope. The values of tractive effort for which the acceleration is determined may be read directly from the tractive effort curve, plotted against speed, as in Fig. 10, or from the curve of tractive effort plotted against current, as in Fig. 11.

The construction of the speed-time curve is now evident. If the train is started with any constant current, I_1 (Fig. 11), the acceleration produced is represented by the slope of QR_1. The line OV_1 is then drawn with the same slope. The constant current can be maintained up to the speed S_1, so that the line OV_1 should be continued until it reaches this ordinate at the point V_1. This line and point are on the speed-time curve to the desired scale. With further increase in speed, the tractive effort will decrease, as indicated by the curve, and the acceleration will be correspondingly less.

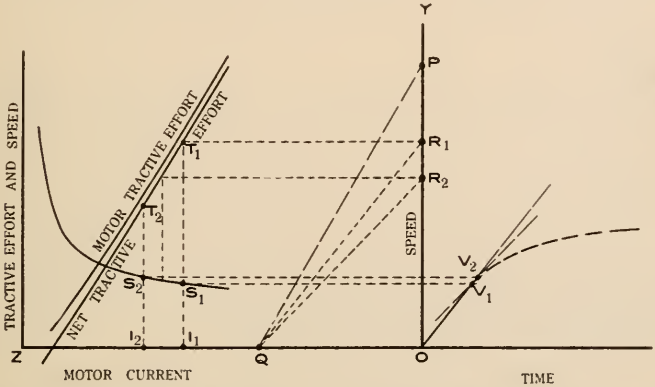

FIG. 11. CONSTRUCTION FOR PLOTTING SPEED-TIME CURVE FROM MOTOR SPEED-CURRENT AND TRACTIVE EFFORT-CURRENT CURVES.

Consider an increment of speed, $\Delta V = V_2 - V_1$. This corresponds to a decrease of tractive effort from T_1 to T_2. If the increment is taken small enough that the variation in force is practically along a straight line, the average tractive effort, acting continuously for a time Δt, will produce an increase in velocity ΔV. If, then, the tractive effort at the mean speed,

$$V_1 + \tfrac{1}{2}\Delta V = \tfrac{1}{2}(V_1 + V_2) \ldots\ldots\ldots\ldots (25)$$

is taken and projected on OY, at R_2, the slope of the line joining this point with Q is the average acceleration during the increment. A line drawn through V_1 parallel to QR_2 will pass through the point V_2 at the end of the increment ΔV. The location may be made conveniently by projecting S_2 parallel to the axis of abscissæ, and noting the intersection V_2. If the increment has been taken sufficiently small, this is a point on the curve, and not a tangent; for the tangent to the curve at the mean ordinate would not pass through the points V_1 and V_2, but would be parallel to the line drawn through them. The magnitude of the error due to this assumption is fully discussed by Mailloux.* It is shown that the error is so small as to be negligible in ordinary calculations if ΔV is not too great.

The construction outlined in the last paragraph may now be continued for the remainder of the acceleration period until the train reaches constant speed. A smooth curve drawn through the points located in this manner is the true speed-time curve; and the accuracy may be made as great as desired by proper choice of the speed increments.

For the coasting portion of the curve, the train resistance may be plotted to any horizontal scale, the ordinates being the same as those for motor tractive effort. In fact, the ordinates representing train resistance, which are plotted down from the gross tractive effort curve, may be stepped off with dividers and transferred to the line *OY* to determine the corresponding retardation. Speed increments may be taken as before, and the coasting curve plotted. For the braking curve an ordinate corresponding to the braking force must be obtained and added to the train resistance. In this manner the entire speed-time curve may be determined.

VIII. PLOTTING DISTANCE-TIME CURVES†

In constructing distance-time curves, a number of methods may be used. Mailloux determines distance by means of the device known as the "integraph," which is a convenient and accurate way. If such an instrument is not available, a planimeter may be used, making partial integrations over portions of the run, so that enough points may be located to draw the curve. This is a much slower process, although of practically the same accuracy as the former. In the absence of any other device, the area of the curve may be determined by making the plot on coordinate paper and counting the small squares included by the diagram. Woodruff uses a series of curves representing distance covered at average speeds, which may be used in estimating the distance passed over during the various increments.

A method which is at least as accurate as any of the purely graphical constructions mentioned is described in the following paragraph:

Assume any convenient scale of distance to be used for plotting the distance-time curve on the same sheet as the speed-time curve. Referring to Fig. 12, let *OB* represent unit distance, say one mile. This same ordinate corresponds to a speed of *V* miles per hour on the speed-time curve. If the train continues in motion at a velocity of V miles per hour for $\frac{1}{V}$ hours, the distance covered will evidently be one mile. Since the speed in such motion is constant, the rate of

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*Transactions A. I. E. E., Vol. XIX, p. 988 (1902).

†The process described for plotting distance-time curves is a general method of graphical integration, and may be used for the construction of integral curves for any function whatever that may be represented by Cartesian graphs.

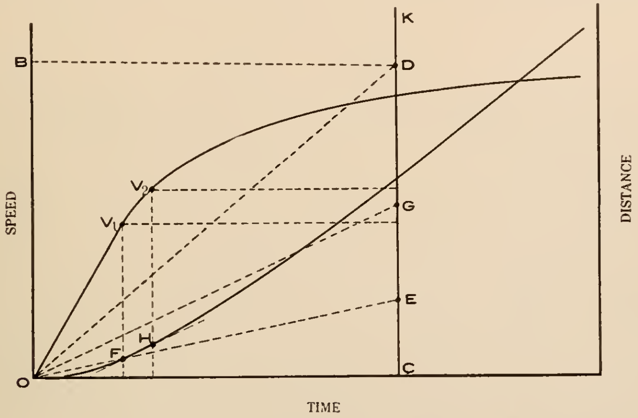

FIG. 12. CONSTRUCTION FOR PLOTTING DISTANCE-TIME CURVE FROM SPEED-TIME CURVE.

covering distance, or the slope of the distance-time curve representing the run, is a straight line. Lay off a length OC on the time axis equal to $\frac{1}{V}$ hours, and erect the perpendicular CDK at C. A diagonal line connecting O and D will then measure the distance traversed when the speed is represented by the ordinate $CD = OB$. In other words, OD is the correct distance-time curve for a constant speed $OB = V$. For any other time, the distance covered will be proportional, and will be represented equally well by the ordinate of the line OD up to that time. Since distance is proportional to the product of speed and time, the distance covered at any other velocity during the time $\frac{1}{V}$ hours will be represented by an ordinate equal to that speed.

This construction may be utilized in plotting the distance-time curve from the speed-time curve as follows. Take the *average* velocity during any time increment and project the ordinate representing it on the line CDK. The intercept on the line joining the projection of this average speed with the origin included within the limits of the time increment measures the distance covered. For instance, the first portion of the speed-time curve, terminating in the point V_1, has been made at a constant acceleration. The average speed during the increment is $\frac{1}{2}(V_1+0)$. Locate the point E on the line CDK so that $CE=\frac{1}{2}V_1$. Connect O and E by the straight line OE. The time increment at the point V_1 intersects this line at F. This is a point on the distance-time curve since, for uniform acceleration, the distance s is

$$s=\frac{V_0+V_1}{2}\Delta t \qquad (26)$$

which is a fundamental relation. For the next increment, from V_1 to V_2, the average velocity, $\frac{1}{2}(V_1+V_2)$, is represented by the ordinate CG, and the line OG determines the slope of the distance curve during this period. A line FH, beginning at the point F and drawn parallel to OG will, therefore, determine the point H on the distance curve at the end of the time increment. This construction may be continued until the entire distance-time curve is located. A smooth graph passing through the points thus plotted is the true distance-time curve. As in the case of the speed-time curve, the points located are actually on the curve and not on tangents. The construction is accurate so long as the deviation of the speed-time curve from a straight line is negligible during each increment under consideration.

It is not claimed that this method of determining distance is more accurate than the use of the integraph or the planimeter, but that it is of more ready application, and gives results which are as accurate as are ordinarily obtainable within the limitations of curve plotting. The error can be made as small as desired by taking increments of time of such magnitude that the speed-time curve is practically straight during any one of them, as explained.

IX. Applications of Graphical Method for Speed-Time and Distance-Time Curves

A problem frequently met with in railway service is the determination of the exact points of cutting off power and of applying brakes in order to make a run of fixed distance in a given time. The solution may be made by the application of the speed-time and distance-time curves. To do this the braking portion of the speed-time curve may be plotted backward from the end of the run and the corresponding distance curve located, while the distance curve for acceleration is plotted forward from the zero point. A period of coasting must be interposed which will satisfy the operating requirements; namely, one which will allow braking to be included at the normal rate and also reach the desired point for the end of the run.

In order to show the method, a complete speed-time and distance-time curve will be drawn. While the entire plot is given for straight level track, the modifications for various combinations of grade and curve may be made as suggested in the foregoing paragraph. A diagram drawn by this method is shown in Fig. 13. The run comprises an acceleration with the motors, followed by a period of coasting, and lastly by a period of braking. This is the simplest form of run ordinarily used, although it is possible to eliminate the coasting, applying the brakes immediately after cutting off the power.

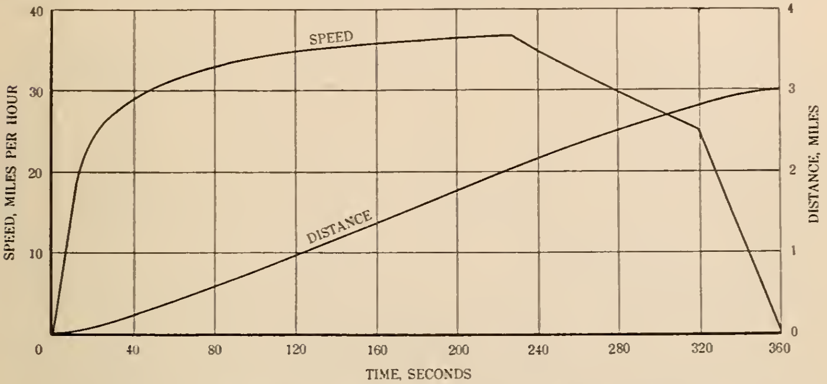

FIG. 13. COMPLETE SPEED-TIME AND DISTANCE-TIME CURVES.

The latter practice, however, is not usual. The various parts of the run are determined independently and afterward connected together as indicated in the following paragraphs. The plot of Fig. 13 checks with that produced by an analytical determination within the limits of accuracy of the cross-section paper used; and the graphical construction has the further advantage of requiring only a set of triangles or a parallel ruler when the same scale of ordinates is used for the speed-time curve as that given on the motor characteristic curve.*

The braking rate is usually assumed constant. A speed-time curve for this portion of the run may be plotted backward from the end, as in Fig. 14, and the corresponding distance-time curve determined.

The coasting speed-time curve is independent of the acceleration and the braking, for during this period the train is acted on solely by the force of train resistance and the incidental resistances present due to the track conditions. For a given profile the coasting speed-time curve may be determined from the weight of the equipment and the train resistance equation. It may be drawn graphically by the methods of Fig. 10 or Fig. 11, the motor tractive effort being replaced by the train resistance per motor (i.e., the total resistance per train divided by the number of motors). It should be remarked that a resistance is a negative force, and should, therefore, be plotted downward from the base. The acceleration produced will be negative unless the force due to a down grade is such as to equal or exceed the negative force of train resistance. A separate speed-time curve for coasting may be plotted on tracing paper or other transparent medium and the corresponding distance-time curve located, as shown in Fig. 15.

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*It is often undesirable to plot the speed-time curve to the same speed scale as that of the motor performance. In such a case the time corresponding to a certain increment of speed may be found directly by laying off a right triangle, the hypotenuse of which is parallel to the acceleration line. Since this triangle may be plotted to any scale whatever, the accuracy may be as great as desired. From the successive speed and time increments thus found, a speed-time curve may be plotted. The distance-time curve may be laid out in a similar manner.

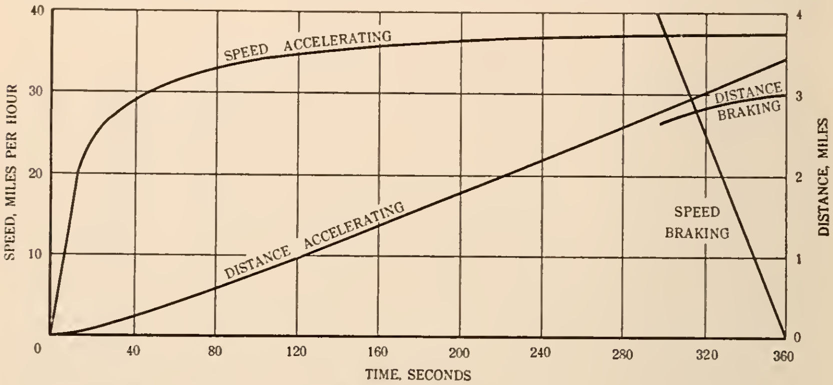

FIG. 14. METHOD OF DETERMINING PROPER POINT FOR CUTTING OFF POWER.

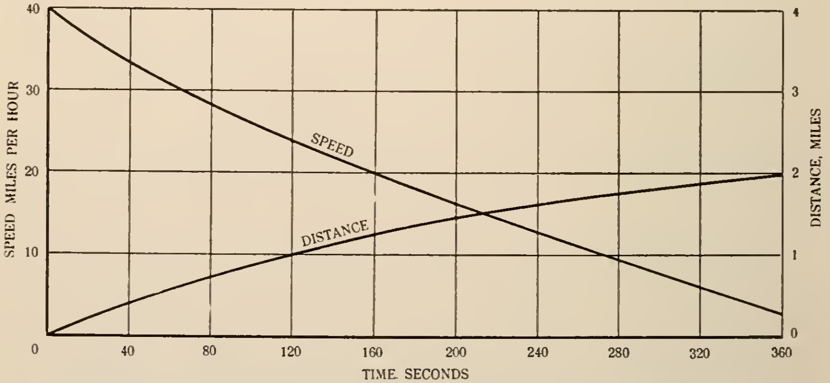

FIG. 15. COASTING SPEED-TIME AND DISTANCE-TIME CURVES.

Since the distance-time curve is the first integral of the speed-time curve, an abrupt change in the slope of the latter corresponds to a point of inflection in the former, or merely to a change in its curvature. The coasting distance-time curve must, therefore, be tangent to both the accelerating and the braking portions of the distance-time curve for the run. For further proof of this it may be noted that, since the *slope* of the distance-time curve is a measure of the speed, this slope must be the same for either curve at the point where the two portions of the speed-time curve join. This fact makes possible the following method of accurately locating the points of cut-off of the current and application of the brakes.

The tracing of the coasting distance curve (Fig. 15) should be laid over the curve of distance while accelerating (Fig. 14) with the axes of coordinate parallel, so that the two curves are tangent at some point. The tracing should then be slid along, *keeping the axes parallel,* until the coasting curve also becomes tangent to the braking distance curve. The points of tangency thus determined correspond to the cut-off of the current and the application of the brakes. These points having been determined and the distance-time curve during coasting transferred to the plot of Fig. 14, the tracing of the coasting curves may be moved *parallel to the axis of ordinates* until the two axes of abscissæ coincide. The coasting line may now be traced on Fig. 14, locating definitely the remainder of the speed-time curve and producing the complete diagram of Fig. 13.

In practice, it is usually convenient to have a number of coasting curves, corresponding to different conditions of grade and track curvature, to cover all the variations liable to occur. Such a series, plotted on a sheet of tracing cloth or transparent celluloid, forms a templet for the location of the principal points on the speed-time and distance-time curves for runs of definite length, making the graphical construction of much greater value in preliminary calculations to determine the size of motors required for a given service.

The graphical method of plotting speed-time and distance-time curves described is equally good for use with any kind of motive power. All that is necessary is to get the relation between speed and tractive effort connected by a graph which can then be used for determining accelerations in the manner outlined. The application is so obvious that it need not be further elaborated.

X. Heating Value of a Variable Current.

The rating of all electrical apparatus depends to a considerable degree on the heating of the active parts. This is especially true in the case of railway motors. One of the principal sources of heating is the resistance of the conductors. The heat produced in a wire carrying a current is proportional to the square of the current multiplied by the time during which it is acting. In general, the value of current in a conductor is not fixed for any considerable period, but is constantly changing. If the variation follows some known law, the effect of the current in producing heat can be found by a comparatively simple mathematical analysis; but if the current is changing in some casual or variable way, the evaluation is not easy.

The latter condition holds true in the case of the electric railway motor cycle. Here the current is a maximum at the instant of starting, after which it gradually falls to a minimum, and is then cut off entirely while the train coasts and comes to rest. The variation is further complicated on account of the occurrence of grades, curves, points where the speed must be reduced, and other special conditions of operation.

Railway motors are usually rated by the current which can be carried continuously, or for a stated period, with a temperature rise above the surrounding air considered safe.* To determine whether or not a motor is large enough for a given service, the variable current must be evaluated to find whether it is above or below a safe amount. The method usually employed is to plot the curve of current taken by the motor against time and from this construct another curve of values of current squared. The integral of the latter curve, divided by the total time of operation, is the square of that current which, applied continuously for the same time, will produce the same loss in the conductors.

As ordinarily applied this method is cumbersome. It requires the use of a table of squares or some similar method of calculation, so that the new curve can be plotted from the original current values. To obviate the necessity of squaring a large number of values, another plan has been devised, which requires the replotting of the current curve in polar coordinates. The effective current can be obtained by this method without the need of squaring the ordinates of the current curve.†

The entire argument in favor of the use of the polar diagram for finding the effective motor current is that it is less laborious than to plot the curve of squared values of current. Two methods, both of them entirely graphical, will now be described for plotting the latter curve, which is more easily prepared by these methods than the polar diagram. The other operations involved in the determination of the effective current are essentially the same for either this or the polar method. The curve of squares of current plotted on a rectangular base has the further advantage that it can be put on the same sheet with the original current curve, thus rendering unnecessary the use of a separate chart and making possible an easier coordination of the values than when the diagrams employed are so different in character as the rectangular and the polar graphs.

In Fig. 16 consider a scale of natural numbers, ON. Corresponding to these it is desired to construct another scale of natural numbers such that a certain ordinate $O'M$ is equivalent to $\overline{ON}^2$. It is evident that the square of ON may be represented by the area

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*For further information regarding the methods in vogue for rating railway motors, see Standardization Rules of the A. I. E. E., 1915 edition.

†For a proof, see A. M. Buck, The Electric Railway, p. 136.

ONBA enclosed by the rectangle having each of its sides equal to *ON*. Since the scale of squares is chosen so that *O′M* is numerically equal to the square of *ON*, it may equally well be stated that it represents the area *ONBA*. The problem is to find the ordinate along *O′M* corresponding to the square of some other value, as *OD* on the original scale. It has been seen that the scale *O′M* may be considered to measure areas, so that the discussion resolves itself into finding the ordinate along *O′M* which will represent the area *ODEH*, which is the square constructed on the side *OD*. If a rectangle with the base *OA* can be found with an equivalent area, its ordinate will be the value sought.

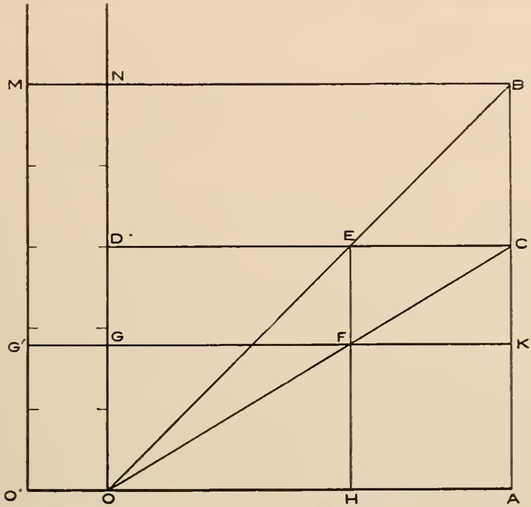

FIG. 16. METHOD FOR SQUARING NUMBERS GRAPHICALLY.

Referring to Fig. 16, construct the diagonal *OB* of the large square, and continue *DE* to meet *AB* at the point *C*. Connect *C* with *O*, cutting *HE* at *F*. The geometrical construction gives

$$ON = AB = OA = NB \quad \ldots\ldots (27)$$

$$OD = HE = OH = DE = AC \quad \ldots\ldots (28)$$

$$OG = HF = AK \quad \ldots\ldots (29)$$

$$\frac{HE}{AB} = \frac{AC}{AB} = \frac{OD}{ON} \quad \ldots\ldots (30)$$

$$\frac{HF}{HE} = \frac{AC}{AB} = \frac{OD}{ON} \quad \ldots\ldots (31)$$

Hence

$$OG = HE \times \frac{OD}{ON}\frac{AC}{AB} = OD \times \ldots\ldots (32)$$

Therefore

$$\text{Area } OGKA = \text{Area } ODEH \quad \ldots\ldots\ldots\ldots\ldots (33)$$

Since

$$\begin{aligned} \text{Area } OGKA &= OG \times OA \\ &= \left(OD \times \frac{OD}{ON} \right) \times ON \\ &= \overline{OD}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34) \end{aligned}$$

Hence $O'G'$, the numerical equivalent of OG, represents $\overline{OD}^2$ to the scale of $O'M$.

The application of the method is obvious. It is only necessary to construct a square at any point on the current-time chart, with a side such that some value of current and its square are represented by the same length of side. Any value of current, corresponding to OD, should be projected on the diagonal OB and also on the side AB of the square. When the projection AC on AB has the point C connected with O, the line CO will intersect HE in the point F. This is the ordinate for the curve of current squared, and may be carried back to the proper position above or below the corresponding value of current. With a small amount of practice the calculation can be made with great rapidity, for the construction lines can be very largely omitted, only the intersections being required to find the proper values.

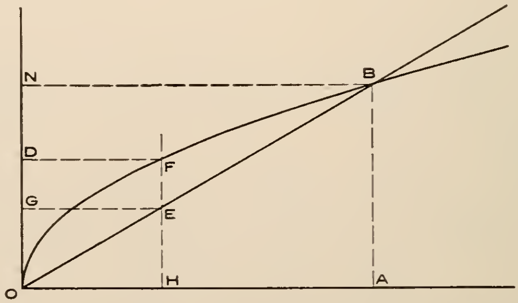

FIG. 17. PARABOLIC CURVE FOR SQUARING NUMBERS.

An alternative method to that just described is to plot a curve between the natural numbers and their squares, the latter values being represented by convenient ordinates. An inspection of Fig. 16 indicates that the locus of the points F is a parabola whose principal axis is ON and which passes through the point B. In practice it is found simpler to make the diagram of the opposite form, as shown in Fig. 17. The parabola OFB is of the form

$$x = k\,y^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

Consider a line OEB drawn through the origin. Ordinates cut off by this line, as HE, are proportional to the abscissæ, as OH. Corresponding ordinates on the parabola are proportional to the square roots of the abscissæ. Therefore HE is equal to $\overline{HF}^2$ on such a scale that $\overline{AB}^2$ is represented by the same ordinate, AB. The construction holds for any line OB intersecting the parabola.

In order to apply the method just described, the parabola OFB and the straight line OEB should be plotted on some transparent medium, such as celluloid. The templet thus made may be slid along the curve of current with the axis OA coinciding with the base line of the current curve, until the parabola intersects the current curve at the proper point; the square of that ordinate will then be found directly under or over the value of current. This can be repeated an indefinite number of times until sufficient points are obtained to plot the curve of current squared. From this the effective current may be obtained, as explained above.

Since the plotting of points as obtained by the parabolic curve may be difficult when the base lines OA coincide, since holes will have to be pricked through the templet, the method may be modified to permit the construction being placed on an ordinary celluloid triangle by moving the axis of the line OEB upward through a suitable distance. This is shown in Fig. 18. Here the base line for the parabola is OA, the edge of the triangle; while that for the diagonal line has been transferred to $O'A'$, at a distance OO' above the other axis. All the ordinates along $O'E'B'$ are therefore displaced by the amount $O'B'$. This will not occasion any difficulty in the subsequent calculations, since the value obtained for the area of the current squared curve will be too great by an amount equal to OO' multiplied by the length of the diagram. As the area is to be divided by the base to find the mean ordinate, the calculation can be made without reference to the constant, and the value of OO' subtracted from the mean ordinate for the current squared curve.

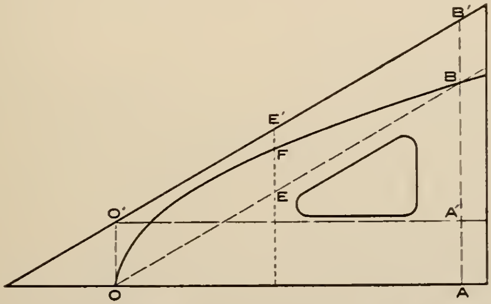

FIG. 18. APPLICATION OF PARABOLIC CURVE TO A TRIANGLE.

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SUBSIDENCE RESULTING FROM MINING

BY

L. E. YOUNG
AND
H. H. STOEK

ILLINOIS COAL MINING INVESTIGATIONS
COOPERATIVE AGREEMENT

(This report was prepared under a Cooperative Agreement between the Engineering Experiment Station of the University of Illinois, the Illinois State Geological Survey and the U. S. Bureau of Mines.)

BULLETIN No. 91

ENGINEERING EXPERIMENT STATION

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 91 AUGUST, 1916

SUBSIDENCE RESULTING FROM MINING

BY

L. E. YOUNG
Mining Engineer for the Illinois Coal Mining Investigations,

and

H. H. STOEK
Professor of Mining Engineering, University of Illinois

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SUBSIDENCE RESULTING FROM MINING

INTRODUCTION.

The subject of the subsidence of the earth's crust as a result of underground excavation due to mining has attracted widespread attention for some years past, but particularly during recent years. With the extension of mining and the increased value of the surface above the mines in many localities, the growth of towns in mining regions, and the extension of railroads over mining properties, the subject is one that will be of increasing interest as time goes on, not alone to those engaged in mining coal and ores, but to the railroads, municipalities, and other owners and users of the surface that may be affected by mining operations.

That the subject is not one of mere local interest is shown by the widespread distribution of surface subsidence as described in the following pages.

This bulletin has been prepared not with a view of bringing forward any new theories in regard to the subject, but it is in the nature of a reconnoissance and a statement of present knowledge of the subject, based upon the literature available up to the present time. It and a companion preliminary Cooperation bulletin on subsidence in Illinois by Dr. Young, which will be issued by the Illinois Geological Survey, are intended as studies upon which to base a detailed cooperative investigation of subsidence conditions in Illinois.

This bulletin represents the result of a study of the literature on the subject. Much of the text is an abstract of this literature, supplemented by extensive private correspondence by the writers, and by a study of conditions in western Pennsylvania, in West Virginia, and in Maryland as given by office data and by an intimate acquaintance with the subsidence problem in the anthracite fields of Pennsylvania.

The authors are particularly indebted to the several anthracite subsidence commissions for the use of unpublished reports, and to a number of engineers for private data, for some of which it has been possible to give due credit in the text, while other data of a confidential nature has had to be incorporated without due credit.

This preliminary study of subsidence literature and of the conditions in Illinois suggests the advisability of undertaking a detailed study of the problem in Illinois. This study may extend over a number of years in the future. To begin such a study, several groups of mines should be selected; one group in northern Illinois, one in the central part of the state, and another in the southern part. At each mine selected monuments should be erected, and the elevation of these monuments taken at intervals. In connection with these surface observations the conditions in the mine should be noted as closely as possible, in the hope that gradually data will be collected upon which it will be possible to determine the probabilities of and the extent of subsidence upon the surface when underground conditions are known, to determine the size of pillars which will most effectively prevent loss of coal due to squeezes and will properly protect a given surface area.

For the preparation of the extensive bibliography, for the preparation of the abstracts of literature, and for the detailed presentation of the data collected, credit is due entirely to Dr. Young, the undersigned having acted mainly by assisting in the gathering of data and in an advisory capacity in the preparation of the manuscript.

H. H. Stoek.

CHAPTER I.

NATURE AND EXTENT OF SUBSIDENCE PROBLEM

The removal of solid minerals from the earth's crust produces cavities, and thus the equilibrium which has previously existed is disturbed. If the cavities caused by the mining operations are not of great extent, or, even if long, are narrow, this disturbance may be apparent only as a local movement and may cause only occasional falls of rock from the roof. If the excavation is wide as well as long, the unsupported strata above the excavation will tend to sag under their own weight and, if their texture will not permit the bending movement necessary for the strata to become adapted to the new conditions, cracks and fissures resulting in extensive falls of roof will occur. Successively, the overlying beds may break and fall until the disturbance extends to the surface.

If the overlying measures bend without breaking and sag until finally they are supported by the floor of the excavation, the strata at greater height may sag successively and in a corresponding manner. Eventually, this movement may extend to the surface, the disturbance generally being less extensive as the vertical distance from the excavation increases.

In estimating the weight upon any coal seam or other mineral deposit due to the overlying rock, it is customary to assume that this weight is distributed more or less uniformly over the entire deposit. When a portion of a bed of mineral is removed, the burden carried per unit of area by the unmined portion becomes greater than the burden carried before any portion of the deposit was mined, because the weight formerly distributed over the deposit is now concentrated upon the pillars. The extent of the increase of burden on the pillars depends upon the extent of removal of the material of the bed, assuming that the overlying rock does not break in such a way as to relieve the stress on the pillars. If the pillars are not strong enough to support the increased load, or if the underlying bed does not have sufficient bearing power to resist the increased pressure, a movement will begin which is commonly called a "squeeze"* or a "creep." Depending upon the depth of the mining operations and the geological conditions, the "squeeze" may cause an extensive vertical movement which may reach to the surface. The removal of coal or other bedded minerals from any considerable area, therefore, at once develops the problem of the support of the surface which involves certain factors requiring careful attention by the mine operator before extensive excavations are made. If the operator, for commercial reasons, meets these problems in a manner that is not in harmony with the prevailing ideas of conservation, a remedy should be sought which will secure for the public the greatest continuing benefit.

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*The Pennsylvania Mine Cave Commission gave the following definition: "A 'squeeze' is caused by the general subsidence of the strata overlying the coal bed, due to a partial failure of the pillars; when this subsidence radiates from origin it is called a 'creep'." Another meaning of "creep" is movement of the floor, due to pressure of pillars.

Upon the opening of a new mine, the following questions may well be asked:

1. Is the owner of the surface, if other than the owner of the mine or mining rights, legally entitled to surface support?
2. Is the material to me mined at such a depth that mining of all of it will not disturb the surface?
3. If the removal of all the deposit will cause surface subsidence, what percentage of the deposit left in pillars will prevent subsidence?
4. What is the ratio between the value of the material in the pillars necessary to prevent surface subsidence and the value of the surface? What would be the charge per ton against this pillar material if the surface were bought outright?
5. What amount and what extent of subsidence may be expected under the conditions of operation most economical at the time?
6. Upon what basis will it be possible to adjust claims for damages?
7. What will it cost to restore the surface for agricultural uses after all the deposit has been removed?

There are certain questions which the public and the state should answer at an early date:

1. Shall the coal or other mineral now in the ground be brought to the surface and used or shall it be left in the ground, serving like worthless rock, only to support the surface?
2. Assuming that the removal of all the material will temporarily prevent the use of that part of the surface overlying the area being mined, will it be better policy for the state to see to it that all the merchantable material is mined and then have the surface restored, or will it be wiser to permit nearly one-half the material to be lost permanently in the effort to avoid temporary injury to the surface?
3. If the mine operator is required by law to protect the surface, shall anything be done to prevent his leaving a large percentage of the deposit in the ground, never to be recovered and simply to support the surface?

Scientists who have investigated the national resources have emphasized the fact that the supply of minerals is not inexhaustible and that at the present rate of increase in production the exhaustion of the supply of some of the most important ones is not far off, as time is measured in the life of a nation. In the case of coal, one of the means by which the life of our supply may be extended is by recovering all, or at least a much greater percentage than is recovered at present, of the coal in the ground. If the extent of the entire area underlaid with workable coal beds be compared with the extent of tillable land not underlaid with coal, it will be noted that the actual area that might be affected by surface subsidence is relatively small. When it is realized that land affected by subsidence may in most cases be restored to service for agriculture after all the deposit has been removed, it may be rightly urged that the mine operators remove much more of the coal than is taken under present conditions, when preservation of the surface is frequently the determining factor in deciding the amount to be mined. Since mineral once lost by improper mining or left in pillars in abandoned mines is lost forever, the maximum recovery consistent with safe mining is of prime importance and is fundamental. The problems, therefore, are to discover what effect mining under the existing physical conditions will have upon the surface, to anticipate and to reduce to a minimum possible surface subsidence and finally to discover the best means of harmonizing and coordinating the various industrial and commercial interests involved.

As will be noted in the discussion of the legal considerations involved in the problem* the legal rights of the several parties interested in the minerals, in the surface, and in the other forms of property in the community must be considered both relatively and absolutely. A study of the subsidence problem from various angles shows the complexity of "conservation" applied to mining, to agriculture, and to other interests at the same time. The complexity seems to increase when efforts are made to coordinate these various interests.

RECORDS OF DAMAGE TO SURFACE.

While the technical press contains many reports of surface subsidence attributed to mining operations there are in America only a few reliable records, available for study, showing the exact amount of subsidence of the surface after the mineral deposit has been mined. However, there are a number of instances in which European engineers have kept records of surface levels extending through long periods of years.

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*See Ch. VI.

Surface movements have in many instances been disastrous; records of damage to property being available both in Europe and in America. In the following section a number of the most important instances of damage to property resulting from mining operations are presented. These instances show that the problem is of widespread interest and is not a local one.

Belgium.

Although serious subsidence adjacent to salt mines was noted in England in 1850* and instances of damage by coal mining are recorded in British technical literature, the problem of surface subsidence due to mining operations seems to have been studied first in Belgium. In the early part of the nineteenth century it was claimed that coal mining about Liege, Belgium, was causing damage to buildings, and in the year 1839 complaints were filed with the city officials on account of damages to property. As a direct result of these complaints, the city appointed a committee to report upon the problem and in filing its report the committee established the necessary restrictions required for the safety of the city and determined the size of adequate safety pillars.

The Belgian engineer, Gonot, formulated a theory of subsidence in 1839 and some years later published a pamphlet dealing with the damage to a row of houses adjacent to the mine of the D'Avroy Bovene Company, claiming the mining company was responsible for the damage done. The mining company published a reply to Gonot in 1858. The Provincial Government appointed two engineers to investigate the cause of the damage to the houses and they reported that the houses were not damaged by coal mining.

By a decree of May 31, 1858, the Minister of Public Works appointed a special committee to report on the influence of mining upon the surface and also to review the rules of the committee appointed in 1839. The committee of 1858 endorsed the recommendations of the committee of 1839.

The disturbance of the surface about Liege continued and G. Dumont was appointed to investigate the matter. In his report† he supported the fundamental principle of Gonot's theory but made certain reservations in its application. He placed the responsibility for the surface disturbances upon the mining companies. The Colliery Owners' Association published a statement* pointing out the fallibility of Gonot's theory but admitted the applicability of the theory to relatively flat seams.

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*Trans. I. M. E., Vol. 19, p. 249, 1899.

†"Des Affaisements Du Sol Produits par l'Exploitation houillère." Liege, 1871.

Since 1875 considerable attention has been given to the problem in Belgium, and the situation has been complicated by the mining of coal from superimposed beds.

England, Scotland, and Wales.

Considerable attention has been given to the subsidence problem in England, Scotland, and Wales, owing to the extent of the coal and salt measures, to the importance of the coal industry, and to the proximity of the mines to centers of population. In the early days of coal mining in Great Britain it was customary to leave pillars, but as mining practice improved a portion of the coal in the pillars was removed. In discussing early methods of working coal, Bulman and Redmayne† refer to surface subsidence resulting from the removal of pillar coal as follows: "The date at which it became customary to remove pillars formed by a previous working has been a point of some importance in determining claims for damage to the surface. and many such claims in which the point arose have led to legal proceedings. That damage of this kind was done at an early date is proved by the records of the Halmote Court for the County of Durham. Early in the fifteenth century there was an inquiry before that court about a case which had occurred in the parish of Whickham in which it is recorded: 'It is found by the jury that John de Penrith is injured by a coal mine of Rogers de Thorton so that the house of the said John is almost thrown down, to the damage of the said John of 200 pounds, assessed by the jury; therefore it is considered that the said Roger repair the said house to the value aforesaid, or satisfy the said sum.' "‡

Since the year 1860 a number of British mining engineers and operators have written upon the general subject of subsidence and support of excavations. Subsidence has resulted from salt mining operations, as well as from coal mining, and owing to the nature and extent of the salt deposits the effect upon the surface has been even more disastrous than the effect of coal mining. Salt mining has been carried on in the vicinity of Northwich, Cheshire, for many years. In a depth of 390 feet there is a total thickness of almost 200 feet of salt in four beds, the thinnest being 5 feet thick and two being each approximately 90 feet. The shallowest bed is covered by 32 feet of soil and by 92 feet of salt marl. Shafts were sunk to the upper bed shortly after 1670 and the pillar-and-room system was used. Pillars from 12 to 21 feet square were left to support the surface, but these pillars were weakened in time by the dissolving action of the water which seeped through the roof and was pumped out as brine. Surface breaks occurred which were generally marked by brine pools. In 1750 the first serious breaks occurred near the main street of Northwich. These old breaks have been filled and buildings have been erected directly over them. Since 1750 numerous breaks have occurred throughout the salt district.* After 1781 all new shafts were sunk to the second bed, which is nearly 92 feet thick and is separated from the upper bed by about 28 feet of hard marl. The most serious subsidence occurred in 1880, and the locality is now covered by a lake about 30 acres in area and of considerable depth.

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*"Des Affaisements Du Sol Attribués a l'Exploitation houillère." Liege, 1875.

†Bulman, H. F., and Redmayne, R. A. S. "Colliery Working and Management," p. 9.

‡"History of Durham." Francis Whellan & Co.

The drilling of brine wells has increased the rapidity with which the pillars have become weakened and has hastened subsidence in the vicinity of the old mines. Brine streams or channels have been formed underground between the wells and old shafts, and subsidence is greatest near these underground streams.

Owing to the seriousness of the subsidence over an area of 600 acres, frame buildings are used, as these may be blocked up and restored after the most serious surface movement has abated.

The local Board of Trade was asked by the Salt Chamber of Commerce in 1871 to have a report made upon the local situation. This request was referred by the Board of Trade to the Secretary of State, who directed Mr. Joseph Dickinson, Inspector of Mines, to make a report. In March, 1873, Mr. Dickinson presented to Parliament a report which was published under the heading, "Landslips in the Salt Districts."

In 1881 there was introduced in Parliament a bill which proposed to give relief to the owners of damaged property in the salt district. This bill failed to pass, but in 1891 a bill was passed providing for Compensation Boards to be formed in the salt districts. These boards were empowered to levy a tax, not exceeding 3d. on every 1,000 gallons of brine pumped, to form a fund to compensate owners of property damaged. This act was put into force, and in 1896 a provision was added, limiting such compensation to private holders of property and excluding all local authorities, gas and water companies, railway and canal companies, and all pumpers of brine, no compensation whatever being allowed them if any of their property were injured by subsidence.

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*Ward, T. "Subsidence In and Around the Town of Northwich in Cheshire," Trans. Inst. Min. Engrs., Vol. 18, p. 241, 1888-1890.

The examples of surface subsidence due to coal mining in England and Wales are very numerous. Much agricultural land has been damaged and also various improvements, including buildings, railroads, bridges, railroad tunnels, canals, reservoirs, and streets and highways.

In one district the Great Western Railway had to fill 60,000 to 70,000 cubic yards annually. A canal in South Staffordshire has been raised 20 feet. Coal mining under the Merthyr tunnel, 1½ miles long, produced a total subsidence of 10 feet in part of the tunnel. The settlement throughout the length of the tunnel was not uniform and part of the line had to be cut down to make the grade uniform.*

In the South Staffordshire district there has been considerable difficulty in securing suitable reservoir sites owing to the fact that coal mining has extended under most of the land and owing also to the fact that the bed is thick and nearly all the coal has been removed. A 3,500,000-gallon reservoir was built on a site which had been undermined thirty-four years before. When the reservoir was filled, the subsidence amounted to 1¼ to 2 inches. The cracks were filled with cement and the reservoir has since given no trouble. Another 43,000,000-gallon reservoir was constructed in the coal district in which there are three workable beds of coal, one being 8 feet 3 inches thick and lying at a depth of 1,200 feet, while 66 feet below it is a 6-foot 3-inch seam, and 80 feet above it is a 6-foot seam. These have been worked and one end of the reservoir has lowered 4 feet more than the other, the great difference in elevation between the ends being attributed to a fault.

In order to reduce the damage to water mains the practice in the English mining districts is to use lead joints instead of the "turned and bored" pipes.†

In the Midland and South Yorkshire coal fields the measures overlying the coal are principally shales, and mining at a depth as great as 2,000 feet has caused some subsidence.‡

In his address as President of the Institution of Mining Engineers, W. T. Lewis called attention to the seriousness of subsidence in Wales, stating that the surface sinks from 10 to 15 feet on account of mining at 1,800 to 2,400 feet.¶ The removal of 4 feet 9 inches of coal and underclay, constituting a shaft pillar, at a depth of 2,108 feet is reported to have caused surface subsidence of 3 feet 6 inches at the South Kirby Colliery.* This amount of subsidence is unusual in the district and was attributed to the crushing of a shaft pillar in the overlying Barnsley bed. A maximum subsidence of 1.74 feet resulted from longwall mining of 5 feet of coal at a depth of 1,595 feet in Derbyshire.† Mr. James Barrow cited an instance of mining coal 5 feet 6 inches to 6 feet 6 inches at a depth of 2,400 feet. The longwall method was used and the debris resulting from the working of the seam and the brushing of the roof was stowed underground.‡ Subsidence caused buildings on the surface to crack, water and gas mains to be broken, and bridges to be squeezed and distorted. Mr. J. Kirkup reported that the mining by longwall of a seam 22 inches thick produced cracks in walls and caused damage in pipes in workings in a seam 279 feet above. Moreover, a careful survey showed that the movement in the upper seam extended in advance of the workings in the lower seam.¶ Mr. I. T. Rees has reported on subsidence resulting from longwall mining in the coal field in South Wales. The lower seam worked was from 3 to 4 feet thick and was well stowed. "Three hundred sixty feet above this seam, workings had been prosecuted in another seam in advance of the seam below, and although there were 360 feet of intervening strata, and the openings caused by working the seams were well stowed, yet the workings of the seam above were affected a distance of 150 feet in advance of the workings of the seam below."§ In 1912 the Wearmouth Coal Company, Ltd. (Sunderland), was forced to stop working the Hulton seam, which employed 400 men, on account of the heavy charges for surface damage resulting from subsidence. In one case the charge was $500,000.**

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*Ingles, J. C. "Subsidence Due to Coal Workings." Proceedings, Inst. of Civ. Engineers, Vol. 135, p. 131, 1898.

†Proc. Inst. of Civil Engrs., Vol. 135, p. 156, 1898.

‡Eng. and Min. Jour., Vol. 84, p. 196, 1907.

¶Trans. Inst. Min. Engrs., Vol. 22, p. 290, 1901.

France.

In France subsidence has been noted in the salt mining district as well as in the coal fields. In French-Lorraine, the salt measures extend under an area approximately 9 by 19 miles. The thickness of the beds varies from 33 to 230 feet and the beds lie at a depth of 300 feet or more. The salt has been removed in part by solution methods, which produce large chambers, and, owing to the great size of these chambers and to the character of the roof, extensive falls of roof rock have occurred. The subsidence has generally taken place slowly, but where the covering is limestone there have been sudden breaks which have caused extensive damage. Among the serious surface movements reported are one in 1879 at St. Nicholas and one at Ars-sur-Meurthe in 1876.*

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*Snow, Charles "Removal of a Shaft Pillar at South Kirby Colliery." Trans. Inst. Min. Engrs., Vol. 46, p. 8, 1913.

†Hay, W. "Damage to Surface Buildings Caused by Underground Workings." Trans. Inst. Min. Engrs., Vol. 34, p. 427, 1908.

‡Proc. South Wales Inst. of Engrs., Vol. 20, p. 356, 1897.

¶Kirkup, J. Discussion of paper on "The Absolute Roof of Mines." Trans. Inst. Min. Engrs., Vol. 31, p. 180, 1905.

§Proc. South Wales Inst. of Engrs., Vol. 20, p. 359, 1897.

**Trans. Inst. Min. Eng., Vol. 44, p. 533, 1912.

Fayol made a number of observations of subsidence at the Commentry Mines, as well as laboratory experiments, and published the results of his observations, including the levels taken at these mines from August, 1879, to May, 1885.† He advanced a theory of subsidence which was essentially different from that formulated by the Belgian engineer, Gonot.

Germany.

Probably the first important German publication on surface subsidence in connection with mining was by A. Schulz, in 1867.‡ He investigated the dimensions of safety pillars and the angle of break in the Saarbruck field. The problem was considered so important that in 1868 the Prussian government appointed a commission of four engineers to investigate the effect upon the surface of mining operations in the coal fields of Belgium, England, France, and Rhenish Prussia. In 1869 von Dechen wrote upon the subsidence in and about the city of Essen. He had previously (in 1866) emphasized the importance of studying the part played by the heavy marl beds overlying the coal measures.

In 1867, von Sparre contributed a paper upon the "afterbreak."¶ In 1894, the project of a canal between Herne and Ruhrort aroused a discussion in regard to the stability of the surface over which the proposed canal was to run. The Board of Mines of Dortmund conducted observations in the Dortmund district and the results were published in 1897.§

In the Dortmund district there have been a number of accidents** due to thrust movements, and in the Ruhr coal fields miniature earthquakes, supposed to have been due to coal mining, have caused considerable damage.

Methods of reducing surface subsidence by hydraulic stowing have received much attention from the mining operators of Upper Silesia and Westfalia.

The coal-beds under Zwickau, Saxony, are situated at a depth of

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*Bailly, L. "Subsidence Due to Salt Workings in French-Lorraine." Annales des Mines, Ser. 10, Vol. 5, pp. 403-494, 1904.

†Bul. de la Société de l'Industrie Minerale, IIᵉ série, Vol. 14, p. 818, 1885.

‡Zeit. für B.-, H.-, u. S.-W., 1867.

¶Glückauf, 1867.

§Zeit. für B.-, H.-, u. S.-W., p. 372, 1897.

**Zeit. für B.-, H.-, u. S.-W., Vol. 51, p. 439, 1903.

600 to 2,500 feet. Beginning in 1885, observations were made at eighty-two points to determine the surface movement resulting from mining operations. After twelve years it was noted that subsidence amounted to 85.2 inches, due to mining at 600 to 900 feet. At 1,500 feet, the subsidence was only 9.17 inches.

The town of Eisleben in the Mansfeld mining district was seriously damaged by earth shocks, fissures, and subsidence during the years from 1892 to 1896. Various theories were advanced concerning the cause of these disturbances. Some held that they were due to the dissolving of various salt deposits by underground water thus producing caverns, and that as these caverns became of great extent, large falls of overlying rock caused the shocks and the subsidence. Others held that in addition to the solution of the salt, carbonated waters were leaching the deeper lying dolomitic formations, and when these became honeycombed, they were unable to support the load concentrated on the natural pillars resulting from the solution of part of the overlying salt beds. At the Mansfeld Copper Mine copper bearing shale from 12 to 20 inches thick was being mined by a longwall method at a depth of from 900 to 1,800 feet. Public opinion blamed the mining company and, as a result of arbitration, the company paid $125,000 damages.*

Potash mining at Stassfurt in beds 50 feet thick and dipping 40 degrees has caused serious subsidence. Stone buildings have sunk as much as 20 feet, rows of houses have been removed to firm ground, and chimneys and towers are standing 5 degrees from the vertical.†

On June 10, 1910, surface subsidence, described as a local earthquake, occurred at the Consolidation Mine. "The part of the coal measures most affected formed part of an undulation or 'saddle.' The forces at work were of such intensity, and so irregular in their action, that steel rails were twisted into corkscrew like shapes, and in a section of the saddle 10 feet in length, two lines of rails, water-pipes, signal-wires, and rope-way were found crushed together into a bundle of about 12 to 16 inches thick."‡

Austria.

The review of the theories of subsidence presented by Austrian engineers, as given by Goldreich,¶ indicates that as early as 1859 there were regulations controlling the mining of coal under railways in Austria. Director W. Jicinsky published a treatise on "The Subsidences and Breaks of the Surface in Consequence of Coal Mining."[*] The publication by Rziha[†] was the first contribution by an Austrian engineer to the theories of subsidence. Most of the Austrian writings on subsidence have been on the problems of the Ostrau-Karwin coal district. Goldreich has studied the problem principally through years of observation in railway engineering.

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*Lang, Otto "Subsidences at Eisleben." Bul. de la Societe Belge de Geologie, Vol. 11, p. 190, 1898.

†Private correspondence.

‡Zeit. f. B.-, H.-, u. S.-W., Vol. 59, p. 68, 1911. Abstracted in Trans. Inst. Min. Eng., Vol. 41, p. 587, 1910.

¶Goldreich, A. H. "Die Theorie der Bodensenkungen in Kohlengebieten." Berlin, 1913.

One of the most serious disasters in Austria resulting from surface subsidence occurred July 19, 1895, at Brux, Bohemia,[‡] where the brown-coal seams lie nearly horizontal at a depth of 325 feet, covered by clay-shale interspersed with quicksand from 10 to 65 feet thick. There are in all four seams having an average total thickness of 80 feet. Some filling had been used, but sand broke into the mine, and it is estimated that two million cubic feet of sand entered the workings. Numerous holes were formed on the surface, rendering sixty-six houses uninhabitable and making 2,000 people homeless.

Another serious disaster occurred at Raibi, Bohemia, at a lead mine, where an attempt was being made to secure an adequate water supply through underground workings. Two short drifts were being driven through the rock toward water-bearing sands, and though a borehole was kept ahead of each drift, a blast so weakened the cover that the roof broke and a rush of sand followed. A large hole was made on the surface and without warning a small municipal hospital dropped forty feet, causing the death of seven of the inmates.[¶]

India.

Coal mining in the Bengal field has caused disturbance of the surface along the outcrop. At the Khoira Colliery the mining of 10 feet 6 inches of coal dipping 30 degrees has caused complete subsidence of the surface where the workings are shallow. At the Barrea Colliery, owing to the value of the rice land, stowing has been used to reduce the amount of subsidence.[§] In the same field mining of thick coal overlaid by thick beds of sandstone has been attended by extensive falls of roof which have produced fatal air-blasts.[**]

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[*] Oestrr. Zeit. für B.-, u. H.-W., p. 457, 1876.

[†] Oestrr. Zeit. für B.-, u. H.-W., 1882.

[‡] Helmhacker, R. "Land Subsidence at Brux, Bohemia." Trans. Inst. Min. Engrs., Vol. 10, p. 583, 1895-96.

[¶] Oestrr. Zeit. für B., u. H.-W., Vol. 58, p. 31, 1910.

[§] Stonier, G. A. "Bengal Coal Fields." Trans. Inst. Min. Engrs., Vol. 28, p. 537, 1904.

[**] Adamson, T. Trans. Inst. Min. Engrs., Vol. 29, p. 425, 1905.

South Africa.

In the diamond mines of South Africa there have been rushes of mud into the mines. These have been due to water softening the steep walls of the open pits which then give way and fill the mine openings.*

In connection with the Rand gold mines there has been surface subsidence similar to that caused by deep longwall coal mining.† The maximum depth from which mining has affected the surface has been 710 feet on the Champ d'Or. Other depths from which subsidence has extended to the surface are 566 feet at the Bonanza mine, 650 feet at the May Consolidated, 480 feet at the Treasury, 340 feet at the New Kleinfontein, 490 feet at the New Heriot, and 425 feet at the Windsor. At the Gueldenhuis Deep an area 1,000 feet on the strike by 620 feet on the dip, at depths of from 650 to 924 feet, with an average stoping width of 15 feet caved suddenly but no sinking of the surface resulted. Similar results at other mines have led South African engineers to conclude that cavings of stopes below 1,000 feet in depth will not affect the surface.

UNITED STATES.

Alabama.

In Alabama little attention has been given to the subsidence problem, owing to the fact that many of the coal mining companies have been operating under land to which they hold the title and of which the surface has relatively little value in comparison with the coal. At least 90 per cent of the mining is at a depth of less than 400 feet. Some cracks have extended to the surface and when damage has been caused to property not owned by the mining companies, it has usually been possible for the mining companies to make settlements not greatly out of proportion to the damage done.‡ At the present time some mining is being carried on where the cover is as much as from 800 to 1,200 feet and very little trouble is being experienced.

Idaho.

In the metal mines of the Coeur d'Alene district, disturbances have been noted which apparently are due to causes similar to those which have produced air-blasts in other ore mines.* Recently at an Idaho ore mine two men were killed and four seriously injured by an air-blast.

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*Williams, G. F. "Diamond Mines of South Africa," pp 400-404.

†Richardson, A. "Subsidence in Underground Mines." Jour. of the Chem. Met. and Min. Soc. of S. A., Mar., 1907; Eng. and Min. Jour., Vol. 84, p. 196, 1907.

‡Private correspondence.

Illinois.

Subsidence of the surface due to coal mining has attracted attention in Illinois for a number of years. In the early days of coal mining, when only the shallow beds were mined, the surface was seriously damaged, but in those days the price of farm lands was low and most of the mining was conducted in sections not thickly populated. The first important suit for damages that was appealed to the higher courts in Illinois was in Sangamon county in 1880 (Wilms vs. Jess, 94 Ill. 464). Since that date but few subsidence cases have been tried in the higher courts in Illinois, most of the claims for damages being settled by arbitration or by decisions of the lower court.

An investigation in 1914 showed that there had been surface subsidence in the most important coal producing counties of the state. Twenty-four of the total of fifty-two counties in which coal is mined produced 94 per cent of the coal mined in the state in 1913, and in twenty-one of these counties, subsidence, in various stages and degrees of intensity, was noted.

The reported damages include injury to farm lands and buildings, to city buildings and streets, to railroads and highways, and to domestic and municipal water supplies. Large tracts of farm land in northern Illinois are reported to be damaged by disturbance of surface drainage due to subsidence. There has been litigation to determine the extent to which mining is responsible for the inundation of lands adjacent to waterways and streams.

Few instances of injury to persons by subsidence of the surface have been reported. Mining at shallow depths has permitted the movement of large bodies of surficial material, at time resulting in a rush of sand, clay, and water into the mine, causing serious damage to the mine. Fortunately there have been but few such instances of personal injury to miners from such rushes. This may be due largely to the precautionary steps which have been taken since the accident in the longwall field in 1883 at Braidwood. The disaster at Mine No. 2 of the Diamond Coal Company near Braidwood, Illinois, was due to the inrush of water through surface breaks caused by subsidence.†

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*Sizer, F. L. "An Air-Blast or Earth Movement." Mines and Minerals, Vol. 33, p. 87, 1912.

†Coal Report of Illinois, p. 97, 1883. Roy, A. "History of the Coal Mines of the U. S." Columbus, pp. 190-194, 3d Ed., 1907.

A horizontal bed, 3 feet thick, was being mined at a depth of about 100 feet. The overlying strata are largely shale and clay. Longwall mining had permitted the surface to sink and, at various points at which the rock cover was thin, cracks and breaks extended for some distance up into the surficial material. In February, 1883, there had been a heavy fall of snow followed by a thaw and rain. On February 15, vast sheets of water were standing on the prairie and on the following day a number of the miners did not go to work, as they feared that the water would break through into the mine. At 11 a. m. on February 16 there occurred a cave which permitted a great inrush of water from the surface. The flow of water cut out a larger inlet to the mine and in a short time all of the low points on the roadways were filled with water so that escape was impossible. In three hours the entire mine was filled and the water rose to the surface. Sixty-one men and boys failed to escape before the mine was flooded.

A comprehensive report upon subsidence in Illinois has been prepared by L. E. Young and will appear as a contribution of the Cooperative Investigation by the Illinois Geological Survey.

Indiana.

The following data regarding subsidence in Indiana have been furnished by H. I. Smith, mining engineer, U. S. Bureau of Mines:

A few squeezes have been reported in the mines near Evansville. At one mine operated under the Ohio River at a depth of 260 feet below the river bottom no trouble from the overlying river was reported and at another mine operated at a depth of about 300 feet no loss of coal due to squeezes was reported when about 55 per cent of the coal was removed. Local squeezes occurred but were stopped by a barrier pillar and the coal was reached from the next set of parallel entries.

Probably the greatest damage from subsidence in Indiana has been in Clay county over the upper and lower block coal beds. In one instance in which there was from 20 to 40 feet of cover, consisting of shale with 2 to 6 feet of clay and soil on top, the overlying material was so yielding that an outline of each pillar or stump could be traced on the surface. After a period of twenty years these sinks are said to have evened up, leaving little or no trace upon the surface. Over recent workings succeeding rooms can be traced on the surface by pit-holes or sinks. In some cases the strata have broken through to the surface and the depth of the hole is the same as the thickness of the coal, that is, about five feet. Local residents state that within five years farm land again becomes tillable and in twenty years the depressions have disappeared. This does not apply to large swags which cover a number of adjacent rooms and which in some cases must be drained. A good example of these swags is found at West Seeleyville in the field north of the interurban stop and between the interurban stop and the Vandalia Railroad. These swags are said to have occurred one year before the mine was abandoned, and the coal is said to lie at a depth of 110 feet.

In Vigo county, about 1½ miles north of Miami, a number of small sinks were observed in a cultivated field. On the opposite side of the road, in Clay county, one of the sinks has broken through and is about five feet deep. Other depressions were observed in the line of these workings, but were not broken through.

A private correspondent reports that in Linton, about six years ago, one side of a concrete block house dropped from 2½ to 3 feet. The break extended from top to bottom and passed through the blocks instead of following the joints. Two or three years ago one section of an L-shaped school house was badly damaged and the front end of a store fell in. The court records show five or six suits at Linton for recovery of damages due to mining. The coal is worked on the room-and-pillar system.

Kansas.

Longwall mining at shallow depths in Kansas in the vicinity of Osage City has caused some subsidence, but no damage has been done to sidewalks, brick buildings, etc. The coal is from 12 to 18 inches thick, and lies at a depth of 70 to 80 feet.[*] Above the coal is a light limestone and upon it rest the upper Pennsylvania strata of alternating shale and limestone.[†]

It is reported that subsidence of surface has resulted from the removal of salt by brine-pumping. The salt measures are about 400 feet thick and are covered by a total thickness of 600 feet of beds of shale, limestone, and sandstone.

In the southeastern part of the state mining is conducted on the room-and-pillar system in coal dipping gently toward the west. There have been subsidences, especially near the outcrop, but no extensive damage has been done.

Near Leavenworth longwall mining is carried on at a depth of about 700 feet in a bed 19 to 24 inches thick. There are no published records of subsidence. The surface is rolling and no damage would be likely

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[*] Private correspondence.

[†] Kansas State Geol. Survey, Vol. 8, p. 70.

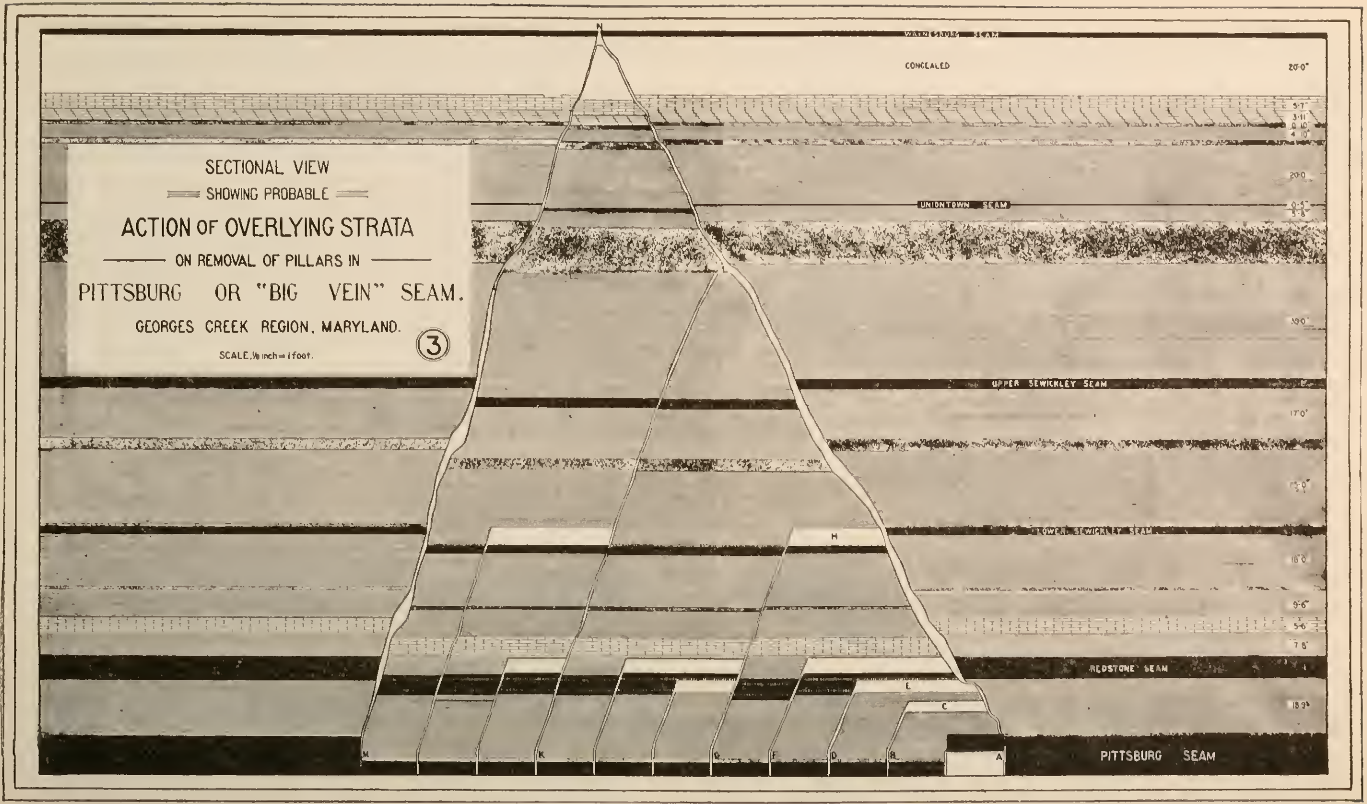

Fig. 1. Probable Effect of Removal of Pillar Coal.

except to buildings, paving, pipes or sewers. The numerous stone and brick buildings of the State Penitentiary at Lansing have been undermined by the State Mine, but show no evidences of subsidence. Mines extended under the Missouri River show no seepage of river water.

Maryland.

In the George's Creek Region, Maryland, the coal seam varies from 6 feet 6 inches to 9 feet 10 inches in thickness and, when the pillar coal is removed, falls occur which extend to the surface.* Fig. 1 indicates the supposed effect of the removal of the pillars under the overlying strata where the surface is 250 feet above the coal seam. Subsidence extends to the surface in such a case after the pillars have been drawn back 220 feet. After the first break occurs at *B* due to drawing the pillar at *A*, the entire block of roof sinks and causes the cavity *C*. The distance from *A to B* horizontally is 40 feet, or 40 feet of pillar have been taken out when the first fall occurs. The second fall occurs at *D* and the fracture line extends to the space *E* at the Redstone seam. This break occurs

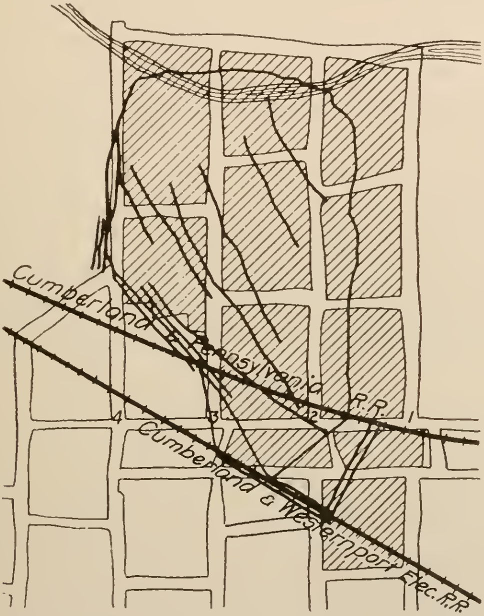
Cumberland & Pennsylvania R.R.
1
2
3
4
Cumberland & Westernport Elec. R.R.

FIG. 2. RELATION OF SURFACE CRACKS TO UNDERGROUND WORKINGS.

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*Reppert, A. E. "Pillar Falls and the Economical Recovering of Coal from Pillars." Proc. West. Va. Coal Mining Institute, p. 110, 1911.

after 60 feet of pillar has been taken out and the break extends almost 40 feet above the floor of the Pittsburgh seam. The third fall extends to *F* and the fourth to *G*, the line of fracture in the latter case extending to the space *H* in the Lower Sewickley seam, 85 feet above the floor of the Pittsburgh seam, the total length of pillars drawn up to this stage being 100 feet. The next break occurs when pillars have been drawn for 160 feet to *K* and the break extends to *L*. When the pillars are drawn back a distance of 220 feet to *M* the fracture extends to the surface at *N*, a height of 250 feet above the bottom of the Pittsburgh seam. This fracture line is approximately correct as shown in Fig. 1 and is based on actual survey and observation of a large number of surface breaks in relation to the mine workings. Fig. 2 indicates the position of surface breaks due to the removal of pillars at a depth of 170 feet over an area 300 feet by 350 feet, the thickness of coal averaging 8 feet. The first surface break occurred between rooms No. 1 and 2 and was about 70 feet from the barrier pillar. The average angle of fracture from the vertical is 22° 30′. The break along the barrier pillar at the top of the rooms was at an angle of 14° from the vertical, while the break along the left hand pillar of No. 4 room was nearly vertical. The conclusion has been drawn that "until a pillar fall extends to the surface, the fracture is conical in shape, but as the pillar line extends down the rooms beyond the first surface break, the strata fracture on a nearly vertical line."*

Michigan.

Copper Mines.—In the deep copper mines of northern Michigan extensive falls of roof have produced air-blasts. At great depths the pillars left to support the roof, or at times masses of poor rock left unmined, show the effect of the tremendous weight upon them. The edges of these pillars fail first and large slabs may burst off and fly some distance. The pillars fail suddenly and the fall of rock may be extensive enough to cause a jar that will be felt on the surface.†

Beginning in 1904 there were a number of caves at the Atlantic mine. The stopes averaged 15 feet in width and extended for a mile along the strike of the lode and for one-half mile down the dip. These falls became so extensive and the pressure on the pillars became so great that the shafts were ruined and the mine was put out of commission, May, 1906.‡

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*Op. Cit., p. 113.

†McNair, F. W. "Deep Mining in the Lake Superior District." Eng. and Min. Jour., Vol. 83, p. 322.

‡Stevens, H. J. Copper Handbook, Vol. 10, p. 377.

Extensive falls of the hanging-wall or roof have caused trouble at the Quincy mine. Stoping averages 20 feet in width and in parts of the mine has been carried on to a depth of more than 6,000 feet on the dip of the lode. The sudden and violent compression of air by falls in the mine has caused damage to the levels and shafts and has produced miniature earthquakes.* The general manager of the Quincy mine, in his report for the year 1914, writes as follows:

"On March 25 air-blasts occurred throughout the mine and continued intermittently for a week or ten days. As a consequence, various cross-cuts and drifts were crushed and closed up. No. 6 shaft timbers were seriously crushed between the 51st and 58th levels, and No. 2 shaft was crushed and closed between the 40th and 50th levels. About 500 feet of the crushed section of No. 2 shaft had to be entirely recovered and retimbered at an expense nearly as great as that of sinking a new shaft. In the remaining portion of the damaged shaft about half of the timbers were replaced.

"Below the 50th level the shaft was not damaged by the air-blasts, though the cross-cuts at the 57th, 64th, 65th and 66th levels were entirely closed, and the levels north were badly crushed.

"In earlier days, when air-blasts were little understood, it was the custom to stope out the lode without reference to the shaft. Going through the upper portions of No. 2 and No. 6 shafts is like going down through open stopes, with practically no pillars left to protect the shafts. It was in the lower part of these sections that the caving and crushing took place with such serious results.

"At the present bottom of the mine, pillars are being left 200 feet on each side of the shaft. The air-blasts have never caused any damage to these sections of the shaft.

"Air-blasts have continued with more or less frequency since July, though they have not damaged or retarded the work to any great extent. In order to meet the air-blasts and prevent as far as possible the damages caused by them, as fast as the mining in each stope is finished, the bottom of the stope along the back of the level is filled with poor rock, constituting what is termed 'rib work.' Experience has taught that these rock-packs are the most effective means yet employed to lessen the damages caused by air-blasts. In order, however, that the highest effectiveness possible may be secured within the limits of profitable mining at greater depth, this rib work should be still further strengthened. It is estimated that the voids in the rock give it a shrinkage of about 20 per cent at the present depth of the mine. In order to lessen this shrinkage and strengthen the rib work, the question of filling the voids with bank sand, stamp sand, or crushed rock is receiving serious attention, inasmuch as provision must be made for better and stronger supports to the back of each level, as fast as stoping is finished. During the year $57,190 was spent for the recovery of the shafts and levels that were damaged by the air-blasts, and $21,487 has been expended in the fight toward preventing damages by air-blasts."

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*Ibid., Vol. 10, p. 1444.

These rock movements appear to be confined to the deeper portions of the mines and no effect is noted at the surface except a vibration, giving the effect of an earthquake. No subsidence is reported.

Iron Mines.—At a number of points on the Lake Superior iron ranges, in Michigan and in Minnesota, the mining of extensive bodies of ore close to the surface has caused the subsidence of large areas. A considerable portion of the iron ranges is covered with glacial deposits and when the bedrock is shattered by mining rushes of sand into the mine may follow.

At several important mines large caves have occurred under important railway lines. Owing to the inclination, volume, and extent of the ore body it was thought that it would be more practical to bring the track to grade by filling than to construct a new line entirely outside the subsiding area. The continued development of the ore body and deeper mining have caused the subsidence to continue from year to year so that now the problem of filling has become a very expensive one for the railroad company.

Missouri.

At Lexington, Missouri, the mining of 20 inches of coal at a depth of 160 feet has caused subsidence amounting in places to the full thickness of the coal. No serious damage has resulted.*

In the Joplin district extensive caves of the surface have resulted from the mining of large bodies of zinc ore at shallow depths, but no detailed study of subsidence has been made.

During the year 1915 a number of mills were damaged through the tailings piles falling into the excavations. One cave-in resulted in the death of several men in the mine by drowning and it seems inevitable that there will be many more caves in the district, particularly in the sheet deposits where small pillars are left.

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*Private correspondence.

In the Flat River district large areas have been mined at depths up to 700 feet. The beds are practically horizontal and have good roofs. "In some mined-out areas where the pillars have been removed and slabbed the back has come in, extended over an area of from one to three acres. In two such caved areas that have been examined it has been found that in each case a natural arch has been formed and the caved material has nearly filled the opening to the back. The largest of the caves of this type has run up about 100 feet into the back, which leaves about 400 feet of undisturbed formation above it."*

Oklahoma.

Coal mining in Oklahoma has caused surface disturbance at a number of places. On September 4, 1914, a serious squeeze occurred in Mine No. 1 of the Union Coal Company at Adamson, which resulted in the death of thirteen miners, complete loss of the mine, and minor surface damage. The mine was opened on the pillar-and-room system. Fig. 3 shows a cross-section through the two thin seams that were worked. The lower seam is 4 feet thick and 45 feet above it is a seam 2 feet 3 inches thick, not worked. The beds dip about 30 degrees. Rooms were turned on 33-foot centers, the room pillars being not more than 9 to 12 feet wide and in places much less. The roof was a sandy shale, 28 feet thick, and above it an equal thickness of hard sandstone and a little fireclay. The squeeze came comparatively quickly, completely

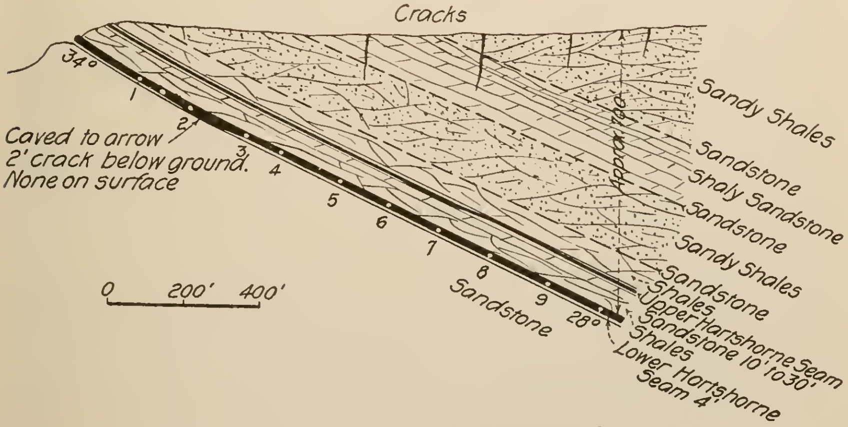

FIG. 3. SECTION THROUGH ADAMSON MINE, OKLAHOMA.

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*Private correspondence.

closing the slope. A number of cracks appeared on the surface, but no serious surface damage was done.*

There are no records available showing the amount of surface subsidence, but the surface cracks have been located fairly accurately. Assuming that the underground break extended to the tenth level, there was about 700 feet of cover, and the angle of break may be calculated for the cracks farthest from the mouth of the slope. It has been assumed that the large crack over the east side resulted when the 7th East entry was lost. Similarly cracks on the west may be correlated with the underground movement on the 6th West and 9th West entries.

George S. Rice, Chief Mining Engineer, U. S. Bureau of Mines, says in regard to this accident:

"What happened was almost inevitable with a strong roof and increasing depth, where so large a percentage of the coal had been extracted in the advance work and the pillars left standing. Estimating from the map, about 75 per cent of the coal had been taken out by the entries and rooms. As a result, in the lowest level of the mine there was a load of over 3,000 pounds per square inch on the 25 per cent of coal which remained in the entry and room pillars. This is more than bituminous coal can sustain. Therefore, I am inclined to think that the main support of the overlying strata had been carried by arch stresses, the arch being buttressed on one side by the strata near the outcrop and on the other by the dipping strata. Then when the fracture occurred at the latter buttress, it threw the entire weight on the mine pillars, causing them to be crushed. The surface cracks, reported by reliable witnesses to have occurred prior to the collapse, running parallel but in advance (horizontally) of the lowest level, indicated that a shear fracture had occurred in a plane roughly at right angles to the plane of the dipping beds, and when this fracture extended laterally to a sufficient distance, it formed a slip plane which permitted the entire weight of the overlying strata less friction to be thrown upon the pillars, resulting in the collapse of the mine."

Pennsylvania.

Pennsylvania Anthracite Field.—In the United States surface subsidence due to mining operations has received most attention in the anthracite field of Pennsylvania, and notably in the city of Scranton. Of the total area of 176 square miles in the Wyoming region, incorporated boroughs and cities cover 101 square miles.† The city of Scranton extends entirely across the coal field, a distance of five miles, and for the same distance along the valley. Beneath a portion of the city are eleven important coal beds having an average aggregate thickness of 58 feet. It has been estimated that during the seventy-five years of active mining under the city 177,000,000 tons of coal have been produced. This output represents a volume of 198,000,000 cubic yards, an amount in excess of the total estimated excavation in connection with the Panama Canal.* In the early years of mining accurate maps of the mines were not made and preserved and a number of the coal companies made little effort to columnize the pillars in the various coal beds that were worked. These conditions have made it difficult to study the problem and to provide adequate and practical remedies.

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*Brown, G. M. "A Sudden Squeeze in an Oklahoma Mine." Coal Age, Vol. 6, p. 588, 1914.

†Report of Pa. State Anthracite Mine Cave Commission, 1913.

While subsidence has occurred in many parts of the city, it was estimated in 1912 that not more than 15 per cent of the area of the city was threatened.

Although surface subsidence had damaged property within the city limits prior to August 29, 1909, the public in general gave the matter little connected attention. This was due largely to the fact that the mining companies hold deeds which permit them to remove the coal without liability for damage to the surface. On the date mentioned, surface subsidence caused serious damage to a school building, which fortunately was not in use at that season of the year.

Following this cave Honorable J. B. Dimmick, then Mayor of Scranton, by approval of the City Council and the Board of Control of the Scranton School District, created a Commission to investigate the physical causes of mine caves and the legal responsibility therefor. The report of this Commission, submitted March 20, 1911, was published in 1912 as Bulletin No. 25, U. S. Bureau of Mines. It was the result largely of the investigations of Eli T. Conner and William Griffith and reviews the existing mining conditions and discusses at length methods for supporting the surface. Considerable attention was given to "flushing" methods of filling and the report contains a chapter by N. H. Darton, entitled "Notes on Sand for Mine Flushing in the Scranton Region." An appendix includes the results of tests to determine the compressive strength of anthracite† and of tests of various kinds of materials for supporting the roof in mine workings.‡

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*Bul. 25, U. S. Bureau of Mines, Washington, 1912.

†These tests were made for a committee of the Scranton Engineers' Club in 1900 in the engineering laboratories at Cornell University, Lehigh University, and the Pennsylvania State College.

‡Fifteen tests on the compressive strength of materials for supporting roof were made for this investigation at the Engineering Laboratory of Lehigh University.

The U. S. Bureau of Mines continued investigations along several lines which were discussed in the report by Conner and Griffith and subsequently published the results as bulletins. Bulletin No. 60 contains the investigation by Charles Enzian entitled "Hydraulic Mine Filling; Its Use in the Pennsylvania Anthracite Fields." Bulletin No. 45 by N. H. Darton is a report upon "Sand Available for Filling Mine Workings in the Northern Anthracite Basin of Pennsylvania."

In 1911 Governor Tener appointed the Pennsylvania State Anthracite Mine Cave Commission to investigate the physical conditions and legal rights of surface support. This Commission consisted of:

W. J. Richards, Vice-President and General Manager, Philadelphia and Reading Coal and Iron Co.

G. M. Davies, Mining Contractor.

J. Benjamin Dimmick, Mayor of Scranton.

E. G. Lynett, Editor, Scranton Truth.

W. L. Connell, Coal Operator, Ex-Mayor of Scranton.

R. A. Phillips, General Manager, Coal Department, Delaware, Lackawanna and Western Railroad.

W. H. Lewis, Retired Coal Operator.

Charles Enzian, Mining Engineer, U. S. Bureau of Mines.

W. A. Lathrop, President, Lehigh Coal and Navigation Co.

(Owing to the death of W. A. Lathrop, he was succeeded by S. D. Warriner, President, Lehigh Coal and Navigation Company.)

The investigation of this Commission covered the period from June 12, 1911, to March 1, 1913, when the report was submitted to the Governor and Legislature, the text of the report being printed in the Journal of the Pennsylvania Legislature for 1913, Volume 5, page 5947. This Journal report contains none of the maps or illustrations essential to an understanding of the report, which is available, therefore, only in typewritten form. In addition to the field investigations, the Commission conducted a series of thirty-four tests upon supporting materials, the tests being made at the Bureau of Mines Laboratory in Pittsburgh, in cooperation with the U. S. Bureau of Standards, under the supervision of Charles Enzian.

The 1913 session of the Pennsylvania General Assembly enacted the Davis Mine Cave Law,* which provides for the protection of public highways, streets, etc., and also provides for the creation by municipalities of a Bureau of Mine Inspection and Surface Support, the duties of which are to investigate the mine workings in their relation to the support of public highways. Thus far the City of Scranton is the only municipality in Pennsylvania that has passed an ordinance for the creation of such a bureau. The City of Scranton Bureau of Mine Inspection and Surface Support has been in existence since August 5, 1913, and has investigated a number of mines and made reports upon such investigations. The reports of this Bureau are not in print.

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*Act No. 857. Approved July 26, 1913.

The City Council of Scranton on October 24, 1913, appointed H. D. Johnson and D. T. Williams to prepare a report upon the mine of the Peoples Coal Company, and in their work they were assisted by Chas. Enzian, Mining Engineer of the U. S. Bureau of Mines. The report of these gentlemen, submitted December 12, 1913, was made under the provisions of the Davis Mine Cave Act. It comprises ninety-three typewritten pages and, in addition to reviewing the mining conditions in three city wards, contains much concise information of general application in the study of the problem of surface subsidence.

The subsidence problem in the Pennsylvania anthracite field has been further complicated by the glacial deposits which occasionally are localized in "pot-holes." These pot-holes may extend to a considerable depth below the glacial sheet, making it dangerous to carry on any mining operations near them. When the subsidence of the coal measures extends to a pot-hole filled with sand and water, the water and some of the sand may seep into the mine, and if the subsidence has shattered the intervening strata or if the roof has been thin and weak, a rush of sand may fill the mine workings. The difficulties of mining under such glacial deposits have been recently presented* to the mining profession and fourteen accidents have been noted.

In most of these a large area of the workings has been filled by the rush of glacial material and water, and in several instances extensive surface subsidence resulted.

On June 10, 1914, at the Sugar Notch Mine, a breast in the Kidney bed broke into the wash. The material entering the mine was largely sand and clay in a semi-fluid state, and its volume was estimated at 20,000 cubic yards. This filled several thousand feet of gangways and tunnels, but no lives were lost. The accompanying illustrations show the important data in connection with this accident. Fig. 4 shows the mine workings, the contours of the top of the rock, the location of drill holes, and the important surface features previous to the accident. The cave occurred at the face of breast 15. Fig. 5 shows the conditions

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*Bunting, D. "The Limits of Mining Under Heavy Wash." Amer. Inst. of Min. Engineers, Bul. No. 97, p. 1, 1915.

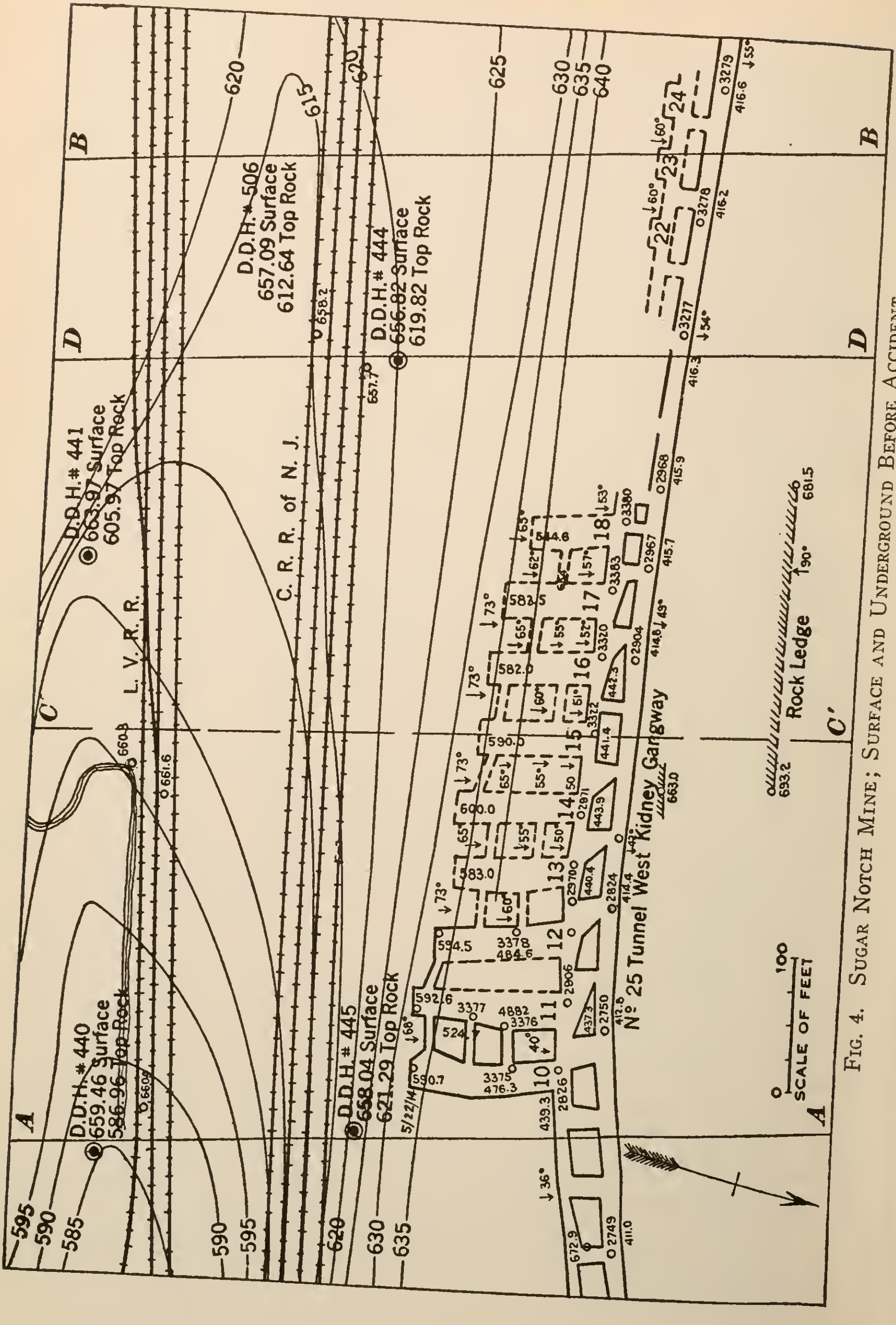

Fig. 4. Sugar Notch Mine; Surface and Underground Before Accident.

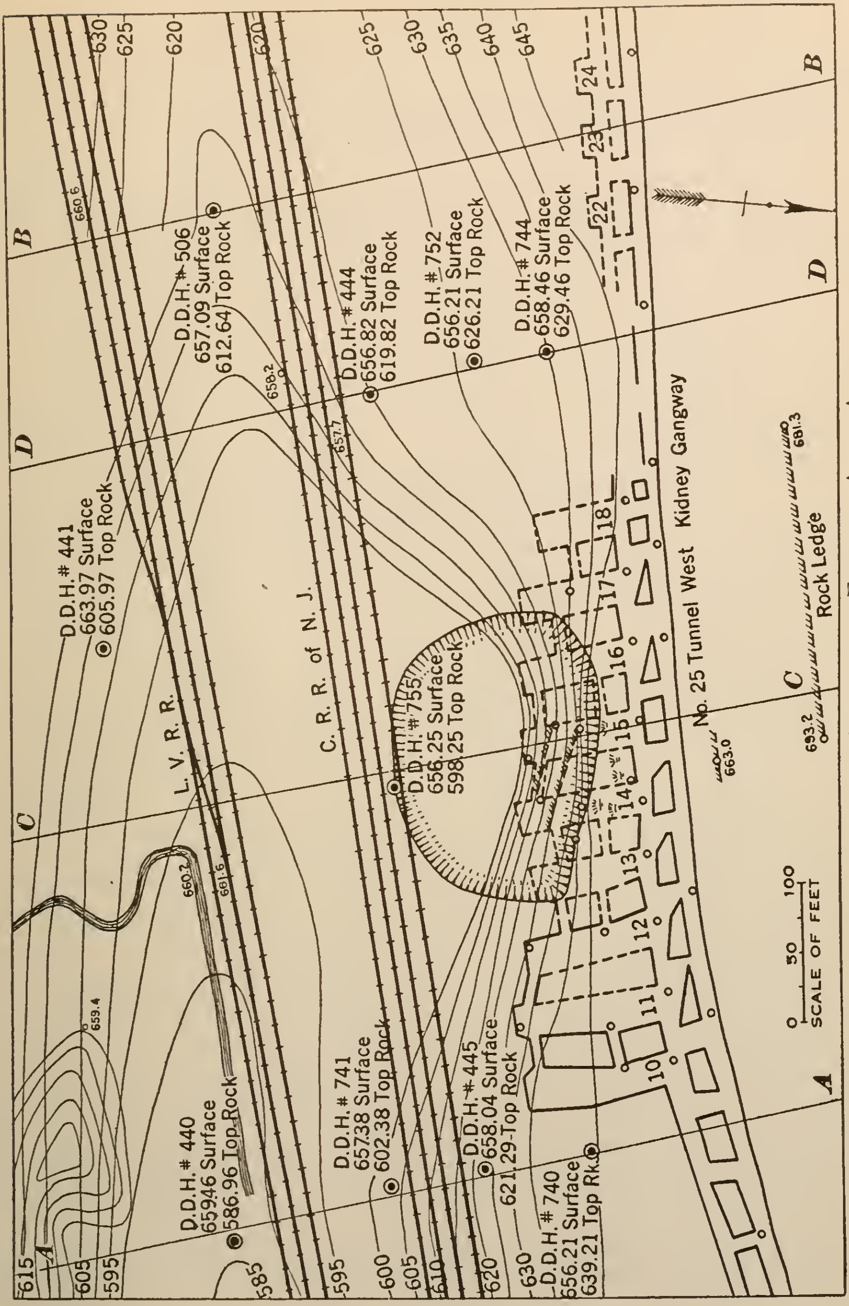

FIG. 5. SURFACE AND UNDERGROUND FEATURES AFTER ACCIDENT.

TABLE 1.

ACCIDENTS IN WYOMING FIELD DUE TO INRUSHES OF SAND AND WATER.

Accident, No.	Date	Mine	Location	Vein	Tapped by	Result
1	July 4, 1872	Burroughs	Plainsville	Hillman	Breast	An inrush of sand and water. A pumpman, the only man in the mine at the time, easily escaped.
2	June 30, 1874	Wanamie No. 18	Wanamie	Red Ash	Breast	Gangway and workings in the vicinity were filled for some distance from the break.
3	Jan., 1882	Maltby	Swoyersville	....	Rock Plane	Gangways were filled; also the shaft for a vertical height of 90 ft.
4	Apr. 23, 1884	Fuller	Swoyersville	Six Foot	Slope	Slope filled to the top for a distance of 900 ft.
5	...., 1884	Ridge	Archbald	Archbald	....	....
6	May, 1885	Ridge	Archbald	Archbald	....	....
7	Dec. 18, 1885	No. 1 Slope	Nanticoke	Ross	Breast	Gangways in the vicinity were completely filled in less than an hour.
8	Aug., 1889	Fuller	Swoyersville	....	Rock Plane	The planes and all workings tributary to it were filled with sand or water.
9	Mar. 1, 1897	Mt. Lookout	Wyoming	Pittston	Breast	A large area of the workings was filled; no men at work at the time.
10	Dec. 30, 1898	Wanamie No. 18	Wanamie	Copper	Breast	Gangway on lower level filled to a height of 2 ft. for a distance of 300 ft. Depression on the surface 100 ft. east and west and 75 ft. north and south.
11	Feb. 2, 1899	Franklin	Wilkes-Barre	Kidney	Breast	Filled gangway to a height of 3 or 4 ft. for a long distance.
12	Apr. 13, 1899	No. 2 slope	Nanticoke	Hillman	Breast	Gangways were filled for several thousand feet. Breasts had been worked 26 years previous; no men at work in the vicinity. Surface depression was 70 to 80 ft. deep.
13	Apr. 25, 1899	Bliss	Hanover	Hillman	Breast	Gangways and tunnel in the vicinity were filled tight to roof. Conical depression on surface 60 ft. in diameter and 40 ft. deep.
14	June 10, 1914	Sugar Notch No. 9	Sugar Notch	Kidney	Breast	Gangways and tunnels were filled tight to the roof. Depression on surface 150 ft. wide, 210 ft. long and 60 ft. deep.

after the cave and indicates the points where additional drill-holes were put down. Fig. 6 shows the conditions which existed in section C-C, before the accident and indicates the supposed limit of the wash. Fig. 7 shows the conditions after the accident and shows the limits of the wash as proven by the additional drill-holes. The face of breast 15 was at an elevation of +590.0 when the break occurred. The elevation of the surface directly above was +657.0 and it was thought that the bottom of the wash was at +630.0. It was planned to carry the breast to +600.0. The bottom of the wash was actually at +600.0 instead of +630.0.

One of the most serious of the accidents noted was the so-called "Nanticoke disaster" of December 18, 1885, which resulted in the loss of 26 men.* The mine workings tapped a pot-hole which was subsequently found to be over 200 feet deep. The hole was 400 feet away from the present stream and was covered by a culm-bank. The thickness of the rock where the cave occurred has been estimated at from 22 to 48 feet. The subsidence produced a hole in the culm-bank 300 feet across.

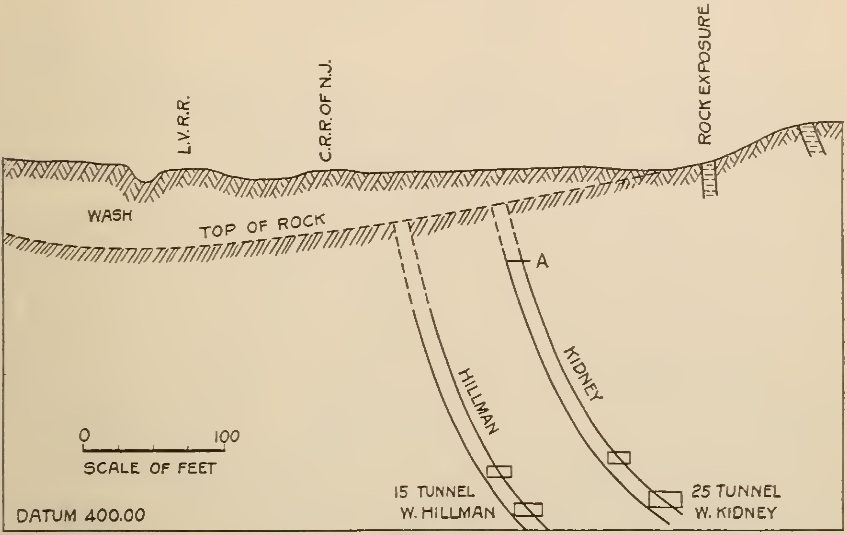

FIG. 6. SUPPOSED CONDITIONS ALONG C-C BEFORE ACCIDENT.

One of the most important occurrences of this nature in the anthracite district, as far as amount of material entering the workings is concerned, was the cave at the Prospect colliery of the Lehigh Valley Coal Co., December 12 and 26, 1915.

It was supposed that the rock over the upper bed of coal was 40 to 50 feet thick and that the surface soil was thin, these being the conditions at points near the break. It was found, however, that the rock where the break occurred was only about 10 feet thick and that the remainder of the cover was loose sand, clay and gravel, apparently glacial material deposited in an old valley. Probably the break was due to the collapse of the thin rock at the bottom of a pot-hole.

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*Williams, G. M. "Dangerous Outcrops," Mines and Minerals, Vol. 20, p. 410, 1900; Ashburner, C. A. "The Geologic Relations of the Nanticoke Disaster." Trans. Amer. Inst. Min. Engrs., Vol. 15, p. 629, 1886-87.

Two breaks occurred, both at times when a small stream flowing over the loose material was flooded, and a large amount of this material was washed into the mine. It was estimated that about 140,000 cubic yards of earth and 350,000,000 gallons of water entered the mine. No lives were lost, but the financial loss was very considerable, as large expense was incurred in changing the channel of the stream, in addition to the cost of pumping out the water and to the loss due to interruption of the work of the colliery.*

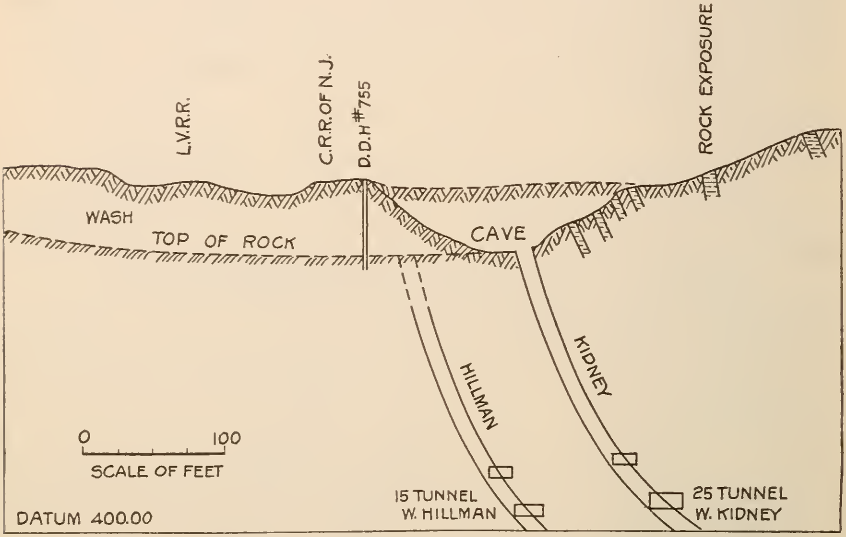

Fig. 7. Actual Conditions Along C-C After Accident.

Pennsylvania Bituminous Field.—In the bituminous fields of Pennsylvania some damage to the surface has resulted from mining operations, but few references to such subsidence are found in the technical press. Generally the deeds to the coal rights have not required the mining companies to support the surface.

In the Connellsville region the surface is of little value as compared with the coal, the topography is rugged and, although cracks extend through from the mine workings to the surface, little attention is paid to them unless they are near important structures. Mining 8 feet of coal at a depth of 600 feet produces cracks as much as 20 inches wide. The recovery of coal varies from 84 to 90 per cent. When it is necessary to protect buildings and railroads as much as 25 to 50 per cent of the coal is left in pillars.* It has been found that at shallow depths, up to approximately 150 feet, subsidence will amount to 50 per cent of the thickness of the coal. At greater depths it will be less, approximating 25 per cent at 300 feet. The attempts to correlate data and to generalize from the data available have not been satisfactory. In discussing the observations made in southwestern Pennsylvania, J. P. K. Miller, Chief Engineer of the H. C. Frick Coke Company, said: "The great difference of strata overlying the coal no doubt contributed largely to the great variation noticed throughout the district. In some parts of this region, the stratum immediately above the coal, between it and the sand formation, varies from a few inches to 16 and 20 feet of shale. Where the sandstone is very close to the top of the coal, the subsidence is considerably greater than it is where the shale thickens; then, too, there is a very heavy percentage of limestone and sandstone in the Leisenring district, while immediately southeast of this, or between Uniontown and Fairchance, the sandstone measures rapidly thin out, and this, too, contributes to the variation in subsidence, or, in a word, where the coal has immediately over it a heavy percentage of sandstone measures, the subsidence is greater than where a thick stratum of shales appears immediately above the coal. Of course, this is only our opinion, but it seems to be the only good reason we can give for the difference in subsidence where the cover is approximately the same (thickness). As an illustration, in the territory in the vicinity of Uniontown, where heavy shales appear above the coal, we have observed 18 inches of subsidence where the cover is 300 feet; in the Leisenring district where heavy sandstone measures appear above the coal and there is a thin layer of shale immediately above the coal, the subsidence is approximately 30 inches. There is another condition that, no doubt, contributes largely in bringing about this difference in subsidences and that is the heavy layer of fireclay immediately beneath the seam of coal appearing in the Uniontown district; while very good bottom conditions—'hard bottom'—appear in the Leisenring district, and the writer believes it may be concluded naturally that this difference in the condition of the bottom section has more to do with the difference in subsidence than the first two conditions above mentioned."

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*Coal Age, Vol. 9, p. 373, 1916.

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*Private correspondence.

The outcrop of the Pittsburgh coal bed extends for many miles in western Pennsylvania, and above these shallow workings many sinkholes have formed. These have attracted very little public attention, as they are considered to be of only a temporary character and most of the buildings above the mined areas are frame and the damage to them has also been only temporary, for if tilted out of line, these buildings have frequently resumed their normal condition after a few months.

Observations made by another company show that the surface subsided 2 feet, 9 inches after practically all of an 8-foot seam had been removed at a depth of 400 feet. The overlying rocks consisted of shales, sandstone and limestone in alternating beds, the thickest limestone bed being 200 feet from the surface and reaching a thickness of 50 feet. There are six other beds of limestone varying from 20 to 30 feet, the total thickness of the seven beds being about 170 feet. The remainder of the column is about equally divided between fireclays, sandstones, and shales. The subsidence took place about twelve months after the mining of the pillars began.*

In the construction of the Greentree Tunnel of the Wabash Pittsburgh Terminal Railway Company in the Pittsburgh district it was found that early mining operations had removed coal from a bed immediately beneath the projected line of the tunnel and that coal had been removed from another bed overlying. W. F. Purdy, Chief Engineer, describes the conditions as follows: "When the heading of the tunnel had proceeded about 500 feet from the west portal we encountered broken ground. The material was fairly solid gray shale which was easy to drill, but none of it could be removed without heavy blasting. At first the only indication of disturbance was that the rock showed soft pockets, and a little later the strata had separated so that large pockets could be excavated without blasting. After having proceeded about 20 feet into the material which had become more or less loosened without our having been able to account satisfactorily for the nature of the ground, the bottom of the heading suddenly broke down about 30 feet back from the face of the heading, permitting partial collapse of the timbering as the settlement was about 2 feet.

"It developed that the broken ground encountered in the heading was at the apex of the mass affected by the subsidence in the mine and the top of the heading was approximately 50 feet above the mine level. Unknown to us at the time, we had been driving the heading for some

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*Private correspondence.

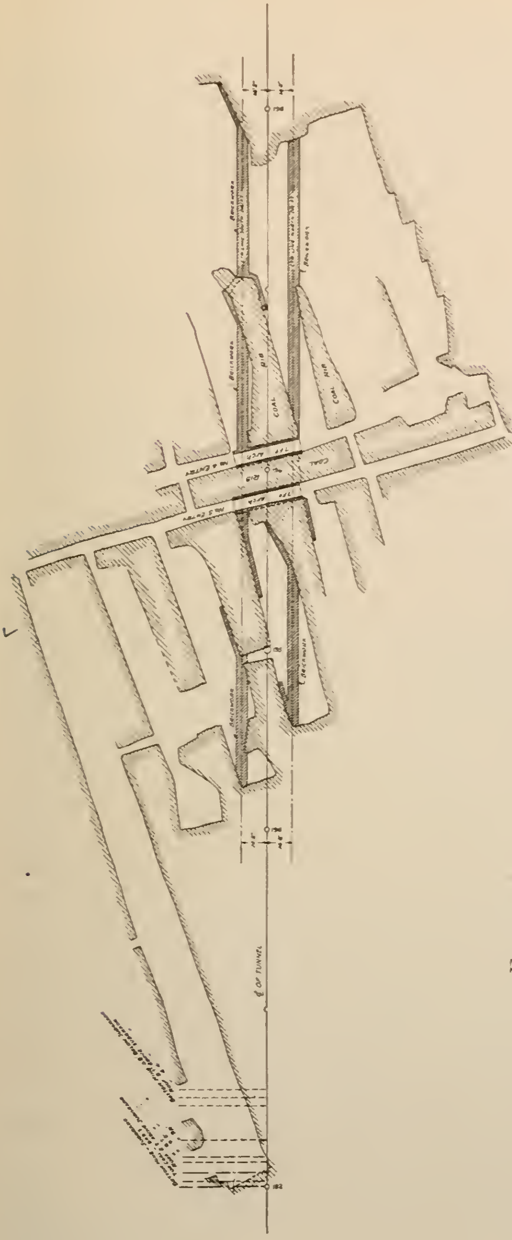

Fig. 8. Plan of Greentree Tunnel and Old Mine Workings.

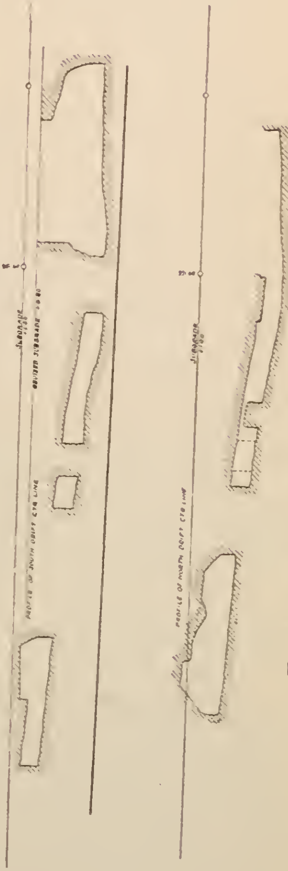
SUBGRADE
REVISED SUBGRADE
PROFILE OF SOUTH DRIFT CTR LINE
PROFILE OF NORTH DRIFT CTR LINE

Fig. 9. Section Through Tunnel and Mine.

distance over the broken ground with a wedge of solid rock between the bottom of the heading and the broken material beneath. When we had reached the point where this continually thinning wedge would no longer support the weight, the ledge broke, practically at right angles to our tunnel, and allowed the timbering to drop as before stated.

"We moved back in the heading to the solid rock and drove a shaft on a steep incline until we reached the floor of the mine about 35 feet below the bottom of our heading. There we found that a considerable area of coal had been mined several years earlier and because of a "swamp" and the apparently heavy expense for pumping the mine was abandoned after having drawn nearly all of the ribs and pillars.

"After reaching the mine level we drove two diverging drifts from the foot of the shaft until we were under the two sides of the tunnel and then carried the two drifts ahead under the prospective tunnel walls. For about 70 feet the ground was entirely broken between the mine and the grade of our tunnel and we built solid brick masonry walls 18 feet in height to provide foundation for the regular tunnel side-walls.

"After leaving the space of about 70 feet already mentioned the ribs and pillars had not been withdrawn and the falling from mine roof was not serious. At some points the roof slate had fallen in to a height about 6 feet above the normal mine roof, and at other places there had been no breakage whatever. We followed up the mine entries and rooms for a distance of about 600 feet at which point on account of the convergence of the tunnel grade toward the mine level the same elevation was common to the floor of mine and the grade of the tunnel.

"The alinement of our tunnel makes an angle of approximately thirty degrees with the rooms in the mine and the work was more complicated on that account. We built solid brick masonry walls diagonally across the mine entries and rooms under each of the two tunnel side-walls. Where there was a space of several feet between the roof of the mine and the normal foundation of tunnel walls, we built the brick walls thick enough to prevent any danger of crushing the slate rock between the brick foundation and the regular tunnel walls. When we reached the point where only two or three feet of natural shale would have been left between the roof of mine and bottom of tunnel wall we cut it all out and carried the brick work up to the grade of the tunnel.

"The tunnel lining is of concrete and the tunnel has been in service for ten years with no sign of settlement or cracking of the concrete

"We also experienced a good deal of trouble, expense and delay on account of another coal mine above the eastern half of the same tunnel. As the top of our tunnel heading approached the bottom of the coal mine and as the intervening wedge of ground became thinner it became very difficult to support the roof of tunnel and we had much trouble on that account, and also because of the mine drainage which poured through the thin ledge of shale between the roof of tunnel and bottom of mine."

West Virginia.

Owing to the character of the topography and the low value of the surface in the coal districts, very few reliable records are available to show the extent of surface subsidence due to coal mining. Some surface movement has been noted where from 7 to 8 feet of coal has been mined and the pillars drawn at depths over 500 feet. When the thickness of the covers is from 200 to 300 feet, the disturbance is greater and "where the cover is light—from 50 to 150 feet—the cracks are sometimes from 2 to 4 feet in width and show a vertical displacement of from 1 to 2 feet."*

The problem of protecting a seam lying from 70 to 80 feet above the seam now being worked has confronted some of the coal mining companies. It is proposed to mine the upper seam before the pillars are drawn in the lower seam, as the subsidence which follows the mining of the pillars in the lower seam greatly disturbs the overlying seams and makes it unprofitable to mine them.

Nature of Damage Due to Disturbance of the Overlying Material.

The damage resulting from the excavation of minerals may be (a) without the mine or (b) within the mine.

In this study the damage external to the mine is the subject of investigation, the internal damage being noted only when it occurs in connection with external damage.

(a) The damage external to the mine may be due to:

1. The vertical, or horizontal, or both vertical and horizontal movement of surface material or surface structures, caused by the subsidence of the strata overlying the excavation.
2. Surface cracks or fissures due to slips, faults, or shear, or to the tension of the surficial beds.

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*Private correspondence.

3. Pit-holes or caves, formed when, instead of gradual and more or less uniform subsidence over a large area, the movement is localized if the excavation is at a shallow depth.
4. Damage by water on account of the lowering of land below the former drainage channels or the high-water levels, and by the derangement of artificial drainage systems, such as sewers in cities or tiling on farms.
5. Interference with or destruction of natural or artificial water-supply.
6. Miniature earthquakes occurring when large masses of rock fall over great areas.

(b) Within the mine, damage may result from:

1. The inflow of water through cracks or breaks caused by subsidence of the strata.
2. The inflow of sand through breaks extending to the surficial material.
3. Local or extensive falls of roof.
4. The failure of pillars, due to the excessive weight of the superincumbent strata.
5. "Air-blasts" or "bumps" accompanying the sudden collapse of pillars and the fall of large areas of roof.
6. Squeezes or creeps.

1. *Nature of Earth Movement.*—Damage to structures on the surface may be the result of either vertical or horizontal movement, or both, and engineering observations in Europe and in America show many interesting facts regarding the extent, the rate, and the duration of surface movements.

"Draw" or "pull" is the variation from the vertical of the line of fracture of rocks that break when the supporting bed or stratum is removed; in other words, the variation from the vertical of the boundary between the disturbed and undisturbed strata. In some cases this is a well-defined plane; in others a zone of indefinite extent. In the case of brittle rocks, the break will be sharp; while in the case of more yielding deposits, such as shales and loose soil, it may be impossible to determine exactly the limits of disturbance.

Several instances of the lifting of objects on the surface have been reported, but no data are available at this time to prove definitely that either a temporary or a permanent elevation of the surface has occurred.

It is claimed that at Northwich, England, where subsidence has resulted from the removal of salt, an elevation of certain streets has occurred.* In Nottinghamshire, England, where a coal seam 3 feet 8 inches thick was mined at a depth of 1,680 feet, it is claimed that there was locally and temporarily an elevation amounting to 4 inches.† Also, it is reported that an "earth tide" is evident in the diamond fields of South Africa, there being noted a rise and fall amounting to 3 inches a day, but this could have no connection with subsidence.‡

The engineer's records at the Warrior Run Colliery of the Lehigh Valley Coal Company in Pennsylvania show that there was a lateral movement of surface monuments as well as vertical movement. (Fig. 10 shows the dates of observation and the amount of the movement.) This resulted from a squeeze in workings extending from 500 to 1,000 feet in depth on a coal seam dipping approximately 30 degrees.¶

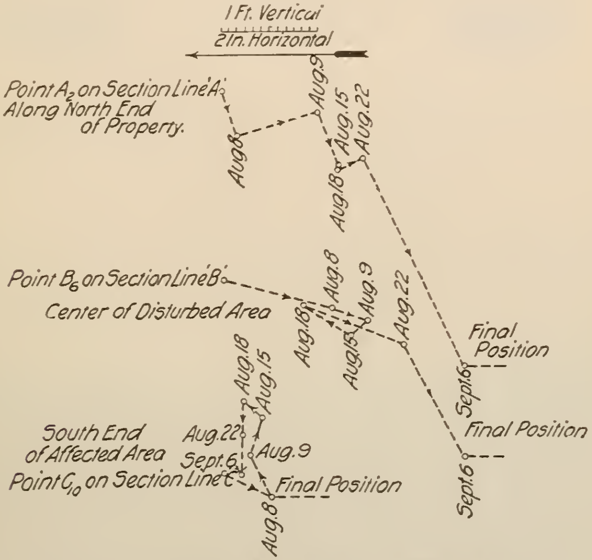

FIG. 10. LATERAL MOVEMENT OF MONUMENTS.

Sags of the surface, or depressions without important breaks or cracks, occur when the movement is due to the bending rather than to the breaking of the strata and when the surficial material, without sud-

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*Trans. Inst. Min. Engrs., Vol. 19, 241, 1899.
†Proc. Inst. of Civ. Engrs., Vol. 135, p. 114, 1898.
‡Jour. Chem. Met. and Min. Soc. of S. A., October, 1911.
¶Enzian, Charles "The Warrior Run Mine Disaster," Mines and Minerals, Vol. 27, p. 439, 1907.

den movement, accommodates itself to the new inclination of the bedrock. Observations in the coal districts indicate that the extent and the gradient of such sags are influenced by the rate of advance of the working face, particularly in longwall mining; by the character and amount of filling; and by the ratio between the depth and the lateral extent of the mine workings; as well as by geological conditions in general.

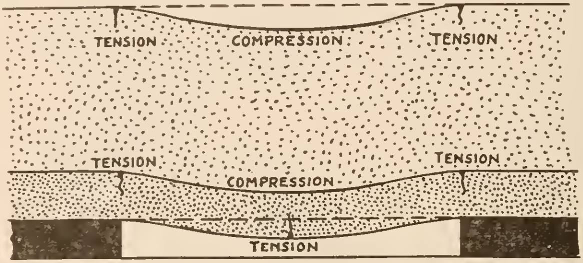

Fig. 11. Tension as a Cause of Surface Cracks.

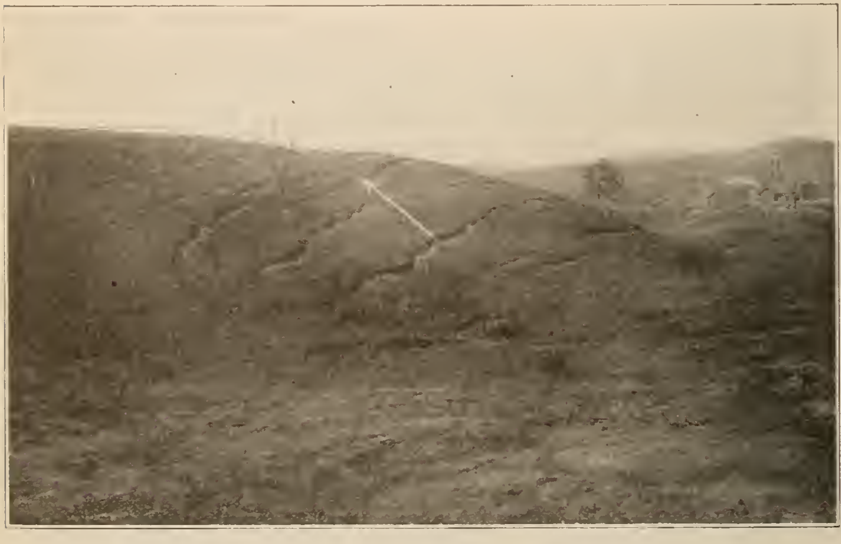

Fig. 12. Surface Cracks in Western Pennsylvania.

2. *Surface Cracks.*—The surface cracks and fissures that appear commonly when mining is carried on at shallow depths may be due to one of several causes. As the mine roof sags over an excavated area the bending action produces compression in the upper part of the strata

near the center of the basin or sag, while around the rim of the basin the upper strata affected are in tension which may be sufficient to cause the surface to break or crack. (Fig. 11.) If the movement is an extensive one and if the height of the surface above the axis of bending is great, the width of the fissure may be considerable. Fissures 2 feet wide have been noted in Illinois and in West Virginia. Fig. 12 shows such surface cracks in western Pennsylvania.

The formation of surface cracks by tension is well demonstrated by an occurrence in Ashland in the anthracite district of Pennsylvania.

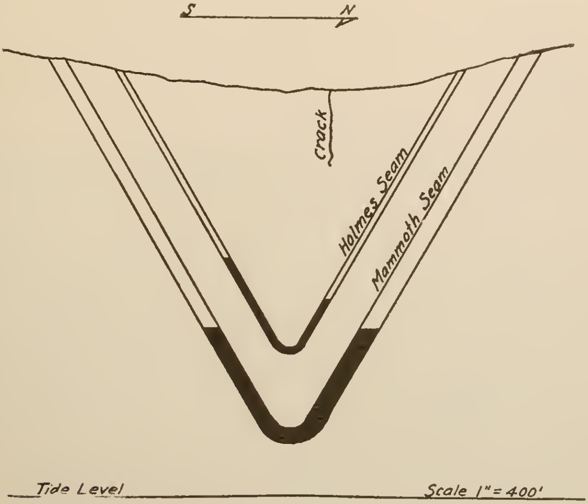

Fig. 13. Surface Cracks at Ashland, Pa.

The crack (Fig. 13) extended for a distance of about a quarter of a mile, and was from an inch to six inches wide, causing considerable damage to property. The vertical distance to the first coal seam was over 800 feet, and later development showed that the crack did not extend to the coal. The coal along the outcrops on both the Holmes and the Mammoth seams had been removed and it is pressumed that the crack was due to tension resulting from the settling of the overlying beds into the worked-out portion.*

The importance of the effect of surface beds upon draw or pull has been pointed out by A. Sopworth.† According to his observations the following classification of overlying beds may well be made:

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*Foster, R. J. Discussion of Paper, Proc. Coal Mining Inst. of America, p. 147, 1912.
†Proc. Inst. C. E., Vol. 135, p. 165, 1898.

(1). "Measures consisting of fairly equal proportions of rocky and argillaceous beds, and containing thick beds of sandstone.

(2). "Measures including a small proportion of rocky beds, say 15 per cent, and only thin beds of sandstone.

(3). "Vāriations between these two."

In the first case the edge of the subsidence will follow or lie over the excavation and in the second case it will lie over the solid coal. In the third case the draw will vary between (1) and (2).

Kay* has emphasized the serious effects which may result from the "pull over" or draw. In his opinion this may cause much greater damage than the actual downward movement. "The strata appear to bend over the goaf in a curve of radius depending on the depth, and thereby subject the strata overlying the recently-worked area to a strain (rendered passive from the movement of the face), coincident with the progress of the working face, and, owing to its great radius and slow movement, doing very little damage to surface structures of ordinary character, as a rule." If the advance of the face is stopped, buildings over the line of the face may be seriously damaged.

R. E. Cooper† called attention to the absence of pull where the overlying beds include strong layers of limestone, shale and sandstone.

Surface cracks may be due to shear. Cracks caused in this way generally are parallel, but they may constitute more than one system. If there are two systems of fissures, generally the openings due to one system are larger and more regular than those due to the other system.

Cracks may be caused by sliding of surficial material particularly where the topography is rough. The shifting of beds of clay may cause subsidence and form a sag or basin, around the perimeter of which tension cracks will appear.

3. *Pit-holes or Caves.*—When the mining is carried on at a shallow depth where there is very little solid rock cover, or when the roof fails under shear, the movement frequently causes a sharp break in the surface, forming pit-holes or caves. (Fig. 14.) Such holes may be caused by the surficial material running into the mine entries beyond the point at which the break actually occurred.‡ This type of disturbance is the cause of much damage to the anthracite mines of Penn-

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*Kay, S. R. "Effects of Subsidence Due to Coal Workings," Proc. I. C. E., Vol. 135, p. 117, 1898.

†Proc. I. C. E., Vol. 135, p. 133, 1898.

‡Fig. 14 is from a photograph of pit-holes in Indiana. In this case, coal 6 to 7 feet thick had been mined at a depth of about 100 feet. The overburden consisted of about 10 feet of shale and 90 feet of drift.

sylvania.* In regions where the surface is valuable for agriculture and for building sites, pit holes are frequently a serious problem because the cost of filling may be great. Subsidence of filled material is likely to continue for some time and the value of such filled ground for building sites is generally low.

Effect of Unwatering Surficial Beds.—Considerable discussion has been aroused by the suggestion that the unwatering of water-bearing beds of clay, marl, and sand may result in subsidence, when no mining has been done. German engineers have had to contend with heavy beds of marl overlying the coal, and have made a number of observations upon the effect of unwatering the surficial beds. There is a difference of opinion, but possibly the majority of the German engineers have thought that unwatering will cause subsidence. It was held by many that when the surficial beds are drained by boreholes or excavation there is a reduction in volume of the beds and that sinking of the

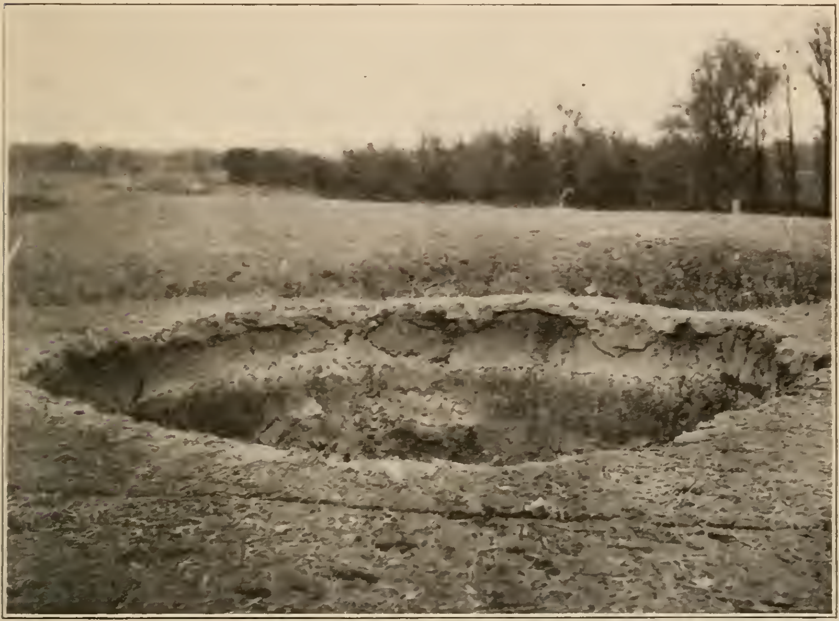

FIG. 14. CAVE IN SOFT SOIL. (Photo by H. I. Smith, U. S. Bureau of Mines.)

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*Bunting, D. "Limits of Mining Under Heavy Wash," Amer. Inst. Min. Eng., Bul. No. 97, p. 1, 1915.

surface will result. The mining industry was held responsible for surface damage, simply because it was acknowledged that unwatering had taken place.

In studying the subsidences about Essen in 1866 and 1868, von Dechen came to the conclusion that the subsidences and surface cracks were not directly the result of the coal workings, but that they were caused by the partial drying of the chalk marl and green sand overlying the coal measures which was caused by unwatering through the mines, boreholes and wells. He also pointed out that there was a shrinkage in volume in the chalk marl, due to the dissolving of carbonate of lime in the marl.

Later investigations led the German engineers to change their views upon the effect of unwatering. Graff made tests and showed* that drainage does not cause any changes in volume in gravel, sand, and quicksand. He concluded that subsidence will not result from unwatering if no solid material is carried away mechanically.

Tests made in the laboratories of the United States Bureau of Mines at Pittsburgh have shown that materials flushed with water do not compress nearly so much as the same material if dry. This would seem to indicate that by unwatering the strata of a mineral deposit, damage may be caused to the surface, even though no solid material is carried away.

F. Bernardi holds that the drying of beds of sand does not cause a decrease in volume or a reduced bearing power.† He reached this conclusion because in "water-soaked sand strata, the grains of sand rest upon grains of sand, and the weight of the surface is carried by these grains of sand resting upon one another and not by the water." If the drying of sand causes a decrease in volume, the wetting of sand should cause an increase.

Of the Austrian engineers Rziha held that unwatering may cause subsidence but the later writers, as Jicinsky and Goldreich,‡ who have had a better opportunity to make observations, hold that no movement occurs if the water does not carry away any solids mechanically or in solution. Data on the shrinkage of beds of loam and clay have been assembled by R. Dawson Hall.¶ "A clay slime, 200 feet thick, will reduce to 50 feet and less, as a result of drainage, and though such a result is rare, . . . , yet the figures suggest what an action drainage has in shrinkage of roof coverings of mines and how even clays of great age may lose bulk by mining operations and let down the rock or surface with its buildings above them. German and English investigations* have been made of the shrinkage in air of flint clays. A flint clay drying in air will shrink in all directions 5 per cent, so that it will measure linearly only 95 per cent as much as before shrinkage. The loss in drying is 14.26 per cent, and this, if the clay were plastic, so as to give laterally with freedom, would reduce the thickness of the bed 14 feet 3 inches in every 100 feet of depth of measure."

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*Graff "Verursacht der Bergbau Bodensenkungen durch die Entwasserung Wasserfuhrender diluvialer Gebirgs schichter," Glückauf, 1901.

†Kolbe, E. "Translocation der Deckgebirge durch Kohlenabbau," p. 63.

‡Goldreich, A. H. "Die Theorie der Bodensenkungen," p. 15, Berlin, 1913.

¶Hall, R. Dawson. "Data on Petro Dynamics," Mines and Minerals, Vol. 31, p. 505, 1910.

4. *Effect on Drainage.*—In the prairie lands and the river-bottom lands of the coal fields of the Middle West, the complete removal of the coal from horizontal beds at comparatively shallow depths has been attended with the problem of the drainage of the surface. Over large areas of prairie land there may be almost no natural drainage, and if the mining of several feet of coal permits the uniform subsidence of the surface, large sheets of water may stand for a number of months over the subsided land, thereby greatly reducing its value for farming purposes. In many instances (as will be noted fully later) the value of the land for farming purposes exceeds greatly the value of the coal in the ground at the present leasing rates.

Satisfactory artificial drainage has been provided in such flat prairie land by the laying of drain-tile at considerable expense. Subsidence may seriously disturb this tiling and may make the entire drainage system of little or no value. In a district such as the Mississippi Valley, where the streams are bordered by extensive bottom lands that are little if any above the high water line, it is claimed that surface subsidence may materially increase the area flooded at a time of high water and may even produce areas that are continually under water or are too wet for farming purposes.

5. *Effect on Water Supply.*—Subsidence of strata generally results in the formation of cracks and fissures in the rock which may be sufficient to permit the escape of water from a water-bearing bed which may have been the source of the water supply of a community or of an industry; thus the fissuring of the rock beneath gravel beds may permit the drainage of the beds which have been the source of water.

Numerous wells and cisterns have been damaged permanently by subsidence due to mining. Instances of only a temporary loss of water in wells have been noted in Illinois, Oklahoma, Maryland, and Pennsylvania, the wells furnishing the normal supply of water after subsidence has ceased if below the wells there are beds of such texture that the fissures will close tightly enough to hold water.

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*Reis, H. U. S. Geol. Survey, 19th An. Report, pp. 404-406, 1897-98.

SUBAQUEOUS MINING.

The subaqueous mining of coal and other minerals may shatter the overlying strata and permit an inrush of water which will destroy life and property. A number of valuable mineral deposits have been opened at the edge of the ocean, and from time to time the workings first made on the shore portion have been extended seaward until the mining of the under-sea portion by a safe method has become the chief problem in the undertaking.

Much attention has been given to the study of pillars and of subsidence owing to the vital necessity of mining in such a manner that water may not enter the mine. Particulars regarding the working of coal seams under the water of oceans, rivers and lakes are given in table 2:*

England, Scotland, and Wales.

Coal is being worked under the sea along the coasts of the counties of Northumberland, Durham, Carmarthenshire and Flintshire in England and Wales and also to some extent off the coast of Linlithgowshire in Scotland.† The coal beds dipping under the Firth of Forth have been mined extensively. Here there are a number of faults parallel to the shore which drop the seams on the seaward side. The bed of the Firth of Forth, although very deep at places, is covered first by a stratum of very hard, stiff unstratified till or boulder clay, which covers the solid rock, while above this is a deposit of reddish plastic clay, from 30 to 40 feet thick and in places finely laminated. This covering forms a waterproof barrier and prevents the sea from reaching the underlying strata. There are four important coal seams having a total thickness of about 15 feet. The lowest one lies at a depth of 340 feet at the shaft and dips rapidly seaward. "Operations of late years have shown that seams can be worked on the longwall system under the sea, with faces from 4 to 8 feet in height, at depths which are small in comparison with those of the workings in most modern collieries. The seams have been worked in three instances to their outcrop against

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*Atkinson, A. A. Trans. Inst. Min. Engrs., Vol. 23. Appendix IV, p. 644, 1901.
†Atkinson, A. A. Trans. Inst. Min. Engrs., Vol. 23, p. 622, 1901.

TABLE 2.

Particulars of Coal Seams Worked Under the Waters of Oceans, Rivers, and Lakes.

No.	Name of Colliery	Name of Coal Seam Being Worked	Depth Below the Water Feet	Thickness of Coal Seam Feet	Water of Oceans, Rivers, and Lakes
	New South Wales				
1	Australian Agricultural Company, Sea Pit or New Winning........	Borehole	145-190	15 to 16	Pacific Ocean (a)
2	Hetton	Borehole	300	7 to 8	Pacific Ocean (b)
3	Hetton	Borehole	165-300	7 to 15	River Hunter (c)
4	Newcastle Coal Mining Company, A and B Pits	Borehole	145-170	6 to 11	Pacific Ocean (d)
5	Stockton	Borehole	240-260	7 to 8	Pacific Ocean (e)
6	Wickham and Bullock Island	Borehole	120-260	6 to 16	River Hunter and Throsby Creek (f)
	Cumberland				
7	Harrington	Main Band	126-168	6½	Irish Sea (g)
8	Workington*	....	90	10	Irish Sea (h)
	Northumberland				
9	North Seaton and Cambois	Low Main	360	4 to 6	German Ocean (i)
	Durham				
10	Ryhope**	Maudlin	1.830	7	German Ocean (j)
11	Seaham**	Maudlin	1,830	5	German Ocean (k)
		Hutton	....	4½	German Ocean (l)
12	Wearmouth**	Maudlin	....	5½	German Ocean (m)
		Hutton	....	4½	
13	Whitburn**	Maudlin	....	....	.... (n)

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*Dunn, M. "A Treatise on the Winning and Working of Collieries," p. 230, 1848.

**Information derived from Mr. Thomas Bell's "Notes on the Working of Coal Mines Under the Sea, and Also Under the Permian Feeder of Water in the County of Durham." Transactions of the Manchester Geological Society, Vol. 36, pp. 366 to 399 and 554 to 559, 1899.

(a) Workings extend about 600 feet beyond high-water mark. The pillars are 96 feet by 36 feet, the bords are 18 feet wide and the cut-through or cross-holings 9 feet wide. About 4 feet of top coal is left next to the roof.

(b) Workings extend about 500 feet beyond high-water mark.

(c) The pillars are made 90 feet by 24 feet; the bords are 18 feet wide, and the cut-through or cross-holings 9 feet wide.

(d) The workings extend about 800 feet beyond high-water mark, including the winnings. About 3 feet of top coal is left next to the roof and a little bottom coal is also left.

(e) Workings extend about 2,500 feet beyond high-water mark. The pillars and bords are of the same dimensions as those at the Hetton colliery.

(f) The pillars and bords are of the same dimensions as those at the Hetton colliery.

(g) About 2 feet top coal and slate is left on, next to roof, in narrow places. The minimum thickness of cover has been fixed at 126 feet. The pillars are left 57 feet by 52 feet, the bords are 14 feet wide, and the walls or cross-holings are 9 feet wide. Thus, 32 per cent of coal is worked, and the pillars are not crushed. About 66 per cent of the overlying strata is compact sandstone. Feeders of water, occurring in workings where minimum cover had been reached, have since become quite dry.

(h) The workings extend 4,500 feet under the sea. The bords were 15 feet wide and the pillars 21 to 24 feet thick. The manager, in order to increase the output of coal, commenced to rob the pillars, this resulting in falls and feeders of salt water. Warnings were given as to what would happen, but these were unheeded. On July 30, 1837, the sea broke in and 36 men and boys and a number of horses were drowned, and the colliery irrecoverably destroyed.

(i) At Cambois bord-and-pillar longwall is being worked under the sea and headings are driven 300 feet in advance to ascertain the existence of any fault or break in the strata.

(j) The workings extend 5,500 feet from low-water mark under the ocean, and over 400 acres of goaf have been formed.

(k) The workings in the Maudlin seam extend 5,000 feet under the ocean and about 85 acres of goaf have been formed.

(l) Twelve pillars, each 120 feet by 90 feet, have been removed in this seam under the goaf of the Maudlin seam, rising seawards from 2 to 2½ inches per yard for the last 1,200 feet.

(m) Workings extend 4,000 feet under the ocean.

(n) Workings extend 4,000 feet under the ocean.

the boulder clay, at depths from 137 to 400 feet below high-water mark, without any accident."* The thickness of the cover under which the whole of the coal seam has been mined is less in this mine than in any other submarine mine in Great Britain.

The workings extending farthest seaward are reported to be those at Whitehaven, which at the William pit extend under the Irish Sea a distance of 19,000 feet (1901) from high-water mark. The coal seam is 10 feet thick and is worked by rooms 18 feet wide with pillars 75 feet square. There is also a higher seam about 7 feet thick which has been worked in places.† North of the William pit is an old mine which has been flooded.

The mining of under-sea coal will become a very important matter in time in Scotland.‡

Restrictions have been imposed upon the working of Crown coal in Great Britain. In the case of one colliery the working of coal under the ocean, unless there is at least 126 feet of strata between the bed of the sea and the top of the seam, and the removal of pillars or the adoption of the longwall system, where there is less than 360 feet of intervening strata are prohibited. Under specified conditions the entire removal of the coal-seam is permitted where the minimum thickness of cover is 270 feet.¶

It has been advised that the workings of coal on the Northumberland coast be limited to areas where there is a minimum of 270 feet of solid strata above the seam. The bed of the ocean generally consists in this vicinity of a stiff clay.§

Australia.

In New South Wales coal mining has been carried on extensively beneath the River Hunter, the Pacific Ocean, and its tidal waters.** Four seams have been worked in parts of this area, the total thickness ranging from 19 to 43 feet. Operations in the vicinity of the outcrop are dangerous because channels in the coal measures become eroded by old streams, and later these channels become filled with alluvial deposits. In general, the coal measures dip slightly toward the ocean, but there are many local dips and faults. The usual dip is given as 1 in 36. There are thick deposits of clay covering the outcrops in places.

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*Cadell, H. M. "Submarine Coal Mining at Bridgeness, N. B." Trans. Inst. Min. Engrs., Vol. 14, p. 237, 1897.

†Moore, R. W. Trans. Inst. Min. Engrs., Vol. 23, p. 660, 1901.

‡Atkinson, J. B. Trans. Inst. Min. Engrs., Vol. 14, p. 253, 1897.

¶Atkinson, A. A. Trans. Inst. Min. Engrs., Vol. 23, p. 629, 1901.

§Robertson, J. R. M. Discussion. Trans. Inst. Min. Engrs., Vol. 28, p. 133, 1904.

**Atkinson, A. A. "Working Coal Under the River Hunter, the Pacific Ocean, and Its Tidal Waters, Near Newcastle, in the State of New South Wales." Trans. Inst. Min. Engrs., Vol. 23, p. 622, 1901.

Owing to the weakness of the roof a number of inundations have resulted at inshore mines from letting down the sand overburden. In consequence of a fall of roof there was a rush of water into the Ferndale Colliery in 1886 and a miner lost his life.* A commission was appointed to investigate this accident and the report submitted included a review of conditions at all the collieries in the district. The title to the coal beneath the River Hunter and the tidal waters resides in the Crown and the leases to these coal lands now include regulations controlling the method of mining beneath bodies of water, with the view of protecting life and also of preventing large volumes of water entering old workings, and thereby interfering with the mining of the coal in the adjacent area.

The mines of the district use the pillar-and-room system. The dimensions of pillars and rooms vary, but in general 50 per cent of the coal is recovered. The practice in a number of the mines is to drive 18-foot rooms, leave 24-foot pillars, and recover part of the pillar coal. When the pillars were left only 18 feet wide on first mining a number of crushes resulted. Owing to the presence of thick, impervious beds of clay no water entered the mines where these crushes occurred, although at equal depths on land the crushes caused surface subsidence and some damage to buildings. In one of the mines the rooms are 18 feet and the pillars 36 feet.

The quantity of water being pumped from the mines varies from 50 to 600 gallons per minute and in most places this water is decidedly salty. Vertical boreholes are put up to determine the thickness and character of the overlying beds.

In determining the safe working limit under the ocean the following conditions have been considered:

(1) The character of the overlying strata, with special reference to loose deposits of alluvium or beds of clay between the bed of the ocean and the coal seam.

(2) The presence of faults and dykes in the strata.

(3) The dimensions of pillars to be left and the width of openings to be made.

(4) The utility of leaving coal next to the roof in some cases.

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*New South Wales Royal Commission on Collieries. Report on the Accidents at Ferndale Colliery, p. 17, Sydney, 1886.

The special conditions of working under tidal waters prescribed in the leases are notably as follows:

(1) The maximum width of rooms shall be 18 feet and the minimum width of pillars 18 feet.

(2) The pillars 18 feet wide shall not be removed.

(3) All headings and rooms shall be driven on sights.

(4) All workings shall be surveyed accurately every three months. All dates of working must be shown on the plan.

(5) The plan of the mine shall contain a faithful record of all dykes, fissures, etc., and shall indicate all excavations as they actually exist.

(6) In one road of every pair of leading headings, a borehole shall be kept going 10 feet in advance, and all leading headings shall be driven at least 150 feet in advance of the working rooms.

(7) When dykes or fissures are stuck in the boreholes, precautions must be taken to protect against possible danger which may result from weakness of roof or flow of water when the dykes or fissures are penetrated by the heading.

(8) The coal under the ocean should not be attacked until after a large goaf has been made by extensive workings under the mainland.

(9) The most accurate information available shall be obtained as to thickness and character of the strata and estuarine deposits overlying the coal seam before commencing to work it.

Similar conditions are specified for working under the sea except as follows:

(1) The minimum width of pillar shall be 24 feet.

(6) All leading headings shall be driven at least 300 feet in advance of the working rooms.

(9) Boreholes penetrating the roof for a height of 30 feet above the coal seam shall be driven on the leading headings 300 feet in advance of the work and 60 feet apart.

Newfoundland.

At Wabana there is a series of iron ore beds which lie in a synclinal trough, one edge of which passes through Belle Isle. The three uppermost beds are mined in both the land and in the submarine areas. The ore beds pass beneath Conception Bay and apparently outcrop in the floor of the bay. The center of the basin is estimated to be about three miles from shore. The lowest bed is from 15 to 30 feet thick.

The method of mining is pillar and room, the rooms being 250 feet long and turned on 35-foot centers with 20-foot pillars.

The development in the submarine territory is sufficient to allow an annual output of 1,000,000 tons, and the total ore reserve has been estimated at practically 400,000,000 tons after proper allowance was made for pillars, faults, and poor zones. The principal ore bed outcrops on Belle Isle and dips seaward so that at high water mark it has a depth of 70 feet; at 3,000 feet from shore the bed is 268 feet deep and has 180 feet of cover. The average grade of the slope is 16 per cent.* According to a private communication from E. E. Ellis, Geologist, Tennessee Coal, Iron & R. R. Co., Birmingham, Ala., 1913, the longest slope at the Wabana mines was 7,500 feet and the end was 6,000 feet under the water.

Cape Breton Island.

The coal measures of Cape Breton Island extend under the ocean, and a number of the coal seams have been worked in these submarine areas. The measures dip at a steep angle, while the sea floor dips at a moderate angle so that the thickness of cover increases rapidly. Owing to the rapid erosion of the outcrop by the sea, some of the seams have been lost. The Mabou mine was flooded from the ocean,† because of a break in the roof in 1909, and the Port Hood Colliery was lost by a flood resulting from the entrance of water through a feeder which was opened when pillars were extracted at a point where 942 feet of solid strata were supposed to lie between the coil seam and the floor of the ocean.

The workings of some of the companies have already been extended seaward a distance of 2½ miles from high-water mark and it is probable that in the future a large part of the coal output will be obtained from these submarine fields. The government has prescribed regulations to control the size of openings and methods of working under shallow cover.‡ Where the cover is less than 180 feet the coal may not be mined; mine openings may be driven where there is not less than 100 feet of cover. Where there is less than 500 feet of solid cover the workings must be divided into sections not more than one-half mile square and a coal barrier not less than 90 feet thick must be left around each section. The barrier may be pierced by not more than four openings, not more than 9 feet wide by 6 feet high. In 1904

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*Cantley, F. "Wabana Iron Mines." Canadian Min. Inst., Vol. 14, p. 274, 1911.
†"Coal Mines Under the Sea," Coll. Eng., Vol. 34, p. 17, 1913.
‡Coal Mines' Regulation Act of 1912, Sec. 54, Nova Scotia.

the government mine inspectors and the management of the Dominion Coal Company agreed upon the size of pillars to be left in the mining of submarine coal.*

The dimensions of rooms and pillars, the percentage of coal left in the form of pillars, and the thickness of cover are shown in Table 3.

TABLE 3.

DIMENSIONS OF ROOMS AND PILLARS, DOMINION COAL COMPANY.

Depth of cover (feet)	Harbour Seam*		Hub and Phalen Seams**		Percentage of coal left in pillars
	Room width (feet)	Size of pillar (feet)	Room width (feet)	Size of pillar (feet)	
200	20	27 x 75	20	30 x 75	51
250	20	27 x 75	20	30 x 75	51
300	20	30 x 75	20	34 x 75	54
350	20	33 x 75	20	36 x 75	56
400	20	36 x 75	20	42 x 75	58
450	20	39 x 75	20	46 x 75	60
500	20	42 x 75	20	50 x 75	61
550	20	45 x 75	20	54 x 75	63
600	20	48 x 75	20	58 x 75	64
650	20	51 x 75	20	62 x 75	65
700	20	54 x 75	20	66 x 75	66
750	20	57 x 75	20	70 x 75	67
800	20	60 x 75	20	74 x 75	67
850	20	63 x 75	20	78 x 75	68
900	20	66 x 75	20	82 x 75	69
950	20	69 x 75	20	86 x 75	69
1,000	20	72 x 75	20	90 x 75	70

*Thickness mined from Harbour Seam, 6 feet.
**Thickness mined from Hub and Phalen Seams, 9 feet.

British Columbia.

A disaster† which may be compared with those occurring in subaqueous mining resulted when the workings of the old Southfield Colliery near Nanaimo, British Columbia, tapped the drowned workings of the South Wellington Mine No. 1 of the Pacific Coast Coal Company on February 9, 1915. The inrush of water resulted in the death of 20 men. It was believed that the new workings were 450 feet away from the water and it was planned that a 100-foot pillar should be left between the water and the new workings. At the time of writing this report no evidence is available showing whether or not the Coal Mines

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*Dick, W. J. "Conservation of Coal in Canada," p. 35, Toronto, 1914.

†"Twenty Men Drowned in Mine Near Nanaimo, B. C." Coal Age, Vol. 7, p. 374., 1915. Watson, R. L. "Coal Mining on Vancouver Island." Mines and Minerals, Vol. 21, p. 249, 1901.

Regulation Act was being complied with; namely, that drill holes shall be kept in advance of the workings.*

Japan.

A large proportion of coal is mined under the ocean in Japan.†

The most serious accident in the whole history of subaqueous mining occurred in this country on April 12, 1915, when 237 men were killed by the flooding of Higashimisome Colliery. The mine is situated in Ube, Yamaguchi-ken and the chief production is from two beds lying wholly under the sea. The output is about 500 tons per day. Four shafts were sunk on the shore, each 119 feet deep, from which two beds are worked to a distance of about 4,000 feet from the coast.

The cause of the accident was the entering of water into the underground workings through a fault in a bed of sandstone 155.4 feet thick, above which there is an alluvial deposit of clay and sand 82.6 feet thick.

A small flow of water occurred when the fault was first reached. The final inrush followed the breaking of a hole about four feet square in the floor of an entry of the upper bed, a few feet back from the fault. Through this the water entered so rapidly that the mine was completely flooded in two hours. The quantity entering was estimated at 392,000 cubic yards. The sea bottom was lowered 60 feet over a small area showing that a considerable amount of solid matter was washed in.

The opening was apparently sealed by the solid material and it was planned that the mine should be reopened by filling the depression in the sea bottom with clay and sand, pumping out the water, and building dams to protect the workings from any future break.‡

INDUSTRIES AND INTERESTS AFFECTED BY SUBSIDENCE.

Surface subsidence involves more than the question of the present value of the land; in many instances the fundamental problem involves the relative present and future importance of various industries and interests. Among the most important of these are agriculture, transportation, and the various interests of municipalities.

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*"Where a place is likely to contain a dangerous accumulation of water, the working approaching such place shall not exceed eight feet in width, or such greater width as may be permitted by the Chief Inspector of Mines, and there shall be constantly kept at a sufficient distance, not being less than five yards in advance, at least one borehole near the center of the working, and sufficient blank boreholes on each side. (British Columbia Laws, 1911, Chap. 160, Part XI, Rule 14.)"

†Trans. Inst. Min. Engrs., Vol. 28, p. 133, 1904.

‡Colliery Engineer, Vol. 36, p. 19, 1915.

1. *Agriculture.*—In the consideration of the agricultural interests involved, attention must be directed to the probabilities of subsequent use for agricultural purposes of land not tilled at present. Probably in no state where mining is important is the value of farm lands in the mining districts higher than in Illinois. It will be shown in another bulletin how the present value of these lands for mining purposes (removing all the merchantable coal) and the present value for farming purposes compare. It has been predicted that the value of the fertile lands of the "corn belt" will increase greatly in fifty years. An agricultural expert has expressed the belief that northern Illinois land will sell for from $400 to $500 per acre, and the best land in the southern counties for $200 by the year 1965.

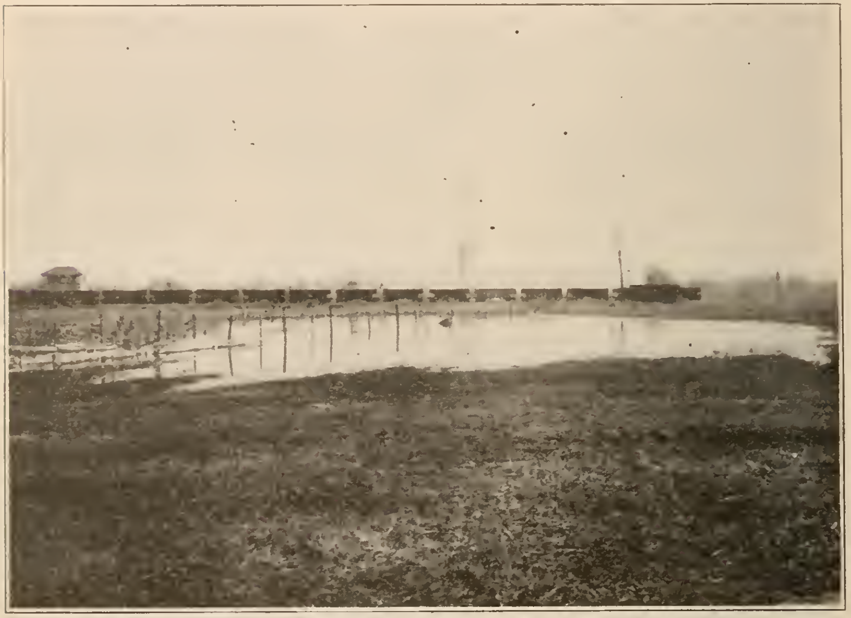

Fig. 15. Pond Formed by Subsidence.

In the longwall field of northern Illinois, where it is claimed that mining has lowered the surface so that drainage is deranged, it is estimated that large drainage projects have cost from $15 to $40 per acre. Fig. 15 illustrates the formation of a pond in a nearly level country by subsidence after mining. Coal of no greater thickness has been mined and is being mined in adjacent states. Estimates of the coal resources of Illinois show that only twenty per cent of the coal occurs in beds more than four feet thick and of the total area (37,486 square miles) underlain by workable coal beds, 32,979 square miles do not contain coal more than four feet thick. Over this great area it is possible that sometime mining by the longwall system may produce subsidence unless a filling system is used that is more effective than any at present in use. This statement regarding the thin coal beds in Illinois applies as well to large areas in Michigan, Ohio, Indiana, Kentucky, Missouri, Iowa, Kansas and several other states, and it is evident that the importance of the subject of subsidence will be even greater in the future than at present.

2. *Transportation.*—Surface subsidence may interfere seriously with transportation by injury to the beds of canals and railroads and the caving of highways and streets. As previously noted, mining in Great Britain and on the continent has necessitated the raising of the banks and the filling of the bottom of many canals. In some instances, canals have been maintained on grade, while the land which they traverse has subsided as much as 20 feet. The necessity for protection of these interests has become so great that laws have been enacted which require that thirty days' notice be given of mining under railways, reservoirs, buildings, or pipes or within a prescribed distance.*

The practice regarding the protection of the right of way of railroads has differed from time to time and has varied also in different countries. The general policy in Europe seems to be to remove all the coal if possible, and the tendency on the continent is to use filling under railways in order to reduce the amount of subsidence.

In the United States many of the great railway systems do not grant the right to mine coal beneath the right of way, if the company has ever owned the coal right. However, coal has been mined under many branch lines and under some of the main lines of railroads traversing the coal districts. Fig. 16 shows the effect of one subsidence in southern Illinois. In the anthracite fields of Pennsylvania many instances might be cited of subsidence of railway tracks. No serious accidents have resulted, as the railway companies have guarded carefully all points where movement is feared. There are no laws regulating mining under railways in the United States. When a pit hole or cave extends to the surface near or under a railway track, the problem of restoration is principally a problem of filling. Good

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*Cockburn, J. H. "Minerals Under Railways and Statutory Works." Trans. Inst. of Min. Engrs., Vol. 39, p. 104, 1909-10.

A large bridge built in 1884 after this plan subsided 3 feet in 1891. The wing walls separated from the abutments, but the abutments themselves were uninjured and subsided bodily, so that they were only 3 or 4 inches out of plumb. When subsidence had ceased the wings were repaired and the bridge was again placed in service.

The effect of subsidence upon railroad tunnels has been noted previously, particularly in the construction of the Merthyr tunnel in Wales, and the Greentree tunnel at Pittsburgh, Pa.

3. *Municipalities.*—As previously noted, many towns in Europe and America have been damaged by subsidence caused by mining. The damages to property in municipalities may include:

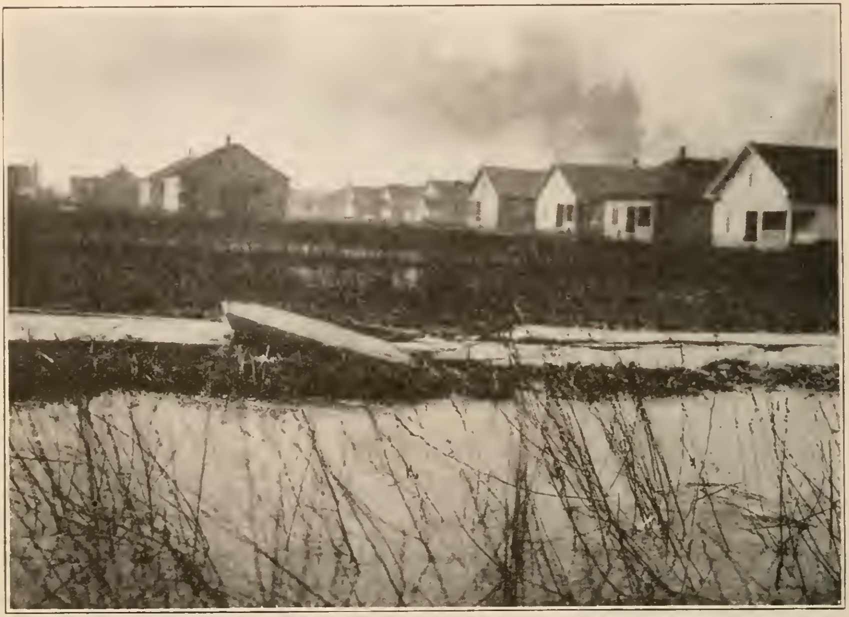

Fig. 17. Break in Sidewalk Due to Subsidence.

(a) *Injury to Streets, Sidewalks, and Transportation Lines.*—When pit-holes or caves occur, it becomes necessary to fill until subsidence has ceased and then reconstruct the street upon the most satisfactory grade. When there is horizontal movement, due to tension or compression, rather than caves, the streets, curbing, and sidewalks may be crushed or heaved (Fig. 17), or there may be tension great enough to cause serious cracks. This trouble has become so severe in certain

German cities that in the sections where compression occurs the gutters and curbs are laid so as to have elastic and waterproof joints. When large gaps are left in construction between curbstones they are covered with strips of sheet iron about 2 inches wide. In order to prevent the overturning of curbing, due to compression occurring transversely, the flagging is made narrower than the sidewalks and a strip of material that will permit compression is laid between the flagging and the curb. Coherent paving, such as asphalt, cement, and concrete, is not used because it would be cracked or crushed.*

(b) *Injury to Buildings, Towers, and Chimneys.*—This may be due to caves, or to tension, compression, or twisting. Large high buildings suffer more than low buildings covering but little ground. Masonry and concrete structures are damaged more than those of wood.

E. Kolbe has discussed at some length the nature of the damages to buildings,† and has pointed out the various factors and conditions with which one must deal in preserving buildings upon land which has subsided as follows:

(1) A building may sink wholly or in part into a surface break.

(2) A building may stand upon the edge of a break and be suddenly and violently twisted or wrenched and shaken.

(3) A building may be located in the mining area and may be subjected to the earth movement and be damaged by the jamming of the adjoining houses.

(4) A building lying over the mined area may sink slowly in the subsidence basin without undergoing greater damage than being placed in an inclined position.

(5) A building may suffer on account of the shock resulting from a fall of roof in the mine.

The types of cracks in brick buildings particularly around and between (Fig. 18) windows have been noted by Kolbe, as shown in the accompanying illustrations. As the illustrations show, the fracture generally follows the joints of the mortar, as these offer the least resistance. When cut stone window sills and lintels are used (Fig. 19), the fracture naturally follows upward around the stone without cracking it. In long brick or tile walls without openings, as for example walls (Fig. 20) surrounding estates, there may be three types of fractures in relation to direction:

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*Nolden "Influence of Mining Upon Buildings and Street Railways." Elektrische Kraftbetriebe und Bahnen, Oct. 24, 1913.

†Kolbe, E. "Translocation der Deckgebrige durch Kohlenabbau." Essen, 1903.

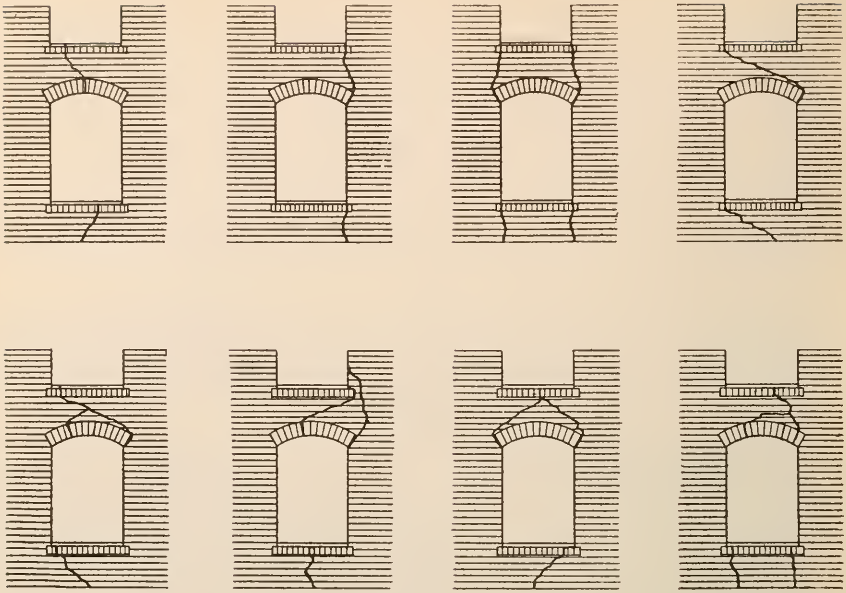

Fig. 18. Cracks in Brick Buildings.

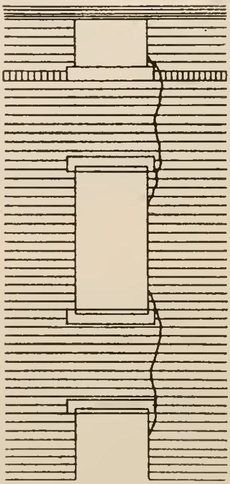

Fig. 19. Effect of Subsidence on Stone Lintels and Sills.

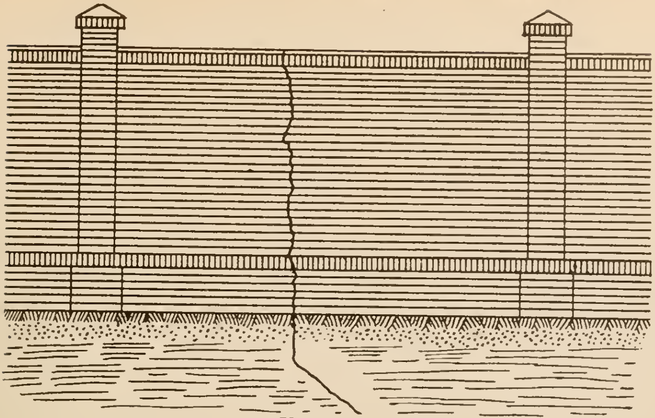

Fig. 20a.

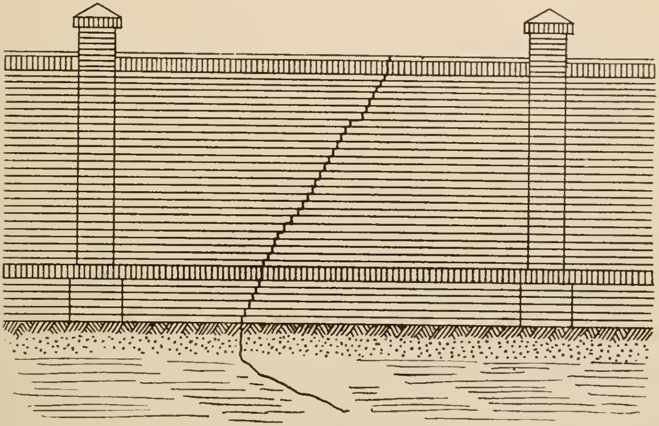

Fig. 20b.

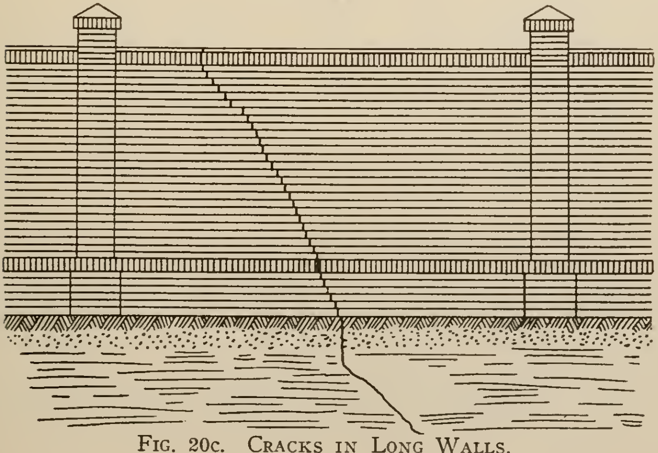

Fig. 20c. Cracks in Long Walls.

(1) The fracture may go perpendicularly up the wall and break the stone coping. (Fig. 20a.)

(2) It may extend diagonally away from the plane of the crack in the ground following the joints in the brick work. (Fig. 20b.)

(3) It may extend along the joints of the brick work diagonally in the same general direction as the plane of fracture in the ground. (Fig. 20c.) The second type is of most frequent occurrence. The same three types of fracturing are characteristic also of high enclosing walls, partition walls, and fire walls and chimneys.

Buildings may be damaged by side movement in which structures are crowded upon each other. When the mortar in masonry walls is cracked, the arches over doors and windows fail and increased pressure is thrown upon adjacent sections of the structure. When buildings are located over the edge of a pillar or on the side of a trough caused by subsidence, the cracks may extend in step fashion diagonally across a masonry wall. Secondary stresses may cause additional cracks in other directions. An example of this type of damage is shown in Fig. 21, which is an elevation of a post office. The cracks extend in the same general direction as the cracks in the ground.

In Germany, where subsidence has been anticipated, large buildings have been erected in sections from 60 to 120 feet long and these sections have been reinforced in all directions by rods and plates so that they will withstand both tension and compression. The joints between the sections have been calked with suitable material or protected with a covering. When buildings are not of great value European engineers have removed the coal as rapidly as possible and completely if possible, advancing the working face in a direction at right angles to the axis of the most important structure. When such precautions were used, the working of two 4-foot seams of coal at a depth of 600 to 780 feet in England caused practically no damage to two rows of 120 cottages.*

When the structures are important and it is estimated that the damage caused by subsidence will exceed the value of the coal, pillars may be left or filling introduced to prevent or reduce the subsidence.†

The problem of protecting important public buildings has received serious attention in Scranton, Pennsylvania. In several instances buildings have been erected on reinforced concrete piles constructed upon the rock underlying shallow coal beds which had been worked by the pillar-and-room method and of which the roof had fallen or seemed likely to do so. Engineers Griffith and Conner made an inspection of the conditions of mining beneath the city and school properties. The tabulated results of their inspection indicate that some coal has been mined under most of the buildings and that in a number of instances mining has been carried on in several beds.* Several of the buildings have been damaged by subsidence. The suggestions (op. cit., p. 60) by these engineers of precautionary measures will be considered later.

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*Longden, J. A. "Effect of Coal Workings on the Surface." Colliery Engineer, Vol. 11, p. 5.

†Spencer, W. "The Support of Buildings." Trans. Inst. of Min. Eng., Vol. 5, p. 188, 1892-93.

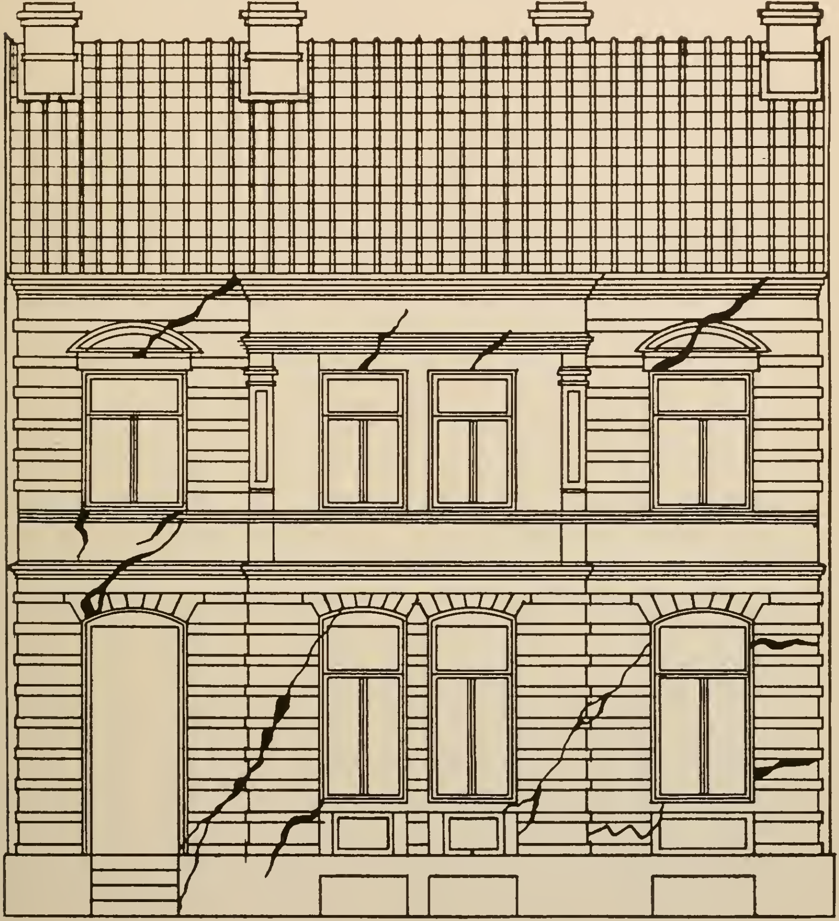

Fig. 21. Cracks in Masonry Wall.

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*U. S. Bureau of Mines, Bul. No. 25, pp. 19-43, 1912.

The report of Enzian, Johnson, and Williams on the extent of damage done, states the results of their examination as follows:

"In the consideration of a plan which might assist in the adjustment of property damaged we considered it important to compile the following information in connection with the properties of the thirteen city blocks which have been more or less subjected to the influence of subsidences that have occurred from time to time. The total assessed valuation of these properties is $1,430,000. The assessed valuation of the properties actually damaged is $411,000. The estimated damage to properties actually affected is $68,700, or about 17 per cent of their assessed valuation. The estimated damage to all the properties in the thirteen city blocks amounts to approximately 4.7 per cent of their assessed valuation. This estimate does not take into consideration any damage that may have been done to public property."*

(c) *Injury to water, gas, and steam lines.*—This type of damage is not unusual in communities in which mining has been carried on extensively. The cracking of water mains has caused damage not only through the direct injury to the main and the temporary failure of the water supply, but also through the escaping water, which in a number of instances has flooded buildings, washed out foundations, and destroyed streets, roads, and earthen structures. Fires have resulted from the escape of gas from broken gas mains. Necessity has brought about the use of expansion and compression joints of various types for preventing or reducing the damage to such lines. The need for frequent inspection of such pipe lines has made it important that they be laid in tunnels or large conduits.

(d) *Injury to sewers and sewage plants.*—Sewer lines as well as steam, water, and gas mains may suffer from subsidence, but in the case of sewer lines the difficulties are even greater, since these lines are generally constructed of materials which are less able to resist tension and compression, and a change in elevation of part of the line may render the entire system useless. An interesting experience regarding the subsidence of sewage works is reported by an English engineer, Malcolm Patterson.†

"At Ravensthorpe, in the Calder Valley, sewage works constructed in 1874 had remained intact for twenty-four years; they lay on the verge of a colliery leasehold. In August, 1897, the effluent outlet submerged

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*Enzian, Johnson, and Williams "Report on Mining Conditions of the Oxford Colliery Workings, Scranton, Pa., Dec. 12, 1913."

†Proc. Inst. Civ. Engrs., Vol. 135, p. 162, 1898.

15 inches below the ordinary level of the stream into which it discharged. At his (the author's) previous visit it was at its normal level, of about 6 inches above the stream. The settling tanks were cracked across the center, and the tank sewer had settled considerably. These settlements arose from getting a 20-inch seam of coal, besides the dirt, about 150 to 160 feet deep, and the boundary of the worked coal terminated in or near the sewage workings. In the same year, a similar disturbance took place at the Castleford sewage works in the same valley. Complete re-levellings of the three roads intersecting the land were taken, and proved an average settlement of 3.3 feet throughout nine-tenths of the 12.5 acres of sewage land, without the surface being broken. In this case the getting of coal, 4 feet to 4½ feet thick, at a depth of 603 feet, was the cause. The contour was singularly constant, the new section being almost parallel with the original section. The strata here were the shales and sandstones of the coal measures, overlaid by the marls and limestones of the Permian formation."

CHAPTER II.

GEOLOGICAL CONDITIONS AFFECTING SUBSIDENCE.

The behavior of the measures overlying the mineral deposit which is being worked depends to a large degree upon the physical character and the structure of the measures themselves. In a recent paper* before the International Geological Congress attention was called to the various geological conditions which influence the effect of underground mining upon the surface as follows:

(1) The general character of the overlying strata.

(2) The presence of faults, fissures, etc.

(3) The dip of the strata.

(4) The direction of the workings with regard to the jointing of the strata.

(5) The compressive strength of the rocks of the various overlying beds.

(6) The bearing power of the underlying beds.

(7) The angles at which rocks break when stressed.

Geological conditions must be studied in each district, as no generalizations can be made which will apply without reservation to all mining fields. The measures overlying a flat seam may be made up of various beds of sedimentary rocks and in places may include sheets or beds of intrusives. The physical character as well as the thickness of each bed may vary over different parts of the same mine, and there may be faults, fissures, rolls, etc., which greatly influence the supporting power of the bed, as well as the manner in which the weight of the bed itself is distributed upon the underlying supports. Unless the thickness and the character of the beds have been proven, and unless it is known definitely that the beds are fairly uniform throughout the field under consideration, it will be impossible to formulate even approximate rules and theories regarding subsidence which will be useful in the study of the problem of surface support.

MINERAL DEPOSITS.

1. *Physical Character.*—Before considering the overlying and the underlying beds, it will be well to note some of the conditions in the deposit being worked which may greatly influence the problem of sur-

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*Knox, George "Mining Subsidence." Proc. International Geological Congress, Vol. 12, p. 797, 1913.

face support. The physical character of the material being mined and of that part, if any, of the deposit which is left in the form of pillars or of filling must be considered. The texture and the structure of the rock left in pillars is of great importance in determining the burden the pillars will carry and in affecting the stability of the pillar after it has been subjected to the action of explosives in the adjacent portion of the deposit and after it has been exposed to the action of the atmosphere and water. In many coal mines, owing to the friability of the coal, it has been necessary to reduce the charges of powder used along the rib and in some instances to avoid the use of powder entirely because the pillars are more or less shattered by the force of the explosives. Rock and coal may be so weakened by jointing or cleats that the pillars offer little support. Moreover, the action of the atmosphere and moisture, particularly upon a deposit jointed as described, may greatly weaken the pillar. Soluble minerals in the pillars may be dissolved by the mine water or the moisture in the air and the pillar thus weakened. Pyrite and other minerals may be oxidized and a deterioration of the pillar will follow. It has been suggested by some that the loss of the included gas in coal beds tends to reduce the strength of the coal. The hydration or the dehydration of minerals may result in the weakening of pillars. The terms "rashing," "slacking," and "slabbing" have been applied to the process of weakening of pillars by the gradual dropping of material from the ribs, due in part to the action of moisture, oxygen, or pressure, or a combination of these agents. In mines operating in soluble minerals the preservation of pillars may be difficult owing to the flow of water in the mine or the moisture in the air. It may become necessary in mines of all types when pillars deteriorate to protect important pillars by a coating of cement or concrete.

Strength tests have been made upon coal and other minerals in order to determine how serviceable they will prove when left in pillars and in order to estimate, in advance of the opening of a mine in a new field, the minimum size of pillar which may be left in safety for the protection of the mine openings themselves and of objects on the surface.

Numerous tests have been made upon rocks used for building purposes, and the data thus secured are of service in determining the size of the pillars to be left in such rock. But more commonly the pillars left in mines are not composed of materials used for building purposes, but rather of coal, ores of the various metals, and rock mineralized more or less with substances which are not permitted in structural materials.

Moreover, the natural structural materials used are generally a selected product. Underground the pillar is frequently made up of the weakest portion of the deposit. Tests upon pillar materials are often of doubtful service, for, as a rule, they indicate the maximum load which can be borne by a unit of the mineral and one that is often a selected unit. A coal bed, for instance, is composed of layers of varying hardness, and frequently it contains streaks of mother coal that would not be included in a sample tested for crushing strength.

TABLE 4.

COMPRESSION TESTS OF ILLINOIS COAL FEBRUARY 6, 1907.

Laboratory of Applied Mechanics, University of Illinois.

Laboratory No.	Specimen from	Equivalent Section, Inches		Height, Inches	Maximum Load	
		Top	Bottom		Pounds	Lb. per Sq. In.
12401	Penwell Coal Co., Pana, Ill.	11¾x12	11¾x12	12½	316,000	2,090
12402	Empire Coal Co.	15 1/5x17 3/5	15 x15 1/3	11.3	540,000	2,170
12403	W. W. Williams, Litchfield, Ill..	13¼x13¼	14 x14	14½	186,000	1,000
12404	Herdien Coal Co., Galva, Ill......	11½x17¼	16 x13	12	208,000	1,020
12405	T. H. Watson, Litchfield, Ill..	13¾x12	13¾x12	15	224,000	1,360
12406	C. N. & V. Coal Co., Streator, Ill.	11¾x 9¼	11 x11¼	13	140,000	1,280

Tests were made in the Laboratory of Applied Mechanics of the University of Illinois upon samples of Illinois coal furnished by the Illinois Geological Survey.* The data regarding the samples and the results of the tests are given in Table 4.

	Dimensions in Inches	Area in Sq. In.	Crushing Strength per Sq. In.
Sample M—			
Parallel with cleavage................	2.01 by 2.02	4,060	3,170
Right angles to cleavage............	1.75 by 1.70	2,975	2,970
Sample B—			
Parallel with cleavage...............	1.95 by 2.01		3,430
Right angles to cleavage..............	1.92 by 1.98	3,925 3,802	3,050

Tests were made by the H. C. Frick Coal Company upon samples of coal from the Pittsburgh seam. These are particularly interesting as they show the strength when compression is parallel to the cleavage and also when it is at right angles to it.

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*Talbot, A. N. "Compression Tests of Illinois Coal." Ill. State Geol. Sur., Bul. No. 4, p. 198, 1909.

A series of tests was made upon Pennsylvania anthracite during 1901 and 1902 by a committee from the Scranton Engineers' Club. In all 416 samples were tested. The samples were uniformly 2 inches square, but were of three different heights: namely, 1 inch, 2 inches, and 4 inches. The results are given in pounds avoirdupois per square inch of horizontal area as presented in the following summary:

Samples	Height of Sample	Grand Average as Per Sq. In.		Number of Tests
		First Crack	Maximum Load	
Northern Field	1	3022	6241	122
	2	2025	4087	116
	4	1875	2854	113
Eastern Middle Field.........	1	4996	7417	7
	2	3343	3857	6
	4	3413	3821	7
Western Middle Field.........	1	3001	8631	3
	2	788	3499	8
	4	1440	2447	3
Southern Field	1	1124	3814	12
	2	1099	2377	12
	4	988	1809	12

From the data obtained the following conclusions have been drawn: "That the squeezing strength of a mine pillar of anthracite whose width is twice its height is about 3,000 pounds to the square inch, and the crushing strength about 6,000 pounds per square inch, or, approximately twice as much. And in general, other things being equal, the crushing strength of mine pillars would vary inversely as the square root of the thickness of the bed.

"The same general rule apparently holds true also for the squeezing strength in all cases in which the height of the pillar is less than the width. In tall pillars, having a height greater than their width, the squeezing strength apparently remains nearly constant while the crushing strength continues to diminish with height according to the foregoing rule."*

Subsequently additional tests were made at Lehigh University on samples of anthracite and of bituminous coal.† Forty-five anthracite specimens were tested. "There seems to have been no uniformity in the amount of compression of the specimens taken as a whole or between the specimens from the same seam." The results of the tests upon twelve bituminous specimens were more uniform. The crushing strength per square inch ranged from 584 to 1,583 pounds, but nine ranged from 1,000 to 1,538 pounds. All of the bituminous specimens were taken from the Pittsburgh seam. Additional data on the crushing strength of anthracite coal have been secured by Bunting[*] and Table 5 shows the crushing strength and the relation between prism strength and cube strength.

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*Mines and Minerals, Vol. 23, p. 368, 1903. U. S. Bureau of Mines, Bul. No. 25, Appendix, 1912.

†Daniel, J., and Moore, L. D. "The Ultimate Crushing Strength of Coal." Eng. and Min. Jour., Vol. 84, p. 263, 1907.

TABLE 5.

AVERAGE RESULTS OF TESTS ON ANTHRACITE SPECIMENS.

Name of Company	Ratio $\frac{h}{b}$	Crushing Strength Lb. per Sq. In.	$\frac{\text{Prism Strength}}{\text{Cube Strength}}$
P. & R. C. & I. Co.			
(25 specimens)	1	2,393	1.00
(25 specimens)	2	2,296	0.96
L. V. C. Co.			
(13 specimens)	1	1,982	1.00
(13 specimens)	2	1,591	0.80
(13 specimens)	3	1,405	0.71
L. & W-B. C. Co.			
(20 specimens)	0.71	3,025	1.22
(20 specimens)	1.07	2,566	1.00
(20 specimens)	1.24	2,393	0.87
(20 specimens)	1.43	2,008	0.81
(20 specimens)	1.77	2,090	0.76
(20 specimens)	2.06	1,880	0.84
D. & H. C. et al.			
(146 specimens)	0.50	5,113	1.63
(146 specimens)	1.00	3,131	1.00
(146 specimens)	2.00	2,234	0.71

b=Least lateral dimension.
h=Height of prism.

The crushing strength of some British coals has been measured and reported by Henry Louis.[†] McNair in discussing the conditions of deep mining in the Lake Superior District[‡] refers to the crushing strength of the trap rock left in pillars as 1,200 tons per sq. ft. Richardson[¶] gives a table of the compressive strength of quartzite cubes taken from the depths of from 1,000 to 3,500 feet in Rand mines. The first fractures appeared in the specimens under a pressure of 1,945 to 6,804 pounds per square inch. The crushing strengths of these specimens were 8,054 and 9,029 pounds per square inch, respectively.

2. *Extent and Dip of Deposit.*—The problem of surface support is naturally different in the case of a deposit which underlies an area of great lateral extent from that of support when the lateral extent is small. When the deposit underlies a small area the geological structure

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[*] Bunting, D. "Chamber Pillars in Deep Anthracite Mines." Trans. Amer. Inst. Min. Engrs., Vol. 42, p. 236, 1911.

[†] Trans. Inst. Min. Engrs., Vol. 28, p. 319, 1904.

[‡] Eng. and Min. Jour., Vol. 23, p. 322, 1907.

[¶] Richardson, A. "Subsidence in Underground Mines." Jour. Chem. Met. and Min. Soc. of S. Africa, Mar., 1907. Eng. and Min. Jour., Vol. 84, p. 196, 1907.

of that area may be worked out fairly accurately and precautions may be taken to protect important structures on the surface.

The dip and the position of the deposit may greatly modify the necessity for and the general policy of surface support.

3. *Uniformity of Mineral Deposit.*—If there is fair uniformity in thickness, structure, quality, and depth over a large area, a systematic plan of support may be adopted, including, for example, pillars of uniform size at regular intervals or a complete removal of the deposit with or without filling. If there is not regularity as to these conditions, it becomes more difficult to employ a system of support or of working which will be economical and at the same time provide support for the surface. Notable examples of such conditions may be found in some of the coal fields of Illinois where rolls, horsebacks, and faults complicate mining, and in some of the lead and zinc fields of the Mississippi Valley, where pillars of barren rock are left in the mines and the rich portion of the deposits is mined out as completely as possible under such conditions. The pillars as a rule are neither uniform in size nor uniformly spaced. While such irregularities in the mineral deposit interfere to a degree with systematic working, yet they at times assist materially in preventing or checking extensive underground movements or subsidence.

UNDERLYING ROCKS.

The physical character of the rocks immediately underlying the mineral deposit is of great importance. Frequently coal beds are underlaid with beds of clay of such consistency that it will not support the pillars when the weight upon them is increased by the opening of rooms. The pillars are slowly pushed into the clay while the clay is forced into the rooms which have been mined. Similarly, when water reaches clay beds underlying the coal, the clay may be softened and forced into the rooms by the weight of the pillars, and a subsidence results. The term "creep" is very commonly applied to such a movement.

Very few tests have been made upon the bearing power of the clays occurring in mines, but numerous tests have been made upon clays and soils upon the surface. Owing to the importance of not placing upon the clay floor of a mine a burden which shall exceed the bearing power of clay, which is usually much less than the compressive strength of coal, the following values are of interest:*

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*Baker, I. O. "A Treatise on Masonry Construction," p. 342, 10th Ed., 1912.

Kind of Material	Safe Bearing Power in Tons per Sq. Ft.	
	Minimum	Maximum
Rock—the hardest—in thick layers in native bed	200.0	....
Rock equal to best ashlar masonry	25.0	30
Rock equal to best brick masonry	15.0	20
Rock equal to poor brick masonry	5.0	10
Clay in thick beds, always dry	6.0	8
Clay in thick beds, moderately dry	4.0	6
Clay in soft beds	1.0	2
Gravel and coarse sand, well cemented	8.0	10
Sand, dry, compact, and well cemented	4.0	6
Sand, clean, dry	2.0	4
Quicksand, alluvial soils, etc	0.5	1

The data on clay given in the table are not for fireclay, and no data have been obtainable which are the results of observations upon the supporting power of such clay of the character and occurring under conditions similar to those found in coal mines.

OVERLYING ROCKS.

The study of subsidence due to mining operations involves particularly a consideration of the rocks overlying the mineral deposit. Lack of uniformity in the overlying measures is the rule, not the exception, and this fact must be recognized in all attempts to formulate theories and rules. The effect of different conditions of the overlying beds is well illustrated by two examples in England. "At Sunderland, where the measures contain 50 per cent of hard-rock beds, seams at a depth of from 1,400 to 1,800 feet have been worked for seventy years without reference to the surface. On the other hand, in the Midland and South Yorkshire coal fields, where the cover is composed largely of soft shales, the effect of workings at as much as 2,000 feet is appreciable on the surface."*

Investigations of the thickness and physical character of each overlying bed are fundamentally necessary to the accurate study of subsidence in any district. Much of the data as to the behavior of various strata that can be secured will be at best only relative. However, the more data that can be secured the fewer will be the variables with which the investigator must deal.

Practically every theory of subsidence which has been advanced, when analyzed, involves some fundamental principle of mechanics. The beds may be subject to tension, compression, bending, or shear. Samples of the various rocks may be tested in the laboratory in order to secure data to be used in the study of each problem. The great difficulty of obtaining specimens which will be representative of the section under investigation is largely responsible for the scarcity of data along certain lines, notably those concerning the strength of rock in tension and in bending.

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*Eng. and Min. Jour., Vol. 84, p. 196, 1907.

Data on the strength of the rocks that are of importance in the study of subsidence have been collected and published by Bunting.*

"Numerous tests of various stones have proved that sandstones take permanent sets for the smallest loads, whereas granite and limestones are nearly perfectly elastic. It has also been proved by tests on various stones that the modulus of elasticity in compression is practically the same as in cross bending, but no fixed relation has been determined of the compressive, tensile, or shearing strength of the various kinds of stone.

"The shearing strength of sandstones and slates per square inch is generally slightly in excess of the modulus of rupture, and the compressive strength of various stones is variable and of comparatively little consequence here, as the compressive strength of even the lightest sandstone ranges from 4,000 to 6,000 lbs. per sq. inch."

The moduli of rupture of various kinds of stone as given by a number of authorities are shown in Table 6.

"Safe unit stresses for various stones have been given by many authorities. Below are given the stresses in pounds per square inch recommended by W. J. Douglas as illustrative of possibly a fair average of such values:

SAFE UNIT STRESSES FOR STONE.

	Compressive Lb. per Sq. In.	Shear Lb. per Sq. In.	Tension Lb. per Sq. In.
Blue stone flagging.......	1,500	....	200
Granite	1,020	200	150
Limestone	800	150	125
Sandstone	700	150	75

"It is to be observed, in the case of sandstone, that a safe tensile strength of 75 lbs. and a shearing strength of 150 pounds per square inch are given. Now, in consideration of the fact that the modulus of rupture is invariably in excess of the tensile strength, also that the resistance to shear slightly exceeds the modulus of rupture, a value of 100 pounds per square inch for the modulus of rupture of standstone would be consistent. . . .

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*Bunting, D. "The Limits of Mining Under Heavy Wash." Amer. Inst. Min. Engrs., Bul. No. 97, p. 1, 1914.

"When sandstones and slates, which generally overlie the coal veins, are considered as beams or slabs spanning mine openings for the support of overlying strata or other superimposed load, their transverse strength is of first importance. The ability of such material to serve as a beam depends upon its tensile strength, since that is always less than its compressive strength." The action of the atmosphere and of water upon

TABLE 6.

Moduli of Rupture of Stones.

	Maximum	Minimum	Average	Authority
Blue stone flagging	4,511	360	2,700	Baker
Slate	9,000	1,800	5,400	Baker
Slate	11,230	7,425	.....	Arsenal tests, 1902
Slate	.....	.....	8,480	Merriman
Granite	2,700	900	1,800	Baker
Granite	.....	.....	1,754	Merrill
Granite	.....	.....	2,610	Arsenal tests, 1907
Granite	.....	.....	1,667	Arsenal tests, 1905
Granite	.....	.....	1,365	Bauschinger
Granite	.....	.....	1,194	Bauschinger
Glass	3,500	.....	.....	Church
Glass	4,132	.....	.....	Fairbairn
Sandstone	.....	.....	1,576	Technology Quarterly
Sandstone	.....	.....	1,200	Merriman
Sandstone	1,273	655	.....	Arsenal tests, 1895
Sandstone	2,243	1,500	.....	Arsenal tests, 1895
Sandstone	2,340	576	1,260	Baker
Sandstone (variegated)	.....	.....	469	Bauschinger
Sandstone (variegated)	.....	.....	718	Bauschinger
Sandstone (variegated)	.....	.....	1,109	Bauschinger
Sandstone (variegated)	.....	.....	341	Bauschinger
Sandstone (carboniferous)	.....	.....	483	Bauschinger
Sandstone (slaty)	.....	.....	249	Bauschinger
Sandstone (slaty)	.....	.....	135	Bauschinger
Sandstone (green)	.....	.....	156	Bauschinger
Sandstone (cretaceous)	.....	.....	597	Bauschinger
Sandstone (cretaceous)	.....	.....	967	Bauschinger
Gray stone	2,200	.....	.....	Kent
Light stone	1,170	.....	.....	Kent
Stone	.....	.....	2,000	Merriman
Quartz conglomerate	.....	.....	654	Merriman

From the results of tests as given in Table 6, the average moduli of rupture of the various stones are as follows:

	Pounds per Square Inch
Blue stone flagging	2,700
Slate	7,736
Granite	1,681
Sandstone	806

rocks which have previously been protected from these natural agents occasionally reduces the strength of rocks.

TABLE 7.

Specific Gravity of Rocks.*

Rocks	Average Specific Gravity	Lb. Wt. per Cu. Ft.	No. of Cu. Ft. per Ton 2,000 Lbs.
Quartz	2.6	162.1	12.3
Andisite	2.9	181.0	11.1
Basalt	2.9	181.0	11.1
Diabase	3.0	187.0	10.6
Diarite	3.0	187.0	10.6
Granite	2.7	168.0	11.9
Limestone	2.7	168.0	11.9
Porphyry	2.7	170.0	11.8
Rhyolite	2.4	149.6	13.4
Sandstone	2.4	149.6	13.4
Schist	2.7	168.0	11.9
Shale	2.6	162.1	12.3

As previously noted, structural features are of great importance and the application of theories and rules will serve only as an indication of tendencies and possibilities. If the various rocks and strata were uniformly homogeneous the problem would be greatly simplified.

Natural processes may give rise to conditions which result in surface subsidence. Possibly the most comparable examples of subsidence due to natural agencies are those of surface sinks which result from the removal of portions of the supporting minerals by natural agencies.

Numerous examples of sink-holes and caves have been noted in the salt districts of Europe and in areas underlaid by calcareous materials which may be dissolved in part by underground waters.† In the United States similar phenomena have been noted. The sink-holes of the Ozark plateau have been studied in Missouri‡ and the information available indicates that they have been caused either by the caving of the roof over solution basins in limestone beds, or by the enlargement through solution of joints leading from the surface to an underground channel. It is probable that the larger sinks are the result of the former cause. Usually these sinks vary in diameter from 100 to 300 feet, although single sinks are known to include as much as 150 acres.

In Illinois, near Millstadt, in the Waterloo Quadrangle numerous sinks have resulted from solution cavities in limestone beds lying at shallow depth.

Apparently the same forces which act during the subsidence of the surface over these cavities caused by nature also cause subsidence over mine workings. It has been suggested that the fissure systems in volcanic areas have resulted from vertical movement or settling due to the transfer of material by volcanic action to the surface; the resulting cavity having probably been closed, in part at least, by the subsequent settling of the surface under the load of extrusive material.* The dropping of a block of the earth's crust tends to produce normal faults, and it may be appropriate to consult the authorities on structural geology regarding the observations which have been made upon faults and fractures which apparently have resulted from forces and processes somewhat similar to those which characterize subsidence due to mining.

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*Herzig, C. S. "Mine Sampling and Valuing," p. 139, San Francisco, 1914.

†Woodward, H. B. "Geology of Soils and Substrata." London, 1912. Quotes Darwin on p. 64, reference to Cheshire salt district, p. 67.

‡Crane, G. W. "Iron Ores of Missouri." Missouri Bureau of Geology and Mines. Vol. 10, 2d sers., p. 34. Lee, Wallace, "Geology of the Rolla Quadrangle." Vol. 12, 2d sers., ch. VIII.

As will be noted later, the investigator of subsidence desires to learn among other things how the strata bend and break when subjected to various forces, and in what direction fracture will occur when various forces act. He desires to learn how rapidly the deformation of rocks occurs and to what depths mining openings may be carried with safety. In the laboratory the angle of break of various rocks may be measured and many other data may be obtained, but the investigator requires also data based upon larger volumes of materials, greater and more slowly acting forces, and conditions more nearly approximating those which result from mining operations.

1. *Cleavage.*—"The planes of cleavage, incipient or pronounced, existing in the overlying roof strata may strike in the same general direction as the planes of cleavage existing in the coal below. The importance of this principle and the necessity of its acceptance justify a reference by way of proof to the natural philosophy of the case. According to geological theory, the cleavage in the coal and in the roof strata was produced by the action of the same force. Assuming that in a given case the planes of cleavage are vertical, the theory is that some force, acting laterally and at right angles to what are now planes of cleavage, was the cause of such cleavage being created in the strata. Such a lateral force is supplied by the shrinkage of the earth's crust. This force, acting with immense energy on the particles of matter in the strata and subjecting them to enormous lateral compression, obliged such particles so to arrange themselves that their longer axes finally lay at right angles to the line of action of the compressing force. The planes of cleavage are thus defined as the planes in which the particles of matter now extend their longer axes."†

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*Lindgren, W. "Mineral Deposits." P. 136.

†Halbaum, H. W. G. "The Action, Influence and Control of the Roof in Longwall Workings." Trans. Inst. Min. Eng., Vol. 27, p. 214, 1903.

2. *Fractures.*—The subject of fractures has been discussed in various works on geology, notably in the recent work of Leith.* "Under tension fractures tend to develop in planes normal to the maximum stress. Tension fractures may develop when a mass is deformed by shearing. Under compressive stresses, fractures tend to develop above the planes of maximum shear, which are inclined to the direction of principal stresses; but the degree of inclination and the direction of dip of the planes away from the direction of maximum stress vary."† Joints in rocks may be due to tension or to compression. Faults, which are "fractures along which there has been some relative displacement of the rock,"‡ may be regarded as the result of tension or of compression. A "gravity" or "normal" fault is generally the result of tension while compression causes "thrust" or "reverse" faults.

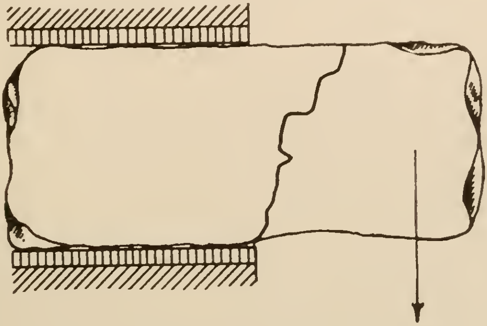

FIG. 22. ANGLE OF FRACTURE OF STONE.

The angle of fracture of rocks under stress has been noted and measured in the field and in the laboratory. Daubrée carried on extensive experiments in 1879 to show the effect of tension and compression.¶ Experiments made upon wax and resin prisms showed that compression causes rupture along a plane at an angle of 45 degrees to the line of force. If there has been preliminary deformation, the angle will be greater than 45 degrees.

Fayol tested pieces of sandstone and shale, as shown in Fig. 22, to discover the angle of fracture when the test piece is held firmly by one end and subjected to a steady and increasing pressure applied upon the projecting portion.

Leith states that data given in United States Geological Survey

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*Leith, C. K. "Structural Geology." 1913.

†In subsidence following the advance of longwall mining strata may first be subject to tension and later to compression.

‡Leith. P. 81.

¶Daubrée, A. "Études Synthétiques de Géologie Expérimentale." P. 179.

Folios show an average dip of 78 degrees for normal fault planes and 36 degrees for reverse fault planes. Faults noted in Illinois have dips ranging from 35 degrees to 75 degrees, the majority, however, approximate 55 degrees.

Lindgren observes* that veins may dip at any angle but "veins dipping 50 degrees to 80 degrees are most common."

Stevens has formulated a law of fissures.† "In a homogeneous mass under pressure, slipping tends to take place only along those planes on which the ratio of tangential stress to direct stress is equal to the coefficient of friction of the material sliding on itself. If the axis of greatest principal stress is vertical, the displacement along the fissure will be that of a normal fault, and the dip of the normal fault which is most favorable to slipping will be 66 degrees. Similarly, when the axis of greatest principal stress is horizontal, the displacement along the fissures is that of a reverse fault, and the dip most favorable to slipping is 24 degrees."

Spencer has studied in the field the veins of southeastern Alaska.‡ There is a systematic arrangement of veinlets in two main sets standing at right angles to each other and dipping in opposite directions. Becker¶ concluded that the fracture had been produced through compressive shearing stresses which were caused by nearly tangential forces acting in a direction normal to the strike of the two sets of fractures. Spencer supports the theory that these fractures were caused by compressive thrust but questions the statement that the thrust was the result of tangential compression. He developed the theory that the general fissuring was a result of "gravitative adjustment in the rock-masses, tending to restore internal equilibrium disturbed during the uplifts which are known to have taken place." A broad mountainous zone rises about 5,000 feet above the interior plateau and 15,000 feet above the plateau bordering the Pacific Ocean. "Standing so far above the neighboring earth blocks, it seems that in this great orographic mass there must even now exist a tendency to bulge toward the unrestrained sides. If so, conditions are favorable for the opening of fractures at a depth dependent upon the crushing strength of the rocks which compose the great mountainous mass."

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*Lindgren "Mineral Deposits." P. 151.

†Stevens, B. "The Laws of Fissures." Trans. Amer. Inst. Min. Engrs., Vol. 40, p. 475, 1909.

‡Spencer, A. C. "The Origin of Vein-Filled Openings in Southeastern Alaska." Trans. Amer. Inst. Min. Engrs., Vol. 36, p. 581, 1906. "The Geology of the Treadwell Ore Deposits." Trans. Amer. Inst. Min. Engrs., Vol. 35, p. 507, 1905.

¶Becker, G. F. 18th Annual Report, U. S. Geol. Sur. Pt. III, pp. 7-86.

The discussion and theory of Spencer is of great interest in this connection as it develops the idea of the settling of a mass of rock under its own weight and when movement is less restrained in one direction than in another. It also emphasizes the question of angle of fracture and systems of fractures which will be referred to later.

The theory of a "dome of equilibrium" developed by Fayol suggests the question of the possibility of removing a layer or bed from the earth without disturbing the surface, owing to the sphericity of the earth. The question of the supporting power of the dome of the earth's crust has been studied by a number of eminent geologists. Chamberlain and Salisbury refer to each portion of the crust as "ideally" an arch or dome. When large areas like the continents are considered, it is the dome rather than the arch that is involved, and in this the thrust is ideally towards all parts of the periphery. According to Hoskins, a dome corresponding perfectly to the sphericity of the earth, formed of firm crystalline rock of the high crushing strength of 25,000 pounds to the square inch, and having a weight of 180 pounds to the cubic foot would, if unsupported below, sustain only 1/525 of its own weight. This result is essentially independent of the extent of the earth's radius.*

The idea that extensive areas can be left entirely unsupported if the curvature of the arch corresponds to the sphericity of the earth is entirely unwarranted, judging from the calculations made and from the experience at many mines.

Various structural features must be noted in determining the cause, effect, and probability of subsidence following mining operations. Among the most important of these are the conformability of the overlying rocks, joints, cleavage, bedding planes, folds, faults, fissures, dikes, and intrusives.

In many mining districts there are heavy beds of surficial material which complicate the problem on account of the water they contain and because they are more or less fluid and have little supporting power. The lateral extent of subsidence is greater when the area is covered with such beds. This is due largely to the smaller sliding angles upon which beds of sand, earth, marl, and gravel will move.

Experiments to Determine Rock Fracture.

Many experiments have been carried on by eminent geologists in order to discover by work in the laboratory fundamental data upon which theories may be based and also to verify if possible, by artificial processes, theories accounting for conditions which may have been the results of complex forces and reactions.

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*Chamberlain, T. C., and Salisbury, R. D. "Geology." Vol. 1, p. 581.

As previously noted, numerous tests have been made to determine the strength of rocks and minerals under various conditions and various properties of rocks have been studied.* Among the most interesting experiments in addition to the tests of materials are the following: Fayol conducted elaborate tests of materials such as those which composed the beds overlying the Commentry Mine and by ingeniously constructed models attempted to measure the lateral and vertical extent of subsidence.† The work of Daubrée has been noted previously. Extensive experiments have been made also by Meade and by Paulcke. In America among the experiments which have attracted most attention are those described by Willis in "Mechanics of Appalachian Structure"; those by Adams and Coker on elastic constants, flowage, and the cubic compressibility of rocks; those by Becker on schistocity and slaty cleavage; and those by Hobbs on mountain formation.

Most of these experiments consider tangenital pressures rather than vertical pressure. Very few of them develop conditions which approximate those which occur when the support of rock is removed.

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*Consult the Bibliography, p. 180, for records of these experiments.
†See page 76.

CHAPTER III.

THEORIES OF SUBSIDENCE—GENERAL PRINCIPLES.

In this bulletin no attempt will be made to discuss theories of mechanics or derive formulæ applying to subsidence, but an effort will be made to state briefly the conditions that exist and to point out the fundamental and controlling factors in a study of the problem.

In order to study the reactions which may exist in the rocks overlying a mineral deposit, it will be necessary to make certain assumptions in order to arrive at some definite conclusions. For example, it must be assumed that the rock is uniformly of a known strength, that it is free from structural weaknesses, and that it exists in masses or beds whose extent, thickness, depth, and dip are known.

The principles of mechanics may be applied to various types of mine openings, notably: (1) The long narrow excavation which may be driven through massive or bedded rocks, or along the strike or the dip of bedded rocks, as tunnels, drifts, crosscuts, and entries. (2) Excavations of greater width, as rooms or stopes. (3) Excavations of great lateral extent, as those of a longwall coal mine, or sections of a pillar-and-room mine after the pillars have been drawn. In these various types of openings the fact must be recognized that maximum pressure may not always be due to a thrust acting vertically downward.

In order to simplify the problem it may be suggested that the rock and mineral overlying and surrounding the excavation be considered as forming one of the following:

(1) A beam of rock lying horizontally or inclined and extending from pillar to pillar or column to column.

(2) A cantilever supported by a pier of rock or mineral.

(3) An arch or series of arches of equal or unequal spans.

(4) A column or pier, either vertical or inclined, supporting (1), (2), or (3).

(5) A dome of the earth's crust.

It should be noted further that when the roof is considered as acting as a beam it may be supported by piers of mineral, of noncoherent filling, of timber, or of masonry, resting upon a more or less yielding floor. With these explanatory statements, the various theories of subsidence that have been formulated will be considered.

Historical Review of Theories of Subsidence.*

Belgian-French Theories.

Belgian engineers were among the first to make a scientific study of earth movements due to mining operations. In 1825 a commission investigating the cause of surface cracks about the city of Liege expressed the opinion that a distance of 300 feet between the mine workings and the surface is more than sufficient to protect the surface. Further disturbance of the surface raised the same questions in 1839. Another commission of mining engineers concluded that there would be no danger to buildings or wells from mining operations at a depth of 300 feet.†

Although credit for formulating the first theory of subsidence is usually given to the Belgian engineer, J. Gonot, it is claimed by L. Thiriart that the fundamental idea of the theory of the normal was first presented by the French engineer, Toillez, in 1838. Gonot studied surface subsidence in the vicinity of Liege in 1839 and formulated a theory which was published in 1858. He claimed that following the removal of coal the overlying strata would sink and the angle of fracture would be perpendicular to the plane of the coal bed. (Fig. 23.) This theory was later referred to as the "Law of the Normal." Mining operators in general and many engineers criticised this theory and, while many later writers accepted the principle as it applied to horizontal

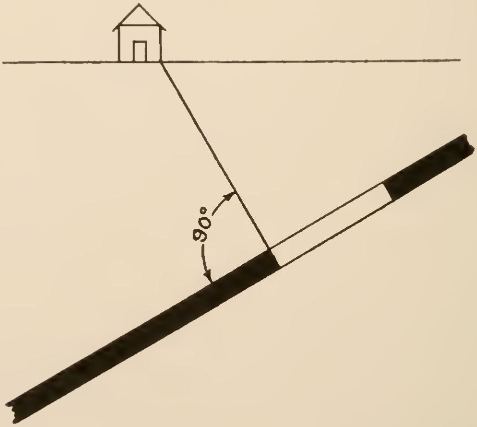

Fig. 23. Diagram Illustrating the "Law of the Normal."

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*Fayol, H. "Sur les Mouvements de Terrain Provoqués par l'Exploitation des Mines." Bul. Soc. Ind. Min., IIe ser., Vol. 14, p. 862; Kolbe, E., "Translocation der Deckgebirge durch Kohlenabbau," pp. 2-51, Essen, 1903.

†Vuillemin, E. and G. Bul. Soc. Ind. Min., IIe ser., Vol. 14, p. 858, 1885.

and slightly dipping beds, various qualifications were suggested in regard to the angle of fracture of steeply inclined beds. (Fig. 24.) Gonot also held that the break extends through to the surface, irrespective of the depth of mining. He based this theory on observations he had made on subsidence at Liege. The Belgian engineer, Rucloux, who was appointed with Wellekens to investigate subsidence about Liege in 1858, called attention to the fact that Gonot's theory undoubtedly could not be applied to vertical and highly inclined beds. While many criticisms were offered, no new theory was presented. The commission held that the observed facts were sufficient to establish the principle that with solid beds of an ordinary thickness and at moderate depths exploitation by contiguous openings and successive fillings up to a considerable extent may be made without affecting the surface. Where the depths are slight, or when for one reason or another the beds lose their solidity, subsidence may be prevented by preserving pillars. The subsidences which are produced on account of the underground work generally follow vertical lines, but may deviate from these lines according to the direction of the beds, more often toward the lower side and often also toward the upper side.

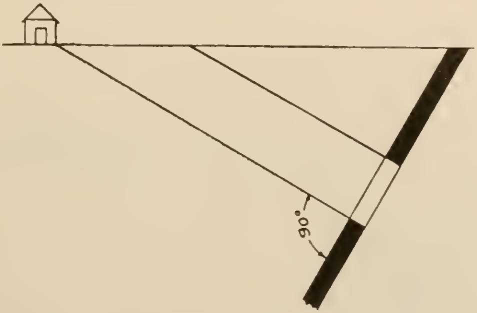

FIG. 24. THE "LAW OF THE NORMAL" NOT APPLICABLE TO STEEPLY DIPPING BEDS.

In 1868 four engineers were commissioned by the Prussian Government to collect information on the question of the "influence that mine workings may have on surface building" in the coal fields of various countries. They found that at that time the majority of Belgian engineers believed that when the coal is entirely removed the most careful packing gives no guarantee against damage to surface building; that the packing only lessens the sinking; and that the surface may be protected by leaving pillars. In order to make this method effective only half the area of the coal seams must be removed.

In 1871 the Belgian engineer, G. Dumont, who had been appointed to make an investigation of conditions in and about Liege, made a careful study of the problem and submitted a voluminous report of 331 pages, in which he supported the fundamental idea of the "Law of the Normal" but limited its applicability to beds dipping not more than 68 degrees from the horizontal. This conclusion was based in part upon upwards of a thousand levels at various parts of the town. He* called attention to the direction and amount of the forces acting on the block of rock overlying the excavation. The broken pieces must fall into the excavation, and on highly inclined seams, according to Gonot's theory, the masses of broken rock would have to move toward the excavation on an angle less than the sliding angle. If *a-b,* in Fig. 25, represents the weight of the rock A-B, and this force is resolved into the forces *a-d* and *a-c,* it is evident that, as the bed becomes steeper, the force corresponding to *a-d* will become less and the force corresponding to *a-c* greater. The tendency, then, will be to create a cavity vertically above the excavation rather than in a direction perpendicular to the plane of the bed.

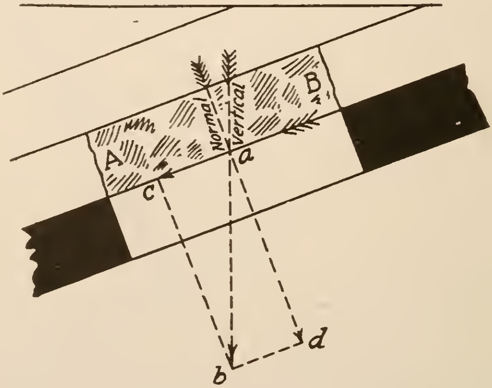

FIG. 25. FORCES ACTING ON ROCK IN AN INCLINED BED.

Dumont held that the "inclination of the strata lessens the depth of the subsidence, but increases the area damaged. Timbering hinders the beds forming the roof of a seam from breaking, and therefore prevents the increase in their volume, which takes place when they break. It thus increases rather than diminishes the subsidence at the surface."†

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*Dumont, G. "Des Affaisements du Sol Produits par l'Exploitation Houillère," Liege, 1871.

†Colliery Engineer, Vol. 11, p. 25, 1890-91.

The period during which the movement of the surface may continue is uncertain. In Belgium it extends generally over ten to twelve years but in certain instances has been known to continue twenty and even fifty years. The draining of old workings or the flooding of a mine may bring about fresh movements a long time after the original movement has ceased.

J. Callon, of the École des Mines, Paris, supported Gonot's theory but with some reservations.* He believed that when the coal bed is overlaid with unconformable beds, the angle of fracture will extend through each bed perpendicular to its plane of bedding. (Fig. 26.) He held that the amount of surface subsidence would depend on the compressibility of the material which fell into the excavation. In hard rocks a cavity narrowing upwards would be formed, while in soft rocks the cavity would be funnel-shaped.

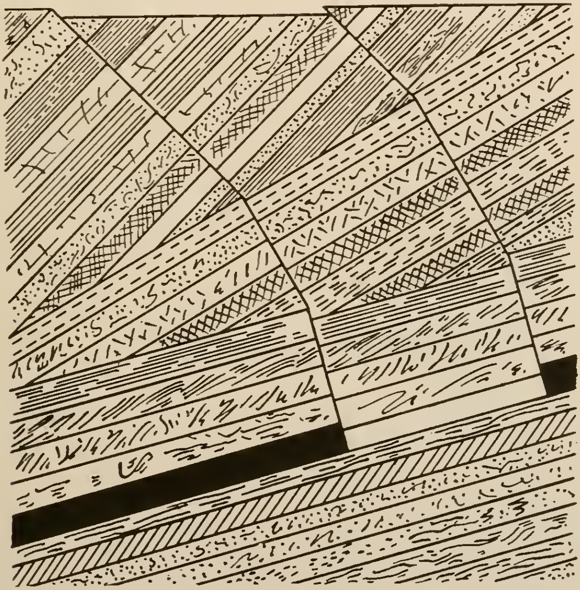

FIG. 26. FRACTURE NORMAL TO BEDDING PLANE.

The Colliery Owners' Association of Liege published a reply to Dumont in 1875.† The validity of Gonot's theory for beds of low dip was admitted, but his claim that the fracture would be normal to highly inclined seams was disputed. They argued that the fracture over the

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*"Cours d'Exploitation des Mines," Vol. 2, p. 334, 1874.
†"Des Affaisements du Sol. Attributés a l'Exploitation Houillère," Liege, 1875.

workings would take place in a series of breaks approximately perpendicular to the bedding plane of each stratum, but that the force of gravity would cause the material to fall from the outcrop side of the excavation, causing the line of fracture to lie between the vertical and the perpendicular to the vein; while on the lower side of the excavation, each bed would tend to support the bed above and there would be an overhanging of slabs of rock toward the excavation. Thus the line of fracture would be between the vertical and the normal to the bedding planes. (Fig. 27.) They also called attention to Coulomb's measurement of the angle of fracture by crushing. "The combination of this force producing crushing with that tending to break the bed by bending induces fracture along a line intermediate between the two directions, and such line goes further from the normal as the inclination of the strata increases."* On the whole the Colliery Owners' Association thought the Dumont's theory was unsatisfactory and often of no practical use and

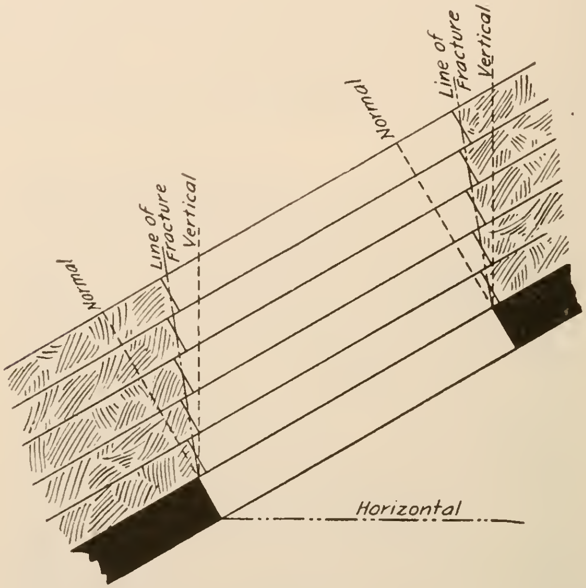

Fig. 27. Line of Break Between Normal and Vertical.

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*Hughes, H. W. "Textbook of Coal Mining," London, 1904.

that the only rule to follow was the examination of the special facts in each particular case.

M. Haton de la Goupilliere (1884), Professor of Mining at the École des Mines, Paris, held views similar to those of Callon. He pointed out the effect of the fallen material, which tends to check subsidence and in fact may stop it at a certain level. With longwall mining and filling he thought the movement would be almost independent of the depth. He held that it would be impossible to have the "Law of the Normal" completely verified in practice.*

The continued subsidence of the surface at Liege and the disagreement among engineers as to the theories of subsidence induced H. Fayol† to make observations of elevations at mines and to conduct laboratory experiments. He first summarized the contradictory opinions of the time as follows:‡

(1) Upon the extension of the movement upwards.

(a) The movement is transmitted to the surface whatever may be the depth of the workings.

(b) The surface is not affected when the workings exceed a certain depth.

(2) Upon the amplitude of the movements.

(a) Subsidence extends to the surface without sensible diminution.

(b) Movements become more and more feeble as they extend upwards.

(3) Upon the relative positions of the surface subsidence and of the mining excavation.

(a) Subsidence always takes place vertically above the workings.

(b) Subsidence is limited to an area bounded by lines drawn from the perimeter of the workings and perpendicular to the beds.

(c) Subsidence can not be referred to the excavation either by vertical lines or lines normal to the beds, but only by lines drawn at an angle of 45 degrees to the horizon, by the angle of repose of the ground, or by some other similar angle.

(4) Upon the influence of gobbing.

(a) The use of packing protects the surface effectually.

(b) Packing simply reduces the effect of subsidence.

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*"Cours d'Exploitation des Mines," 1883.

†Director, Commentry and Montvicq Mines in France.

‡Bul. Soc. Ind. Min., IIe ser., Vol. 14, p. 805, 1885.

(c) Subsidence is greater with stowing than without it.

Fayol conducted a long series of investigations and experiments* and came to the conclusion that the movements of the ground are limited by a kind of dome which has for its base the area of the excavation and that their amplitude diminishes by degrees as they extend further away from the center of the area.

"Fayol's rule agrees with all the facts observed; absence of subsidence, more or less important subsidences, movements limited to the vertical above the perimeter of the excavations, those limited to the normal or to other inclinations, and so on. It has the disadvantage of being indefinite; but in a question which embraces so many elements, many of which are unknown or not well known, such as the nature of the rocks, the thickness of the beds, irregularities in geological structure, the action of water, etc., we cannot hope to arrive at absolutely accurate formulæ; we shall have accomplished much when we get to know very nearly the true form, the direction, and the relative amplitude of the subsidences, and are in a position to combat false ideas successfully."†

According to Fayol the disturbance of the strata is greatest over the center of the area excavated and it diminishes in amount toward the perimeter of the excavated area. As the vertical distance above the excavation increases, the amount of the movement decreases, and, if the workings are at great depth, there will be a depth beyond which the movement will cease. When graphically represented the limits of the movement are depicted by a dome; outside of this dome there can be no disturbance whatever. However, Fayol called attention to the possibility of movement if there should be a series of these domes in close proximity to each other, and to the effect of dip, rock structure, etc. upon the practical application of this theory. As a result of his experiments and observations, Fayol concluded:

(1) If excavations were stowed in a thoroughly tight and efficient manner with incompressible materials there would be no subsidence, but ordinary stowing is not done under these conditions, because the materials employed are all more or less compressible and the excavations are never perfectly filled up. When the roof settles the stowing resists feebly at first, after which the resistance rapidly increases and finally arrests the downward movement.

(2) The amplitude of the subsidence diminishes in proportion to

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*See page 138.

†Galloway, W. "Subsidences Caused by the Workings in Mines," Proc. South Wales Inst. of Engrs., Vol. 20, p. 311, 1897.

the depth of the workings below the surface, the diminution being proportional to the increase of depth.

Leon Thiriart in 1912 called attention* to the theory of Banneux, which Thiriart called the "Law of the Tangent." Thiriart's theory is a modification of Banneux's, and Banneux's theory resembles that of Hausse.† The bending moment is considered for each bed successively, beginning with the one immediately overlying the coal. By elaborate calculation, based on observations of subsidence, formulæ are derived by which a table showing the angle of break for various dips has been compiled.

German Theories.

A. Schulz, one of the first German engineers to study surface subsidence due to mining operations, in 1867 published his ideas on the angle of fracture and the size of pillars necessary to protect objects on the surface.‡

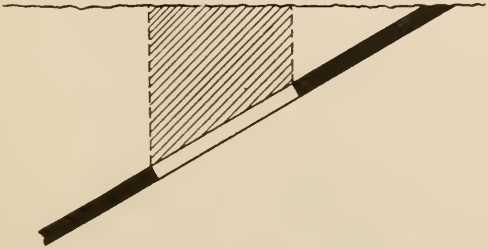

FIG. 28. VERTICAL FRACTURE OF DIPPING BEDS OF SHALE.

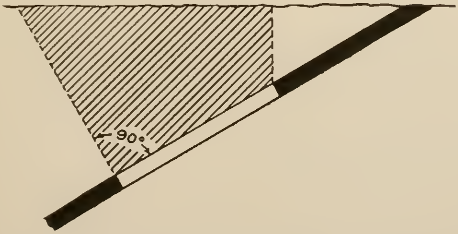

FIG. 29. SCHULZ'S IDEA OF FRACTURE OF SANDSTONE BEDS.

He criticised Gonot's "Law of the Normal" and held that in dipping beds of shale the fracture will occur along vertical planes (Fig. 28), while in sandstone the fracture on the dip side will approach the normal, but on the outcrop side it will be vertical. He held in general that the fracture would occur between the vertical and the normal to the bed.

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*Thiriart Leon "Les Affaisements du Sol Produits par l'Exploitation Houillère," Ann. des Mines de Belgique, Vol. 17, p. 3, Bruxelles, 1912.

†See page 97.

‡Schulz, A. "Investigations on the Dimensions of the Safety Pillars for the Saarbuck Coal Mining Industry and on the Angle of Fracture at Which the Strata Settle Into Worked-Out Rooms." Zeit. für B.-, H.-, u. S.-W., 1867.

During the same year that Schulz published his paper on the angle of fracture, Mining Assessor von Sparre published a criticism of Gonot's theory.* He held ideas similar to those of Schulz; namely, that the fracture will occur between the vertical and the normal. He suggested the consideration of the separate beds and showed that for each bed the fracture would occur between the vertical and the normal on both sides of the excavation in a dipping coal seam. As shown in Fig. 30, the bounding planes of the break will be not *ab* and *lh* nor *ac* and *hm* but midway between.

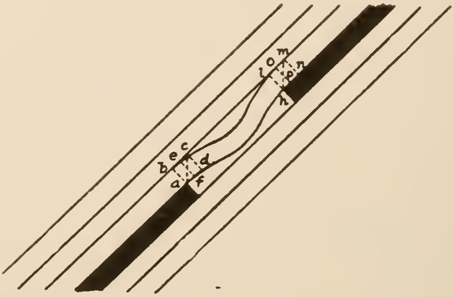

Fig. 30. Fracture in Dipping Beds According to von Sparre.

Von Dechen called attention in 1866 to the importance of studying the part played in subsidence by the heavy marl beds overlying the coal measures. Some engineers, in fact, held that subsidence was due entirely to the unwatering and drying of these marl beds. Von Dechen noted also that the "Law of the Normal" could not be applied to very steeply inclined or vertical beds.†

In 1894 the project of constructing a canal between Herne and Ruhrart caused an investigation of the stability of the surface over which it was proposed to build the canal. A survey of conditions in the Dortmund district was made by the Board of Mines of Dortmund; levels were run and maps were made, and a very complete report was submitted in 1897.‡ It was concluded from observation on the Westphalian mines and for the conditions of that district that there would be no "harmless depth." The lateral extent of subsidence was found to increase greatly with thickness of the marl covering. It was noted that careful filling of the mine workings will greatly reduce the amount of vertical subsidence but will not affect greatly the lateral extent, in fact, in some instances it was found that with filling the lateral extent of subsidence is greater than when no filling whatever is used.

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*Von Sparre, J. "On the Angle of Fracture of Strata of the Coal Measures," Glückauf, 1867.

†Von Dechen, H. "Opinion on the Surface Subsidence in and About the City of Essen," Manuscript, 1869.

‡"On the Influence of Coal Mines Under Marl Capping Upon the Earth's Surface in the Dortmund District." Zeitschrift für B.-, H.-, u. S.-W., Vol. 45, pp. 372-392, 1897.

From the data collected an effort was made to determine the angle of fracture in rock and also the angle at which the limiting plane of subsidence of the marl extends from bedrock to the surface. In rock dipping not more than 15 degrees, the angle of fracture was found to be about 75 degrees, measured from the horizontal (Fig. 31), while in steep seams the angle approaches the natural slope, generally not less than 55 degrees.

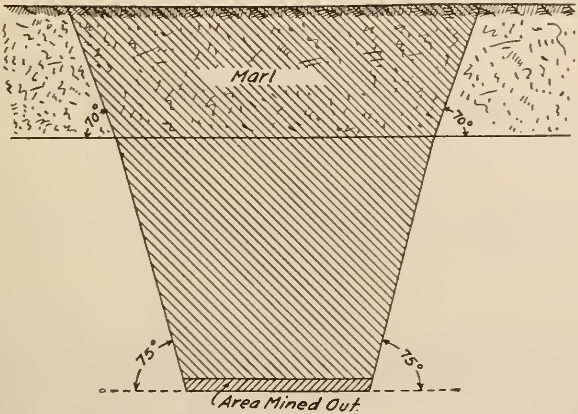

FIG. 31. SUBSIDENCE BEYOND ANGLE OF BREAK ACCORDING TO WACHSMANN.

For dips up to 65 degrees the vertical amount of subsidence may be found by the formula:

$$S = f.\ m.\ \text{Cos}\ a$$

in which $S =$ vertical amount of subsidence,
$m =$ thickness of coal worked,
$a =$ angle of dip of coal seam.

f is a coefficient whose maximum values are as follows, if filling is complete:

0.40 for dips 0—10°,
0.30 for dips 10—35°,
0.25 for dips over 35°.

When no filling is used f may be as much as 0.80.

Wachsmann* held that when an underlying coal bed is mined, the lowermost strata collapse, the next higher strata sink and crack if the excavation is of sufficient extent, while the uppermost strata sink without breaking or cracking. There is subsidence beyond the actual angle of break, as shown in Fig. 32.

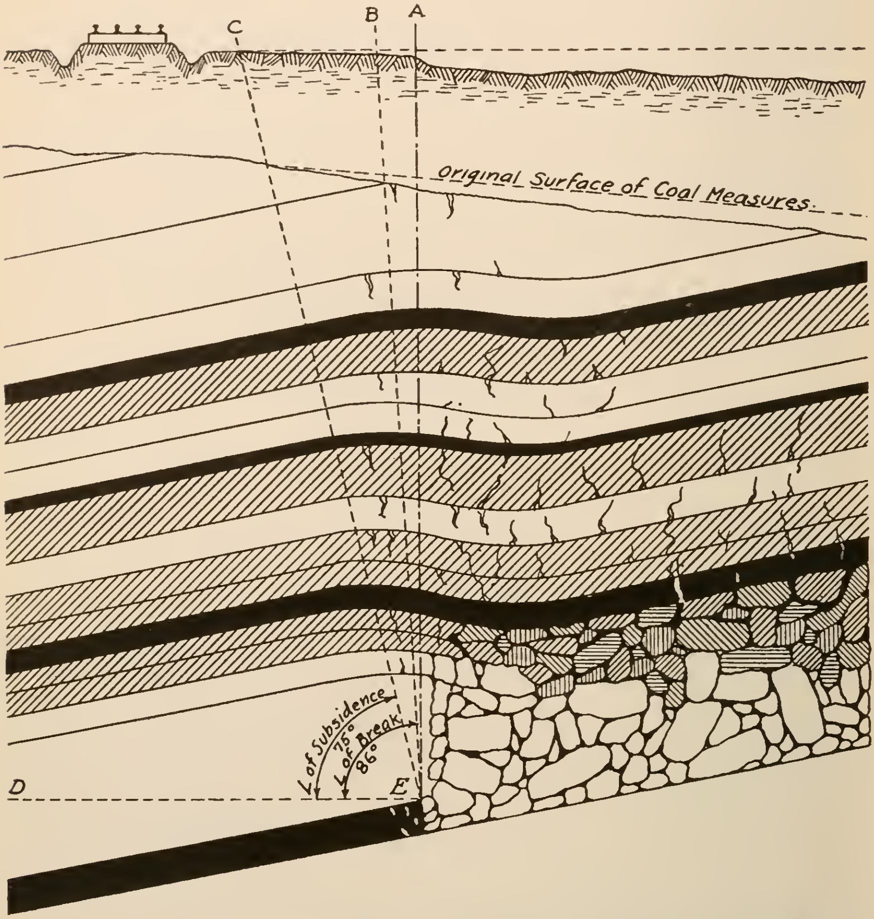

Fig. 32. Angles of Fracture in Rock and of Subsidence in Marl.

In 1885 Mining Engineer R. Hausse published some observations on the angle of break.* Owing to the great interest in the problem of subsidence, Hausse directed his entire time to the scientific investigation of various phases of the problem in Germany, particularly in Saxony, and published the results of his work in 1907.† In his preliminary chapter Hausse discusses the behavior of rocks over excavated areas and distinguishes between the "first" or "main break" and the "afterbreak." Over a horizontal bed he found that the main fracture is vertical and that the after break extends upward over the unmined coal. He pointed out that after the first falls occur, filling the cavity, the overlying strata subside, compressing the fallen material as filling is compressed by the roof in longwall mining. He stated that the plane of fracture lies between the vertical and the plane of sliding for the rock in question, and generally the plane of fracture will bisect the angle between the two limiting planes (Fig. 33). He discussed the relation between the directions of the main break, the after break, and the angle of inclination of the seam, as follows:

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*Über die Einwirkung des Oberschlesischen Steinkohlenbergbaues auf die Oberfläche. Zeit. f. Obersches B.- u., Hüttenmannischen Verein, p. 313, 1900.

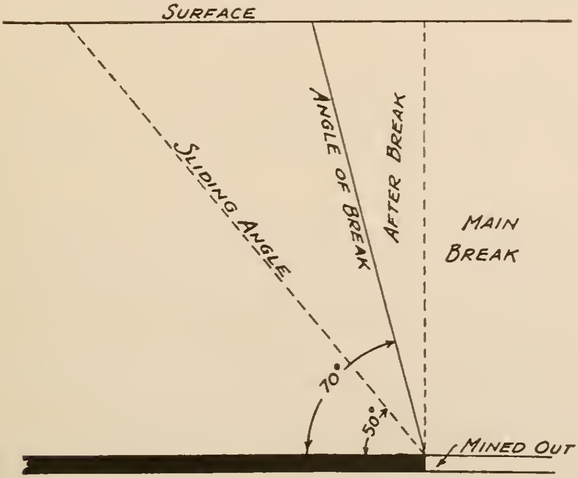

FIG. 33. HAUSSE'S THEORY; ANGLE OF BREAK BETWEEN VERTICAL AND SLIDING ANGLE.

The angle of main fracture on the upper side $= 90 \frac{a}{2}$, in which a=angle of dip. The angle of after break is assumed to be either constant or equal to 20°, or it is taken as decreasing from 20° to 10° in proportion to the increase of the dip from 0° to 45°. In a series of tables, Hausse presents the angle of fracture for various dips, making certain assumptions. He determined coefficients of increase of volume for mining with filling and without filling on the basis of observations made in the Royal Colliery at Plauen under the Dresden-Tharandt Government Railway. Where there was no filling, the coefficient was found to be 0.01 and with filling the coefficient was found to be 0.002.

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*Hausse, R. "Von der Niedergehen des Gebirges beim Kohlenbergbau und den damit Zusammenhangenden Boden-und Gebaudesenkungen," Ziet für das Berg-Hütten und Salinenwesen, pp. 324-446, 1907.

†Hausse, R. "Beitrag zur Bruch Theorie; Ehrfahrungen über Bodensenkungen und Gebirgsdruckwirkungen," Jahrbuch für das Berg-und Hüttenwesen in Sachsen, 1885.

Starting from Rziha's assumption of sliding angles, Trompeter* determined by the use of Hausse's observations the breaking zone with regard to the expansive power or increase in volume of broken rock. From his experience he found this for the Rhenish-Westphalian coal district to be an increase of 12 meters for every 100 meters in depth.

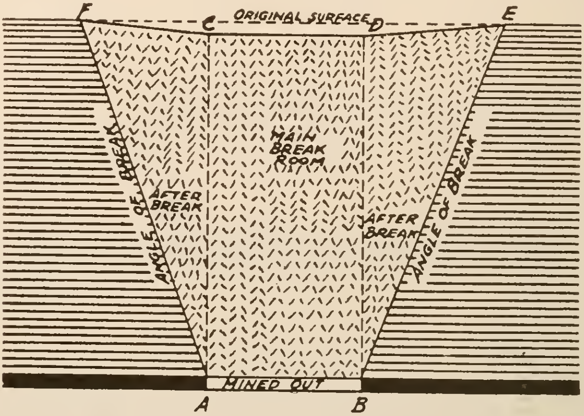

Fig. 34. "Main Break" and "After Break." (Hausse.)

Puschmann has described the subsequent working of overlying seams in the coal district of Upper Silesia.†

According to many engineers, the unwatering of the surficial material has been the immediate and sole cause of surface subsidence. Those who hold this opinion claim that surface movement has resulted from the sinking of shafts and the driving of boreholes alone and without any actual removal of the mineral deposit. There is, however, a wide difference of opinion on this matter. Von Dechen held that the subsidences[*] near Essen in 1866 and 1868 were caused by the partial drying of the marl and green sand overlying the coal measures. Graff[†] demonstrated by tests that drainage does not cause any change of volume of sand or quicksand and held that, when the water does not carry away any solids, there can be no subsidence resulting simply from unwatering.

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*Trompeter, W. H. "Die Expansivkraft im Gestein als Hauptursache der Bewegung des den Bergbau Umgehenden Gebirges," Oestreichishe Zeit., für Berg.-und Hüttenwesen, 1899.

†Puschmann Über den Nachträglichen Abbau Hangender Flöze beim Oberschlesischen Steinkohlenbergbau," Zeit. für d. B.-, H.-, u. S.-W., Vol. 58, p. 387, 1910.

The principal value of the work of the Germans has been in determining the angle of break, extent of surface subsidence relative to mine workings, and the practical coefficients of increase in the volume of the measures overlying the mined area, as these measures sink into the worked out area. Data upon these phases of subsidence have been collected in a systematic and accurate manner during a long period of years by engineers in the coal mining districts of Germany.

Austrian Theories.

Subsidence of surface due to mining operations has attracted considerable attention in the Ostrau-Karwin coal region in Austria. Ministerial regulations controlling the removal of coal under railways were formulated and became effective January 2, 1859. In 1876 Mining Director W. Jicinsky[‡] published a treatise on subsidence.

The question of the applicability of the government regulations for the protection of railroads to the railroads leading to the mines and serving the mines almost exclusively aroused discussion in the late 70's and the early 80's. The Board of Mines of Olmutz framed a regulation to modify the existing regulations so that they would not apply so strictly to mining railroads. F. Rziha, Professor of Railroad Construction at the Technical School of Vienna, was engaged to advise on the proposed regulation. He expressed his opinion on this question, formulated a theory of subsidence, and presented regulations to govern the exploitation of the coal under the mining railway.[¶] Rziha formulated a theory covering (1) the direction of fracture and (2) the amount of the vertical sinking of the earth, a brief statement of which follows:

(1) The Direction of Fracture. When rock is undercut, there is a tendency for it to fall or sink in proportion as gravity exceeds cohesion. The action may be falling or tearing, or both. He distinguished what he

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[*] Goldreich Die Theorieder Bodensenkungen in Kohlengebieten, p. 20.

[†] Graff "Verursacht der Bergbau Bodensenkungen durch die Entwasserung Wasserfuhrender diluvialer Gebirgs-schichten," Glückauf, 1901.

[‡] Jicinsky, W. "The Subsidences and Breaks of the Surface in Consequence of Coal Mining." 1876. Published later as a monograph of the Ostrau-Karwin coal district, 1884, "The Effects of Coal Mining on the Surface."

[¶] Oest. Zeit. f. B.-, u. H.-W., Vol. 29, 1881, and Vol. 30, 1882.

called a "falling space" and surrounding it, more or less concentrically, a "friability" or "tearing space." He found the falling space to approximate the shape of a paraboloid. First the rock becomes loosened and afterwards falls when gravity exceeds cohesion. In time, limited by the structure of the rock, a dome-shaped space *abc* is formed (Fig. 35), working laterally and vertically from the center of disturbance. Outside this space and more or less concentric is the dome of tearing, indicated by the dotted line *amc*. If the tearing sphere extends to the surface, it will cause surface disturbance within the area bounded by *mn* (Fig. 36), and the overhanging wall will gradually change its slope, depending upon the lateral extent and degree of the tearing and the relation between gravity and cohesion (see Fig. 37). Rziha thought that the stratification of the beds did not have much effect upon the angle of break. It may be noted that he did not make a detailed study of the subsided area in the field. The Mining and Metallurgical Society of M-Ostrau held that actual subsidence in the Ostrau-Karwin district did not conform to Rziha's theory.

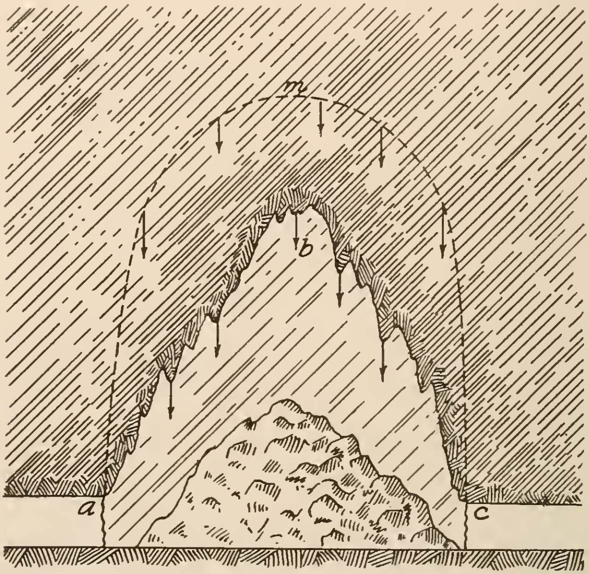

FIG. 35. "ZONE OF FALLING" AND "ZONE OF TEARING."

(2) Vertical Movement. Rziha treated this subject under two headings: (a) The collapse into the underground excavation, and (b) the unwatering of the roof, causing a decrease in volume. He held

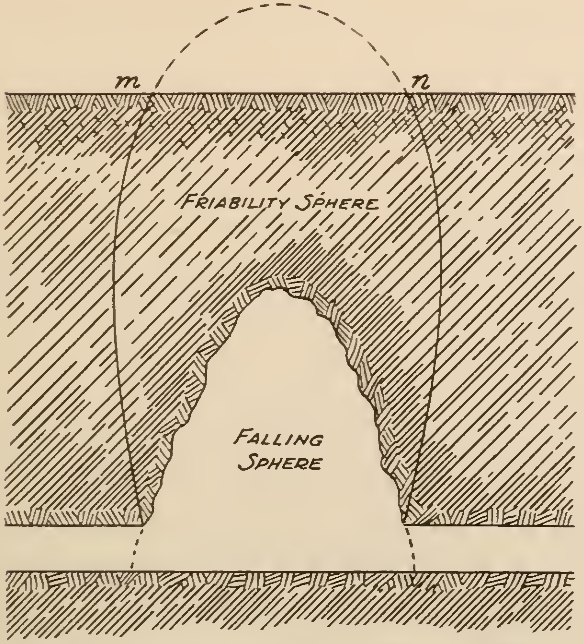

FIG. 36. "ZONE OF TEARING" EXTENDED TO SURFACE.

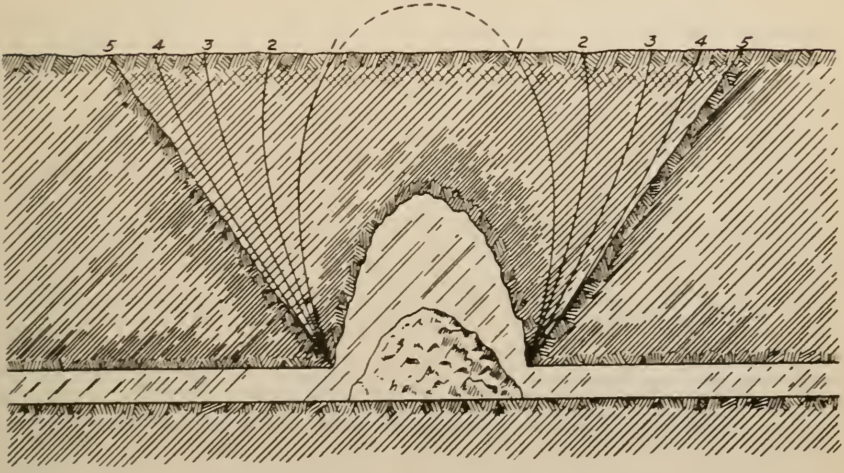

FIG. 37. SUBSIDENCE OUTSIDE UNDERMINED AREA.

that when mining is carried on at a great depth there may be no disturbance of the surface, and attempted to determine this depth by the formula:

$$h = \frac{M}{a}$$

in which h = harmless depth,
a = coefficient of increase of volume,
M = vertical seam thickness.

When pillars are left, he assumed that they will reduce subsidence, and the formula used to determine the harmless depth is:

$$h = \frac{M\,B}{a}$$

in which B is a coefficient for pillars and the filling, varying from 0.50 to 0.60. Rziha presented coefficients for the increase in volume of six kinds of rock. Goldreich* is of the opinion that an additional factor, overlooked by Rziha, is height of excavation. The less the height, the greater is the probability that the overlying rock will sink without complete or extensive crushing. Moreover, the completeness of the compression of the packing under the roof weight depends somewhat upon the height and extent of the excavation. These coefficients were not secured by mine investigation.

Goldreich criticises Rziha's suggestion of leaving pillars, and states: "If Rziha had been acquainted with the shape of the surface depressions, he would never have recommended the working methods given in his report." Objection is made also to the idea of a harmless depth as it "logically implies there ought to be the possibility of creating cavities of unlimited size beneath the harmless depth without giving rise to land movements on the surface." The Mining and Metallurgical Society of Ostrau (Moravia) to which organization Rziha's regulation was submitted, published its opinion in 1882.† Observations over a period of thirty years in the Austrian coal fields lead Goldreich to support Jicinsky's statement that no movement occurs if the water does not carry away from the volume under observation any solids mechanically or in solution.

Goldreich states‡ that the efforts to study theoretically the causes of subsidence in Austria date from the year, 1882. The Committee of the Mining and Metallurgical Society of Ostrau, Moravia, consisting of W. Jicinsky, J. Mayer, and von Wurzian, attacked Rziha's theory on the basis of observations in the Ostrau-Karwin district. Rziha failed to consider the probability of the overlying rock bending and sagging without breaking to fill the worked-out space, as shown in Fig. 38. In the event that sagging takes place, the increase in volume will be much smaller than that figured by Rziha, who assumed a breaking up of rock. (Fig. 39.)

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*Goldreich Op. cit., p. 53.

†Oestrr. Zeit. für B.-, u. H.-W., Vol. 30, 1882.

‡Goldreich Op. cit., p. 45.

The Committee preferred to use the term "undangerous depth" in addition to Rziha's phrase "harmless depth" for those depths at which mining will produce a gradual subsidence of the surface without damage to small objects or structures.

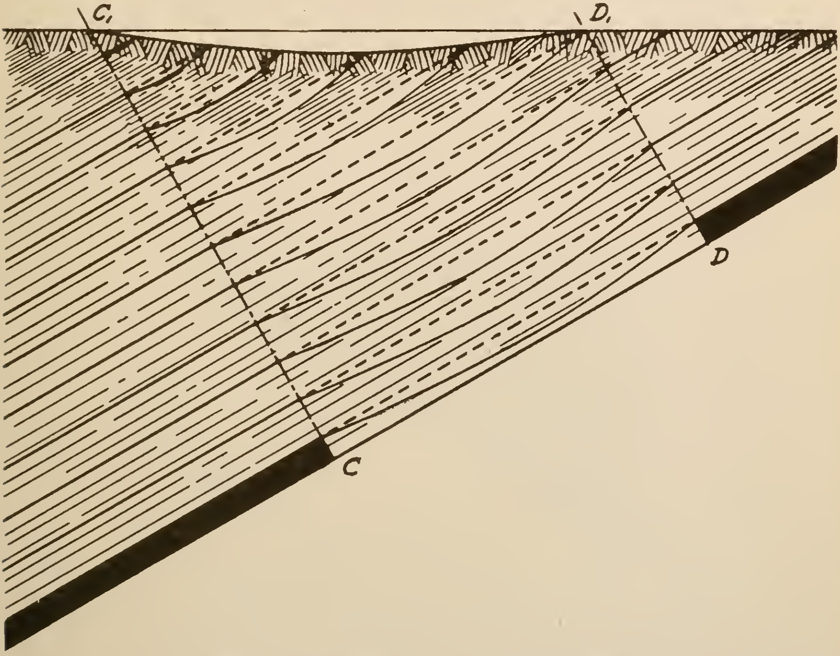

Fig. 38. Large Subsidence in Case of Bending of Rock.

Jicinsky, a member of the committee, found the average increase in the volume of the rock of the coal measures and calculated the harmless depth from the maximum surface subsidence observed, according to the formula:

$$s = t + m - a\,t$$

$$\text{or } a = 1 + \frac{m - s}{t}$$

in which

s = surface subsidence,
t = thickness of coal rock exclusive of the coal bed,

m = thickness of coal bed,

a = average coefficient of increase of volume of the coal rock considered as a whole.

The term "coal rock" means the bed of rock overlying the coal which is broken in the course of subsidence, with consequent increase of volume. If s is made 0, the formula shows the thickness of overlying rock, exclusive of the marl, necessary to prevent surface subsidence. The entire mass of material above the coal rock is believed to settle without increase in volume and is not taken into account in the formula.

The average coefficient of increase of volume of the coal rock is found to be 0.01 (i. e., $a = 1.01$) for several cases of subsidence indicated by Jicinsky. His conclusion is then, as expressed by the formula, that the surface subsidence is equal to the thickness of the bed taken out minus 0.01 of the thickness of the overlying rock which is shattered by movement.

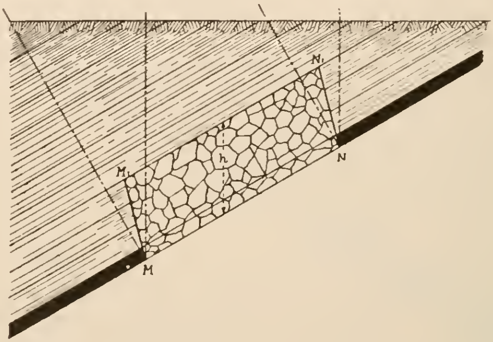

FIG. 39. SMALL SUBSIDENCE IN CASE OF BREAKING OF ROCK.

The Committee also noted the vertical amount of subsidence and the duration of the movement, and special protective devices for shallow depths were considered. It was estimated that filling was compressed to 0.6 of the thickness of the bed so that in determining the harmless depth only 0.4 of thickness of the coal bed should be used in the formula. The committee agreed that "the interests of national economy demand that the leaving of coal pillars shall be prescribed in the rarest cases" and "the cost of the surface objects to be protected is to be compared with the coal losses." The Committee prepared regulations for coal mining under the mining railways, and these later became the basis of the regulations adopted.

Jicinsky's* monograph on subsidence appeared in 1884 and contained a complete statement of the principles accepted at that time by the leading Austrian engineers, and in it he formulated the following principles:

"The subsidence depends: (1) on the thickness of the seam, (2) on the dip of the seam, (3) on the depth of the mine, and (4) on the constitution of the roof rock of the seam. It may be regarded as a rule: (a) That the depth of the land subsidence is directly proportioned to the thickness of the seam, and the extension of subsidence is in direct proportion to the mined area; (b) that the depth of the surface subsidence increases with the dip of the seam, whereas its extension decreases, and for this reason in vertical seams the land subsidence is deep but manifests itself in the form of a kettle-shaped pit."

Jicinsky held further: "During every collapse of a solid rock there takes place a piling up of the broken masses. It follows that such a break causes an increase of volume and at a certain height in consequence of this increase in volume the entire empty space is so filled that no further after-break is possible; hence the effects of the break upon the surface must decrease with increasing depth. Every rock, even every single rock stratum, has its own hardness and toughness, and for this reason not all the kinds of rock behave in the same way during their collapse."

The subsidence formula of Jicinsky can be used to determine the harmless depth. Jicinsky discussed this point at length. He also made a study of the direction of fracture in the overlying rocks, and held that along the strike fracture is always normal to the coal bed. He objected to Gonot's and Schulz's theories for fracture on the dip and held that the angle was midway between the normal and the vertical. His views have been supported by 80 per cent of his observations.

He also considered in his monograph the amount of surface subsidence resulting from the mining of several superimposed seams, the various stages of rock movements, and safety pillars. In his opinion arbitrary rules cannot be used for determining the size of pillars, but each case must be studied separately and figured according to the local conditions.

C. Balling made a study of the angle of fracture in the northwest Bohemian brown coal basin and found that, for a depth of not more than 300 feet and a dip of 8 degrees, the angle varied from 68 to 74 degrees.*

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*Jicinsky, W. "The Influences of Coal Mining Upon the Surface." Monograph of the Ostrau-Karwin Coal District, 1884.

Chief Mine Inspector Anton Padour also made a report upon subsidence in the northwest Bohemian brown coal district.† He noted the vertical amount of subsidence and found the following relation:

$$H = 46 \sqrt[3]{h^2}$$

in which

H = vertical height of subsidence,
h = height of excavation.

He found that in the Bruch district, where the covering is firm marl, the angle of fracture is as follows:

(1) Thickness of capping from 1,000 to 1,200 feet, angle of seam from 0 to 8 degrees.

(a) towards the dip, angle varies from 72 to 69 degrees.
(b) towards the rise, angle varies from 72 to 74 degrees.

(2) Thickness of from 1,000 to 1,100 feet, angle of seam from 27 to 30 degrees.

(a) towards the dip, angle varies from 63 to 60 degrees.
(b) towards the rise, angle varies from 78 to 77 degrees.

In 1911 Goldreich delivered a lecture before the Austrian Society of Engineers and Architects of Vienna on the "Theory of the Railway Subsidences in the Mining District, With Special Consideration of the Ostrau-Karwin Coal District." Since that date he has made several important contributions to the literature of this subject.

In 1912 Franz Bartonic discussed the causes of subsidence, but made no contribution to the theory.‡

In 1913 Goldreich published his work on "The Theory of Land Subsidence in Coal Regions with special Regard to the Railway Subsidences of the Ostrau-Karwin Coal District," and followed this with a volume entitled, "Land Movements in the Coal District and their Influence on the Surface."¶

Goldreich criticises Jicinsky's contribution and theory in a number of points. He takes no exception to the fundamental principles but objects to the formula. Goldreich questions the assumption of a coefficient of increase of volume that will be applicable to all cases.

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*Balling, C. "Die Schatzung von Bergbauen nebst einer Skizze über die Einwirkung des Verbruches unter-irdischer, durch den Bergbau geschaffener Hohlraume, auf die Erdoberflache," A. Becker, 1906.

†Padour, Anton Chapter on "Damage to the Land and Buildings" in the "Guide Through the Northwest Bohemian Brown Coal District," 1908.

‡Bartonic, Franz "Die Ursachen von Oberachenbewegungen im Ostrau-Karwiner Berg-Revier. Montanist, Rundschau, Feb. 16, March 1 and March 16, 1912.

¶In press.

He points out the fact that the so-called "after-slide" of surficial material is not considered in the formula. He accepts Jicinsky's determination of the fracture in the coal measures, but objects to the formulating of general rules for determining the size of pillars. The principal portion of Goldreich's work is given to a discussion of the phenomena of subsidence in connection with railways. "From the profiles of sunken railway sections of the Ostrau-Karwin coal district it can be seen that these profiles have a parabolic form, that the maximum subsidences are found in the middle of these depressions, and that the amounts of subsidence decrease almost regularly towards the two ends of the curves until they finally become equal to zero." This regularity of the depressions caused Goldreich to undertake to formulate a theory of subsidence applicable to conditions such as those which exist in the Ostrau-Karwin district, the most distinctive feature being the surficial bed of plastic marl as much as 1,200 feet thick in places. Where the coal measures outcrop, the regularity of the surface depressions disappears and Goldreich takes refuge in the statement that we must depend merely upon experience.

Goldreich's observations developed the fact that following the subsidence of bed rock there is a vertical subsidence of the marl directly overlying and a lateral after-sliding of the adjacent and outlying marl. In discussing the subsidence of the roof strata he emphasizes the effect of the elasticity of each stratum. "When the elasticity of the subsiding roof strata is so great that the latter reach the floor of the worked out room without any disturbance in the coherence of the superimposed strata, then the volume of these subsiding strata remains unchanged." The subsidence of roof strata without increase of volume will occur in the case of the extraction of thin and flat seams. "The increase of volume which takes place during the first stage of the subsidence process is not enduring; for in consequence of the weight of the following roof strata the broken rock is again compressed, so that at the end of the rock movement there results a decrease of volume which is certainly not identical with the initial increase of volume." Only by observing the amount of surface subsidence caused by an underground working can a basis for estimating the coefficient of increase of volume under actual conditions be obtained. When the overlying beds are elastic there will be little increase in volume as the movement proceeds upward; under such conditions the term "harmless depth" cannot properly be used. "It cannot be pointed out strongly enough how absurd is the establishment of a harmless depth which should be valid for all workings; the harmless depth has rather a theoretical character because the presuppositions required for the actual existence of the harmless depth are very seldom true in practice."

British Theories.

As previously noted, subsidences resulting from the mining of salt and coal in the British Isles were observed at an early date and were the cause of investigation by British engineers, who in general have supported the important principles of Belgian-French theories, although certain persons have taken exception to particlar points in these theories. Numerous observations have been made upon subsiding areas and considerable valuable information has been collected, the data have been correlated and arranged, and empirical formulæ have been constructed so that adequate pillars may be left for the protection of surface structures and property of various kinds.

In 1868 a commission of Prussian engineers investigated subsidence in the various coal mining districts of England, and found that in England the opinion was approximately as follows:

(1) The working of coal at every known depth may affect the surface, but at depths greater than 400 meters (1,300 feet) it can cause damage only to certain buildings, such as cotton mills.

(2) In the case of complete extraction, filling may be a means of effective protection against movements of the earth.

(3) The practice of leaving pillars constitutes an efficient protection against the effects of exploitation upon the surface. The extent of these pillars of safety should be determined by the surface to be protected, the depth being known and the angle of rupture being assumed.

Observations carried on by J. S. Dixon demonstrated that the wave of maximum subsidence regularly followed the advancing face and that a wave of disturbance was just as regularly projected in advance of it;* that is, the wave of disturbance preceded the working face, but the maximum subsidence followed it. Joseph Dickinson called attention to the similarity between earth movements due to natural causes and those resulting from mining operations. He considered that "the direction of subsidence may be judged by analogy from the slopes taken by faults and mineral veins. The slope of a fault in horizontal strata averages about 1 in 3.07 from the perpendicular, varying according to the hardness and cohesion of the strata from about 1 in 5 in hard rock to 1 in 3.75 in medium, and 1 in 2.5 in soft. He considered that for horizontal seams not exceeding 6 feet in thickness, and with strata of the average hardness of those in Lancashire, ordinary subsidence may be taken as extending on all sides to one-tenth of the depth, and that to obtain security a margin should be added. This margin is limited by some engineers to an additional one-tenth of the depth, while others add an arbitrary amount. When the strata are softer the extent of the subsidence is sometimes taken as one-sixth, or even one-fourth of the depth of the working, while, on the other hand, for hard siliceous rock, such as is found in South Wales, reductions are needed. He also agrees with other writers, that in seams of moderate inclination larger areas are required for support on the rise side than on the dip."* T. A. O'Donahue in discussing subsidence† endorses the observations of J. Dickinson in the following language: "Joseph Dickinson is practically the only writer who has succeeded in connecting the threads of what was apparently a mass of contradictory evidence and in showing that the majority of cases approximately agree with a more or less definite rule." In O'Donahue's opinion, which is the result of considerable experience in studying the effect of surface subsidence, including the taking of levels, the "breaking lines of strata may be estimated within narrow limits with average conditions." He enumerates the important factors affecting the position of the breaking lines and the ultimate extent of the subsidence as (1) the relative hardness of the strata, (2) the inclination, and (3) the thickness of the coal seam. He also mentions the influence of surface deposits. He considers the various angles of draw that have been noted and points out that for safety the maximum angle for given conditions must be taken as the limit for safety. For coal beds 6 feet thick and overlying strata of moderate hardness, he has found that the angle of draw is from 5 to 8 degrees beyond the vertical. This means that if a pillar is to be left to protect objects on the surface, a margin of one-twentieth to one-tenth of the depth should be left in order to provide against the draw. With inclined strata the draw increases roughly in proportion to the degree of inclination of the strata. He accepts the normal theory as correct when applied to dips of 18 to 24 degrees, but only for dip workings. When the mine workings are on the *rise* the maximum draw is estimated at 8 degrees for strata nearly horizontal and the draw is taken to cease with strata at an inclination of 24 degrees.

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*Dixon, J. S. "Some Notes on Subsidence and Draw." Trans. Min. Inst., Scotland, Vol. 7, p. 224, 1885.

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*Hughes, H. W. "A Textbook of Coal Mining," 5th Edition, p. 185, London, 1904. Dickinson, J. "Subsidence Due to Colliery Workings," Proc. Man. Geol. Soc., Vol. 25, p. 600, 1898, and Colliery Guardian, Oct. 28 and Nov. 11, 1898.

†O'Donahue, T. A. "Mining Formulæ," p. 244, Wegan, 1907.

The ideas of O'Donahue are expressed in his formula for shaft pillars as follows:

M = Margin of safety, say from 5 to 10 per cent of the depth,

D = depth of shaft,

X = distance at the seam, between two lines drawn from a point at the surface, one line being vertical and the other at right angles to the seam.

Shaft pillar in horizontal strata

$$\text{Radius of pillar} = M + \frac{D}{7}$$

In inclined strata

$$\text{Rise side} = M + \frac{D}{7} + \frac{2}{3}X$$

$$\text{Dip side} = M + \frac{D}{7} - \frac{1}{3}X$$

For seams less than 6 feet thick the size of the pillar may be decreased, while for thick seams it is suggested that the size of pillars determined for a 6-foot seam be multiplied by the "square root of the thickness of the seam in fathoms."

In discussing the effect of the thickness of the seam upon the amount of subsidence, O'Donahue calls attention to the effect of the material stowed in the goaf or gob. He makes the point that, other conditions being the same, the mining of a 6-foot seam would result in more than twice the vertical subsidence caused by mining a 3-foot seam, owing to the fact that little material is thrown into the gob in mining the 6-foot seam, while in mining the 3-foot seam undoubtedly much "brushing" would have to be done and, therefore, there would be considerable material left in the goaf or gob. Therefore, the total subsidence per foot of coal removed will be greater in the case of the thicker seam.

He objects to the statement that mining at depths of 1,800 to 2,000 feet will not cause subsidence, because careful levellings will show that the complete removal of the coal at even greater depths will cause a sinking of the surface. "When the whole of the mine is taken out subsidence of the surface follows at all workable depths. The writer's observations show that the working of a seam, for instance 4 feet thick, will cause the surface to subside about 3 feet if the seam be not greater than 600 feet in depth, and will cause a subsidence of from 12 to 18 inches at a depth of 2,400 feet.

H. W. G. Halbaum has made a careful study of roof pressures in longwall work and has made some notable contributions to the theories of subsidence. In his study of the action of the roof in longwall mining* he called attention to the *locking of the roof* due to the lateral thrust in great roof sections. "The great roof sections, by successive slips, have descended a few inches. The motion has been arrested for the time being by the lateral thrust, and the great body of strata remains securely gripped in the powerful jaws of its natural clamps." Subsequently Halbaum formulated the following proposition: "Contained in the total force of the roof action, there is a horizontal component, the action of which is contrary to the direction of working, and the power of which is sufficient to deflect the roof action from the vertical line."†

After discussing carefully the planes of strain, Halbaum considers the cantilever idea. He likens the unsupported roof strata to a huge cantilever whose load consists primarily of its own weight. "It is evident that if the cantilever were homogeneous and if the neutral surface were at half-depth, and if efficiencies of the compressive and tensile stresses to propagate their respective strains were equal, we should, under such conditions, obtain a mean line which would be vertical; for the tensile and compressive components would be equal in length and equal, though opposite in obliquity. The obliquity of one component would thus exactly balance the opposite obliquity of the other, and the mean line would be vertical." He then points out that such a balancing of components is unlikely, for the resistance of ordinary coal-measure strata to compression is usually greater than their resistance to tension. The neutral surface of the cantilever must generally lie below the half-depth of the beam. Moreover, the beam is not homogeneous. "Viewed from the broader standpoint of internal nature and external environment combined, there must be little or no exaggeration in the statement that our cantilever is immeasurably stronger to resist compression than to resist tension; and hence we are bound to infer that its neutral surface is very low indeed and probably not many feet above the roof-line itself." "It follows that by far the greater portion of the absolute line of elementary strain is supplied by the tensile component, that by far the greater portion is projected over and towards the solid, and that the mean elementary line must, therefore, possess a normal obliquity little less in magnitude than that of the tensile component itself." Stated in brief the idea is this: "We start with a thick unloaded cantilever and we end with a thinner but loaded beam; thinner, because from the standpoint of their efficiency, the upper layers are gone; and loaded, because from the standpoint of their dead weight, the upper layers remain only as a true load on the effective beam beneath. This simultaneous thinning and loading of the effective cantilever seems probable for several reasons: The principal one perhaps is to be found in the fact that the original beam is a composite beam formed by an aggregation of smaller beams (strata) in superposition. The whole of the composite beam is an effective beam only so long as its several layers firmly adhere at their conterminous horizontal planes or boundaries. As soon as the uppermost layer (or series of layers) separates from its subjacent layer, or tends to slide thereon, it ceases at once to form any part of the effective cantilever, to which cantilever it must thenceforward sustain the relation of a load only." He calls attention to the fact that "when we examine the cases of natural subsidences of the earth's crust, we find that the great planes of strain, in the normal case, are always projected over and towards the solid (or unsubsided) strata."

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*Halbaum, H. W. G. "The Action, Influence and Control of the Roof in Longwall Workings." Trans. Inst. Min. Engrs., Vol. 27, p. 211, 1903.

†Halbaum, H. W. G. "The Great Planes of Strain in the Absolute Roof of Mines." Trans. Inst. Min Engrs., Vol. 30, p. 175, 1905.

In a paper before the International Geological Congress, Professor George Knox summarized the various points "which may be considered sufficiently well established to form a basis for further investigations—namely:

(a) That surface subsidence invariably extends over a greater area than that excavated.

(b) The angle of pull is determined by the ratio between the excavated and subsided areas.

(c) That this ratio is determined by a large number of factors, among which may be included the following:

1. The amount of permanent support left in the unmined area.
2. The thickness of the seam worked.
3. The depth of the workings from the surface.
4. The method of working adopted.
5. The direction of working in relation to the jointing of the strata.
6. The rate at which the workings advance.

7. The nature of the strata overlying the workings.
8. The presence of faults, fissures, etc., in the strata.
9. The permeability of the overlying rocks.
10. The dip of the strata.
11. The surface contour.
12. The potential compressive forces existing in the strata containing the workings."*

He concludes that the ratio between subsidence and draw must be the joint result of the forces liberated by the withdrawal of support from underneath the strata in the mined area. The larger the proportion of settlement resulting in subsidence the less can occur in the form of draw, and vice versa "The number of factors that may influence the results produced by the settlement of undermined strata is so great that only a wide and comprehensive inquiry by geologists and mining engineers in those countries where mining is conducted on a large scale can be hoped to provide sufficient evidence to establish a definite theory or theories to assist in overcoming some of the more common dangers due to subsidence."

Alexander Richardson, in a paper before the Chemical, Metallurgical, and Mining Society of South Africa, took up the question of stresses in deep masses of rock unsupported for hundreds of feet horizontally. "Where the strata are unfaulted, one would be justified in considering the mass as a huge slab supported on two or more sides or as a lever hinged at the bottom of the workings. Over extensive areas the pressure on the roof of an excavation, assuming the bed to be horizontal, will become in time equal to the weight of the superincumbent strata; under no circumstances is it immediately so, since the overlying beds must have some carrying strength."†

Opinions of American Engineers.

While no new theories have been advanced by American engineers it may be profitable to review their opinions as given to the public through papers, investigations, or testimony.

As previously noted, a number of prominent engineers have made investigations as to the nature, extent and cause of the damage to property resulting from surface subsidence in Scranton, Pennsylvania. The published and the special reports noted on page 28 include expressions of opinion, but little discussion of the principles and theories of subsidence.

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*Knox, George "Mining Subsidence." Proc. Int. Geol. Congress, Vol. 12, p. 798, 1918.

†Richardson, Alexander "Subsidence in Underground Mines," Jour. Chem. Met. and Min. Soc. of S. Africa, Vol. 7, p. 279, March, 1907; Eng. and Min. Jour., Vol. 84, p. 196, 1907.

Douglas Bunting, who has made a study of the "Limits of Mining under Heavy Wash" in the anthracite region of Pennsylvania,* considered the various sedimentary rocks of the coal measures and determined the minimum thickness of rock cover for various depths below the surface and for rooms of various widths. He had previously made a study of chamber pillars in deep anthracite mines, and had calculated the width of rooms for various depths upon the basis of the compressive strength of anthracite.†

In discussing subsidence in the longwall district of Illinois, G. S. Rice said, "The roof settles most in the first few months, but it is several years before it is entirely settled, by which time the gob has been squeezed down to one-half or one-third its original thickness." The roof is very free from slips and vertical cracks or joints until the coal has been mined below it, but when the coal is brought down in a long strip, it marks the roof just where the break of coal has occurred, and along these marks the roof afterwards breaks. These breaks seem to run up indefinitely, and oftentimes they can be followed up to the black slate, 8 or 10 feet above. As a result of mining the seam, which varies in thickness from 2 feet, 10 inches to 4 feet, or an average of 3 to 3½ feet, "the settling of the roof is appreciable at the surface even when the seam is at a depth of 400 or 500 feet; but so gradual is it and without vibration that the deep mines have caused no trouble in going under railroad tracks, and even under brick buildings, as has been done at La Salle."‡

Charles Connor believes "If we extract all the coal we, naturally, will have a subsidence of the surface. That must inevitably follow because, when the support is all removed, the rock settles down on the floor of the mine." He cited observations made in the county of Lanark, Scotland, where the mining of seven seams approximating 30 feet in thickness and lying at depths of from 900 to 2,700 feet necessitated the raising of canal banks 18 feet. The sinking was gradual and no water was lost out of the canal.¶

In discussing the action of beds overlying mine workings

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*Amer. Inst. Min. Engrs., Bul. No. 97, p. 1, 1915.

†Bunting, Douglas "Chamber Pillars in Deep Anthracite Mines." Trans. Amer. Inst. Min. Engrs., vol. 42, p. 236, 1911.

‡Rice, G. S. "System of Longwall Used in Northern Illinois Coal Mines," Columbia Univ. School of Mines Quart., Vol. 16, p. 344, 1894.

¶Connor, Charles "Discussion of Paper." Proc. Coal Min. Inst. of America, p. 149, 1912.

W. A. Silliman expressed the opinion that these beds do not act as a monolith, but that the beds may have little adherence to each other. Where the measures are weakest the strain will be greatest, as in the fireclay bottom.*

S. A. Taylor has pointed out the fact that the tendency of one measure to slip on another is "counteracted by the fact that the roof is a continuous mass, so that as soon as any part tends to give, it has to slide to get past adjacent surfaces and the surfaces are pressed together with no little weight. If any part is weak, it is nevertheless held in place and the strain goes to the other measure." In his opinion the roof rock acts as a monolith in most cases.† He also believes that the occurrence or absence of subsidence depends on the height and character of the overlying strata. "You cannot set any hard and fast rule; the rule set down by the English authority 'one to ten' (there will be no breakage on the surface if the rock cover is ten times the thickness of the coal worked) will not hold. It may be true in some cases, but it will not serve as a universal rule, as its truth or falsity in any instance depends on the character of the overlying strata."‡

R. D. Hall has suggested that when the roof sags down over the edge of a pillar the curve of the roof tends to follow back over the solid coal, criticising the general notion that the roof lies flat upon pillar and then sags down over the edge of the pillar.¶ He has also shown to what extent in his opinion shear is the cause of the failure of mine roof.§ He concludes his discussion of shear, "In the case of mine roof, everyone seems to be confident that we have a structure which invariably fails from shear. The idea is contrary to all the evidence and should be dismissed. The raggedness of roof fractures disproves it if other reasoning does not."** In discussing the strength of mine roofs R. D. Hall has presented a series of sketches, showing conditions producing breakage of roof.†† He suggests that the roof over rooms acts after the first fractures not like a beam but like an arch, and that continuous beams or plates are replaced by disconnected arches or vaults. He concludes by suggesting a "progressive advance in demolition: First, a condition, as yet unnamed, symbolized by the tunnel in solid rock in which roof and sides and floor all partake of the beam strain. Second, a horizontal shear which converts the sides into mere supports and the roof into a true beam or plate; Third, a rupture of the roof which converts it into an arch, and finally, a failure of the arch or vault by one of the many weaknesses to which such structures are subject."*

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*Proc. Coal Mining Inst. of America, p. 84, 1911.

†Op. cit., p. 85.

‡Taylor, S. A. In discussion of R. D. Hall's paper, "Effect of Shear on Roof Action." Proc. Coal Mining Inst. of America, p. 146, 1912.

¶Hall, R. D. "Action of the Roof." Proc. Coal Min. Inst. of America, p. 64, 1911.

§Hall, R. D. "Data of Petrodynamics." Mines and Minerals, Vol. 31, p. 210, 1910.

**Hall, R. D. "Effect of Shear on Roof Action." Proc. Coal Min. Inst. of America, p. 144, 1912.

††Hall, R. D. "The Strength of Mine Roofs," Mines and Minerals, Vol. 30, p. 474, 1910.

The tendency of the roof to arch has long been noted, and the mechanics of natural rock arches has been discussed by a number of engineers. However, there has been little agreement among engineers as to the portion of the burden of the overlying beds which is actually borne by such natural arches. The strata acting as a uniformly loaded horizontal beam cannot support a great load, and as the strata sink the upper measures tend to arch and eventually the entire mass may be supported by the arch.

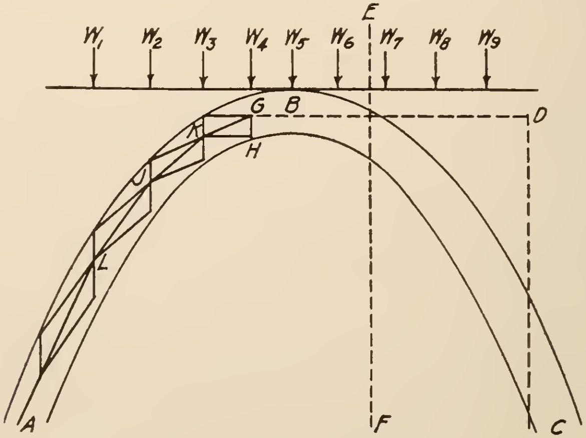

FIG. 40. STRESSES IN ARCH.

The theory of the arch as applied to this problem has been discussed by B. S. Randolph† as follows: In the arch *A B C*, Fig. 40, the two sides *A B* and *B C* are mutually supported at *B* where the thrust is horizontal. Assuming the load to be evenly distributed over the arch, it is found that the points *B, G, K, J, L* all lie in the line of stress. This line of stress "when lying in solid material over an excavated cavity will constitute, for all practical purposes, an arch supporting all the material above it and allowing the removal of all the material below it up to the point where this material becomes effective in resisting the stress. There will, of course, exist along and on each side this line of stress a zone of material under more or less pressure, depending for its width on the total stress and the elasticity of the material. The position and character of the forces acting on the arch will vitally affect the shape of this line of stress. In an arch under a perfect fluid, where the pressures are all radial acting toward a common center, the line of stress becomes the arc of a circle. With an excess of load toward the center, it takes the shape of the parabola, the focal distance shortening as the central load exceeds that on the side. With the excess of pressure on the sides, say at an angle of 45 degrees, it assumes the shape of an ellipse, the focal distance shortening as the pressures at the side exceed those in the middle." In the arch formed over rooms, as the load for all practical purposes is equally distributed, "the curve will be a parabola with a longer or shorter focal distance

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*Hall, R. D. "The Last Stand of the Mine Roof." Coal Min. Inst. of Amer., 1914, and Coal Age, Vol. 6, p. 982, 1914.

†Randolph, B. S. "Theory of the Arch Applied to Mining." Coll. Engineer, Vol. 35, p. 427, 1915.

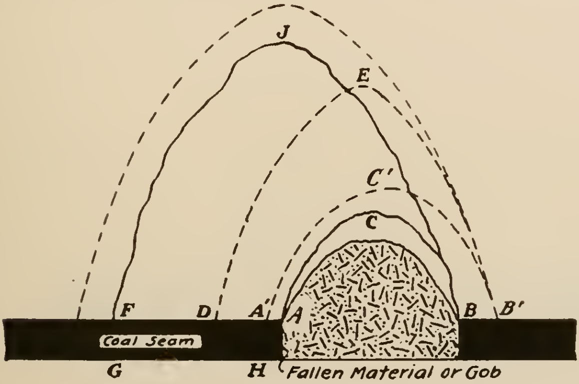

FIG. 41. ARCH STRESSES IN MINE ROOF.

depending on the nature of the strata." "Let Fig. 41 represent the section of a coal seam from which the coal has been removed between *A* and *B*, the roof having fallen to the irregular line *A C B*. The dotted line *A′ C′ B′* will indicate the line of stress. This, it will be seen, impinges on the coal close to the edge at *A*. The stress at this point represents half the weight of the strata overlying the span *A B* which is assumed to be sufficient to crush the coal about the point *A*. The integrity of the arch being destroyed, the line of stress must seek a new position such as *D E B*. Naturally this movement will be no greater than is absolutely necessary to gain a solid footing for the arch, which will again be so near the edge of the coal already crushed that it will fail again in a short while, necessitating a further adjustment of the position of the arch. With this continuing failure and readjustment we have the well-known phenomena of a crush or squeeze advancing slowly over the workings, destroying coal as it goes.

If now a considerable body of the seam, as *A H G F,* is quickly removed a "fall" may result which will reach high above the seam, say to the line *F J B,* which will cause the line of stress to move quickly and reach the coal well back from the point *F,* where it is sufficiently solid to give the needed support, and the working will be said to have "gotten ahead of the crush," when in fact the crushing force has gone ahead of the working. This explains the common experience of the relief at the working end of the pillar caused by an extended break in the roof over the exhausted area.

Under other conditions, especially when the arching line of stress has a wide span, thus carrying a large amount of weight, the crushing force may prove too much for even the solid coal well back from the end of the pillar and cause the phenomena of crushed coal, broken timbers, creeping floor, etc., well down the room or stall, while the ends of the pillars will be free from any trouble, as they carry only the small amount of material which is below the line of stress. This condition will sometimes be cured automatically by the material falling from the top of the cavity over the exhausted area in such a manner that the space between the material already down and the undisturbed measures will be filled and the opposite limb of the arch (the right hand in the figure) will find support on this already fallen material and thus shorten the span of the arch and lessen the total weight, as illustrated in Fig. 42.

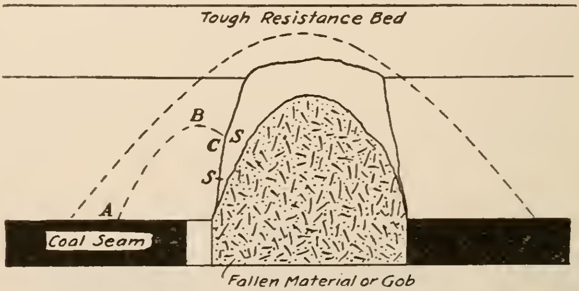

FIG. 42. SPACE SHORTENED BY FALLING OF ROOF.

When the break has reached the surface, this filling takes place more rapidly owing to the fracture of the overhanging beds along the edge of the break and, since the arch has a new point of support for its inner or right hand limb, the conditions are ripe for further working undisturbed under the smaller arch.

While the general shape of the line of stress in the cases under consideration is the parabola, for all practical purposes the ratio between the span and the rise or height of the arch will vary much as the material varies in which it exists. After the fall of the first mass the cavity grows through the crushing and falling of the material along the top of the arch, due to the pressure along the line of stress, and by the splitting off along the joint planes of the material on the sides due to the same cause. Since the pressure along the upper portion of the line of stress is manifestly less in a high sharp arch than in a low flat one, the shape of the arch in this respect may be expected to vary with the capacity of the material to withstand this stress. Hence, there will be a high arch in a soft material with numerous joints and a flat arch in tough material with fewer joints. This may be verified practically by the examination of old drifts or tunnels where the overlying material has had an opportunity to fall and take the shape due to such conditions without regard to other influences.

If, then, the cavity in its upward progress encounters a bed of tough resistant shale or sandstone, it may fall so slowly that a large area may be opened by continuous mining during the delay, resulting in a heavy weight along the line of stress due to the wide span and crushing the coal either at the working end of the pillar or at such point along the course of the room as the line of stress may meet the coal as shown in Fig. 42. Such a crush is not likely to find relief until the overlying measures are sufficiently broken down to fill the space *S S,* and allow the development of a new smaller arch of stress *A B C,* which, having less span and consequently less load, will transmit less load to the point *A.*

Dr. F. W. McNair has reviewed the question of pressures and support in the deep copper mines of the Lake Superior region. In a lode dipping 38 degrees and with pillars 50 feet wide, having on either side an open space of 150 feet, the pressure on the pillar at a depth of 5,000 feet would be 1,239 tons per square foot, allowing for neither rigidity nor arching and supposing the weight on the pillar evenly distributed. The pillar would fail under this pressure if it were mainly trap rock. "As a matter of fact, in such a case, the rigidity of the mass distributes a large part of the load out over the rock beyond the walls of the opening. That this rigidity may be considerable is illustrated in several cases in which areas of hanging wall as wide as 200 feet or more have no support between walls and yet have stood up for several years. As the rock between pillars and walls bends downward the tendency is to concentrate the load at the edge or face of the pillar or walls. The outer parts of the pillar may thus become overloaded and fail by the splitting off of pieces of rock, that break from the base as well as the top, and like any hard rock under a crushing load, the pillar usually fails suddenly. The hanging rock mass moves, of course, when the pillar crushes, and the vibration due to the sudden though slight displacement is often conveyed to the surface. The result is a miniature but perfectly genuine earthquake that may be felt over a distance several times that of the pillar from the surface. With the crushing of the pillar and the movement of the hanging wall, a readjustment of the weight takes place, and the process begins over again. Eventually, at great depths, the hanging and foot walls must come together.

"The readjustments that take place when a pillar fails sometimes put an enormous longitudinal thrust on the foot wall, and in places its surface portion has buckled under such stress. Experience seems to show that at the great depths recently reached it is useless to expect to hold up the hanging rock mass for a long time by any scheme of pillars unless far too much of the lode is left in place, and that the only feasible method is to cut away the entire lode and permit the hanging to cave as rapidly as it will to the point where the broken rock fills again the whole space and redistributes the weight over the foot wall."*

C. T. Rice objects to the general statement that stopes will cave until filled, except in the case of running ground. In the few caved stopes which he has inspected he has "always found an open space between the arched roof and the pile of caved rock. In general, such a large stope opening is necessary before caving commences; the self-supporting dome is assumed before the stope fills itself. The caving action is progressive, and as the slabs accumulate in the stope they so support the sides that caving ceases. Finally, owing to the weakening of other stopes, the faulting stage is reached; not until then does the opening become completely filled.

"In supporting the roof of a stope, only that portion of the roof that is below the line of the dome of equilibrium requires support; the rock above this dome sustains itself. If, therefore, the shape of this dome of equilibrium in each kind of rock were known, it would be easy to calculate the weight of rock hanging below the dome, and so timber the stope as to hold up this weight." C. T. Rice is under the impression that the shape of this dome is fairly constant in each kind of rock; especially in the same rock in the same district. "Of course, slips and joints, sudden changes in chemical composition, the dip of the strata in sediments, and many other facts, would affect the shape of the dome, but as long as these were small their effect would also be small. If investigation of the shape of this dome should suggest any formula to determine the strength of timber necessary to support the ground below the dome, the effect of these joints, etc., could easily be included by the factor of safety used."*

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*McNair, F. W. "Deep Mining in the Lake Superior Region," Min. and Sci. Press, Vol. 94, p. 275, 1907, and Eng. and Min. Jour., Vol. 84, p. 322, 1907.

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*Rice, Claude T. "The Dome of Equilibrium and the Caving System of Mining." Mining and Sci. Press, Vol. 95, p. 85, 1907.

CHAPTER IV.

ENGINEERING DATA AND OBSERVATIONS.

In America very few data have been collected on subsidence due to mining operations, at least the data, if collected, have not been made available for scientific purposes.

In order that observations may be of value the following correlated data are desirable:

(1) The elevations of a number of points on the surface for a period of years both prior to, during, and following the mining directly beneath.

(2) The position of these points with regard to permanent stations located outside of the mining field or upon ground which has not been or will not be subject to the influence of the mining operations.

(3) The position of the working face in the mine on the various dates of survey.

(4) An accurate location and description of the character of the portions of the mineral deposit left unmined.

(5) An accurate location and a description of the supporting materials placed in the excavated area.

(6) The thickness and dip of the material mined.

(7) The thickness and character of the bed immediately underlying.

(8) The thickness, dip, and character of the overlying rocks and all available information in regard to structure.

(9) The thickness and character of the surficial material.

(10) The quantity of water removed from the mine.

(11) The location, extent, and data of underground movements of rocks overlying the mineral deposit.

In Europe records have been kept for many years in various districts in order to determine the vertical amount, lateral extent, rate, and duration of subsidence.

Among the first surveys made to determine the movement of the surface were those of Fayol.* At Commentry Mine from 1879 to 1885, as shown in Figs. 43 and 44, surveys were made to correlate surface movement and the advance of the working face. The seam which was almost 48 feet thick was worked in horizontal slices of about 8 feet in ascending order. The thickness of rock cover was 321 feet. Some filling of shale and sandstone quarried on the surface was used. It was observed that:

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*Fayol, H. Bul. Soc. Ind. Min., II ser., Vol. 14, p. 818, 1885. Coll. Eng., Vol. 11, p. 26, 1890-91.

(1) During the removal of the first slice, the lowering of the surface gradually grew greater, and was further increased considerably by the working of the second.

(2) The area of subsidence was about four times larger than the area worked.

(3) The maximum sinking was 3 feet 5 inches or one-fifth of the height of the two slices.

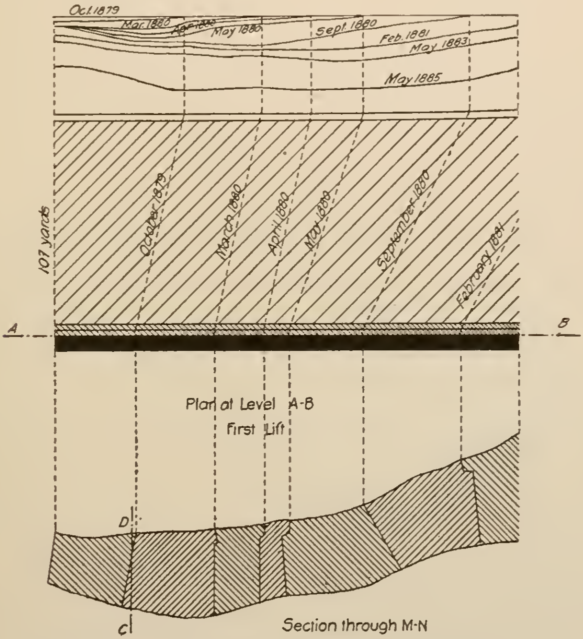

Fig. 43. Subsidence at Commentry Mine.

(4) The movements of the ground appeared at first at a certain horizontal distance in advance of the working faces and this distance remained nearly constant.

(5) The subsidences increased during a certain time while the working proceeded.

(6) The second lift caused a total subsidence almost equal to that of the first lift. This subsidence was 2 feet 1 inch for the first and 1 foot 11 inches for the second, in all 4 feet.

(7) The area of subsidence cannot be determined, either by normals or verticals from the bed worked.

The surveyor's records showing surface movement in connection with the Warrior Run Mine disaster have been noted previously.* The ratio between the volume of subsidence as noted on the surface and the volume of excavation has been noted for the caving system of mining on the Gogebic range.† "When a slope caves, and the dome above it runs up into sand or loose rock, the depression formed is usually in the shape of an inverted cone; but where the ore body is wide, or deep below the surface, the subsidence usually takes the form of terraces. Sometimes comparatively large areas will break through cleanly and the whole surface will drop suddenly and as a unit, but this is exceptional.

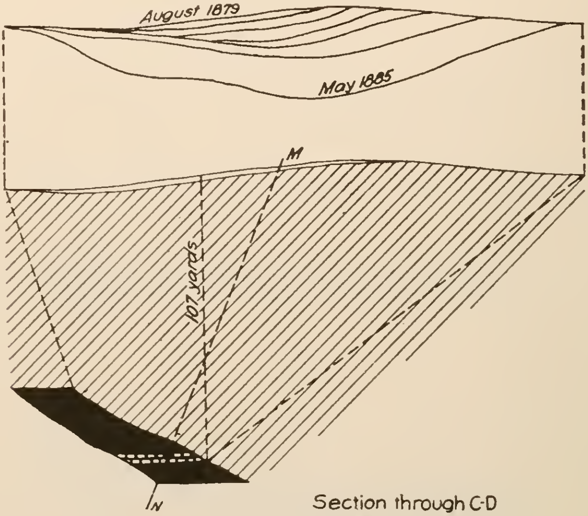

FIG. 44. SUBSIDENCE AT COMMENTRY MINE.

After the back has once started to cave, the surface usually sinks in terraces." In the area under observation the deposit consisted of a lense of soft hematite about 40 feet in average width and 150 feet high, with a length of nearly 1,600 feet on the incline, lying in a trough between a dike of diorite and a thick band of slate. The trough pitched 11 degrees, the hanging wall was hard jasper, and the ore was mined first by square-set rooms and pillars, about 60 per cent of the ore being secured on first mining, but later the pillars were robbed. The hanging wall has dropped from 15 to 75 feet and subsidence has extended to the surface. The proportion between the volume of surface subsidence and the volume of excavation is shown by the following figures:

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*See page 43.

†Eaton, L. "Surface Effects of the Caving System," Min. and Sci. Press, Vol. 97, p. 428, 1908.

	Cubic Feet
Original volume of ore body	7,680,000
Present volume of ore body, including rock, timber, etc	2,360,000
Volume of material removed	5,320,000
Volume of material removed under cave	2,300,000
Volume of material removed under subsidence on surface	3,020,000
Volume of cave on surface	790,000
Volume of subsidence on surface	1,430,000
$\frac{\text{Volume of cave on surface}}{\text{Volume of ore taken from under it}} = \frac{1}{2.91}$	
$\frac{\text{Volume of subsidence on surface}}{\text{Volume of ore taken from under it}} = \frac{1}{2.11}$	

J. S. Dixon made observations at Bent Colliery* A line was selected at right angles to the advancing workings and as nearly as possible on the level course of the coal. Stations were put in every 100 feet and afterwards every 50 feet. The excavation was 5 feet 6 inches high and the overlying strata were allowed to fall and fill it. The surface was principally boulder clay. The original level of the surface before the pillars were drawn is shown in Table 8.

The pillars were removed for a distance of 240 feet back from the solid coal on Jan. 21, 1882, and no subsidence of the surface had ensued. On May 27, 1882, the levels showed the maximum subsidence to have been 1.80 feet at station 1650, which was 145 feet back from the face, and the draw extended to 60 feet in advance of the working face. On November 14, 1882, the face was 610 feet from the solid, and the subsidence was as is shown in the table. On April 15, 1883, the face was 750 feet from the solid; on November 27, 1863, it was 1,060 feet distant; and on October 23, 1884, the removal of the pillars had been completed for some months, and the face was 1,230 feet from the solid. On June 17, 1885, the workings had been in the same position for about a year. On December 4, 1885, it was found that subsidence had practically ceased and the draw had not altered.

The conclusion arrived at is that subsidence from the removal of coal in this case attains its maximum towards the center of the excavated space, and gradually decreases in each direction. The maximum subsidence, 4 feet, was 73 per cent of the thickness of the coal, and the average, 3.76 feet, was 68 per cent. The wave of maximum subsidence regularly followed the working face at an average distance back of 186

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*Trans. Mining Inst., Vol. 7, p. 224, Scotland, 1886. Cit. in Brough, B. H., "A Treatise on Mine Surveying," pp. 241-245.

TABLE 8.

OBSERVATIONS AT BENT COLLIERY.

Peg.	Original Level of Surface	Subsidence From Original Level at					
	1881	Nov. 14, 1882	April 5, 1883	Nov. 27, 1883	Oct. 23, 1884	June 17, 1885	Dec. 4, 1885
600	640.6	....	....	....	0.25	0.35	0.45
650	648.9	....	....	....	0.35	0.60	0.60
700	657.2	....	....	....	0.77	0.94	0.94
750	664.6	....	....	....	1.18	1.27	1.40
800	667.5	....	....	0.23	1.37	1.75	2.00
850	673.1	....	....	0.63	1.50	2.24	2.34
900	675.6	....	....	1.13	2.57	2.74	2.82
950	676.0	....	....	1.61	2.97	3.14	3.22
1000	677.1	....	....	2.10	3.27	3.49	3.60
1050	677.3	....	....	2.43	3.32	3.64	3.75
1100	678.8	....	0.50	2.80	3.52	3.80	4.00
1150	679.7	....	0.70	2.93	3.57	3.81	3.92
1200	680.6	....	1.20	....	3.52	3.52	3.52
1250	680.9	0.60	1.60	3.08	3.45	3.45	3.45
1300	679.2	0.40	2.00	3.03	3.42	3.42	3.42
1350	677.9	1.60	2.25	3.00	3.27	3.27	3.27
1400	677.5	1.60	2.45	3.83	3.17	3.17	3.17
1450	680.8	2.30	2.90	3.42	3.42	3.42	3.42
1500	680.1	....	3.05	3.05	3.05	3.05	3.05
1550	677.1	2.90	3.20	3.20	3.20	3.20	3.20
1600	675.5	3.00	3.00	3.00	3.00	3.00	3.00
1750	672.8	2.80	2.80	2.80	2.80	2.80	2.80
1850	663.4	1.70	1.70	1.70	1.70	1.70	1.70
1950	648.6	0.60	0.60	0.60	0.60	0.60	0.60
2000	649.7	0.04	0.04	0.04	0.04	0.04	0.04

feet, or 1 foot horizontal for each 3½ feet perpendicular. The permanent lengths of the draw may be taken as 100 feet on one side and 83 feet on the other. At these points the depth of the coal was 650 and 646 feet, representing a draw of 1 horizontal for each 7.14 feet perpendicular on the average. The coal dips at 1 in 20.

Surveys were made at the South Kirby Colliery in order to determine the extent and amount of surface movement due to the removal of a shaft pillar.* No observations were made until two years after the mining of the pillar was commenced. The seam in which the pillar was removed lies at a depth of 2,108 feet, is 3 feet 9 inches thick, and dips 1 in 18. Above the coal is one foot of clod. Some movement of the surface was noted before the surveys were made. At depths of 1,600 and 1,800 feet occur the Beamshaw seam of 3 feet and the Barnsley seam of 9 feet which had been worked previously. The data of the surveys are given in Table 9. The unusually large ratio of subsidence (3.47 feet) to the total thickness excavated in removing the pillar (4.75 feet) is attributed to the failure of the shaft pillar in the overlying Barnsley seam.

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*Snow, Charles "Removal of a Shaft Pillar at South Kirby Colliery." Trans. I. M. E., Vol. 46, p. 8, 1913.

TABLE 9.

DATES OF LEVELING AND PARTICULARS OF SUBSIDENCE AT SOUTH KIRBY COLLIERY.

Dates of Levelings	Subsidence Since Previous Leveling	Total Subsidence
	Feet	Feet
November, 1903	Nil.	Nil.
July, 1904	Nil.	Nil.
March, 1905	Nil.	Nil.
October, 1905	0.74	0.74
April, 1906	0.17	0.91
October, 1906	0.22	1.13
June, 1907	0.77	1.90
April, 1908	0.17	2.07
December, 1908	Nil.	2.07
May, 1909	Nil.	2.07
May, 1910	0.82	2.89
May, 1911	0.18	3.07
June, 1912	0.40	3.47
June 1913	Nil.	3.47

Levelings made by Chas. Snow at the Hickleton Main Colliery (details not given) showed that subsidence was evident 433 feet in advance of a rapidly advancing longwall face, and that total subsidence occurred 666 feet back from the face, the amount of subsidence being 4.5 feet. At the edge of the shaft pillar, 500 feet back from where the greatest subsidence occurred, the subsidence was 1.28 feet. Subsidence extended for a distance of approximately 600 feet over the shaft pillar.*

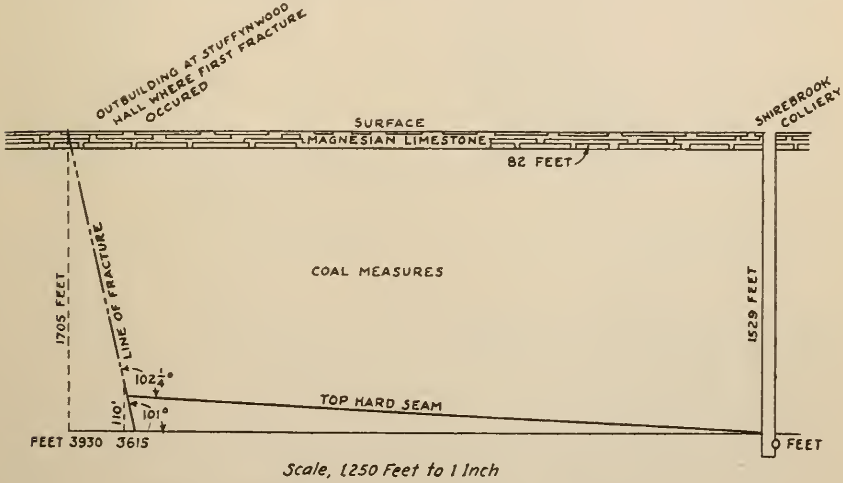

FIG. 45. ANGLE OF FRACTURE AT SHIREBROOK COLLIERY.

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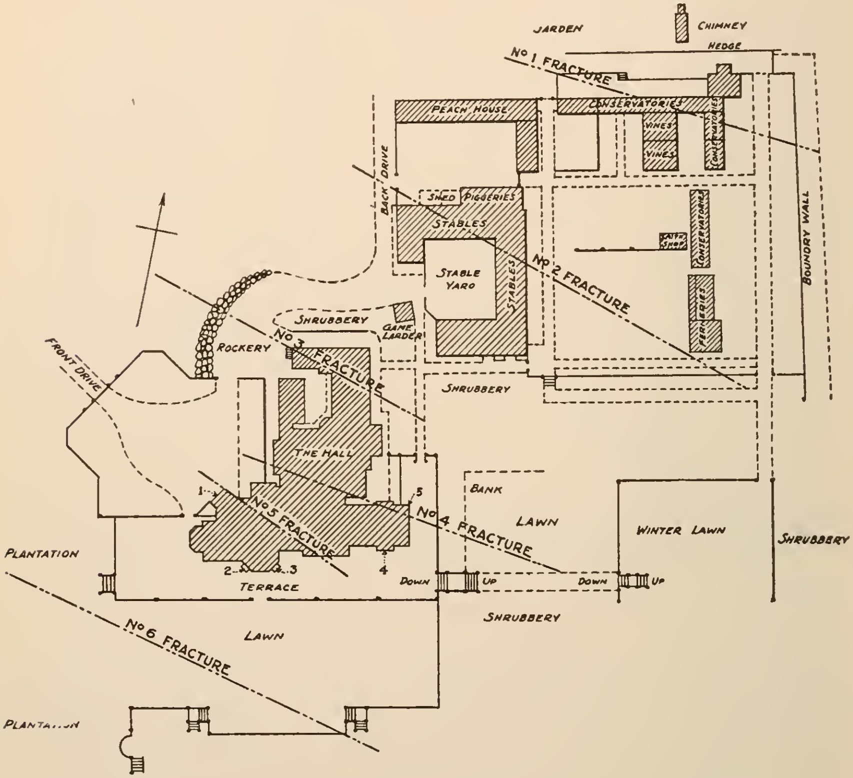
*Trans. I. M. E., Vol. 46, p. 21, 1913.

For a period of 16 years surveys were made at the Teversal and Pleasley Collieries by J. Piggford,* but details of the surveys have not been published. The angle of draw or fracture was estimated to be approximately 16 degrees from the vertical and toward the unworked coal. The depth of the coal seam was approximately 600 feet and the coal was from 5 to 6 feet thick.



Scale. 120 Feet to 1 Inch

FIG. 46. FRACTURES AND SURVEY STATIONS, SHIREBROOK COLLIERY.

Records were kept at Shirebrook Colliery and reported by W. Hay.† The coal lies at a depth of from 1,500 to 1,700 feet, dips 1 in 24, and is 5 feet thick. When the longwall face was 240 feet from Stuffynwood Hall cracks were noted in the surface, the direction of fracture varying as much as 15 degrees from the direction of the coal face. Fig. 45 shows the angle of fracture in section. When the working face was almost vertically beneath, the cracks had attained their maximum width and thereafter commenced to close. When the face had advanced 300 feet farther, the walls of the buildings had assumed practically their normal position.

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*Trans. I. M. E., Vol. 38, p. 128, 1909.

†Hay, W. "Damage to Surface Buildings Caused by Underground Workings." Trans. I. M. E., Vol. 36, p. 427, 1908.

Levels taken at regular intervals are given in Table 10. The survey stations are indicated on Fig. 46.

TABLE 10.

SUBSIDENCE AT STUFFYNWOOD HALL.

Date	Time in Months From First Levels	Station No. 1a	Station No. 1	Station No. 2	Station No. 3	Station No. 4	Station No. 5
March 7, 1906	0	0.00	0.00	0.00	0.00	0.00	0.00
May 18, 1906.	2	0.00	0.00	0.00	0.00	0.00	0.00
June 11, 1906.	3	0.00	0.00	0.00	0.00	0.00	0.00
June 20, 1906.	3	0.00	0.00	0.00	0.00	0.00	0.00
July 10, 1906.	4	0.02	0.00	0.01	0.02	0.02	0.03
Aug. 27, 1906.	5½	0.11	0.09	0.16	0.18	0.18	0.20
Oct. 25, 1906.	7½	0.19	0.17	0.17	0.19	0.18	0.20
Dec. 10, 1906.	9	0.20	0.18	0.17	0.19	0.17	0.21
Feb. 6, 1907..	11	0.22	0.18	0.17	0.19	0.23	0.29
Mar. 9, 1907.	12	0.26	0.26	0.17	0.25	0.31	0.37
Mar. 29, 1907.	12½	0.28	0.26	0.26	0.25	0.31	0.37
June 20, 1907.	15½	0.67	0.43	0.44	0.44	0.52	0.63
Aug. 21, 1907.	17½	0.86	0.67	0.67	0.68	0.80	0.91
Nov. 12, 1907.	20	1.09	0.84	0.83	0.84	0.99	1.10
Jan. 30, 1908.	23	1.28	0.96	0.91	0.97	1.15	1.26
July 9, 1908..	28	1.56	1.23	1.21	1.34	1.41	1.51
Nov. 12, 1908.	32	1.74	1.34	1.35	1.37	1.54	1.63

The maximum subsidence was 1.74 feet and the minimum 1.34 feet, the average being practically 30 per cent of the total height of excavation.

Levels extending over a period of five years were taken by S. R. Kay on the surface of a portion of two collieries mining at depths of 360 feet and 990 feet.* Where the levels were run, the surface was fairly level, the strata were nearly horizontal and were free from faults of any magnitude. The strata consisted of alternating beds of shale, sandstone and limestone, none being massive. Figs. 47 and 48 show the data secured. The working of the 5-foot seam at a depth of 360 feet resulted in subsidence amounting to practically 70 per cent of the thickness excavated. Similar effects resulted from mining the 3-foot, 6-inch bed. At the greater depth subsidence began about six months after the coal had been mined and continued for years.

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*Kay, S. R. "Effect of Subsidence Due to Coal Workings." Proc. I. C. E., Vol. 135, p. 115, 1898.

A report by C. Menzel* showed that since 1885 observations of the rate of settlement had been made at eight-two points in the vicinity of the collieries of Zwickau, Saxony. The depth of the coal beds varies from 600 to 2,400 feet. A maximum subsidence of 7.1 feet was noted twelve years after three seams had been mined out at a depth of from 600 to 900 feet. At a depth of 1,500 feet the subsidence was only 0.6 feet. By the use of filling subsidence was greatly reduced, it being noted that on an average the filling was compressed to one-half of its volume when stowed. The ratio of subsidence to thickness of seam excavated was found to vary from 1:1 to 1:7, the average being 1:2. Frenzel suggested this latter ratio for shallow seams.

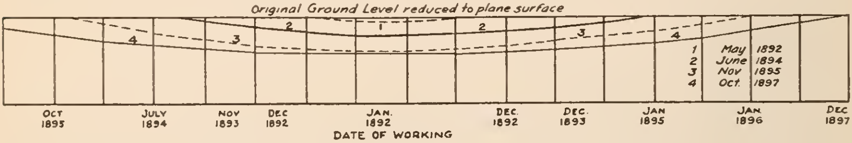

Fig. 47. Data Obtained by S. R. Kay.

Numerous observations have been made in Germany during the last thirty years. R. Hausse has reported upon the angle of break, angle of draw, and the coefficient of increase of volume. Jicinsky, Goldreich, and others have reported upon subsidence in Austria-Hungary but in general these data have been secured in districts where the coal measures are covered with heavy beds of marl. From the foregoing statements of observations the following may be presented as representative in so far as general statements can be made to apply to mining operations each of which is conducted under different geological conditions.

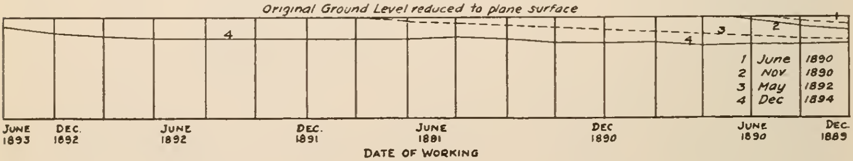

Fig. 48. Data Obtained by S. R. Kay.

ANGLE OF BREAK AND DRAW.

Dr. Niesz has made many observations upon subsidence, particularly on the angle of fracture in various kinds of rock and on the compressibility of filling. He states that "the angle of fracture of limestones, conglomerates, etc., is found to be from 45 to 48 degrees—nearly the angle of repose. In quicksand the angle is greater, while in clay, slate, and marl it may be 60 degrees, and in stone under favorable conditions even 75 degrees. Sandstones with silicious binding material are ranked as nonplastic strata. Initial subsidences in these are followed by others, but at longer intervals than in plastic strata. The angle of fracture is generally not less than 82 degrees."[*]

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*Menzel, C. "On the Relation of Surface Subsidence to the Thickness of Worked-Out Coal Seams at Zwickau." Abs. Proc. I. C. E., Vol. 140, p. 331. Jahrbuch für B.-, u. Hüttenwesen im K. Sachsen, p. 147, 1899.

Dr. J. S. Dixon reported, "In a level seam about 6 feet thick, by careful leveling on the surface prior to and after working, it was found that the draw or angle of subsidence of the strata was about 76 degrees from the horizontal plane."[†]

H. F. Bulman says that in a seam dipping 1 in 10, the lines of break extended over the solid coal forming an angle of 45 degrees with the horizontal on rise workings, 50 degrees in level workings, and 56 degrees on dip workings. In a wide goaf area the average inclination of the planes of fracture was 68 degrees from the horizontal plane; and at the rise side of a shaft pillar, the inclination was roughly 58 degrees from the horizontal plane over the solid coal.[‡]

S. R. Kay[¶] has presented the following formula for determining the radius of support:

$$r=\frac{3\sqrt{d}\;x\;\sqrt[3]{t}}{0.8}$$

r = radius of support in feet,
d = depth in feet,
t = thickness excavated in feet.

This allows for the angle of break or draw.

Joseph Dickinson says, "the direction of subsidence may be judged of from the slopes of faults and mineral veins." He gives these slopes as 1 in 5 for hard rock, 1 in 3.75 for medium rock, and 1 in 2.5 for soft rock.[§]

O'Donahue says that the angle of break will be from 5 to 8 degrees beyond the vertical for horizontal beds, and that the maximum draw on dip workings will be 24 degrees; he finds the same angle to be the limit for workings to the rise.[**]

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[*] Zeit. für Berg.-, Hütt.-, u. Salinenwesen, Vol. 58, p. 418, 1910.
[†] Trans. Inst. Min. Eng., Vol. 34, p. 416, 1907.
[‡] Trans. Inst. Min. Eng., Vol. 34, p. 417, 1907.
[¶] Proc. Inst. Civ. Eng., Vol. 135, p. 149, 1898.
[§] Trans. Manchester Geol. Soc., Vol. 25, p. 600, 1885.
[**] O'Donahue, T. A. "Mining Formula," p. 248.

O'Donahue* offers two formulæ to determine the angle of draw:

$d = 8 + \frac{2}{3} D$
$d' = 8 - \frac{1}{3} D$, in which
D = inclination of seam in degrees,
d = angle of draw toward dip workings,
d' = angle of draw toward rise workings.

E. H. Roberton gives a rule for shaft pillars (used in Northumberland and Durham) which allows for the angle of break and draw:

$$\text{Radius of shaft pillar in feet} = \frac{D}{6} + 2\sqrt{Dt},$$

D = depth of shaft in feet,
t = thickness of seam in feet.

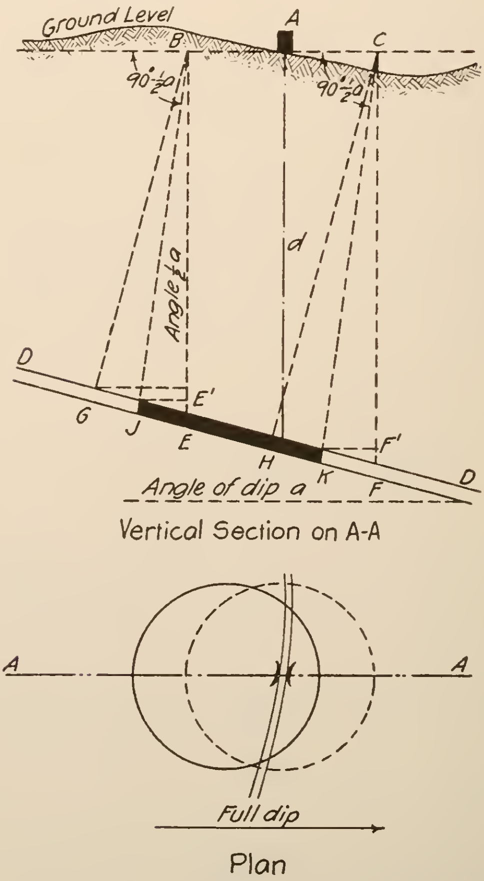

FIG. 49. LOCATION OF SHAFT PILLAR IN DIPPPING BED. (O'DONAHUE.)

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*O'Donahue, T. A. "Mining Formula," p. 248.

Hausse estimated that in general the line of fracture will be between the vertical and the normal to the seam. In addition to the line of main fracture, Hausse refers to the secondary break or draw. He says that in case of horizontal beds this line of secondary break is situated along the bisector of sliding materials of the supplementary angle of the natural slope.

The effect of the dip of the strata has been considered by many authors in their discussion of the simplest cases, in fact, most of the formulæ for angle of break consider the dip of the strata.

Gonot's law of the normal and Schulz's rule, the earliest of the theories, considered the angle of dip. As previously noted, Hausse, following Jicinsky, supports the theory that the angle of break will fall midway between the normal to the seam and the vertical. From a careful study of the subsidence occurring in the Saxon coal field R. Hausse determined the direction of the plane of fracture by the following formula:[*]

$a =$ angle of fracture,
$d =$ dip of strata,

$$\tan a = \frac{1 + \cos^2 d}{\sin d \cos d}, \text{ in which,}$$

$$\text{if } d = 0^\circ, \quad \tan a = \infty \quad \text{and } a = 90^\circ$$
$$\text{and if } d = 90^\circ, \quad \tan a = \infty \quad \text{and } a = 90^\circ.$$

S. R. Kay suggests that for inclined strata the angle of fracture will be midway between the perpendicular to the seam and the vertical. If the angle between the perpendicular to the seam and the vertical is a, then the pillar necessary to protect a given object on the surface must be shifted, on account of the dip, from a position directly beneath the object by an amount equal to $d \tan \frac{1}{2} a \cos a$, in which d equals the depth.

Goldreich gives Table 11 showing the angle of break according to the most important theories.[†]

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[*] Results actually obtained in practice confirm this theory. Thus, for supporting the glass works at Doehlen, in Saxony, a 76.8-foot pillar was left; nevertheless the surface sank considerably. The coal seam dipped 12° and was 540 feet deep. Calculated from the depth and size of the pillar, the angle of fracture was found to be 82°, or 2° 20′ less than the result obtained from the theoretical formula. In another case in the same district the value of a was found to be 82° 30′, or 1° 50′ less than that found theoretically. (Brough, B. H. Proc. Inst. Civ Engrs., vol. 135, p. 150, 1898.)

[†] Goldreich, A. H. "Die Theorie der Bodensenkungen in Kohlengebieten," p. 42.

TABLE 11.

Angle of Break.

Dip	Rule of Hausse		Rule of Thiriart	German Rule		Rule of Jicinsky	English Rule
	First Hypothesis	Second Hypothesis		First Hypothesis	Second Hypothesis		
			Toward the Dip				
0	70	70	70	75	70	90	81¼
10	65	67	67½	75	70	85	73¾
20	60¼	64¼	65¼	70	70	80	66¾
30	56	62	63	60	60	75	64
40	52¾	60¾	61¼	55	55	70	64
50	50¾	60¾	60¼	55	55	70	64
60	51	61	60½	55	55	75	64
70	54	61	62	55	55	80	64
80	61½	65½	65¼	55	55	85	64
90	70	70	70	55	55	90	64
			Toward the Rise				
0	70	70	70	75	70	90	81¼
10	75	77	72½	75	70	85	83¾
20	79¾	83¾	74¾	75	70	80	88½
30	84	90	77	75	70	75	90
40	87¼	95¼	78¾	75	70	70	90
50	89¼	99¼	79¾	75	70	70	90
60	89	99	79½	75	70	75	90
70	86	93	78	75	70	80	90
80	78½	82¼	74¾	75	70	85	90
90	70	70	70	75	70	90	90

AMOUNT OF SUBSIDENCE.

Various writers have attempted to express the amount of subsidence as a percentage of the thickness of the seam worked. In Table 12 data from various districts are assembled, showing the depth of the workings, the thickness of the coal mined, and the vertical amount of subsidence expressed as a percentage of the thickness of the material removed.

TABLE 12.

Amount of Subsidence Expressed in Percentage.

Depth in Feet	Percentage Subsidence	Thickness of Coal Required Feet	Filling	Locality	Authorities
360	70.0	5.0		England	S. R. Kay
990	64.0	3.5		England	S. R. Kay
....	....				
....	46.0	29.36	Stowing	Bully-Greenay,	
432	44.4	30.0		France, England	
600	75.0	4.0			O'Donahue
2400	25.0	4.0			O'Donahue
650	68.0	5.5		England	Dixon
748	19.0	7.5	Stowing	France	Fayol
2600	00.0	13.0	Harmless depth without stowing	France	Fayol
1040	00.0	13.0	Harmless depth with stowing	France	Fayol
390	40.0	7.0	33% of seam put in gob	England	Gresley
325	87.0	30.0		England	Grazebrook
1500	30	5.0	Stowing	England	Hay
600-2400	50			Germany	Menzel

Attempts have been made to formulate rules by which the amount of subsidence may be predicted in advance. Some of the formulæ are based upon the thickness of coal and depth of workings. Most of them include factors for character of rock and filling, but few introduce factors for inclination of the beds.

The discussion of the relation between the depth of workings and the vertical amount of subsidence has brought to the foreground the question as to whether or not subsidence will result irrespective of depth. According to the formulæ of Jicinsky and Menzel there is for each thickness of coal bed a depth beyond which mining will not affect the surface. In 1884 Jicinsky suggested the following:

$$\begin{aligned} S &= m + t - 1.01t \\ &= m - .01t \end{aligned}$$

in which S = vertical subsidence,
m = vertical thickness of coal,
t = thickness of overlying beds.

Menzel suggests the formula

$$S = \frac{t + 350}{350\,m}$$

in which S = subsidence in yards,
t = depth in yards,
m = thickness of seam in yards.

The factor 350 must be increased to 400 for depths greater than 350 yards. This principle that there is a harmless depth has been supported by Fayol, Banneux, Thiriart, Rziha, Jicinsky, and Menzel.

Fayol formulated two rules as follows:

(1) The height of the zone of subsidence where sandstone predominates and the beds have an inclination less than 40 degrees, and where the area is infinite, does not exceed 200 times the height of the excavation.

(2) When the area is limited, the height of the dome is about twice the breadth excavated for excavations less than 6 feet and up to four times the breadth excavated for seams more than 6 feet.

In general the Germans say that the "dead point" or "harmless depth" has not been reached in Westphalia and question whether or not the term should be used. Callon said that there is no harmless depth, and the majority of the British engineers hold that the removal of all the coal over extensive areas will produce subsidence.*

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*The efficiency of filling in reducing subsidence will be considered in Ch. V, see p. 138.

TIME FACTOR IN SUBSIDENCE.

In a study of subsidence it is frequently important to know (1) how soon after the movement shows in the mine workings it will manifest itself upon the surface; (2) the period during which the movement is most severe, and (3) the duration of subsidence.

Upon all of these points there seems to be a great difference of opinion, which is due undoubtedly to the great variety of conditions under which the observations have been made. Fayol wrote, "The period during which movement of the surface may continue is very uncertain. It is allowed to be ten or twelve years in Belgium and at Saarbruck. In other places it has been as long as twenty and even fifty years."*

The committee of the Mining and Metallurgical Society of Ostrau, Moravia, reported in 1881, "The land subsidence manifests itself within one to three months after the collapse observed in the mine. It manifests itself most intensely during the first half year, and then becomes less noticeable. According to our experience it may be assumed that after two years, or more safely, after three years, there do not occur any measureable land subsidences in consequence of a collapsed working."†

S. R. Kay reported that, in working a 5-foot seam at 360 feet, subsidence began about six months after the coal was removed and continued four years.‡

Elevations taken at the Bent Colliery by J. S. Dixon showed that the greater part of the subsidence took place within the first year and that the maximum subsidence came within three years. The depth to the seam was approximately 650 feet.¶

In observations made by W. Hay at Shirebrook Colliery, in which mining was being conducted at 1,700 feet, the maximum subsidence appeared in two years.§

G. E. J. McMurtree reported that the mining of 8 feet of coal at a maximum depth of 800 feet caused subsidence continuing fifteen years.**

In discussing the timbering of roadways in longwall mines in Illinois, S. O. Andros says, "Permanent timbering can be extended

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*Colliery Engineer, 1890, Vol. 11, p. 25, 1890.
†Goldreich, p. 63.
‡Proc. Inst. Civ. Eng., Vol. 135, p. 115, 1898.
‡Trans. Min. Inst. of Scotland, Vol. 7, p. 224, 1886.
§Trans. Inst. Min. Eng., Vol. 36, p. 427, 1908.
**Proc. Smith Wales Inst. of Engrs., Vol. 20, p. 367, 1897.

only to that point where the first rapid and violent subsidence has ceased, and it is not usual to extend permanent timbering to any point until the face has been advanced beyond it for at least two years."*

George Knox says:

"When workings advance rapidly the tendency will be for the strata to bend without fracturing; whereas if the opposite is the case, the force of the motive zone has time to break through, as is frequently shown on the working face after a prolonged stoppage."†

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*Illinois Coal Mining Investigations, Bul. No. 5, p. 32, 1914.
†Knox, George "Mining Subsidence," Int. Geol. Congress, Vol. 12, p. 804, 1913.

CHAPTER V.

Laboratory Experiments and Data.

TESTS AND EXPERIMENTS FOR SECURING DATA.

In the laboratory various experiments and tests can be made to secure data which will be of assistance in the study of subsidence. Among these may be noted the following:

General tests of the materials entering into the problem.

Effect on superimposed material of the removal of part or all of the supports.

Probably the most extensive experiments along this line which have been described in scientific publications have been those made by H. Fayol.*

His experiments to demonstrate subsidence included a variety of materials, as iron, fibre, canvas, rubber, sand, clay, and plaster. He placed iron bars 1.9 inches by 0.19 inch (50 millimeters wide by 5 millimeters thick) one above the other horizontally, the whole being supported by blocks of wood, *A, B, C, D, E, F,* Fig. 50. These blocks rested upon an iron table *G*. A strong iron rule *H* was placed upon the upper bar of iron, and by means of stays *I,* and bolts, the rule and bars were fastened together and to the table. The wooden blocks *B, C, D, E,* were removed over a length of about 4 feet, and the sagging of the iron bars was noted.

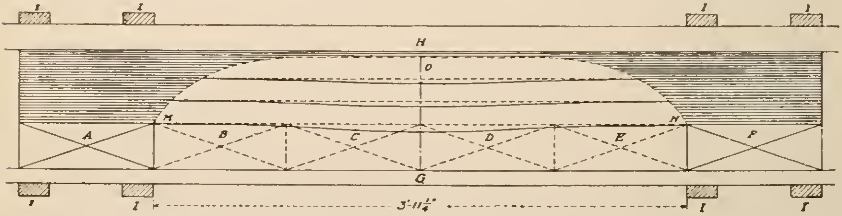

Fig. 50. Sagging of Iron Bars.

It was found that the deflection of the lower bar was 5 millimeters (0.19 inch), of the tenth bar from the bottom 3.25 millimeters, of the twentieth 1.75 millimeters, and that after the thirtieth bar there was no more bending. The limit of the deflections is the curve *MN* shown in Fig. 49.

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*Fayol, H. "Sur les Mouvements de Terrain Provoqués par l'Exploitation des Mines." Bul. de la Société de l'Industrie Minerale. IIe ser., Vol. 14, p. 818, 1885. Translation Coll. Eng., Vol. 11, p. 25, and Vol. 23, p. 548.

The same experiment was tried with flat aloe ropes and with straps of canvas and India rubber in place of the iron bars. With straps of canvas and India rubber the curve of the limits of deflection, that is to say, the limit of the zone of subsidence, had a height nearly equal to the distance between the points of support. This height was about one-third of the same distance for the ropes and one-sixth for the iron bars. Wood and rocks also bend in a manner similar to the materials mentioned.

In order to study the movement in beds of loose materials and in strata that might have been crushed by subsidence, Fayol used artificial beds of earth, sand, clay, plaster, or other materials, and constructed boxes of various dimensions having one side of glass. The box usually employed was 2 feet 7 inches (.80 meter) long, 1 foot (.30 meter) broad, and 1 foot 7 inches (.50 meter) deep. On the bottom of the box were placed, side by side, small pieces of wood of equal thickness, a few centimeters in width, and as long as the breadth of the box. Experiments were made both with one row of these little pieces of wood, and with several placed one above the other. Upon them were laid successive layers of artificial strata, varying from 1 millimeter to several centimeters in thickness. To note the movements, small pieces of paper about ¾ inch (2 centimeters) in length and ⅜ inch (1 centimeter) in width, were put into the planes of stratification, and, on the glass, lines were marked in ink, covering exactly the lines formed by the paper. These lines enabled the least movement to be followed.

By withdrawing the little pieces of wood, excavations were formed and movement produced in the artificial strata.

Fig. 51 represents the movements by taking away, in the order indicated by the numbers, the upper row of wooden pieces, where there were three rows each 0.3937 inch (1 centimeter) in thickness.

The first bed (dry sand), which rests directly on the pieces of wood, falls in as each pillar is withdrawn. The second bed commences to sink only when a certain number of pillars have been taken away. The sinking is shown at first by a slight curve, which has its greatest deflection toward the center of the excavation. Then the third bed follows the second. The movement gradually extends in depth, and reaches the upper bed after the removal of the twelfth pillar. After the removal of the seventeenth, the beds have become bent, as shown in the sketch, the limits of the deflection being the curves Z_{13} and Z_{17}. (The index figure of the curves is the number of the last pillar taken away; namely, the curves $Z_8 Z_4$ indicate the extent of the movements after the removal of pillars 4 and 8.)

It is apparent that the zone of sinking is a sort of expanding dome, which grows in proportion as the excavation extends.

The bending of the first bed, hardly observable at first, is considerably increased. The second bed sinks rather less than the first, the third less than the second, and the sinking of each diminishes regularly in proportion as it is higher above the excavation. This sinking takes the form of a basin, the center of which is on the vertical axis of the excavation.

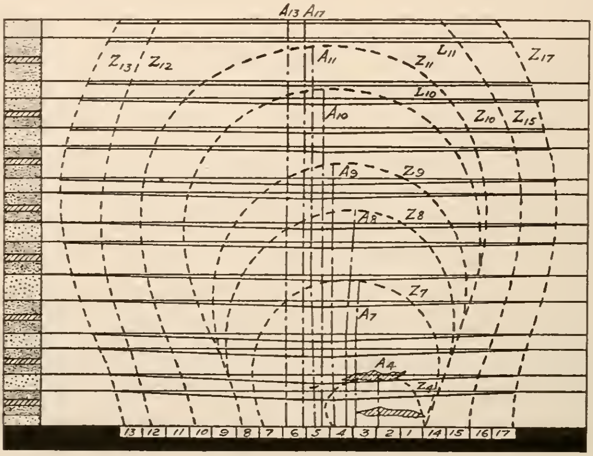

Fig. 51. Subsidence of Artificial Beds.

The lines A_4, A_7, A_8, A_9, A_{11}, A_{13}, A_{17}, are lines followed by the greatest deflections of the sunken beds after the removal of the pillars *4, 7, 8, 9, 11, 13, 17.* These lines nearly coincide with the axes of the domes, which show the limits of the movement.

Throughout the experiments it was evident after the removal of a certain number of the pillars that the pressure of the superincumbent mass was strong at the center and weak at the circumference of the excavation.

The second row of wooden pillars was taken away and thus the depth of the excavation was doubled. The sinking of the lower beds increased; some of them fell in; and the broken ground occupied much more space. The disturbance was greater below, but not at the surface. The line of maximum deflection did not remain vertical, and some of the limiting domes were inclined.

Removal of the third row increased the disturbance caused by the removal of the two former; the fractures of the beds and the spaces between the strata were multiplied; some opened more, others closed. As before, the movement started at the lower beds and reached the upper as the excavation extended. The removal of each row of supports results in a new state of stability, which continues if no more pillars are taken away.

Similar experiments were made with beds at various inclinations, and it was found that the line of greatest deflection was between the vertical and the normal, and that it departed further from the normal (that is, the perpendicular to the inclination of the beds) in proportion as the beds became more inclined. Whatever the inclination, the subsidence of each bed had always the form of a basin.

When horizontal beds were covered over by beds dipping at various inclinations; that is, resting unconformably on them, the zone of settlement took the direction of the inclination of the beds and its axis tended to become perpendicular to the beds affected. The lines drawn through the maximum bend of each bed were no longer continuous, but in passing from one set of beds to another were broken and shifted in the direction of the dip of the new set. In all cases the sinking of each bed and of the surface was in the form of a basin.

An experiment was made with horizontal beds, which showed that a block of coal left between two worked-out places may be of no use to protect the surface above it, because the zones of subsidence due to the excavation on either side, which, as already seen, take the form of domes, may overlap each other between the coal and the surface.

As the area of subsidence increases in proportion as the excavation is extended, it may be asked whether there is any limit in depth to the propagation of the movement when the excavation extends indefinitely. To answer this, a mass of horizontal beds was isolated round about by a space being left between them and the vertical sides of the box, and then the wooden pillars (in this case .03937 inch thick) were taken away from under the whole area of the mass. Being entirely free at the sides it might be considered to represent a mass of strata lying over the middle of a working of large extent.

On taking away the pillars, the zone of sinking was seen to increase little by little, and to stop at a certain depth; the movement did not reach the surface. The expansion of the lower beds filled the space excavated and the upper beds rested on the fallen rock. The pressure exerted by the upper strata was very much greater in the middle than at the circumference, and in this case, too, the sinking of the strata was in the form of a basin.

The effect of faults was tested by inserting in a mass of horizontal beds a thin plate of metal, placed at an inclination, and extending the whole width of the beds. This broke the continuity of the beds and represented a fault without throw. Its tendency was to stop the movement from extending above it, though the sinking occurred as usual on its low side, leaving an opening in the plane of the cut, which extended to the surface.

Fayol also made experiments upon the angle of fracture of rocks, the increase in volume of crushed rock, and the compressibility of crushed rock of various sizes.

Effect of Lateral Compression Upon Stratified Materials.

Elaborate experiments were made by Willis* in order to study the deformation of strata by compression. The substance used was beeswax with plaster of Paris to harden it and Venice turpentine to soften it so that by using different proportions of these materials, beds of a wide range of consistency could be constructed. A load of shot was applied upon the beds when constructed, in order to approximate the conditions under which strata at depth are deformed. The machine used for compressing the piles of strata endwise was a massive box of oak provided with a piston which could be advanced by a screw. The pressure chamber was 3 feet 3⅜ inches long by 6 inches wide. The depth of the box was 1 foot.

T. M. Meade† made a number of experiments, and considered in detail the types of surface which may be developed. He used various kinds and combinations of bars and applied pressure in various ways. An elaborate set of experiments was made to demonstrate circumferential compression. He used for this purpose discs of clay placed within a circumferential band which could be shortened.

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*Willis, B. "The Mechanics of Appalachian Structure." 13th An. Rep. U. S. Geol. Sur., Part II, pp. 211-281, 1891.

†Meade, T. M. "Evolution of Earth Structure," p. 146, London, 1903. "The Origin of Mountains," p. 331, London, 1886. Cadell. Trans. Royal Soc. of Edin., Vol. 35, part 7, 1888.

Effect of Vertical Compression Upon Beds of Stratified Materials.

Various tests upon bedded materials used for filling in mines have been made by the United States Bureau of Mines. Incidentally these tests have demonstrated the movement or flow of material in beds under pressure. Fig. 52 illustrates the bending of shale under pressure in a mine. In this case, however, the bending is accompanied by fracture because of the large movement allowed by the absence of restraint on the under side.

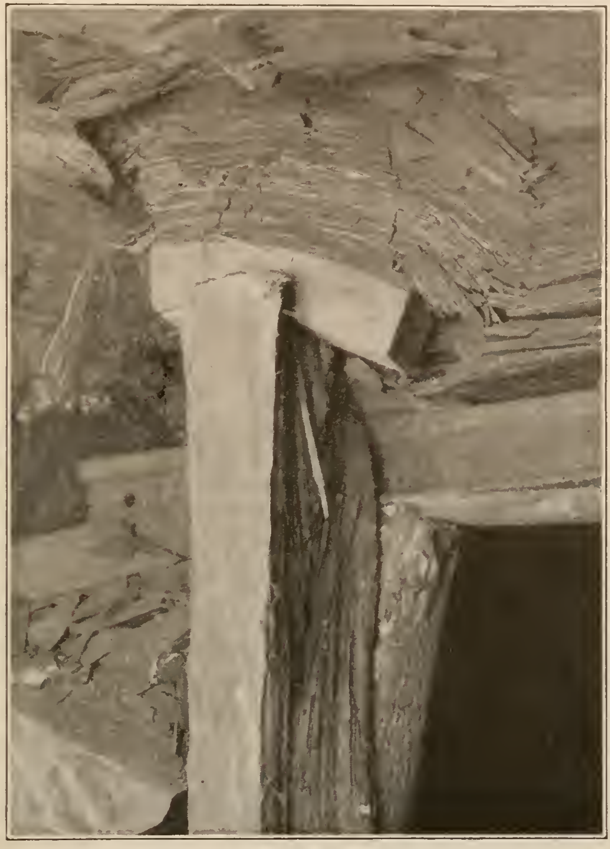

FIG. 52. BENDING OF SHALE UNDER PRESSURE.
(Photo by H. I. Smith, U. S. Bureau of Mines.)

Effect of Lateral Tension Upon Stratified Material.

Not very much work has been done to determine the tensile strength of rocks and practically nothing has been done upon beds of stratified material.

General Experiments.

General experiments to illustrate geological phenomena and to discover the properties of rocks under conditions of pressure and temperature which may exist at great depths, have been conducted by Daubrée, Adams, and Coker, and various other scientists working at times privately and at other times under the auspices of scientific bureaus of governments and of societies.

The Behavior of Various Types of Artificial Supports.

Extensive tests have been made by the United States Bureau of Mines in various government laboratories and by various mining companies in order to determine the actual and the relative strength of different types of supports.*

SUGGESTED EXPERIMENTS AND TESTS.

(1) In order to study surface subsidence resulting from the removal of supports, it is suggested that a model be constructed, say on a 1/100th scale, both horizontal and vertical, approximating relatively the geological sequence of beds in a given district. The beds should have the same strength relatively in proportion to their weight, or the weight applied, as exists in the geological section which the model represents. The model should be of sufficient extent laterally to represent several panels of a pillar-and-room mine laid out on the panel system. Provision should be made for removing supports so that conditions such as would exist when pillars are drawn may be created.

Observations should be made upon the height of surface from time to time and, after surface movement has ceased, the model should be dissected so that the effects of subsidence below the surface may be noted. Similar models should be constructed to demonstrate working beds of various thicknesses, depths, and dips, and under other systems of mining.

(2) Strength tests of roof materials should be made. The tensile strength and the angle of fracture in bending tests should be determined.

(3) The bending power of the various materials which constitute the mine floor should be measured.

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*See Bibliography on Prevention of Subsidence.

(4) In typical mines and under normal working conditions, the pressure or weight of roof should be measured and recorded over as long a period as possible at each point selected.

(5) A study should be made of the composition and physical properties of the rock strata between the beds mined and the surface and also immediately below the beds mined.

CHAPTER VI.

PROTECTION OF OBJECTS ON THE SURFACE.

The surface may be protected by the use of natural or artificial supports. Probably the most general method of preventing subsidence and of protecting objects on the surface is by leaving unmined a portion of the mineral deposit, with the idea that the pillar thus left will have sufficient strength to support the overlying rocks.

In considering the service which a pillar may render and in determining the size of the pillar or other support for protecting specific mine openings or objects on the surface, it will be necessary to consider some of the following factors, and in some cases all of them:

(1) The unit strength of the material forming the pillar.*

(2) The height of the mine opening.

(3) The dip of the mineral deposit.

(4) The angle of break of the overlying rock.†

(5) The angle of draw or drag or pull over the pillars, as observed in the district or under similar conditions.

(6) The strength of the overlying rocks.‡

(7) The nature and amount of filling in the mined-out area adjacent.

(8) The depth at which mining may be carried on without affecting the surface.

(9) The bearing-power of the bottom or floor.

(10) The weight of overlying materials which must be supported.

To determine the size of pillar necessary to protect mine openings of a given width, it is customary in some textbooks to assume a span of roof and overlying rock to be supported, to estimate the total weight of such a block for the depth of workings, and then, with the known or assumed unit crushing strength of the material to be left in the pillar, the cross-section may be calculated. Such calculations are seldom used in practice and they are open to the objection that they assume a pillar to be uniform throughout, while, as a matter of fact, all bedded deposits are composed of a large number of layers that may vary widely in hardness. For instance, some beds of very hard coal contain

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*See p. 70-76.
†See p. 130.
‡See. p. 76.

thin layers of mother coal which reduce the strength of the bed, thus vitiating any calculated results for strength of pillar based on tested specimens taken from the solid part of the bed.

SHAFT PILLARS.

Numerous rules have been formulated for the calculation of shaft pillars in flat seams. Among the best known are the following:*

Merivale. $S = 22 \sqrt{\frac{D}{50}}$

in which S equals length of side of pillar in yards and D equals depth of shaft in fathoms.

Andre. Up to 150 yards deep, a pillar 35 yards square. Up to 175 yards deep, a pillar 40 yards square. Up to 200 yards deep, a pillar 45 yards square, and so on, increasing 5 yards for every 25 yards of depth.

Dron. Draw a line enclosing all the surface buildings, such as engine houses, fans, etc. Make the shaft pillar of such a size that solid coal will be left in around this line for a distance equal to one-third the depth of the shaft.

Wardle. The shaft pillars should not be less than 120 feet square, and the deeper the shaft the larger the pillars. Supposing the minimum to be 120 feet for a depth of 360 feet, 30 feet should be added for every 120 feet in depth.

Hughes. Leave one foot in breadth for every foot in depth; that is, a shaft 600 feet in depth should have a pillar 300 feet in radius.

Pamely. For any depth to 300 feet, it may be sufficient to have a pillar 120 feet square. Adopting this size as a minimum, we may fix any size of pillars for greater depths by increasing the pillar one foot for every four feet in depth.

Foster, R. J. To include the factor of thickness of seam, when conditions are normal, the following formula is suggested:

Radius of pillar $= 3\sqrt{Dt}$, in which
$D =$ depth of shaft,
$t =$ thickness of seam.

Mining Engineering (London). For shallow shafts a minimum of 60 feet radius should be adopted,† and for deeper shafts this should be increased by one-tenth of the depth multiplied by the square root of one-third the thickness of the seam in feet.

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*Colliery Engineer, Vol. 17, p. 538, 1897. Coal and Metal Miners' Pocket Book.
†Colliery Engineer, Vol. 18, p. 117, 1897.

$$R = 60 + \frac{D}{10}\sqrt{\frac{t}{3}}$$

Roberton, E. H. In Northumberland and Durham the practice is shown by the following formula:

$$R = \frac{D}{6} + 2\sqrt{Dt}$$

$R =$ radius of the shaft pillar in feet,
$D =$ depth of shaft,
$t =$ thickness of seam.

Scotch engineers, in order to protect buildings have pillars from 1/3 to 1/5 larger than the floor plan of the building. This diversity of opinion among engineers is well shown by Fig. 53.*

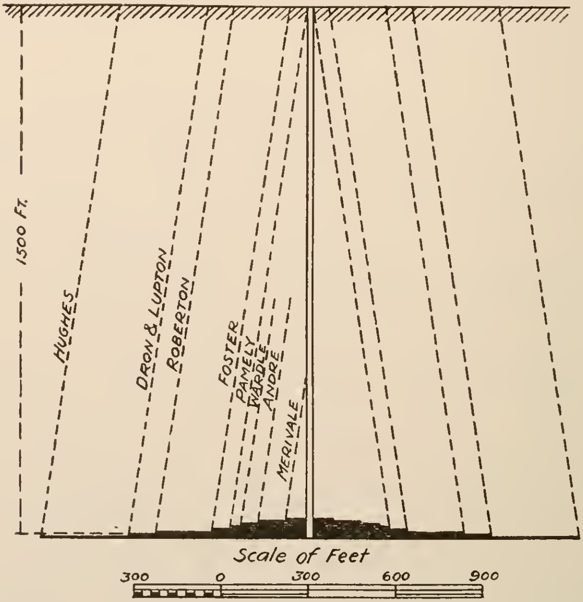

Fig. 53. Sizes of Shaft Pillars According to Different Formulas.

The Central Coal Basin Rule, presumably founded upon the experience of mining men in Illinois and surrounding states, is: "Leave 100 square feet of coal for each foot that the shaft is deep. If the bottom is soft, the result given by this rule is increased by half. For 5 or 6-foot coal beds, the Central Basin Rule may be used unless it has been shown by other operating mines in the district that a larger pillar is needed. With thicker coal a larger pillar should be left."*

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*Knox, G. Proc. Int. Geol. Cong., Vol. 12, p. 798, 1913.

The practice of some coal companies in the Connellsville region of Pennsylvania is to leave pillars under buildings so that there is a margin of from 25 to 30 feet of coal around the building. If the tract is large, from 50 to 60 per cent of the coal is removed, the remainder being left in pillars proportioned so that they will serve in the most advantageous way to protect the building. This is the practice for depths from 150 to 300 feet.

In determining the size of the pillar necessary to protect objects upon the surface, as has previously been noted, the ability of the pillar to carry the load is not the only question to be considered. Among the most important of the other problems is that of draw or pull over the pillar previously noted, and the ability of the underlying bed to sustain the load concentrated upon it by the pillar. Quite frequently the underlying bed is less stable and has less crushing strength than the pillar. It seems logical then to proceed as follows in determining the size of pillar necessary to protect an object upon the surface:

(1) Determine the lateral extent of pillar necessary in order to prevent damage by draw.

(2) Determine whether the pillar thus outlined is sufficiently large to support, without crushing, the burden of the overlying beds.

(3) Determine whether the load upon the pillar will cause the pillar to be forced down into the underlying beds, or cause a flow of the underlying material.

ROOM PILLARS.

In his discussion of methods of protecting the surface, M. Fayol referred to the use of pillars between the working places. "The meshes of the network consisting of pillars with working places between them should be made smaller as the workings are shallower. As the depth becomes greater the size of the meshes can be enlarged and dimensions of the areas worked can be increased relatively to the sizes of the pillars that are abandoned, regard being had to the height and width of the zones of subsidence so that the various zones may be kept distinct from each other. This general rule is susceptible of many combinations according to the thickness, the inclination, the number and depth of the seams worked. If the excavation is of small dimensions the subsidences which take place above them are restricted in size and become enlarged both in width and height as the excavation increases in area. If each of the pillars, 1, 3, 5, and 7 (Fig. 54) be taken out singly, zones of subsidence similar to Z_1, Z_3, Z_5, and Z_7, would be produced; but when pillar 2 is taken out the line of roof subsides on to the floor, and the zone of subsidence rises to Z_2. The same thing happens when No. 6 pillar is taken out, and if No. 4 pillar is taken out, the space comprised between the zones Z_2 and Z_6 is set in motion and determines the formation of zones Z_4."*

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*Illinois Miners' and Mechanics' Institutes, Instruction Pamphlet No. 1, p. 49.

It follows from this statement of Fayol that if the room pillars are properly proportioned and properly spaced, the disturbance of the strata may be limited to the volume within the zones. The material outside these zones throws no weight upon the material within the zones. Necessarily, then, any vertical pressure must fall upon unmined material forming the pillars and the pillars must be large enough to withstand the pressure.

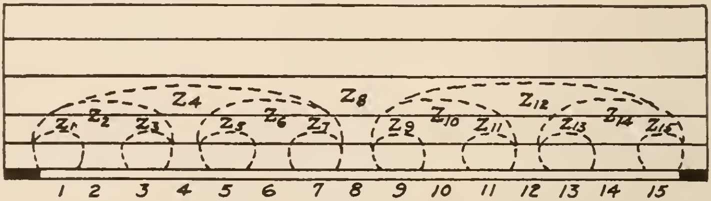

FIG. 54. EFFECT OF EXTENT OF EXCAVATION ON AMOUNT OF MOVEMENT.

In a paper before the Pennsylvania State Anthracite Mine Cave Commission, 1913, Douglas Bunting said: "The application of a formula for determining the safe size of coal pillars for various thicknesses of veins and depths can be considered practical for depths greater than 500 feet, but it is doubtful if the same formula would be of any practical value for application to veins at less depth and certainly of diminishing practical value with reduction in depth and thickness of veins for the reasons that the variable conditions of vein, top, bottom, etc., are of more consequence with small pillars than with large pillars."†

D. Bunting‡ made a careful study of chamber pillars in deep anthracite mines on light dips. He considered the crushing strength of coal which for anthracite was found to average 2,500 pounds per square

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*Proc. South Wales Inst. Eng., Vol. 20, p. 340, 1897. It should be noted that these zones outline the dome through which the movement extends, and not the limit of the *falling zone*, as described by Rziha.

†Bunting, D. "Pillar and Artificial Support in Coal Mining, With Particular Reference to Adequate Surface Protection." Pa. Legislative Journal, Appendix, Vol. 5, p. 5988, 1913.

‡Bunting, D. "Chamber Fillers in Deep Anthracite Mines." Trans. A. I. M. E., Vol. 42, p. 236, 1911.

inch for cubes. The ratio between the strength of prisms and cubes was taken as follows:

$$\frac{\text{Strength of prism}}{\text{Strength of cube}} = 0.70 + 0.30\,\frac{b_1}{h}$$

in which

b_1 = width of pillar,
h = thickness of vein.

The weight of overlying strata was taken at 144 pounds per cubic foot.

(1) Load per square foot on a pillar $= \dfrac{144\,y\,z\,b_1}{b_1^2}$, in which

y = depth below the surface,
b_1 = width of pillar,
z = distance between chamber centers.

With 1,000 pounds per square inch as the safe load for a cube we obtain by substituting in equation (1):

$$\frac{144\ y\,z\,b_1}{b_1^2} = 144{,}000\ \left(0.70 + 0.30\,\frac{b_1}{h}\right)$$

$$\text{or } y\,z = 1{,}000\ \left(0.70 + 0.30\frac{b_1}{h}\right)\ b_1$$

By making proper allowance for the crushing strength of the pillar material and the weight of overburden, this formula may be used generally for flat beds.

The relative widths of rooms and pillars are determined largely by practice. For bituminous coal of medium hardness and good roof and floor, the following rule is sometimes used: "Make the thickness of room pillars equal to one per cent of the depth of cover for each foot of thickness of the seam, according to the expression:

$$W_p = \frac{tD}{100}\text{, in which}$$

W_p = pillar width,
t = thickness of seam,
D = depth of cover,

and then make the width of room or opening equal to the depth of cover divided by the width of pillar thus found, according to the expression:

$$W_o = \frac{D}{W_p}$$

in which W_o is the width of room.

"Frail coal and coal that disintegrates readily when exposed to the air, and a soft bottom, may increase the width of pillar required as much as 50 per cent of the amount found above; also, a hard roof may increase the same as much as 25 per cent; while, on the other hand, a frail roof or a hard coal or floor may reduce the width of pillar required 25 per cent."*

"As to the thickness of pillars in the Pittsburgh seam with strata 100 to 500 feet thick, the following rule should be a safe one to follow, in which the pitch is from 1 to 5 per cent:

Thickness of Surface Feet	Thickness of Pillars George's Creek Feet	Thickness of Pillars Fairmont Feet
100	25	18
150	32	20
200	40	25
250	50	30
300	60	35
350	70	40
400	80	45
450	90	50
500	100	55

These figures are based on experience in this seam, where the floor or bottom is hard and not affected by water. For a fireclay bottom somewhat thicker pillars would be necessary to withstand any extraordinary weight. Rooms should be not more than 14 feet in width in the Georges Creek region and 20 feet in the Fairmont region."†

The average dimensions of pillars and rooms in ordinary pillar-and-room mining in Illinois are shown in Table 13.‡

TABLE 13.

Dimensions of Pillars and Rooms in Pillar-and-Room Mining in Illinois.

District	Average Depth in Feet	Room Width in Feet	Pillar Width in Feet	Average Thickness of Coal in Feet
II	140	26	19	top bench 2 ft. bottom bench, 3 ft. 9 in.
III	90	22	18	4 ft.
IV	201	25	9	4 ft. 8 in.
V	243	26	16	4 ft. 8 in.
VI	270	22	18	9 ft. 5 in.
VII	227	31	30	7 ft.
VIII	174	27	8	Seam No. 6—6 ft. Seam No. 7—5 ft.
Average of 48 Representative mines	208	26	19	

The question of the thickness of cover is an important one in connection with the size of the room pillars and particularly when the drawing of pillars is considered. This has been emphasized by F. W. Cunningham as follows: "The topography of the surface relative to hills and vales should be considered when starting to draw pillars and relative to this subject a question may be asked, which is an important one, viz., How many coal properties have contour maps of the surface? Suppose, for example, the rocks at the surface rise abruptly on each side of a narrow valley to say 200 or 300 feet. Would it be proper to commence pillar drawing under this valley?"*

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*Coal and Metal Miners' Pocket Book, 9th Ed., p. 286, 1907.

†Reppert, A. E. "Pillar Falls and the Economical Recovery of Coal From Pillars." W. Va. Coal Min. Inst., p. 116, 1911.

‡Ill. Coal Min. Investigation, Bul. No. 13, p. 76, 1915.

STRENGTH OF ROOF.

In determining the limits of mining under heavy wash, D. Bunting considered the strength of slabs of roof rock supported by pillars. "In deriving a formula for computing the breaking load of a slab of stone from the formula $\frac{pI}{e} = M_m$, let W represent the distributed loading plus the weight of the beam itself in pounds, b, d, L represent the breadth, depth, and span, respectively, in inches, and R equal the modulus of rupture in pounds per square inch.

Bunting suggests that "the modulus of rupture does not express the actual stress in the extreme fiber of the beam of rock, but is a quantity useful only as a basis of comparison." He gives the following safe unit stresses for stone, recommended by W. J. Douglas as illustrative of possibly a fair average of safe stresses:

	Compression Lbs. per sq. in.	Shear Lbs. per sq. in.	Tension Lbs. per sq. in.
Granite	1200	200	150
Limestone	800	150	125
Sandstone	700	150	75

The maximum bending moment for a constrained or prismatic beam is equal to $\frac{WL}{12}$. By substituting in the formula for flexure $\left(\frac{pI}{e} = M_m\right)$ we obtain the formula $W = \frac{2\,bd^2}{L} R$. Likewise, the maximum moment at the center of such beam being equal to $\frac{WL}{24}$, the formula becomes $W = \frac{4\,bd^2}{L} R$.

It is evident that failure of flexure would theoretically take place at the points of support and not at the center of the span.

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*Cunningham, F. W. "Methods of Removing Coal Pillars." Proc. Coal Min. Inst. of America, p. 35, 1911.

In applying the formula $W = \frac{2\,bd^2}{L} R$ to the case of a slab spanning a breast or other mine opening, the weight of the overlying material will be taken at 108 pounds per cubic foot, and the depth of the opening below the surface will be designated by d' in feet.

Then, $W = \frac{108\,Ld'}{12}$, which would be the loading of the slab with a breadth of 1 foot. Substituting this value of W in the equation $W = \frac{2\,bd^2}{L} R$ and simplifying, the equation $d^2R = \frac{3}{8}\,L^2\,d'$ is obtained. If, however, the weight of the overlying material per cubic foot be represented by w, the expression becomes $d^2R = \frac{w\,L^2d'}{288}$

In the use of the formula derived for determining the minimum safe thickness of rock over mine openings for various depths below the surface, consideration must be given to a number of conditions, the more important of which are:

1. Nature of the top immediately above the coal seam and its comparative strength and liability to disintegration upon exposure to the atmosphere.
2. Nature and thickness of the bed, the ability of the pillars to resist squeezing, and the liability of disturbance of the overlying strata, due to covering or squeezing in underlying beds.
3. Probable errors in relative vertical location of top of rock and mine workings.
4. Possibility of the existence of deep gorges and pot holes.*

In order to arrive at a brief solution in calculating pillars of quartzite for Rand mines, Richardson† made use of the following formulæ:

Bending

1. $F_b = 106\,\frac{Kt}{l^2w}$
2. $L = 10.2\sqrt{\frac{Kt}{w}}$
3. $W = 106\,Kt^2 - l^2tw$

Shearing

1. $F_s = \frac{34.2\,dk}{l^2w}$
2. $L = 5.85\sqrt{\frac{dk}{w}}$

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*Bunting, D. "Limits of Mining Under Heavy Wash." Amer. Int. Min. Engrs. No. 97, p. 18, 1915.

†Richardson, A. "Subsidence in Underground Mines." Eng. and Min. Jour. Vol. 84, p. 196, 1907.

3. $W = (34.2\,dk - l^2w) \quad t$

in which F_b = factor of safety for bending,
F_s = factor of safety in shearing,
l = length of side of slab or distance from center to center of pillars,
L = length of side of a slab which will only support its own weight,
W = total distributed load which the slab will carry in addition to its own weight,
K = compressive strength of pillar material pounds per square inch,
t = thickness of slab in feet,
w = weight of a cubic foot in pounds,
d = diameter of pillars in feet.

He presumes that "where the slope areas are not extensive the weight of the upper masses will be supported by their own strength," and calculates the size of pillar which will support continuous slabs of rock, homogeneous and uniformly loaded. By use of the formula he prepared a table of sizes of pillars for various spaces and concluded that slabs usually break up by shearing and that the strength to resist this depends on the size and distance apart of supports.

FILLING METHODS.

Various materials and methods are employed to protect the surface if it is deemed advisable to remove all the material deposit, or if the material left in the forms of pillars is found inadequate to support the surface.

Waste material resulting from the regular mining operations or broken for this particular purpose may be stowed or packed into the excavation. If sufficient or suitable material is not available under ground it may be lowered or dropped from the surface and stowed where needed. Crushed materials may be introduced from the surface and transported through pipes and stowed by water or compressed air. Timber, steel, or various forms of masonry may be employed to support areas upon which important structures may be erected.

This entire subject has been studied by the engineers engaged upon investigations of subsidence and surface support in the Pennsylvania Anthracite field, who say:

"Most coal beds consist of interstratified layers of coal, fireclay, slate, and bony coal, the latter three composing the principal refuse material of the mine. In these beds, in which it is necessary to remove some of the roof rocks or take up some of the floor of the mine in order to obtain height sufficient for the mules and the men to travel along the roads, much mine refuse is produced, which is stored in the chambers. In beds less than four feet thick many chambers are filled with mine refuse or gob from floor to roof. In places this gob is merely thrown in carelessly or is shoveled in; in other localities it is packed as tightly as possible by hand. When there is much interstratified fireclay or bone in the coal beds there will be larger quantities of the gob, and the thinner the bed the greater will be the quantity of mine rock raised or taken down for roads. The supporting value of stored gob depends upon the compressibility of the material of which it is composed."[*]

Griffith's Method of Filling.—It has been suggested by William Griffith that worked-out portions of mines be filled by blasting up the bottom and shooting down the roof. The suggestion was made in connection with a report to the Scranton Mine Cave Commission and Mr. Griffith has secured patent (U. S. Patent 1,004,418) covering this method. W. Griffith and E. J. Conner say: "It is a well known fact that loose rock occupies from $1\frac{2}{3}$ to twice the volume of the same weight of rock in place. Your engineers have conceived the idea of taking advantage of this fact, well known to engineers, for the purpose of cheaply producing an adequate support of the rock and surface above certain classes of coal beds under the city of Scranton. So far as we know, this method, in its entirety, has never been used before in any coal mining district, and the suggestion is here made for the first time.

"The process is applicable to beds less than 6 feet in thickness and consists simply in blowing up the floor and shooting down the roof of the mine, each to a depth equal to the thickness of the coal bed. This produces a total thickness of loose rock equal to three times the thickness of the coal. The rock would be well packed together and have great supporting power, and, moreover, the desired ends would be attained in a comparatively inexpensive manner.[†]

The method of blasting stowing material from the hanging or foot walls is commonly used in metalliferous mines.

GOB STOWAGE IN LONGWALL MINING.

In longwall mining "the rock obtained from brushing the roof, that which remains after building the pack walls, and the clay obtained from undermining the coal are thrown behind the pack walls lining the roads.

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[*] Griffith, Wm., and Conner, Eli T. "Mining Conditions Under the City of Scranton, Pa." U. S. Bureau of Mines, Bul. No. 25, p. 52, 1912.

[†] Griffith, William, and Conner, Eli J. "Mining Conditions Under the City of Scranton, Pa." U. S. Bureau of Mines, Bul. No. 25, p. 57, 1912.

The gob area is usually filled with rock and clay to within 2 to 5 feet of the coal face. This loose rock and clay helps to support the roof and control the roof weight on the coal face. The waste should fill the gob sufficiently to allow the roof to come down gradually without breaking off short at the face of the pack walls, but should not fill the gob so completely that it carries too much of the roof and does not throw enough weight on the face of the coal. The width of the pack wall, called 'building,' necessary to prevent the walls from squeezing out and filling the roadway when the roof weight comes on them depends upon local conditions. The Third Vein District Agreement between the Illinois Coal Operators' Association and the United Mine Workers of America provides: 'The miner shall build 4 yards of wall at each side of his road, and if he has more rock than is required therefor he shall not load any of it until he has filled his place therewith.' Room centers at the longwall face (in Illinois) are usually 42 feet apart."*

GOB PIERS.

In some cases, especially when the prevention of any movement of the surface is especially desirable, gob piers are used. These are pillars of waste rock, either laid up by hand throughout, or having the outer wall carefully laid while the interior is filled with refuse shoveled in. The resistance of such supports to compression depends upon the material used and the care with which they are built.

CONCRETE AND MASONRY PIERS.

These forms of support are more expensive than those previously mentioned and are likewise more substantial. Masonry has frequently been used to support the roof below important structures and occasionally to support the walls of inclined beds and the overburden.

One of the earliest and also one of the most notable examples of the extensive use of masonry in metal mines was the construction at the Tilly Foster Iron Mines.† The total masonry constructed amounted to 20,189 cubic yards.

Whenever possible the concrete used is introduced from the surface through boreholes. An interesting example of such use of concrete is reported by Mr. Temple Chapman of Webb City, Missouri. In a zinc mine six concrete piers were constructed, 35 feet high by 16 feet wide and

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*Andros, S. O. "Mining Practice in District I." Illinois Coal Mining Investigations. Bul. No. 5, p. 20, 1914.

†Engel, L. G. "Masonry Supports for Hanging Walls at the Tilly Foster Iron Mines." Columbia School of Mines Quarterly, Vol. 6, p. 289, 1885.

20 feet long. The measures were horizontal and the distance from the surface to the roof was 150 feet. First a 6-inch hole was drilled from the surface to the roof with a churn drill at a cost of $0.90 per foot. A large pile of tailings was close at hand, consisting of crushed rock passed through a half-inch hole and containing some finer material and sand. The mixture was six parts of tailings to one part of cement, which is about equal to four parts of gravel, two of sand, and one of cement. This was mixed mechanically and discharged direct from the mixer into the drill hole. Underground two men were kept busy building up the form, which was made of 1 by 12 inch board laid on edge and 2 by 6 inches set vertically at 2-foot intervals and wired together across through the form. Worn perforated trommel screen jackets cut in strips 10 feet long by 4 inches wide were used to reinforce the concrete. These were laid east and west a foot apart and the concrete was poured. A foot higher similar strips were placed at right angles to the first, and so on. A few 60-pound rails were put into the tops of the piers, projecting from pier to pier where possible. These piers were placed between ore pillars, the plan being to remove these ore pillars. The piers were built at a cost of $3 per cubic yard at a time when the ore in the pillars was worth $12 per cubic yard.*

A novel method of using concrete in connection with packing or stowing was employed in France and reported by J. H. Piffaut.† The coal bed, quite thick and highly inclined, was worked in 8-foot slices in descending order. Upon the floor of a slice was spread a layer of coal dust from 1 to 1½ inches thick; then a layer of concrete from 8 to 10 inches thick; and upon this was placed the ordinary packing. As the working place had previously been timbered, the concrete surrounds the base of the posts. When the next slice is removed the concrete floor of the upper slice acts as a roof for the lower slice, which is timbered in the regular manner in order to support the concrete loaded with packing. It is claimed that this has proved satisfactory in the mining of thick beds.

COGS.

Cogs are cribs of timber filled with waste rock. They may be erected quickly and they have great strength. They find some use in the ordinary course of mining, but they are especially useful in preventing an impending squeeze, or in stopping one that has already started by supplying such support that the overlying strata break through to the surface. Their strength is, of course, lost when the timber decays.

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*Correspondence.

†Piffaut, J. H. "The Use of Cement-Concrete in the Working of Thick Coal Seams." Trans. Inst. Min. Eng., Vol. 29, p. 274, 1904.

SPECIAL TYPES OF COGS AND PIERS.

William Griffith has recently developed a cog which it is expected will be many times as strong as the ordinary timber cog and both stronger and more durable than the common concrete pier. The objection to concrete cogs or piers is that when the compressive strength is exceeded the mass of concrete will go to pieces and will give no support whatever. With rock and timber piers, even though the percentage of compression may be large, the piers do not go to pieces but have some supporting power. The concrete pier will collapse suddenly while the other types of piers will be gradually deformed. Mr. Griffith says that what is needed is something that will bear up under the heaviest weight, that will "give" to a certain extent and will then withstand the continuing burden. In his new pier, concrete is the basic material with timber to reinforce it. The piers are constructed so that it is impossible for the timber to pull away and for the concrete to be crushed. "The timber should be creosoted and after the pier is constructed it should be coated on the outside with cement by the use of the cement gun."

Tests show that a cog or pier, forty days old, will sustain for each square foot of horizontal area:

7 tons with a compression of 1 per cent.
130 tons with a compression of 3 per cent.
140 tons with a compression of 14 per cent.

IRON SUPPORTS.

From time to time various types of metal supports have been tried in the working places of mines. Where iron props or posts have been installed in the Scranton district no subsidence occurred and it is the opinion of the local engineers that the effectiveness of such props has not been demonstrated. Rolled steel shapes are being quite extensively used as legs and collars and as beams for the support of wide openings, such as shaft bottoms. Iron supports have also been tried in metalliferous mines, but, except for the support of the shafts, stations, and passageways, they have never found extensive application. Iron props have been used in foreign mines.

HYDRAULIC FILLING.

One of the most important methods of protecting the surface above mine workings is by filling the workings with fine material carried by water through pipes. In his report upon this method as used in the Pennsylvania Anthracite fields, Charles Enzian says: "Heretofore the process has been termed and even at present is known in the Pennsylvania Anthracite region as 'slushing,' 'flushing,' and 'silting.' As a result of various suggestions from men of long experience in this work, the name 'hydraulic mine filling' was adopted for the use of the report."* The process has been used in (a) extinguishing mine fires, (b) arresting mine squeezes, (c) supporting the surface, (d) reclaiming pillars and increasing the yield of coal, (e) disposing of spoil banks, and (f) in lessening stream pollution.

According to the Colliery Engineer, Vol. 33, p. 537, flushing was first used August, 1884, by John Veith, General Inside Superintendent of the Philadelphia & Reading Coal and Iron Company, who employed it to extinguish a fire in the Buck Ridge slope near Shamokin, Pennsylvania.

The second use of flushing and its first use to support or control overlying strata is credited in the same reference to Frank Pardee of Hazleton, Pennsylvania. In 1886 F. Pardee used the system to stop a squeeze which threatened the slope and breaker of the Laurel Hill colliery at Hazleton. He accomplished this by flushing adjacent breasts with culm. The breasts were steeply pitched. The squeeze was stopped by means of natural pillars, each 10 yards wide, and two breasts filled with culm, each 10 yards wide, and the subsiding rock broke off.

The most extensive early use of flushing was at the Kohinoor colliery at Shenandoah, Pennsylvania. When this colliery was taken over by the Philadelphia & Reading Coal and Iron Company, January 1, 1884, it was found that because of workings in the thick Mammoth seam a large part of the town of Shenandoah was likely to be affected by a subsidence of the surface. The Mammoth seam was from 40 to 60 feet thick, thus making timbering impossible. The coal was about 400 feet from the surface. After various methods had been suggested, the officials of the company decided to flush culm into the workings, none of those engaged in the enterprise knowing of the previous use of culm for roof support by F. Pardee.

A very detailed description of the method used in flushing the culm into the workings can be found in the above reference in the Colliery Engineer and in Bulletin No. 60 of the U. S. Bureau of Mines.

The materials that have been used or may be available for hydraulic mine filling include culm, ashes, crushed refuse from coal washing plants, sand, gravel, clay, loam, granulated slag, and crushed rock.

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*Enzian, Charles "Hydraulic Mine Filling." U. S. Bureau of Mines, Bul. No. 60, 1913.

The methods employed and results accomplished have been described by Davies, Griffith, and Enzian.*

The process consists of conveying culm, sand, ashes, etc., to the desired place by means of water, the method used depending upon conditions. If the pipe line can be laid on a steep grade from end to end, the material will flow easily and little water will be required. On the other hand, if the grade is light or if it must be reversed over part of the line a larger quantity of water is required and, of course, a larger pipe. There must always be sufficient velocity to prevent settling of the solids and this can be obtained only by having sufficient head. Naturally the whole operation is easiest when the grade is steep, the pipe short, and the curves and connections few.

To avoid blockage of the pipe, clear water should be allowed to flow for a few minutes before filling is added, in order to establish a current throughout the pipe and when the flushing is to be interrupted, the addition of filling should be stopped some time before the water is shut off, so that the solid matter may be washed out of the pipe.

The proportion of water required depends upon the velocity of the current and the nature of the filling material. In general practice about 90 per cent of the material carried by the line is water.†

Good practice requires absolute control of the filling until it is deposited at the desired place. This necessitates carrying the pipe line to the place of deposit, no allowance being made for flow in chambers.

As the filling should be interrupted after 200 to 400 cubic yards have been deposited and the material be allowed to settle for fifteen to eighteen hours, it is desirable that branch lines be laid to different points, so that the process, as a whole, need not be interrupted. During the period of settling, water seeps out and the material shrinks from 1 to 10 per cent in volume. It is necessary that the drainage be so controlled that the least possible solid matter will be carried away. The finest part of the filling has an important part in the cementation of the mass.

The process requires careful and continuous attention, though the number of men employed need not be large. Generally, there should be one man for the surface, one to patrol each 1,000 feet of pipe line, and one to inspect the filling.

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*Davies, J. B. "Flushing Culm." Mines and Minerals, Vol. 18, pp. 342, 389. 1898. Griffith, William. "Flushing Culm." Mines and Minerals, Vol. 20, p. 388, 1900. Enzian, Charles "Hydraulic Mine Filling." U. S. Bureau of Mines, Bul. No. 60, pp. 58-60.

†Wilson, H. M. "Irrigation Engineering." Pp. 61-69, 338, 344, Revised edition, 1910.

The results obtained have been very satisfactory and a large amount of material formerly deposited on the surface is now washed back into the mines.

In discussing wastes in Illinois coal mining, G. S. Rice commented upon the feasibility of employing hydraulic fillings. He noted the use of culm for filling in Pennsylvania and stated that "In Illinois, the substitute would have to be surface sands and gravel. That this would be impracticable in the great majority of cases throughout the State is self-evident, particularly if water, the usual vehicle for transportation, is employed, inasmuch as the majority of the thick seams in Illinois have clay under them which water would soften, and thus tend to cause a 'squeeze.' Aside from this, much farm land would be destroyed in getting the filling material."*

In longwall mining the application of hydraulic filling under present practice does not seem to be generally feasible. Hydraulic filling in flat seams worked on the longwall plan was inaugurated near Liege, Belgium, in 1913, but has not been employed on a sufficiently large scale to justify a statement that it is practicable for flat seams.†

Over a hundred collieries in Upper Silesia have employed hydraulic filling‡ in seams varying from 4 to 40 feet in thickness. Subsidence has been reduced from 30 to 70 per cent to 0.3 to 7.8 per cent of the height of the seam. In 1914 twenty-seven collieries, employing forty equipments, used hydraulic filling. The sand commonly used in Silesia for filling is mined with steam shovels and then transported by railroad, sometimes for considerable distance, to the mine, where it is dumped on a grizzly to remove the boulders and then mixed with a suitable amount of water to flush it into the mine. At one mine, at least, the boulders are crushed and mixed with sand filling. A detailed description of the methods used in Upper Silesia will be found in the reports of the Upper Silesia Mining Association. In the Saarbrücken district there are on state-owned lands more than twenty independent hydraulic-filling installations, costing $350,000. This method is employed for iron and potash mines as well as in the coal mines.

"The only fairly extensive installations at work in Britain is that of the Wishaw Coal Mining Company, Motherwell. There are other installations in a small form, or under consideration, but nothing yet has been adopted on an extensive scale. A small trial outfit has been installed at one of the Fife pits, and there is a proposition to use hydraulic stowing where the seams run under the sea. There is a small installation at the Crowgarth iron-ore mine."*

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*Rice, G. S. "Mining Wastes and Mining Costs in Illinois." Ill. State Geol. Sur., Bul. No. 14, p. 220, 1909.

†See Trans. Inst. Min. Eng., Vol. 46, p. 439, 1913-1914.

‡Trans. Inst. Min. Eng. Vol. 46, p. 534, 1912. Report of British Consul-General, Westphalia, p. 25, 1911.

"In France it has been used, especially at the collieries in the Department of the Pas-de-Calais, and also in the coal fields of St. Etienne. In Belgium it is used at several collieries. In Spain the Penarroza Colliery is erecting a plant, and several collieries in Austria, as well as Poland and Russia, are employing the system. It is used also at lignite mines in Manchuria and in the gold mines of Australia and the Transvaal."†

Gullachsen reports that in order to avoid the great expense of pumping to the surface the water used in hydraulic filling the Cinderella Deep mine introduced a system by which sand is sent into the mine in a dry condition. A wooden box-launder was constructed measuring 12 by 11 inches in inside cross-section. This launder was carried down the vertical shaft to a depth of 3,900 feet to the level at which the filling material was required. The sand, which should not contain more than 7 per cent of moisture, is stored in a surface bin, from which it is taken on a conveyor belt to the top of the shaft and there discharged into the launder. On reaching the bottom of the launder, it falls on a steeply inclined iron plate, at which point jets of water are turned into the sand, which is then carried away as a pulp. The great objection to this system is the difficulty of securing a constant supply of dry sand. As soon as the sand contains more than 7 per cent of moisture, it is inclined to adhere gradually to the sides of the launder, which in time becomes choked. The launder was connected to a Roots blower and jets of compressed air introduced, the idea being to assist the drying of the sand and to increase the velocity of the falling stream, but this device was found to result in only a very slight improvement."

PNEUMATIC FILLING.

The stowing of crushed rock by means of compressed air has been successfully employed in the Lake Superior copper district at several mines, having been developed at the Champion mine of the Copper Range Company by F. W. Denton. Stamp sands or tailings from the concentration plant are hauled in railroad cars a distance of eighteen miles and discharged through pipes into the worked-out stopes. It is claimed that by the use of this material a saving is made over the method of support formerly used. In order to provide sufficient material for filling the stopes, waste rock secured from sorting in the stopes was supplemented by rock blasted out of the walls. At present the sand is used in addition to the waste material discarded in the stopes.*

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*Paton, J. D. "Modern Developments in Hydraulic Stowing." Trans. Inst. Min. Engrs., Vol. 47, p. 468, 1914. See also Trans. Inst. Min. Engrs., Vol. 48, p. 134, 1914, and Iron and Coal Trades Review, Vol. 89, p. 454, 1914.

†Gullachsen, B. C. "Hydraulic Stowing in the Gold Mines of the Witwatersrand." Trans. Inst. Min. Eng., Vol. 48, p. 122, 1914. See also Trans. Inst. Min. Eng., Vol. 41, p. 586, 1910, and Trans. Inst. Min. Eng., Vol. 43, p. 632, 1911.

SUPPORTING POWER OF FILLING.

The problem of support of surface by filling suggests two important points, in addition to the controlling factor of the cost of filling. When the worked-out portions of the mine are filled by the natural process of caving, the factor of increase of volume of material should be known. Moreover, as the overlying beds sink upon this filling the factor of compressibility of the filling must be considered. Fayol made extensive and careful investigations along these lines, and his determinations of the increase of volume are shown in Table 14.

TABLE 14.

INCREASE IN VOLUME OF MATERIALS IN FILLING.

Relative Volumes

Nature of Rock	Unbroken	Crushed to Powder	Grains .078 to .118 inch (2-3 mm.)	Grains .393 to .59 inch (10-15 mm.)	Grains .59 to .787 inch (15-20 mm.)	Mixtures, Grains and Fine Dust
Clay	100	196	209	226	225	216
Shale	100	213	210	221	224	229
Sandstone ...	100	219	214	211	310	214
Coal	100	207	224	199	223	202

The mixture of large and small pieces of sandstone and shale commonly used for stowing increases in volume about 60 per cent. The greater the increases in volume, the more easily is the crushed material compressed. Fayol's results of tests of compression upon crushed material are given in Table 15.

The pressures noted in Columns I, II, III, and IV correspond to depths of strata of 1,638, 3,276, 8,190, and 16,380 feet, respectively.

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*"Sand Filling at Champion Copper Co., Painesdale, Michigan." Min. and Eng. World, Vol. 41, p. 1194, 1914.

TABLE 15.

Results of Tests of Compression Upon Crushed Material.

Nature of Rock	Space Occupied Before Being Broken	Rocks Having Been Previously Crushed or Broken, the Space Occupied under Pressure of			
		I. 1422 lb. per sq. in. 100 kgm. per sq. cm.	II. 2844 lb. per sq. in. 200 kgm. per sq. cm.	III. 7110 lb. per sq. in. 500 kgm. per sq. cm.	IV. 14,220 lb. per sq. in. 1000 kgm. per sq. cm.
Clay	100	100	90	75	70
Shale	100	128	116	110	97
Sandstone	100	136	125	120	105
Coal	100	130	125	118	109

Fayol concluded that the material which ordinarily fills the goaves of mines always occupies a larger space than it did originally, and after an expansion of about 60 per cent it appears to undergo in workings of from 300 to 900 feet in depth a compression of about 30 per cent, which leaves a volume about 12 per cent larger than the volume of the unbroken rock.*

The supporting strength of dry filling as studied in connection with the problem of surface support at Scranton, Pennsylvania, is shown in Table 16.†

Anton Frieser reports that in coal mining in Bohemia hydraulic filling has been carried on extensively and that, with such filling at depths of from 60 to 200 feet, the roof pressure compresses one volume of ordinary stone-and-sand packing to 0.6, clay packing compresses to 0.5, and puddled sand and ashes to 0.8 or 0.9.‡

In the Ruhr coal district of Germany, filling has been used extensively and the amount of compression has been noted carefully.¶ This has been possible as new openings were driven through workings which had been filled from two to eight years previously.

Dr. Niesz has found that gobbing, under pressure, may lose four-tenths of its height, small-grained pit-heap material 25 per cent, and pure loose sand 8 per cent.§

The commission reporting upon the slide at Turtle Mt., Frank, Alberta, Canada, commented upon the efficiency of various kinds of filling in mine workings. The general statement was made that under

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*Colliery Engineer, Vol. 33, p. 548, 1913.

†U. S. Bureau of Mines, Bul. No. 25, p. 59, 1912.

‡Oestrr. Zeit für Berg.-und Hüttenwesen, Vol. 43, p. 253, 1895.

¶Oberhausen, J. "Compression of Slope Fillings," Glückauf, p. 1146, Nov. 22, 1902. Translation in Columbia School Mines Quarterly, Vol. 26, p. 271, 1904.

§Zeit für Berg.-, Hütt.-, u.-Salinew., Vol. 58, p. 418, 1910.

TABLE 16.

SUPPORTING STRENGTH OF VARIOUS FORMS OF DRY FILLING.

Kind of Material Comprising the Artificial Supports	Approximate Depth, in Feet, of Column of Coal Measure Rock, 1 Foot Square, Necessary to Compress Artificial Roof Support					
Per Cent. of Compression	1	3	5	10	20	30
	Feet	Feet	Feet	Feet	Feet	Feet
1. Rectangular gob piers, ordinary construction	....	10	12	36	125	*306
2. Circular piers of mine rock, well constructed	....	46	75	146	292	*512
3. Timber cogs filled with gob, average construction	....	8	68	182	270	*419
4. Loose pile of broken sandstone through 1¾-inch ring, 40 per cent. voids	....	....	20	53	124	*298
5. Pile broken sandstone, 40 per cent. voids, voids filled with sand	....	....	21	53	186	*465
6. Loose pile large size broken sand rock, 45 per cent. voids.	....	48	66	121	351	*492
7. Mine room filled with large broken sand rock, 50 per cent. voids	12	27	45	117	434	a615
8. Mine room filled with broken sandstone, 40 per cent. voids	....	44	74	177	619	1,310
9. Mine room filled with broken sandstone, 40 per cent. voids filled with sand	....	46	77	325	6,000	b8,860
10. Mine chamber filled with dry coal ashes, 64 per cent. voids	....	13	25	70	143	332
11. Mine room filled with dry river sand	12	40	70	442	1,715	6,640
12. Mine room filled with river sand flushed in with water	111	522	891	2,310	....	c8,860
13. Mine chamber filled with coal culm flushed in with water	32	118	190	472	1,822	5,905
14. Concrete pier, 1 part cement, 7 parts sand and gravel; 5 months old	117	1,092	(e)	....	....	....
Resistance of flushed culm	1.0	1.0	1.0	1	1	.†.......
Resistance of flushed sand	3.5	4.4	4.7	5	4	.†.......
Concrete pier	3.6	9.0	(d)	(d)	(d)	†(d)

a 27 per cent. settlement.
b 23 per cent. settlement.
c 20¾ per cent. settlement.
d Worthless.
e Gradually cracked to pieces under continuous load equal to 600 feet of rock.
*Free to expand laterally.
†Comparative.

average conditions the settlement would be 5 per cent of the thickness of the bed if ordinary sand were used; an inappreciable amount if granulated slag were used; 10 to 15 per cent with loam, sandy clay, and ashes; and 40 to 60 per cent with dry packing. Under the conditions at Frank the coal pillars left merely serve "to delay the process (of movement) for under the great pressures due to depth, shales, such as here constitute the hanging wall, will 'flow' and seal all openings."*

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*Daly, R. A., Miller, W. G., and Rice, G. S. "Report of Commission Appointed to Investigate Turtle Mt., Frank, Alberta, 1911." Can. Dept. Mines, Geol. Survey Branch, Memoir No. 27, p. 30.

TABLE 17.

EXTENT OF FILLING IN RUHR COAL DISTRICT, GERMANY.

Per Cent. of Compression Referred to Original Thickness	Area Worked Out Square Metre	Average Depth from Surface	Age of Workings at Time of Reopening	Composition of Filling
29	14,400	370	8	Waste rock, slates, and sandstones from surface.
28	20,800	450	2	Granulated slag and waste rock (clay and slate).
37	104,000	360	5	Waste from seam and from roof and footwall.
39	26,400	300	4	Waste from seam and from roof and footwall.
60	36,000	270	5	Waste rock from bottom of gangways.
21	21,000	380	2	Waste rock from surface, granulated slag and clay slate.
25	25,000	440	2	Same as preceding.

CONSTRUCTION OVER MINED-OUT AREAS.

When a building is threatened by subsidence resulting from mining operations, or when it is planned to erect a structure upon land which has been undermined and which does not offer sufficiently stable material for a foundation, various steps may be taken to prevent damage to the structure erected or proposed.

Owing to the danger of surface subsidence, the Central Railroad of New Jersey introduced sand into the old mine workings beneath the site of a proposed depot in Scranton in 1911. The Diamond and the Rock seams had been worked and after investigation of the workings it was decided that it would not be necessary to fill the entire area of the workings, but only to reinforce sufficiently the smaller pillars in both seams and fill the wider areas in the Diamond seam so as to prevent any further caving of the roof. In an 8-inch borehole, drilled for this special purpose, a 6-inch pipe was placed. The depth to the lower seam was 80 feet. Sand was brought in railroad cars and flushed into the workings, a total of 9,400 cubic yards being placed at a cost for labor of 29 cents per cubic yard of sand filling.*

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*Bunting, D. "Pillar and Artificial Support in Coal Mining." Penn. Legis. Jour., Appendix V, p. 5989. 1913.

The problem of constructing a six-story building, 60 feet wide by 157 feet 7 inches long, on Wyoming Avenue, in Scranton, Pa., was solved by constructing a series of concrete columns. The Big, or 14-foot, bed was close to the surface and had been mined beneath the property, but no maps were available to show the exact location and size of the pillars, and the old workings were inaccessible. Beneath the 14-foot bed other thinner beds had been worked. Five lines of holes were drilled to the rock under the 14-foot bed, the average depth being 40 feet. They were spaced 14 feet 10 inches in one direction and 16 feet 4 inches in the other. Twelve-inch steel pipes were driven into the holes and filled with concrete and, on the top of these, reinforced concrete beams were built.*

The Scranton Electric Company flushed ashes into the old workings under its new power house on Washington avenue. At the present time it is sinking a shaft to be used for dumping ashes into these workings, thus avoiding the expense of hauling them away.

In Pittsburgh, Pennsylvania, the residence section of the city extends over areas from which coal has been mined and it has been thought advisable to construct special foundations under buildings which might be endangered by surface subsidence. Exploratory holes at Beacon Street and Shady Avenue showed that the mine workings were 35 to 55 feet below the surface. Some of the roof had fallen, but some pillars had been left and it was anticipated that subsidence might not be uniform. A pillar of coal extended under one corner of the site for a house. Holes 10 and 14 inches in diameter were drilled to the rock below the coal and six concrete columns were constructed in order to provide support for that part of the house which would be unaffected by caving over the rooms in the mine. No column was constructed under the corner of the house where the coal pillar was located. The concrete columns were 8 inches and 12 inches in diameter inside the galvanized-iron lining which was placed in each hole. The lining was slightly smaller than the hole so that the rock might sink without disturbing the columns. Each column was reinforced and upon these columns were erected reinforced concrete girders which served as a foundation for the house.†

When it is proposed to remove all the mineral in a horizontal bed beneath a structure, it is advisable to mine out the coal in advance in a direction at right angles to the longer axis of the structure and to advance the face at a uniform rate as rapidly as possible so that the structure may be subjected to stress for as short a period as possible.

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*Stevenson, G. E. "Founding a Building Over Coal Mine Workings." Eng. News, Vol. 71, p. 791, 1914.

†"Concrete Column Foundation for a Building Over Coal Mine Workings." Eng. News, Vol. 117, p. 632, 1912.

Reference has previously been made to the special types of construction employed in buildings* and bridges† when surface movement is anticipated. Foundations may be reinforced, long buildings may be divided into units, joints permitting expansion and contraction may be provided, expansion pieces may be placed in railroad tracks, pipe line, cables, etc. In cities in German coal mining districts gutters and curbing are laid with elastic and waterproof joints. Asphalt, cement, and concrete pavements are not used because they are not easily repaired.

RESTORING DAMAGED LANDS.

When subsidence causes breaks and pit holes in agricultural lands, the surface may be rendered temporarily almost valueless for certain kinds of tilling. When the land is of great value for farming, these holes may be filled with waste rock from the mine, cinders, and other refuse, to within four feet of the surface. The remainder of filling necessary to restore a regular surface slope should consist of good soil. At a number of mines in Illinois where such surface damage has resulted from mining operations, the mining companies cooperate with the farmers in filling the pit holes with mine rock.

When subsidence does not break the surface but simply causes shallow basins below the general drainage levels, large ponds form during the spring and may result in the permanent flooding of valuable land. In Northern Illinois, in the longwall field, the topography is such that tile drains have been laid to permit the use of the land. Longwall mining frequently causes a surface movement sufficient to destroy the usefulness of such artificial drainage systems. Referring to the problem in Northern Illinois, G. S. Rice said: "It may be solved to a certain extent through draining the sunken lands by pumping, but even with such a method, aside from the expense, there is a serious difficulty from storm water. When the subsidence is from 2 to 4 feet it will render previously level lands of little use for raising crops until the particular area has come to full settlement and has been retiled. If it were possible to systematize mining so that the land nearest the water courses was first undermined and then in succession the land further away, the damage done to farming would be minimized."‡

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*P. 63.

†P. 60.

‡Rice, G. S. "Mining Wastes and Costs in Illinois." Geol. Survey, Bul. No. 14, p. 48.

CHAPTER VII.

Legal Considerations.

RIGHT OF SUPPORT.

The title to the minerals, and the right to work them may be held separately from the surface. Under the common law the owner of the surface is entitled to surface support, even though the owner of the minerals finds it impossible to remove them without disturbing the surface. Moreover the owner of the surface is entitled not only to vertical support, but also to lateral support from his neighbors even to the extent that minerals upon adjoining lands cannot be removed in such a manner or to such an extent that the surface of adjoining properties is disturbed.

Leases of coal rights often state distinctly that the lessee shall not be liable for damage to the surface, and where surface rights only are sold, the deed often states that the title to the surface does not include the right to surface support if the owner of the mineral rights mines out the mineral. In spite of such clauses in deeds and in leases suits are of common occurrence when surface and mineral rights are owned by different parties.

MINING UNDER MUNICIPALITIES.

The problem of the claims of municipalities in the coal districts has aroused considerable discussion. In many instances coal mines have been opened upon lands remote from towns and upon which no buildings other than the mine structures were erected at the time. Later mining villages have grown up near the mines and residences and other buildings have been constructed upon the land which had previously been undermined. In many instances, owing to the importance of locating near an abundant fuel supply, industrial plants have been erected in these mining villages or in other towns in the coal district. Eventually large cities have grown up on the lands on which coal mining was the pioneer industry. Similarly mines have been opened outside the limits of important cities and mining operations have been confined to the area which was outside the limits of the city when the mine was opened, but in the course of years the city has extended its limits to include the mine and the area undermined.

The claims of the municipality upon the mining interests, which may have a right by contract and under the law to mine all the coal and to be exempt from liability for damages to the surface, were forcibly presented by Mayor B. Dimmick of Scranton before the Pennsylvania Anthracite Mine Cave Commission, as follows:

"I am of the opinion that there is no constitutional barrier against the inclusion in the general police power of a state or a community of the specific power to declare as null and void and as against sound public morals any and all contracts that waive the right to a reasonable support of the surface which is to be occupied and used for community purposes. I would recommend submitting to the Legislature an act that would declare null and void and as being contrary to public policy any and all contracts that waive the right to, or release from responsibility for, reasonable support of the surface wherever such surface is actually devoted to community life.

"Fortunately this problem has been attacked at a period in which public opinion is slowly but surely crystallizing in favor of acceptance of two general principles, the first being in the direction of such qualifications to the ownership and use of property as are exacted through the increasing interdependence of modern life, such qualifications being in no sense a redistribution of property, in no sense a taking away from one and giving to another, but simply such restrictions and regulations as are demanded, not only in the carrying out of the ancient rule, 'So use your own as not to injure another,' but also for the general welfare of the community. The second principle is that Society must accommodate itself to such costs as are incident not only to fair return to both capital and labor, but also to all the accidents and burdens that result from any activities that Society desires or is compelled to enjoy, and this principle is being regarded as so clearly equitable that its enforcement is being demanded, all private contracts to the contrary notwithstanding. If this principle can and should be enforced when the health of the community or individual is at stake, surely it can and should be enforced when, as in the case of support of the surface, the very lives of men, women, and children are jeopardized.

"The maintenance of the surface, upon which are located the communities that extract the coal, should be regarded as a necessary factor in the cost of mining and should be paid for by the consumer. Such inclusion of the cost of the support of the surface in the general cost of production will be fairer than any fixed tax to be imposed by the State and then paid out, say to municipalities, to be expended in securing such support.

"It is possible that even under the existing welfare clauses of the acts governing municipalities of Pennsylvania, the proposed exercise of police power might be upheld, but certainly the hands of these municipalities in the anthracite region would be greatly strengthened by such proposed legislation.

"In contemplating this exercise of police power, I realize that there is possibly no exact precedent therefor, yet such exercises would clearly fall within not only the modern but even the ancient definition of the power. So eminent a judge and publicist as Jeremiah S. Black once said that "the police power of the State, of which she cannot disarm herself if she would, enables her to regulate the use even of private property in such manner that neither the general public nor particular individuals can be made to suffer by it unjustly.' "

With these claims of Mayor Dimmick many eminent lawyers have taken issue.*

Opinions upon a number of the points under discussion are given in the following citations:

"In the natural state of land one part of it receives support from another, upper from lower strata, and soil from adjacent soil." (Per Lord Selborne in Dalton v. Angus, 6 A. C. 791.)

"Where the surface belongs to one and the minerals to another, no evidence of title appearing to regulate or qualify their rights of enjoyment, the owner of the minerals cannot remove them without leaving sufficient support to maintain the surface in its natural state." (Wilms v. Jess, 94 Ill. 464, 1880.)

The same principles hold between the owners of different minerals lying in separate beds. If one bed lies above another the owner of the lower bed must give support to the upper bed. (MacSwinney p. 301.)

A coal mine operating beneath a clay mine is liable for injuries to the upper mine caused by failure to leave sufficient pillars in the coal mine. (Yandes v. Wright, 66 Ind. 319, 1879.)

The right of support is not affected by the nature of the strata nor by the difficulty of propping up the surface. (MacSwinney p. 292.)

The right of support is wholly independent of the comparative values of the substance receiving and the substance giving support. (*Op. Cit.*, p. 292.)

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*For a complete statement of American cases on the various points of discussion between surface and mining rights, see Lindley on mines, Title IX, Ch. II and III, 3d Ed., 1914. For British cases, see MacSwinney, R. F. "The Law of Mines, Quarries and Minerals." Ch. XIV, 4th Ed., 1912.

The right of lateral support is an absolute one. The obligation to respect it is in no way affected by the question of negligence. (50 Mo. App. 525.)

"Every owner of land in its natural state has a *prima facie* right to support, lateral as well as vertical; and the adjacent or subjacent owner has no right, *prima facie,* in order to win his minerals, to withdraw such support. The burden, both in pleading and in proof, is upon him who asserts that the position is different from that existing as of common right." (*Op. Cit.*, p. 299.)

EXEMPTION FROM LIABILITY FOR DAMAGE TO SURFACE.

A conveyance of the right to mine all the underlying minerals implies that in so mining such minerals the surface land shall be sufficiently supported and that so much of such minerals may be mined as can be obtained without injury to the surface. A waiver of the obligation to support the surface must be made by the owner of the surface land by language clear and unequivocal. Such a waiver does not follow a conveyance of all such minerals, nor from the use of language in such a conveyance to the effect that the mining operations shall be "conducted with as little damage to the surface as conveniently may." (Seitz v. Coal Valley Mining Co., 149 Ill. App. 85, 1909.)

Where a land owner sells the surface, reserving to himself the minerals with power to get them, he must, if he intends to have power to get them in a way which will destroy the surface, frame the reservation in such a way as to show clearly that he is intended to have that power. (Wilms v. Jess, 94 Ill. 464, 1880.)

When an instrument excludes the right of the surface owner to support, the mine owner may be liable, if he works negligently, or contrary to the custom of the country. (MacSwinney p. 311.)

The right of support by land in its natural state may also be excluded, wholly or in fact, by statute. Examples of this may be found in various English Acts. (See MacSwinney p. 312.)

In an investigation of the surface damage in a section of Scranton, Pennsylvania, it was found that 48 per cent of the titles contained a clause completely waiving surface support, in the following language: "All the coal in, under and upon said lot, together with the sole right and privilege to mine and remove all the coal under said lots without incurring in any event whatever any liability for injury or damage done to the surface of said lots or improvements thereon or that may thereafter be put thereon caused by mining or removal of said coal."

Fourteen per cent of the titles contained waivers which are more or less conditional in their nature: "All the anthracite coal lying underneath, also half the width of streets adjoining. It being understood and agreed that at least one-fourth thereof, properly distributed, shall be left for surface support and the coal shall be mined in a workman-like and skillful manner, it being understood that all the coal is to be mined and paid for except so much left thereof as may be necessary to be left for pillars to support the surface thereof, and it being possible that there may be a difference of opinion relating to the fulfillment of this provision it is agreed that the matter shall be submitted to a board of competent and skillful engineers, each party to select one and, in case of failure to agree, said engineers are empowered to call in a third mining engineer and the decision of the majority shall be final."

Ten per cent of the titles contained the following clause: "All the coal and minerals under said lot, together with the right to mine and remove all of said coal and minerals, provided also that in removing the coal the second party shall leave one-fourth thereof in place for the protection of the surface."

The remainder of the titles examined by the investigators contained the following clause: "All right, title, etc., to all coal in and under said lots, also the coal under the surface in front of said lots to the center of the street."

In the report of the Pennsylvania State Anthracite Mine Cave Commission excerpts of 42 deeds are given showing the various forms in which reservations have been made when the title to the surface has been severed from the mining right.*

In the bituminous fields a customary form of exemption clause in deed for coal, separate from the surface, is as follows: "All the coal underlying and within the described lands together with the right to take the entire quantity, or a less quantity of said coal, without leaving any support for the overlying strata, and without liability for any injury or damage which may result from the breaking of said strata." Another type of exemption clause employed in Illinois is as follows: "Releasing and surrendering any and all claims for damages and all liability by reason of damages either to person or property which may in any way be caused or occasioned at any time hereafter, directly or indirectly, by the mining or removing of coal or other minerals."

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*Pa. Legislative Journal, Appendix, Vol. 5, p. 5953, 1913.

PROTECTION OF SURFACE BY GRANTS AND BY LEGISLATION.

The right of support for land in its non-natural state may be acquired by express or implied grant.* Where land is severed from adjoining land for a particular purpose, and such purpose is known at the time the mineral right is severed from the surface, there is, *prima facie,* an implied grant of a reasonable degree of support for carrying out the particular purpose under consideration.

In order to protect railways, canals, waterworks, sewers, etc., the British Parliament has enacted legislation which guarantees that such structures shall not be undermined, if liable to be damaged, without notice. In accordance with these acts, the mining company may be required to leave a pillar, but the mining company is compensated for the coal left in the ground and for any damage that may be sustained by the interruption with the system of mining. In the case of the Railway Act, it is specified that "the (railway) company shall from time to time pay to the owner, lessee, or occupier of any such mines, extending so as to lie on both sides of the railway, all such additional expenses and losses as shall be incurred by such owner, lessee, or occupier by reason of the severance of the lands lying over such mines by the railway, or of the continuous working of such mines being interrupted, or by reason of the same being worked in such a manner and under such restrictions as not to prejudice or injure the railway, and for any minerals not purchased by the company which cannot be obtained by reason of making and maintaining the railway; and if any dispute or question shall arise between the company and such owner, lessee, or occupier as aforesaid, touching the amount of such losses or expenses, the same shall be settled by arbitration.†

Similar sections are included in the Wat. Cl. Const. Act, 1847; the Pub. Health Act, 1875 (Support of Sewers); Amendment Act, 1883; the Local Government Act, 1894; the Small Holdings and Allotments Act, 1908; and the Housing and Town Planning Act, 1909.‡

Under these acts, as noted, the intention of a mining company to remove the mineral beneath any of the legally specified structure must be announced through a regular "notice of intention to work." This notice is given in most cases thirty days in advance of the intended working. If, after notice has been given, the owner of the structure does not agree to negotiate with the mining company, it is lawful for the mining company to proceed in the regular manner of working. If damage results, it shall be repaired by the mining company.*

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*MacSwinney Pp. 315, 316.

†Railway Clauses Consolidation Act, 1845. Sec. 81. 8 and 9 Vict., c. 20.

‡MacSwinney P. 370.

Gas works and gas mains are not within the British mining code excepting in so far as they are vested in local authority. Private gas companies are not entitled to support if the mineral right has been severed, but the colliery company would be liable for damages to gas pipes and leakage of gas. In the absence of special provisions, owners, lessees, and occupiers of mines are not liable for damage caused to tramways by working mines or minerals in the usual and ordinary course.† This mining code of Great Britain is not applicable to burial grounds, school sites, public highways, bridges, nor canals.

In several states of the United States there are statutes in regard to support, particularly in western states in which the lode mining law permits extralateral mining. The statutes of Colorado, for example, prescribe that "when the right to mine is in any case separate from the ownership or right of occupancy to the surface, the owner or rightful occupant of the surface may demand satisfactory security from the miner, and if it be refused, may enjoin such miner from working until such security is given." No person shall have the right to mine under any building or other improvement unless he shall first secure the parties owning the same against all damages, except by priority of right."‡

Other states, such as Idaho, North Dakota, South Dakota, and Wyoming, have similar laws. In commenting on this type of legislation, Lindley says, "We are not aware that this class of legislation has been the subject of judicial investigation. It seems to us that such legislation is not altogether free from constitutional objections."¶

Arkansas has a law, approved Feb. 28, 1907, forbidding the mining of coal or any other mineral substance from beneath a cemetery or burial place. No openings whatever may be driven under or through the mineral directly beneath the cemetery under penalty of a fine of $5,000 or imprisonment of from one to five years.*

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*Cockburn, J. H. "Minerals Under Railways and Statutory Works." Trans. Inst. Min. Engrs., Vol. 39, p. 104, 1909.

†Op. cit. P. 128.

‡Rev. Stat., Colorado, p. 4213, 1908.

¶Lindley "American Law Relating to Mines and Mineral Lands." Vol. 3, p. 2016. 3d Ed.

In Pennsylvania the Davis Mine Cave Act, approved July 26, 1913, provides regulations governing the mining of coal and other minerals and the "support underlying and beneath the surface of the several streets, avenues, thoroughfares, courts, alleys, places, and public highways within the limits of the several municipal corporations, and authorizing the creation of a Bureau of Mine Inspection and Surface Support," by any municipal corporation within the anthracite coal fields. Members of the Bureau of Mine Inspection and Surface Support have the right and power to enter, examine, and survey any mine within the limits of the municipality. Mining companies are required to furnish accurate and complete maps of the workings and to keep the same up to date.

The mining companies are required to "maintain, uphold, and preserve the stability of the surface" of the various streets, etc. The officers of mining companies are made responsible for the violation of the provisions of the act and for violation are subject to a fine of $1,000 or imprisonment for ninety days, or both.†

An ordinance was enacted by the Borough of Plymouth, Luzerne County, Pennsylvania, forbidding mining within 200 feet of the street lines and as the borough is platted in 400-foot squares, this prohibited any mining whatever. The county courts in Borough of Plymouth v. Plymouth Coal Co. restrained the coal company from mining under the streets.

REMEDIES.

In the event that the owner of the surface is entitled to surface support and is sustaining damages by the mining operations beneath or adjacent to his land he may recover damages or if the damage is irreparable or immeasurable he may apply for an injunction to restrain mining operations. If the mining operations are being conducted by parties whose financial resources are not adequate to insure the payment of damages in case such are assessed, an injunction may be issued.

The right of support is not infringed by excavation, but by subsidence and damages do not exist until subsidence has actually occurred. (Catlin Coal Co., v. Henry Lloyd, 109 Ill. App. Rep. 122, 1902.) The Pennsylvania court now holds, however, that the cause of action accrues when the support is removed and is barred after the lapse of six years from such removal. It is said by the Pennsylvania court that the adoption of any more onerous rule "would encourage the purchase of surface over coal mines for speculation in future law suits." (Noonan v. Pardee, 200 Pa. 474, 86 Am. St. Rep. 722, 55 L. R. A. 410.)

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*Arkansas Acts of 1907, Sec. 566 d-f.

†Pa. Acts of General Assembly, 1913, No. 857.

"The owner of the subsidence estate is not liable to the surface proprietor for a subsidence caused by excavations made by his predecessor in title, although damage does not occur until after such owner came into possession. This results from the fact that, while the subsidence gives the cause of action, the responsibility therefor attaches to him whose acts and ommissions have brought about the mischief."*

If the owner of real estate which has been injuriously affected or damaged by a permanent structure has not brought an action to recover damages and conveys the land to another, the cause of action does not pass with the title nor inure to the benefit of the depreciation in value in the price paid. (La Salle County Carbon Coal Co. v. Sanitary District of Chicago, 260 Ill. 423, 1913.)

Depreciation in the value of the surface caused by the mere apprehension of future damage gives no cause of action.† Only damage which has actually occurred may be considered by a court, but each fresh subsidence constitutes a basis for a new claim for damages. (Catlin Coal Co. v. Henry Lloyd, 124 Ill. App. 394, 1906.)

"The right of support is not infringed unless the subsidence is substantial. There must be some real sensible interference with the land. The right of support in ordinary cases is infringed where the subsidence is substantial, but the damage is inappreciable; and it is now settled that an injunction may be obtained where the subsidence is substantial, although the damage is inappreciable. The right of support is analogous to a right of property, and is a right to have the surface kept securely at its ancient and natural level."‡

A mine owner cannot avoid liability by showing that his workings have been proper and in the customary manner. "The act of removing all support from the superimcumbent soil is, *prima facie,* the cause of its subsequently subsiding." (Wilms v. Jess, 94 Ill. 464, 1880.)

Where land has been artificially burdened by a building and no contract or prescription is available to regulate its right to support, no right to support, lateral or vertical, exists for the building. In an

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*Lindley P. 2021.
†MacSwinney P. 294.
‡MacSwinney P. 297.

action for removing support from land artificially burdened, the plaintiff has always been obliged, as a matter of pleading, to show that he is entitled to have the weight supported.*

Where the injury to the surface would have resulted from mining operations if no buildings existed upon the surface, the act creating the subsidence is wrongful and renders the owners of the mine liable for all damages that result from mining to the buildings as well as to the land itself. (Wilms v. Jess, 94 Ill. 464, 1880.)

As a general rule, the measure of damages in actions for injuries to real property is the difference in market value before and after the injury to the premises. But to this rule there are exceptions, and it has been held that the cost of repair or of restoring the premises to their original condition is the true and better rule to apply. The valuation should be adopted which will be most beneficial to the injured party, for he is entitled to the benefit of the premises intact. (Donk Bros. Coal and Coke Co. v. Slata, 133 Ill. App. 280; 135 Ill. App. 633.)

A bill in equity to restrain the mining of coal was dismissed as there is a remedy at law in case damage is done to surface by subsidence. (Henry Lloyd v. Catlin Coal Co., 109 Ill. App. 37, 1902.)

What amount of coal may be safely mined and what amount must be left for necessary support of the soil are largely engineering questions, and it is only in rare cases, where the remedy at law is so inadequate as to render such course necessary, that a court of equity will direct the work by injunction. (Henry Lloyd v. Catlin Coal Co., 210 Ill. 460, 1904.)

As previously noted† considerable damage has resulted in England from the pumping of brine. Under the Brine Pumping Act of 1891 (54 and 55 Vict. c. 40), upon application, compensation districts may be formed within the pumping fields. For every compensation district, under the act, there is established a compensation board. This board is incorporated and consists of representatives of the various interests concerned; one-third shall be persons not interested in the brine business and appointed by the county council; one-third elected by the brine pumpers; and one-third (not interested in brine pumping) appointed by the local sanitary authority. A compensation fund is maintained in each district; upon each pumper is levied a tax not exceeding 3 pence per 1,000 gallons pumped for a twelve-month period. Out of this fund damages allowed by the compensation board are paid.

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*MacSwinney P. 304.
†Ch. I.

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THE TRACTIVE RESISTANCE ON CURVES OF A 28-TON ELECTRIC CAR

BY
EDWARD C. SCHMIDT
AND
HAROLD H. DUNN

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ENGINEERING EXPERIMENT STATION

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THE TRACTIVE RESISTANCE ON CURVES OF A 28-TON ELECTRIC CAR

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and

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THE TRACTIVE RESISTANCE ON CURVES OF A 28-TON ELECTRIC CAR

I. INTRODUCTION

The tractive resistance of cars running on curved track is greater than their resistance on straight track of like grade and construction. This excess is generally termed curve resistance or resistance due to curvature. A knowledge of its magnitude is needed in many problems which present themselves in connection with steam and electric railway design and operation. While these problems are more important and more numerous on steam roads, they are not unimportant nor infrequent on electric roads. Nearly all the existing information regarding curve resistance has arisen from tests and experience with trains on steam railways, and it is doubtful whether the values of curve resistance thus derived are valid for the single self-propelled cars used on electric lines. Such considerations led the Railway Engineering Department of the University of Illinois to make the tests, the results of which are presented in this bulletin.

The tests were undertaken to measure the curve resistance of a 28-ton electric car, owned by the department, with the view of determining its value at various speeds and on as great a variety of curves as were available on the lines of the Illinois Traction System, upon which the experiments were conducted. The results established for this car the relations between curve resistance and speed and between curve resistance and rate of curvature.

The test conditions, test methods, and final results are presented in the body of the bulletin, while the details concerning the apparatus, test data, methods of calculation, and intermediate results are given in the appendixes. Throughout the bulletin the term ''curve resistance'' or ''resistance due to track curvature'' means the tractive force needed by the car on curves in excess of the force needed to move it over straight track. This tractive force is the force required at the wheel rims to keep the car moving at uniform speed on level track and in still air. It is expressed in pounds per ton of car weight.

Acknowledgment is gratefully made of the interest and coöperation of the officers of The Illinois Traction System, which rendered it possible to conduct the tests on that road; and to Mr. D. C. Faber, formerly a Fellow in the Department of Railway Engineering, who was in charge of the test car during some of the tests and who made some of the preliminary calculations.

II. Summary

At the expense of some duplication there is presented at this point a summarized statement of the conditions, methods, and results of the tests, which may serve to provide a general view of the work and to facilitate an understanding of the more detailed explanations which follow.

1. *The Car, Track, and Equipment.*—The tests were made with a car such as is commonly used on interurban electric roads; it has a body 45 feet long of the double-end type with round vestibules. This body is carried on four-wheeled trucks which are spaced 23 feet 3 inches from center to center and which have a wheel base of 6 feet 4 inches. The car is equipped with four 50-horsepower motors and weighs approximately 28 tons.

The tractive resistance of this car was determined when running upon each of seven curves whose curvature varied from 2 to 14½ degrees, and also when running upon adjacent tangent track. The superelevation of the outer rail on the curves varied from 0.75 inches on the 2-degree curve to 5.9 inches on the 14½-degree curve. The track was laid with 70-pound rails on ties spaced about 24 inches between centers in gravel or cinder ballast. Judged by the standards which prevail on electric interurban roads built for moderate speed, the track was well constructed and well maintained. It was surveyed especially for the purposes of the tests. With a few exceptions the tests were made on dry rail in fair weather. The average air temperature varied during the tests from 25 to 65 degrees F. and the maximum average wind velocity was 18 miles per hour.

2. *Methods.*—In making a test, the test car was run first in one direction and then in the other over one of the curves and its adjacent tangent. During each such pair of runs the car speed was maintained as nearly constant as possible. Similar pairs of runs were then made at other speeds, until sufficient data had been accumulated to define the resistance at various speeds on the curve and on its corresponding tangent. Under this procedure each pair of runs results in two values of resistance: one with such wind as prevailed helping the car, the other with the wind opposing it. From the final curves which define the mean between these values, the influence of the wind is, therefore, nearly, if not quite, eliminated, and the results relate to movement in still air. The results of the tests provide mean values of resistance on each curve and on its tangent. The difference between these values of resistance is the desired resistance due to track curvature.

3. *Results.*—The tests demonstrate that for the car in question curve resistance varies directly with both track curvature and speed. For a particular speed, the curve resistance increases as the curvature increases and in direct ratio with the curvature, as shown in Fig. 16. This implies that at a particular speed the curve resistance, when expressed in pounds per ton per degree of curve, is a constant for all curvatures; a relation which is in accord with the results of previous experiments. On the other hand, the value of curve resistance expressed in pounds per ton per degree is shown to be different at each different speed, and it varies in such a way that for a curve of a particular curvature the curve resistance increases in direct ratio with the speed, as shown in Fig. 17. The concurrent relations between curve resistance, track curvature, and speed are shown in Fig. 18, and they are defined by the formula:

$$R_c = 0.058\ S\ C,$$

in which R_c is the curve resistance expressed in pounds per ton, S is the speed in miles per hour, and C is the degree of curve. Values derived by means of this equation appear in Table 4.

III. Means Employed in Conducting the Tests

4. *The Test Car.*—The car used for the tests is owned by the Railway Engineering Department of the University. It is a standard 45-foot car similar to those commonly used on interurban roads built for moderate speed. Its general design is shown in Figs. 1 and 2.

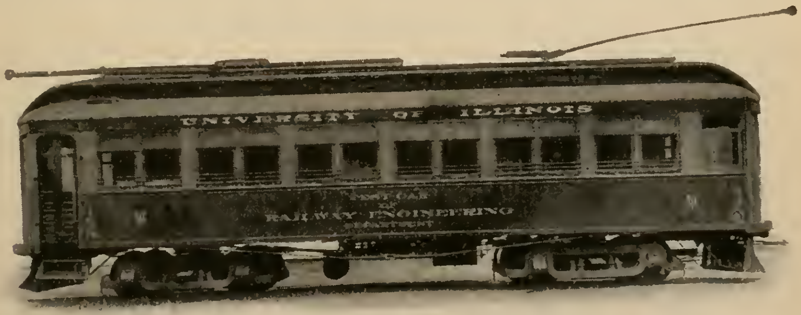

Fig. 1. The Test Car

The car weighs 55,150 pounds, although during the tests this weight was subject to certain corrections due to changes in equipment and in the number of passengers. The sectional area of the car body and trucks is 90 square feet.

The trucks are of the standard Motor Truck Company's C-60 type and weigh 7,824 pounds each, without the motors. The truck journals are 4¼ by 8 inches, the wheel diameter is 33 inches, and the truck wheel base is 6 feet 4 inches. One of the trucks is equipped with rolled steel wheels and the other with chilled cast iron wheels, all of which have standard Master Car Builders' tread and flange contours. During these tests the car was equipped with ball-bearing center

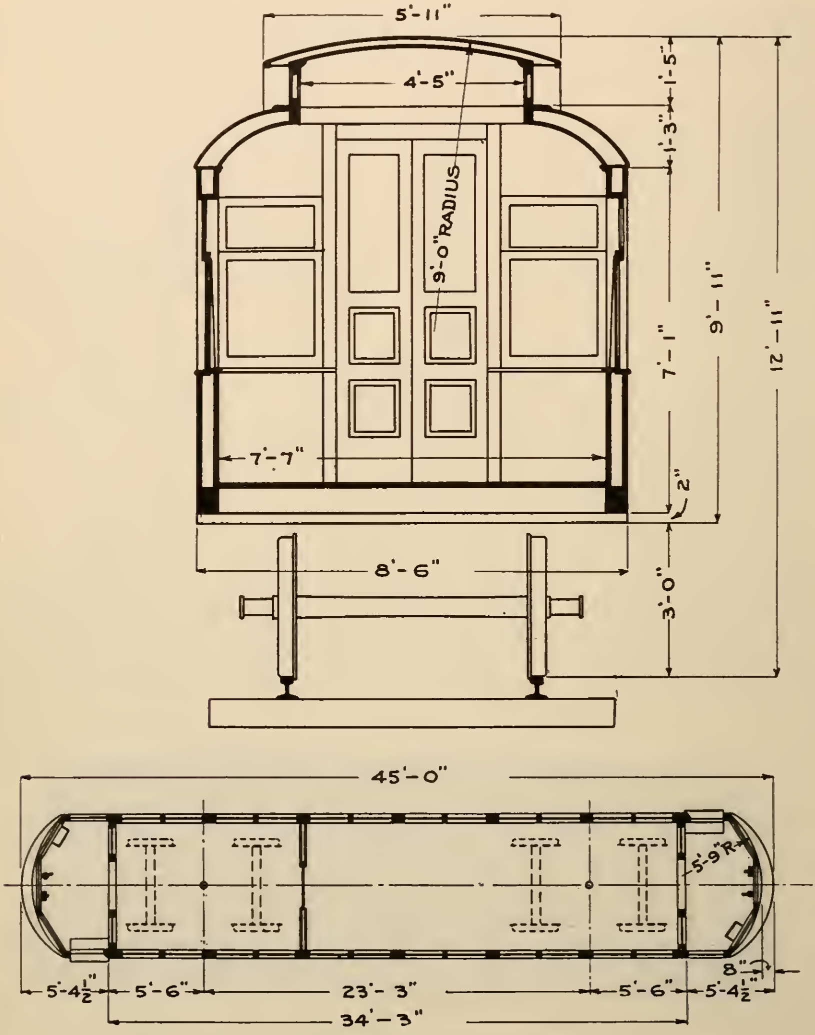

FIG. 2. PLAN AND CROSS SECTION OF THE TEST CAR

plates. Each truck is provided with two Westinghouse 101-D, 500-volt, direct current motors, which have a commercial rating of 50 horsepower each. The motors are mounted on the axles and geared to them in the ratio of 22 to 62. They are controlled by the Westinghouse unit switch system of multiple control.

An especially designed recording apparatus within the car offers a means for measuring and recording the current consumed, the voltage, speed, time, distance traversed, location on the road, and brake cylinder pressure. Continuous graphical records of these data are drawn upon a chart which is made to travel at a rate proportional either to time or to the distance traveled by the car. A more complete description of the car and of this recording apparatus appears in Bulletin 74 of the University of Illinois Engineering Experiment Station. In addition to the data above enumerated, there were recorded for each run the average wind velocity and direction, air temperature, rail condition, and the gross car weight.

5. *The Track.*—The tests were made on the lines of the Illinois Traction System between Danville, Urbana, Champaign, Decatur, and Springfield. The oldest portions of this track were laid in 1903, the latest in 1907. Judged by the standards which prevail on interurban electric roads built for moderate speed, the track was well constructed and well maintained. On both the curves and the tangents used in the tests, the track was laid with 70-pound A. S. C. E. section rails carried on hard-wood ties spaced about 24 inches between centers. The ballast was either gravel or cinders.

The curves chosen were seven in number varying in curvature from 2 to 14½ degrees, as great a range in curvature as was presented by the track available for test purposes. The gauge on all but the 8-degree and the 14½-degree curves was 4 feet 8½ inches, while on these two curves it was 4 feet 9 inches. A summary of the facts relating to the seven curves is given in Table 1. All the track used for

TABLE 1

DATA RELATING TO THE SEVEN CURVES

Average Curvature	Length of Track Section	Superelevation			Grade	Weight of Rail
		Average	Maximum	Minimum	Difference in Elevation	
Degrees	Feet	Inches	Inches	Inches	Feet	Lb. per Yd.
2°–0′	500	0.7	1.2	0.0	1.47	70
2°–50′	1737	3.0	4.1	1.3	8.52	70
3°–40′	1125	1.9	3.3	0.2	4.81	70
5°–0′	714	2.8	3.8	0.4	0.15	70
6°–30′	365	4.5	5.4	2.2	3.22	70
8°–0′	176	5.3	5.5	5.0	0.65	70
14°–30′	276	5.9	6.9	5.0	0.17	70

the tests was surveyed especially for the test purposes, and the results of these surveys are presented in Appendix I, together with further details.

IV. Test Conditions and Test Methods

6. *Test Conditions.* — The tests were all made in moderate weather. The lowest air temperature recorded during any test was 15 degrees F., and the lowest average temperature throughout the duration of any test was 25 degrees. The corresponding highest temperatures were 70 degrees and 65 degrees, respectively. With five exceptions the tests were made on dry rails. The highest average velocity of the wind prevailing during any of the tests was 18 miles per hour, whereas the maximum component of this velocity parallel

Table 2

A Summary of Test Conditions on the Curves and on Their Corresponding Tangents

1 Curve or Tangent	2 Test Number	3 Degree of Curve	4 Weight of Car and Load	5 Approx. Average Air Temp.	6 Rail Condition	7 Average Wind Velocity
			Pounds	Deg. F.		M.P.H.
Curve	117–118	2°–0′	56200	25	Wet	12.0
	123–124	2°–0′	56750	30	Dry	4.0
Tangent W	117–118		56200	25	Wet	12.0
	123–124		56750	30	Dry	4.0
Curve	119–120	2°–50′	56750	45	Dry	3.5
	121–122	2°–50′	56750	30	Dry	3.0
Tangent S	119–120		56750	45	Dry	3.5
	121–122		56750	30	Dry	3.0
Curve	125–126	3°–40′	57350	35	Dry	15.0
	127–128	3°–40′	57500	25	Wet	0.0
	129-130	3°–40′	57350	65	Dry	15.0
Tangent R	125–126		57350	35	Dry	15.0
	127–128		57500	25	Wet	0.0
	129–130		57350	65	Dry	15.0
	141–142		57800	40	Dry	3.8
	153–154		57900	60	Dry	0.0
Curve	125–126	5°–0′	57350	35	Dry	15.0
	127–128	5°–0′	57500	25	Wet	0.0
	129–130	5°–0′	57350	65	Dry	15.0
Tangent R	*					
Curve	133–134	6°–30′	57300	35	Wet	7.5
Tangent D	109–110		56950	55	Wet	18.0
	111–112		56350	55	Wet	18.0
	113–114		56200	55	Dry	10.0
Curve	141–142	8°–0′	57800	40	Dry	3.8
	153–154	8°–0′	57900	60	Dry	0.0
Tangent R	*					
Curve	141–142	14°–30′	57800	40	Dry	3.8
	153–154	14°–30′	57900	60	Dry	0.0
Tangent R	*					

* Same tangent as for 3°–40′ Curve

to the tangent track or to a chord connecting the ends of the curve was 15 miles per hour. The wind velocity and direction were obtained by means of a portable wind vane and an anemometer set up beside the test track. A summary of the conditions for each test is given in Table 2.

7. *The Selection of the Track.*—The primary consideration in the selection of the curves was to have them include as great a variety and range in degree of curvature as the circumstances would permit. When available, those curves were chosen which had at either end a long and comparatively level stretch of straight track, in order that the resistance on the curve and on the tangent might be measured simultaneously and, therefore, under like conditions of wind and weather. In three out of the seven cases, however, the straight track adjacent to the curve was, by reason of its construction or maintenance, or by virtue of the operating conditions which there prevailed, unsuitable for the purposes of the tests, and in these instances the tangent track was chosen elsewhere. In such cases the determination of tangent resistance was made as far as possible under conditions similar to those which obtained during the tests on the related curve. The curves chosen were fairly regular in curvature, and all the track on both the curves and the tangent sections was well ballasted.

8. *General Methods.*—By means of the data which were recorded during each run and which have been enumerated in Chapter III, it is possible to calculate the gross resistance offered to the motion of the car over a given track section. This gross resistance is composed of the resistance due to grade, that due to acceleration, that due to wind,* the net resistance on level straight track at uniform speed, and, on the curved track, the resistance due to curvature. The purposes of the tests require the elimination of the first three of these five elements of gross resistance. The grade and acceleration resistances may be easily eliminated by calculation, but the wind resistance is neither controllable nor to be eliminated by calculation from the data at hand. The elimination of wind resistance may, however, be accomplished by the method of making the tests, and with this end in view the following method of test was used.

Over each track section the car was run in one direction at a predetermined speed which was maintained as nearly uniform as possible. It was then immediately run over the same section at the same speed, but in the reverse direction. Where curved and tangent track were adjacent they were both included in each of these runs. This process was repeated at various speeds until enough data had been accumulated to define for this track section the relation between resistance and speed up to the maximum speed of operation. The whole group of these pairs of runs constitutes what is designated in the report as a test.

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*Throughout the bulletin wind resistance is distinguished from the resistance due to still air, the latter being an inseparable part of inherent or net resistance.

The wind in one case opposes and in the other helps the motion of the car, and each pair of such runs results, when grade and acceleration resistance have been eliminated, in two values of resistance; one value equal to net resistance* plus such resistance as was offered by the wind, the other equal to net resistance minus the resistance offered by the wind. In runs in which the direction of the wind was parallel to the direction of motion of the car, a properly determined mean between these two values is the net resistance itself with the influence of wind entirely eliminated. In runs in which the direction of the wind made an angle with the direction of motion of the car, only that portion of wind resistance due to the component of wind velocity parallel to the track would disappear from the mean value of resistance, and there would remain embodied in this mean† a certain resistance due to the increased wheel-flange friction caused by that component of wind velocity which is normal to the direction of motion of the car. All the resistance curves resulting directly from the tests represent the mean between paired values determined in the way just described. The curves themselves, therefore, define values of resistance in which there remains only that effect of the wind produced by its velocity component normal to the track; that is, they are in error by only a slight excess in flange friction. Under the low wind velocities which prevailed this error is small, and doubtless much less than the casual variations which occur in inherent resistance itself. These conclusions apply to the immediate results of the tests; that is, to the resistance-speed curves shown in Figs. 3 to 13, inclusive. For reasons stated in the next paragraph the error referred to does not, however, appear in the final results.

The simultaneous operation of the car over the curve and its adjacent tangent ensures that the resistances on the curve and on the tangent were obtained under practically identical wind conditions. Whatever error or excess in mean resistance arises from the normal component of wind velocity is, consequently, likewise identical on the curve and on the tangent. Since, however, the final result sought, the resistance due to curvature, is the difference between the total resistance on the curve and the resistance on the tangent, this error, by the process of subtraction, disappears from the final results of the tests. In those cases in which tangent resistance had to be measured on track not adjacent to the corresponding curve, the wind conditions were nearly the same as during tests on the curve, and the same conclusion, therefore, holds in these instances also.

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*As used in this connection the term net resistance is assumed to include on the curves the resistance due to curvature.

†The method of determining this mean is explained in Appendix II.

From the data recorded in the test car the gross resistance was first determined for each run over each track section. This was then corrected for acceleration by means of data provided by the car records, and next for grade resistance by reference to the track profiles. The methods of making these calculations are stated in Appendix II, together with further details concerning the car records and the test methods.

V. The Immediate Results of the Tests

The calculations made for each run result, for the track section under consideration, in a value of car resistance at a particular speed. For all the runs these resistance values, together with the corresponding speeds, are set forth in the tables given in Appendix III. Using these values as coördinates, the points shown in Figs. 3 to 13 have been plotted; each point defines the resistance-speed relation during one run, and the whole group of points in each figure characterizes this relation for a particular track section. By a method which is explained in Appendix II, a line has been drawn among the points of each of these figures which represents the mean resistance discussed in Chapter IV, and which in each case has been accepted as defining the relation existing between resistance and speed for the track section in question. Figs. 3 to 13, inclusive, constitute the immediate results of all the tests.

The points indicated by circles in these figures pertain to runs in which the wind opposed the motion of the car, while those represented as black dots pertain to runs in which the wind helped the car motion. The few cases in which there was no wind are represented by circles half filled in. Under the test procedure previously outlined, there should appear in each figure equal numbers of circles and dots. This condition, however, is not exactly met in any of these figures, because during the process of calculation a few points were rejected on account of discrepancies or inadequacy in the record. In general, however, the balance implied by the test procedure is substantially realized.

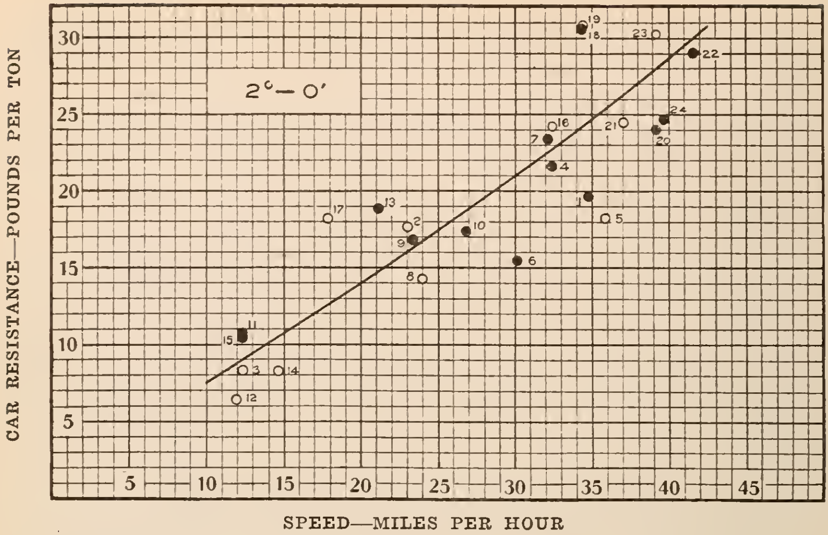

Fig. 3. The Relation of Resistance to Speed on the 2°–0′ Curve

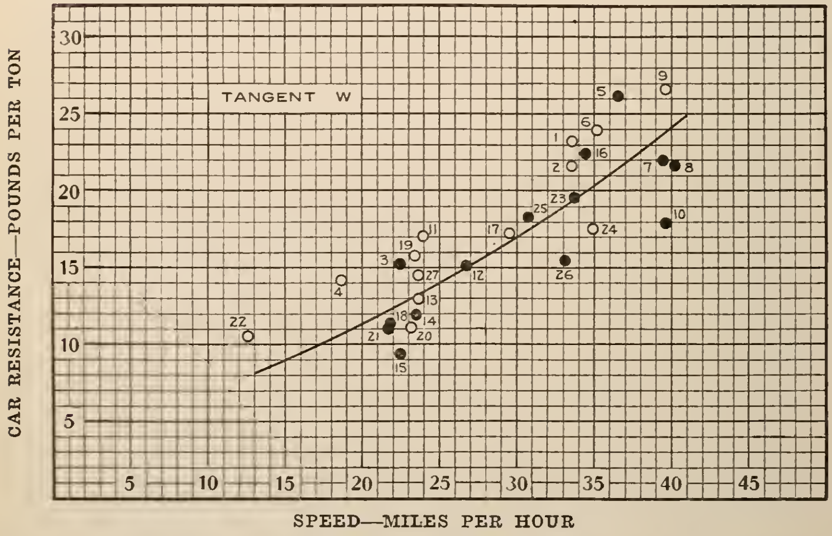

Fig. 4. The Relation of Resistance to Speed on Tangent W, Which Pertains to the 2°–0′ Curve

There is considerable variation among the points in these figures and also a considerable variation from the mean defined by the curves. Some of this is due to the influence of wind, but even among the points

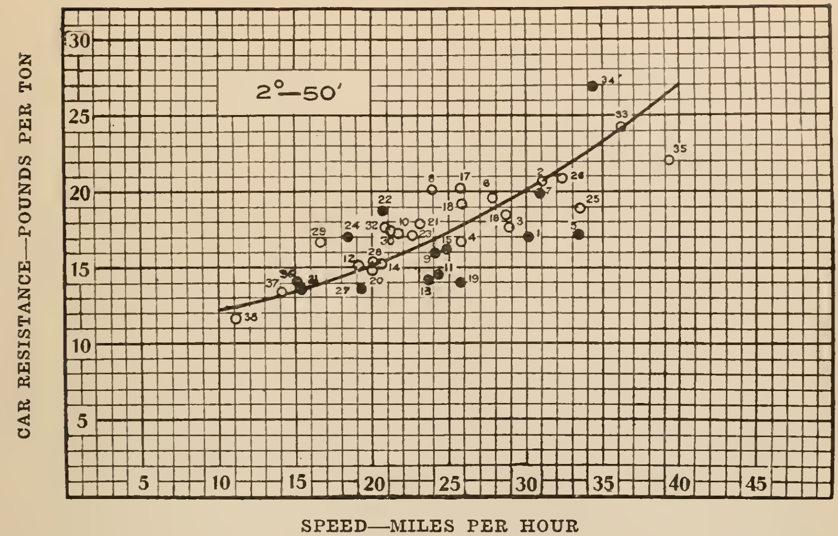

Fig. 5. The Relation of Resistance to Speed on the 2°–50′ Curve

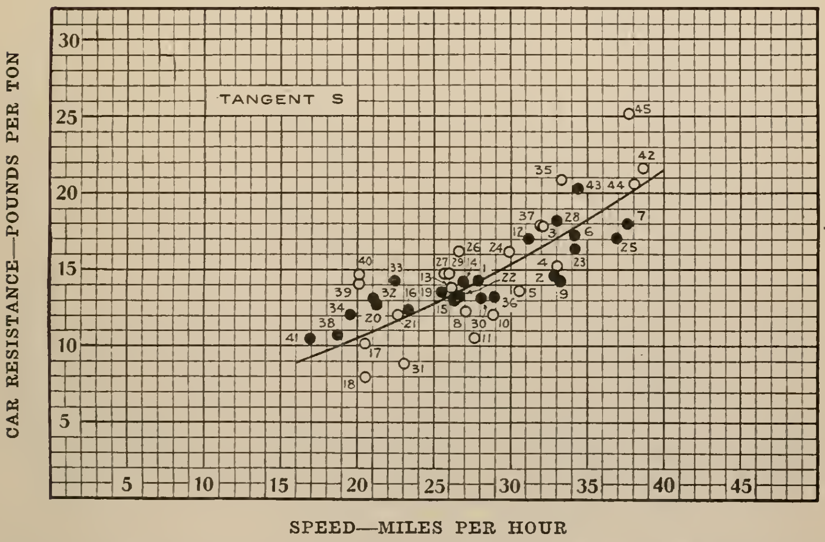

Fig. 6. The Relation of Resistance to Speed on Tangent S, Which Pertains to the 2°–50′ Curve

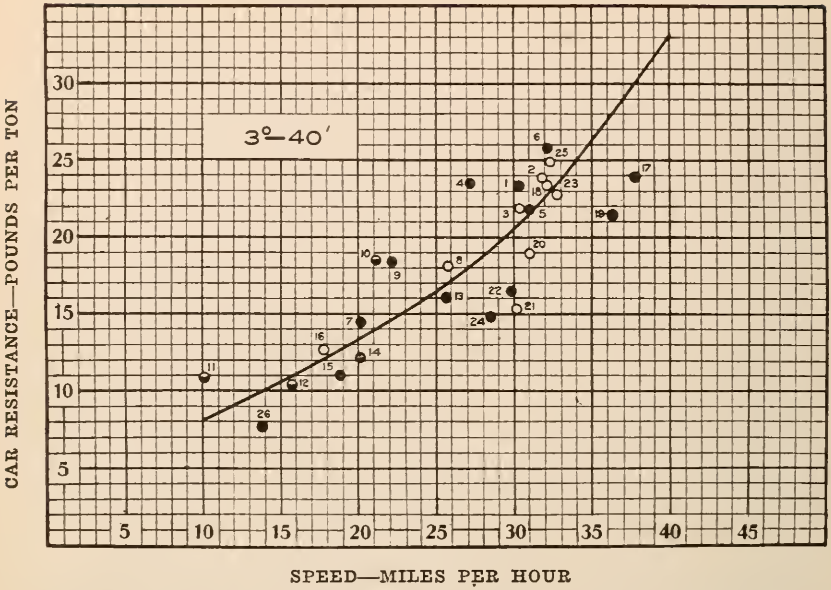

FIG. 7. THE RELATION OF RESISTANCE TO SPEED ON THE 3°–40′ CURVE

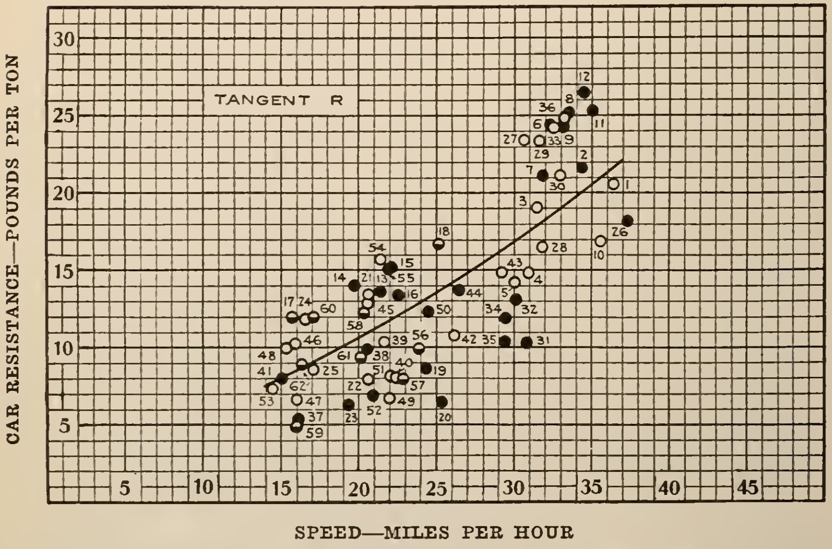

FIG. 8. THE RELATION OF RESISTANCE TO SPEED ON TANGENT R. WHICH PERTAINS TO THE 3°–40′, THE 5°–0′, THE 8°–0′, AND THE 14°–30′ CURVES

relating to like wind conditions there is similar disagreement. Part of this variation may be due to accumulated errors in instruments or in the calculations; but, since every precaution was taken to avoid such errors, they are undoubtedly rare and small in amount. The reasons for the differences referred to must be sought chiefly in the casual changes which occur in such elements of inherent resistance as flange friction, journal friction, and gear friction, and perhaps also in instantaneous changes in wind velocity and direction. The discordance shown is not greater than that usually encountered in measurements of car or train resistance.

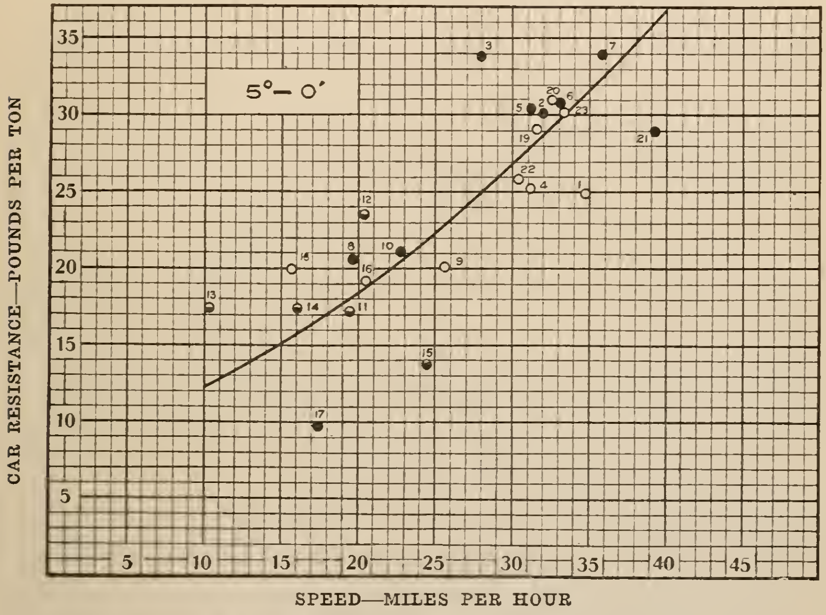

FIG. 9. THE RELATION OF RESISTANCE TO SPEED ON THE 5°–0′ CURVE

All the figures show a continuous increase of resistance with speed, except Fig. 13, applying to the 14°–30′ curve. In this case the resistance decreases until a speed of about 12½ miles per hour is reached and then increases as the speed is increased beyond this point. This peculiarity, which showed itself during the earliest runs on this curve, led to repetitions of the tests, but always with the same result, and Fig. 13 must be accepted as representing the facts in the case. The superelevation on this curve is 5.9 inches, and the component of car weight parallel to the plane of the rails amounts to about 5,800 pounds. It was assumed that perhaps this component, up to the

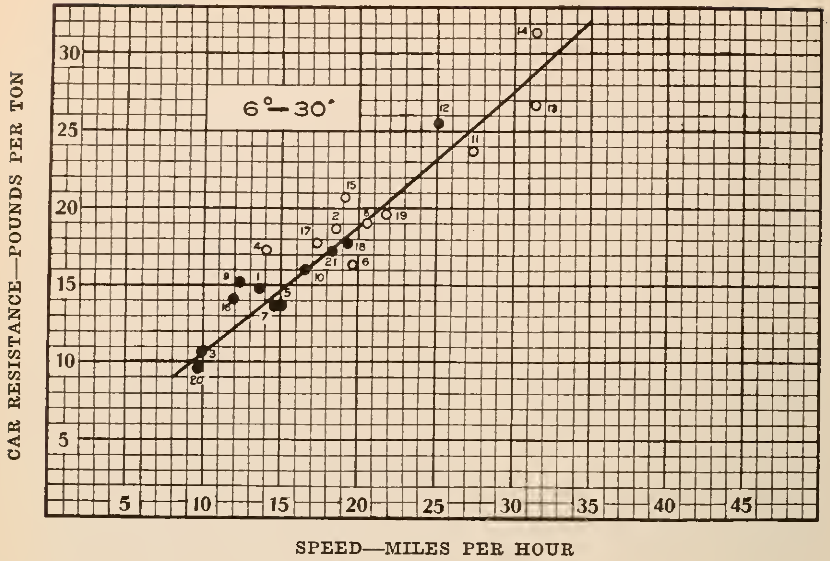

FIG. 10. THE RELATION OF RESISTANCE TO SPEED ON THE 6°–30′ CURVE

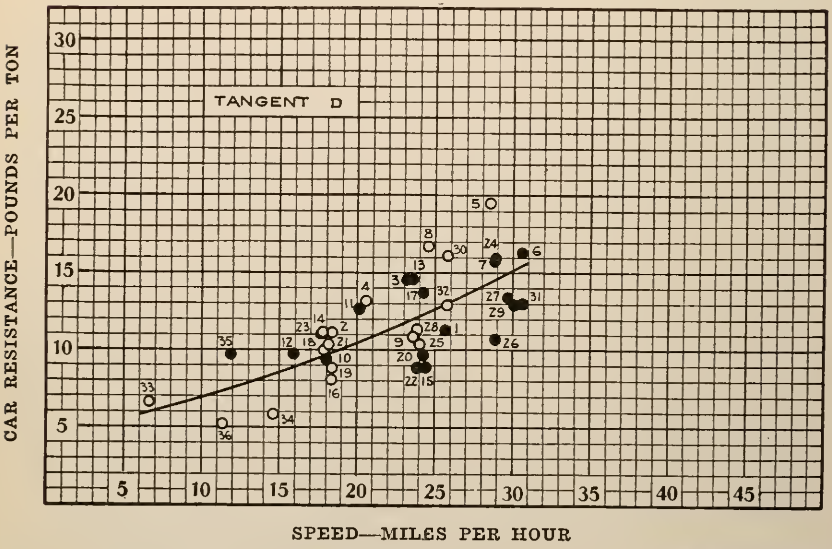

FIG. 11. THE RELATION OF RESISTANCE TO SPEED ON TANGENT D, WHICH PERTAINS TO THE 6°–30′ CURVE

speed of minimum resistance, had produced an excessive flange friction on the inner rail and had bound the trucks by keeping the inner side bearings in continuous contact. Since this component is opposed by the centrifugal force developed by the car in rounding the curve, these effects should continuously diminish as the centrifugal force increases up to the speed where this component of car weight equals the centrifugal force. Here the point of minimum resistance would be expected to occur. Calculations for this case, however, show this speed to be at 16 instead of at 12½ miles per hour. This discrepancy is not in itself enough to discredit the assumption; but investigation of the other curves shows that this speed, at which the centrifugal force and the weight component are in balance, occurs at from 23 to 26 miles per hour, and, if the assumption were valid, similar minimum points should occur at these speeds in the resistance curves for these cases. No such minimum points occur, however, in any of the other figures, and the explanation on these grounds must consequently be abandoned. The condition presented in Fig. 13 is accepted as unexplained by the information at hand.

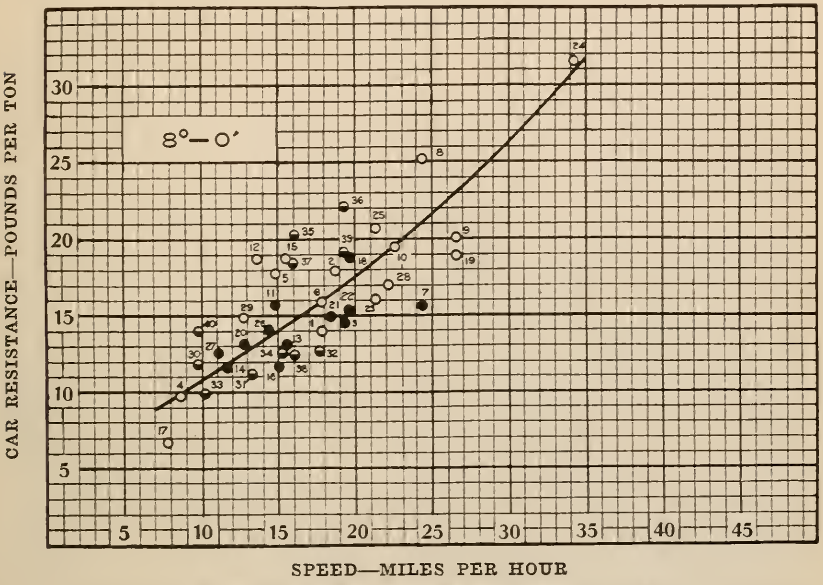

Fig. 12. The Relation of Resistance to Speed on the 8°–0′ Curve

From the results shown in Figs. 3 to 13, grade and acceleration resistance have been eliminated, and, as previously explained, the effect of the component of wind velocity parallel to the track has also been eliminated; the influence of the normal component of wind velocity, on the other hand, is still embodied in these results. The resistance defined by the line drawn in Fig. 3, for example, which relates to the 2-degree curve, consequently comprises net resistance on level track at uniform speed, resistance due to the component of wind velocity normal to the track, and the resistance due to curvature. The resistance defined by the line in Fig. 4, which relates to the tangent adjoining the 2-degree curve, comprises net resistance on level track at uniform speed and the resistance due to the normal component of wind velocity. The ordinates of the curves in these two corresponding figures, therefore, differ only by an amount equal to the resistance due to curvature on the 2-degree curve. Similar statements apply to the curves in Figs. 5 to 13.

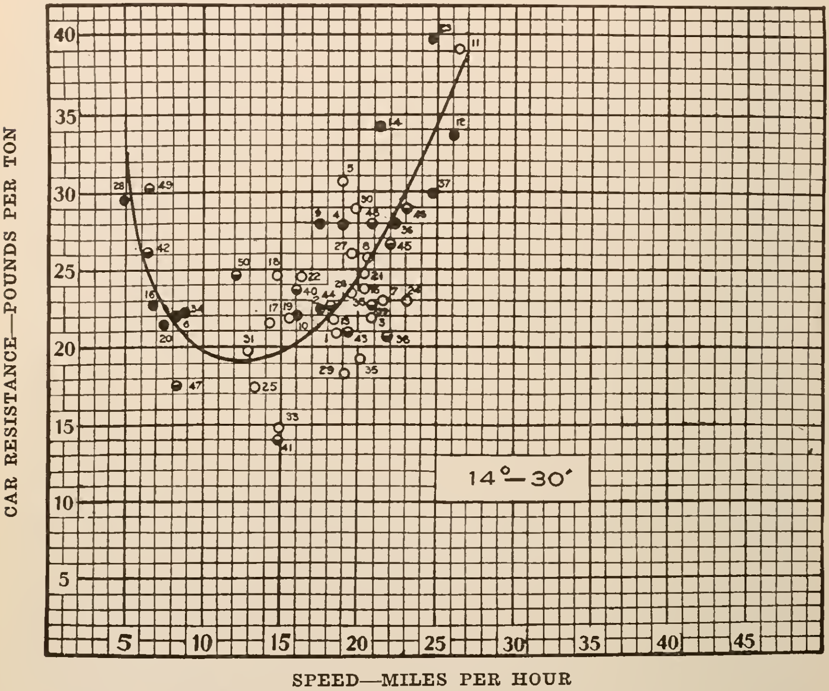

Fig. 13. The Relation of Resistance to Speed on the 14°–30′ Curve

In the further consideration of Figs. 3 to 13 it will be convenient to keep in mind their relationship. The 2°–0′, 2°–50′, 3°–40′, and 6°–30′ curves (Figs. 3, 5, 7, 10) have each their own tangent (Figs. 4, 6, 8, 11), the diagram for which immediately follows the diagram for the corresponding curve. The 5°–0′, 8°–0′, and 14°–30′ curves (Figs.

9, 12, 13) have all the same tangent as the 3°–40′ curve. The relation between these figures is as follows:

Fig. 3, the 2°–0′ curve, is used with Fig. 4, Tangent W.
Fig. 5, the 2°–50′ curve, is used with Fig. 6, Tangent S.
Fig. 7, the 3°–40′ curve, is used with Fig. 8, Tangent R.
Fig. 9, the 5°–0′ curve, is used with Fig. 8, Tangent R.
Fig. 10, the 6°–30′ curve, is used with Fig. 11, Tangent D.
Fig. 12, the 8°–0′ curve, is used with Fig. 8, Tangent R.
Fig. 13, the 14°–30′ curve, is used with Fig. 8, Tangent R.

VI. The Final Results of the Tests

9. *The Resistance Due to Curvature.*—The resistance-speed curves of Figs. 3 to 13, properly paired, are all brought together in Fig. 14 in which seven pairs of lines appear, one for each of the seven curves. The first pair of lines in Fig. 14 relates to the 2-degree curve, the upper line marked *C* shows the relation between resistance and speed on the curve, while the lower line marked *T* shows this relation on the adjacent tangent. These lines are reproduced from Figs. 3 and 4, respectively. Fig. 14 presents also six additional pairs of curves similarly marked pertaining to the six remaining curves, and all reproduced from Figs. 5 to 13.

As has been stated in Chapter V the difference between the ordinates of these pairs of lines at any speed is the value of curve resistance at that speed for the curve in question. For example, for the 2-degree curve (see Fig. 14) the resistance on the curve at 20 miles per hour is 14.0 pounds per ton, and the resistance on the tangent is 11.3 pounds per ton. The difference between these values, 2.7 pounds per ton, is the curve resistance at 20 miles per hour on the 2-degree curve. At speeds varying from 10 to 40 miles per hour the values of resistance on the curve, resistance on the tangent, and curve resistance have been thus determined for each of the seven curves and set forth in Table 3. This table summarizes, therefore, the direct results of the tests as presented in Figs. 3 to 14, inclusive, and it presents also the derived values of curve resistance whose determination was the immediate purpose of the tests. The values of curve resistance given in the table are accepted as the average values at the various speeds. As has been previously explained these values are nearly, if not quite, freed from any effect of wind.

10. *The Relation of Curve Resistance to Curvature.*—For each of the speeds in Table 3 there appears for each of the seven curves a

TABLE 3

VALUES OF RESISTANCE ON THE CURVE, RESISTANCE ON THE TANGENT, AND RESISTANCE DUE TO CURVATURE FOR EACH OF THE SEVEN CURVES AND AT VARIOUS SPEEDS. THESE VALUES ARE DERIVED DIRECTLY FROM FIGURES 3 TO 13 INCLUSIVE. THEY ARE EXPRESSED IN POUNDS PER TON

Curve	Speed—M.PH.							
	10	15	20	25	30	35	40	
2°–0′	7.50	10.78	14.00	17.45	21.00	24.70	28.70	Res. on Curve
	6.95	9.00	11.30	13.93	16.88	20.25	24.15	" " Tang.
	0.55	1.78	2.70	3.52	4.12	4.45	4.55	Curve Res.
2°–50′	12.20	13.40	15.05	17.30	20.00	23.25	27.00	Res. on Curve
	6.95	8.55	10 44	12.75	15.30	18.21	21.50	" " Tang.
	5.25	4.85	4.61	4.55	4.70	5.04	5.50	Curve Res.
3°–40′	8.00	10.50	13.25	16.40	20.50	26.20	33.00	Res. on Curve
	5.80	8.02	10.60	13.45	16.70	20.47	24.50	" " Tang.
	2.20	2.48	2.65	2.95	3.80	5.73	8.50	Curve Res.
5°–0′	12.25	15.00	18.38	22.30	26.83	31.62	36.70	Res. on Curve
	5.80	8.02	10.60	13.45	16.70	20.47	24.50	" " Tang.
	6.45	6.98	7.78	8.85	10.13	11.15	12.20	Curve Res.
6°–30′	10.60	14.65	18.75	23.00	27.50	32.05	36.55	Res. on Curve
	6.87	8.62	10.55	12.65	15.00	17.80	20.95	" " Tang.
	3.73	6.03	8.20	10.35	12.50	14.25	15.60	Curve Res
8°–0′	10.80	14.05	17.55	21.60	26.25	31.75		Res. on Curve
	5.80	8.02	10.60	13.45	16.70	20.47		" " Tang.
	5.00	6.03	6.95	8.15	9.55	11.28		Curve Res.
14°–30′		19.70	24.60	34.45				Res. on Curve
		8.02	10.60	13.45				" " Tang.
		11.68	14.00	21.00				Curve Res.

value of curve resistance. At 15 miles per hour, for example, there are given in the third column of the table seven values of curve resistance: 1.78 pounds per ton on the 2-degree curve, 4.85 pounds per ton on the 2°–50′ curve, and so on. These values are plotted in the second diagram of Fig. 15 as seven points which show the relations existing during the tests between curve resistance and curvature at the speed of 15 miles per hour. Using the remaining values from Table 3, similar groups of points have been plotted in Fig. 15 which show this relation at the other speeds.

An inspection of the points in Fig. 15 discloses at all speeds a fairly regular relationship between curve resistance and curvature. With perhaps the exception of the first, pertaining to 10 miles per hour, these diagrams show for each speed an increase in curve resistance which is approximately directly proportional to the increase in curvature. There have been drawn among the points in these diagrams straight lines which have been accepted as defining the average of the relations between curve resistance and curvature represented by the points derived from the individual tests. For obvious reasons these lines are all made to pass through the origin of coördinates.

At speeds up to 25 miles per hour the points in Fig. 15 which relate to the 2°–50′ and to the 5-degree curves lie above the mean values represented by the lines, while at higher speeds they correspond closely with the mean. At speeds up to 25 miles per hour the points for the 3°–40′ and for the 8-degree curves correspond well with the mean, but lie below it at higher speeds. The test data offer a probable explanation of these variations only in the case of the 5-degree curve, whose poorer construction might be assumed partially to account for its high resistance. In the other cases, however, no such explanation is available. In drawing the lines in Fig. 15 due consideration was given to the relative weight of the points there plotted; that is, to the number of tests represented by the various points.

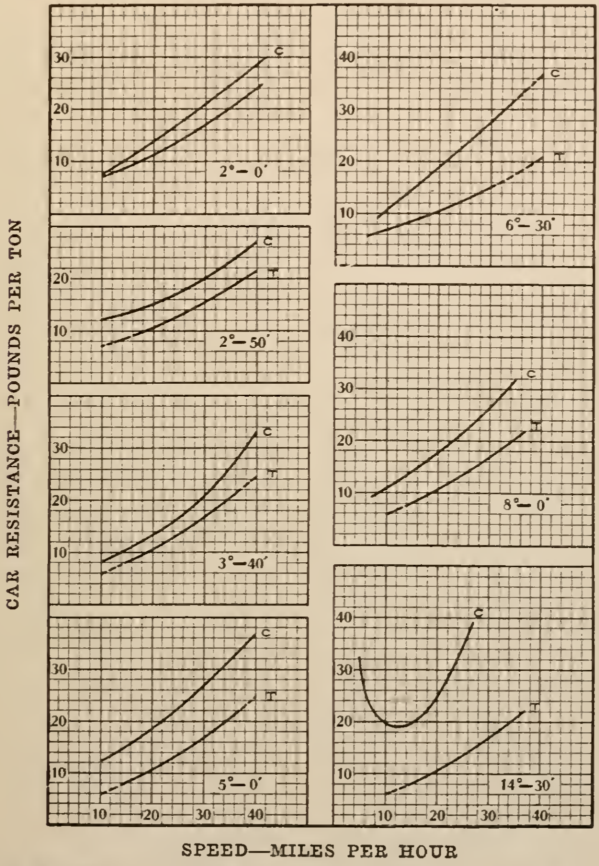

FIG. 14. THE RELATION OF RESISTANCE TO SPEED ON ALL THE CURVES AND ON THEIR CORRESPONDING TANGENTS

The lines drawn in Fig. 15 are brought together and reproduced to a larger scale in Fig. 16, in which, as before, each line defines for a particular speed the average or general relation between curve resistance and curvature. The whole group defines this relation for the entire series of tests. For each speed curve resistance increases in direct proportion with the curvature. At 15 miles per hour, for example, the curve resistance expressed in pounds per ton is 4.35, 8.70, and 13.05, for curves of 5, 10, and 15 degrees, respectively. This implies at this speed a constant curve resistance of 0.87 pounds, when expressed in pounds per ton *per degree*. The direct proportionality

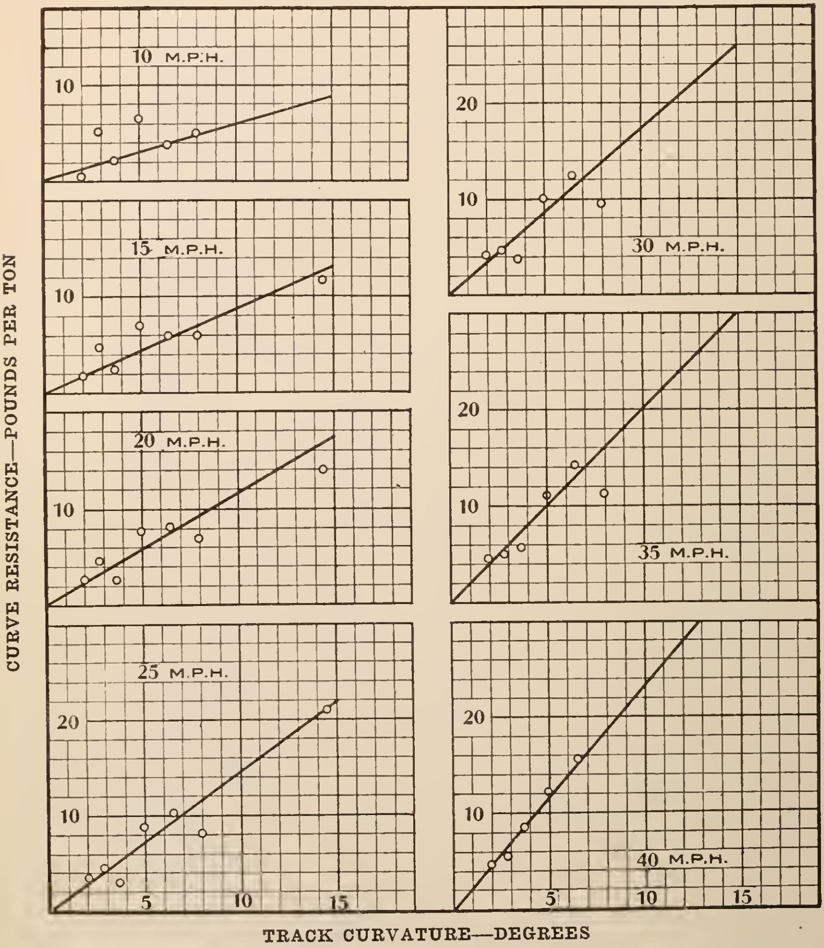

FIG. 15. THE RELATION BETWEEN CURVE RESISTANCE AND CURVATURE AT VARIOUS SPEEDS

between curve resistance and curvature exhibited in Fig. 16 is in accord with the results of previous experiments and with current practice.

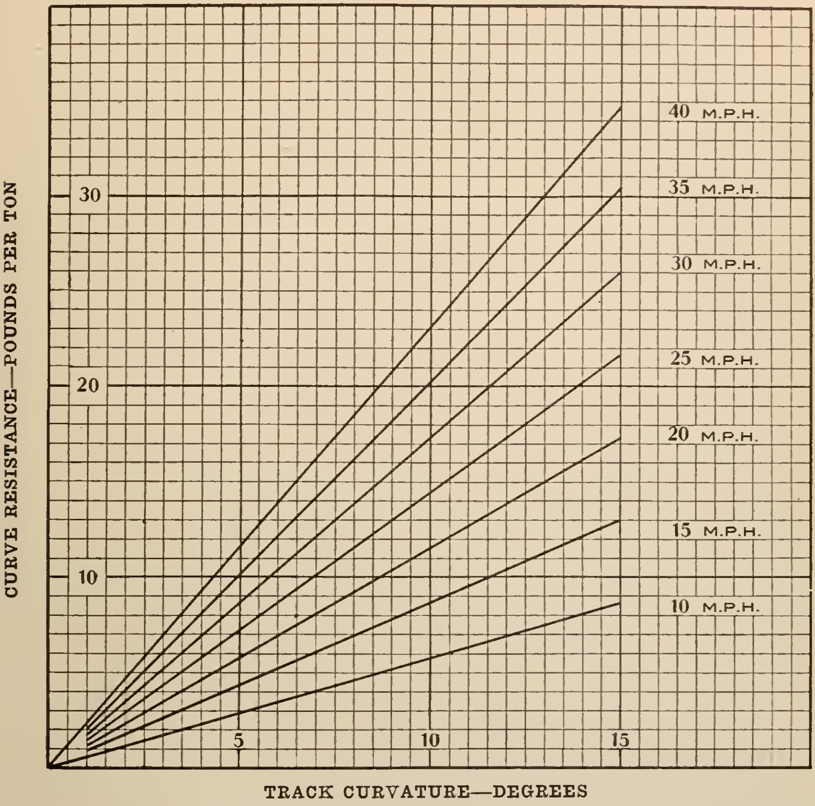

Fig. 16.—The Relation Between Curve Resistance and Curvature, at Various Speeds. The Lines Shown Are Assembled from Fig. 15

11. *The Relation of Curve Resistance to Speed.*—The lines of Fig. 16 present values of curve resistance for seven speeds which cover the speed range of the experiments. The figure offers, therefore, a means of determining the general relation between curve resistance and speed. If in Fig. 16 the ordinates of the seven lines are measured at a curvature of 5 degrees, seven values of curve resistance are obtained: 2.90, 4.35, 5.80, 7.25, 8.70, 10.15, and 11.60 pounds per ton, which correspond respectively to speeds of 10, 15, 20, 25, 30, 35, and 40 miles per hour. These corresponding values of resistance and speed are plotted in Fig. 17 as the seven points there shown for a curvature of 5 degrees. These points are found to lie on a straight line as shown

in the figure. The two other straight lines corresponding to 10 degrees and 15 degrees curvature were obtained by a like process. That the lines connecting the points in Fig. 17 are straight is due, of course, to the relation which exists between the lines in Figs. 15 and 16; that is, to their relative slope. It is proper to explain that the lines originally drawn in Figs. 15 and 16 did not precisely satisfy this condition in the derived lines of Fig. 17. They did so so nearly, however, that, for the sake of the resulting simplicity, it seemed justifiable to modify them, and the slopes of a few of the lines in Fig. 15 were adjusted so that the lines connecting the derived points in Fig. 17 would be straight. An inspection of Fig. 15 shows, it is believed, that this adjustment has done no violence to the experimental data. It has, on the other hand, resulted in a simplicity of the relations shown in Fig. 17 and of the formula given beyond, which seems amply to warrant it.

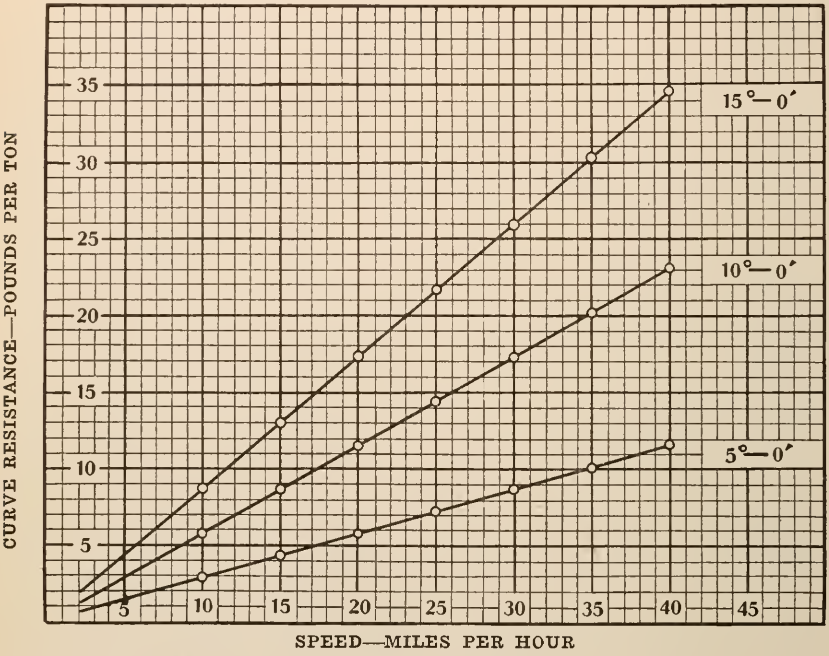

FIG. 17.—THE RELATION BETWEEN CURVE RESISTANCE AND SPEED FOR CURVES OF VARIOUS CURVATURES

The lines drawn in Fig. 17 apply to curvatures of 5, 10, and 15 degrees, and by the process above explained similar lines for other rates of curvature might have been included. Fig. 17 is derived directly from Fig. 16 and, like it, presents the average results of the whole research. From Fig. 17 it is obvious that for constant curvature, curve resistance was directly proportional to speed; that is, on any given curve the resistance due to curvature varied in these experiments directly with the speed. On the 5-degree curve, for example, its average value at 10 miles per hour was 2.90 pounds per ton, and at 30 miles per hour 8.70 pounds per ton, three times as much. In previous experiments there has been occasional evidence that curve resistance might be greater at high than at low speeds, although in some discussions of the subject the contrary is held to be true. Neither opinion has had definite or adequate support, and there has hitherto been no conclusive experimental evidence of any definite relation between curve resistance and speed. In practice the influence of speed has been generally ignored.

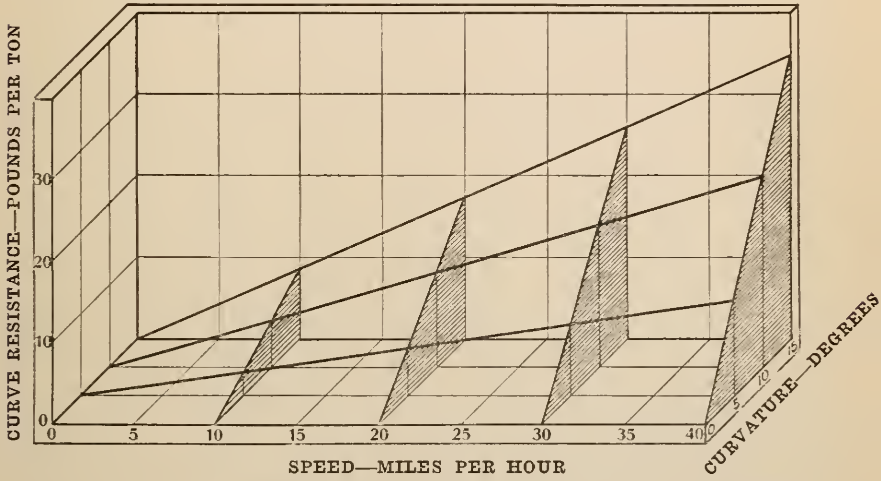

Fig. 18. The Concurrent Relations Between Curve Resistance, Speed, and Curvature

12. *The Concurrent Relations of Curve Resistance, Curvature, and Speed.*—For the car used in the tests curve resistance varies directly with both curvature and speed, the rate of variation in each case being as shown in Figs. 16 and 17. These two figures have been combined in the diagram of Fig. 18, which exhibits the concurrent relations of the three variables. These relations are also defined by the equation:

$$R_c = 0.058\ S\ C \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which R_c is the curve resistance on level track at uniform speed expressed in pounds per ton, S is the speed in miles per hour, and C is the degree of curve. This formula represents exactly the mean relations between curve resistance, speed, and curvature, which are defined by the lines drawn in Figs. 15, 16, and 17, and it embodies, consequently, the generalized results of all the tests.

By means of Formula 1 there have been calculated the values of curve resistance which are presented in Table 4. The values are given for curves varying from 1 to 15 degrees and for speeds ranging from 10 to 40 miles per hour, corresponding approximately to the range in curvature and in speed during the tests. Inspection of Table 4 shows curve resistance to vary from 0.58 pounds per ton on a 1-degree curve at 10 miles per hour to 34.8 pounds per ton on a 15-degree curve at 40 miles per hour. Expressed in pounds per ton *per degree of curve,* curve resistance varies from 0.58 at 10 miles per hour to 2.32 at 40 miles per hour. Although for the sake of exact correspondence between the table and Formula 1 and Figs. 16 and 17 the tabular values are given to the second decimal place, it should not be assumed that the last figures are significant.

TABLE 4

VALUES OF CURVE RESISTANCE AT VARIOUS RATES OF CURVATURE AND AT VARIOUS SPEEDS. THESE VALUES ARE DERIVED FROM FORMULA 1 AND REPRESENT THE FINAL RESULTS OF THE TESTS

Curvature Degrees	Curve Resistance — Pounds Per Ton							Curvature Degrees
	Column Headings Indicate Speed in Miles Per Hour							
	10	15	20	25	30	35	40	
1	0.58	0.87	1.16	1.45	1.74	2.03	2.32	1
2	1.16	1.74	2.32	2.90	3.48	4.06	4.64	2
3	1.74	2.61	3.48	4.35	5.22	6.09	6.96	3
4	2.32	3.48	4.64	5.80	6.96	8.12	9.28	4
5	2.90	4.35	5.80	7.25	8.70	10.15	11.60	5
6	3.48	5.22	6.96	8.70	10.44	12.18	13.92	6
7	4.06	6.09	8.12	10.15	12.18	14.21	16.24	7
8	4.64	6.96	9.28	11.60	13.92	16.24	18.56	8
9	5.22	7.83	10.44	13.05	15.66	18.27	20.88	9
10	5.80	8.70	11.60	14.50	17.40	20.30	23.20	10
11	6.38	9.57	12.76	15.95	19.14	22.33	25.52	11
12	6.96	10.44	13.92	17.40	20.88	24.36	27.84	12
13	7.54	11.31	15.08	18.85	22.62	26.39	30.16	13
14	8.12	12.18	16.24	20.30	24.36	28.42	32.48	14
15	8.70	13.05	17.40	21.75	26.10	30.45	34.80	15

13. *Conclusion.*—The final results of the tests are summarized in Formula 1 and in Table 4, which present mean or average values of curve resistance at various rates of curvature and at various speeds. They apply to the particular car tested under conditions of weather and track which have been fully defined. They are useful for predicting the curve resistance to be expected from similar cars running under like conditions, but if they are so used these conditions should be borne in mind. It is probable that the formula applies to cars

whose weight varies considerably from the weight of the test car; but if the truck wheel base is materially different, the formula should be used with caution. Its use should probably not be extended to speeds much in excess of 40 miles per hour. How far the formula is applicable beyond a curvature of 15 degrees cannot be stated, but it is significant that it represents the actual test results on the 14½-degree curve with considerably greater accuracy than on some of the curves of lower curvature. Whether the results apply closely to other than self-propelled cars is open to question.

Formula 1 and Table 4, as well as Figs. 16 and 17, are generalizations which define only average values of curve resistance. They are derived by processes which preclude their precise agreement even with all the test values themselves, as is disclosed by an examination of Fig. 15. Consequently, in an individual case their use in predicting curve resistance cannot reasonably be expected to give results which correspond with the actual resistance any more exactly than do the averages upon which these generalizations rest correspond with the results of the individual tests; no closer correspondence, that is, should be expected than exists between the mean curves and the points of Figs. 3 to 13 which are fundamental to the whole process. Such limitations are common to all generalizations of train or car resistance and do not seriously impair their usefulness for the purposes for which they are generally employed.

Bulletin 74 of the Enginering Experiment Station gives the results of experiments in which the resistance of this car was determined when running on level tangent track at uniform speed. This resistance is given by the formula:

$$R_t = 4 + 0.222S + 0.00181\frac{A}{W}S^2 \quad \ldots\ldots\ldots\ldots\ldots (2)$$

By combining this equation with Formula 1 above, we obtain for this car the following formula which gives its total resistance when running on a level curve at uniform speed.

$$R = R_t + R_c = 4 + 0.222S + 0.00181\frac{A}{W}S^2 + 0.058\ SC \ldots\ldots (3)$$

In Formula 3, R is the total resistance expressed in pounds per ton, S the speed in miles per hour, C the curvature in degrees, A the cross-sectional area of the car in square feet, and W its weight in tons. If the car is on a grade and its speed changing, the usual corrections for grade and acceleration must be applied to Formula 3.

APPENDIX I

THE TRACK

Information regarding the track has been given in Chapter III. This appendix is intended to supplement Chapter III by the presentation of further details concerning the track on the curves.

The results of the track surveys are presented in Figs. 19 to 25, inclusive, in which are given for each curve the profile on each rail, elevations having been taken every 100 feet; the superelevation of the outer rail, also at 100-foot intervals; and the degree of curvature. For the 2°–0′, 2°–50′, 3°–40′, and 5°–0′ curves the curvature was determined at 50-foot intervals by measuring mid-ordinates, for the 6°–30′ curve it was determined at 50-foot intervals by measuring deflection angles, and for the 8°–0′ and 14°–30′ curves by measuring deflection angles at intervals of 25 feet. Each curve has been designated by the average of these values of curvature. Distances are shown in the figures by means of stations which were 100 feet apart. In the tables given in Appendix III the section limits are defined by trolley line pole numbers. The location of each pole is indicated on the alignment diagrams of Figs. 19 to 25 by means of a short dash beside which is recorded its distance in feet from the preceding station. For example, Pole 20, Fig. 19, is located 80 feet from Station 23.

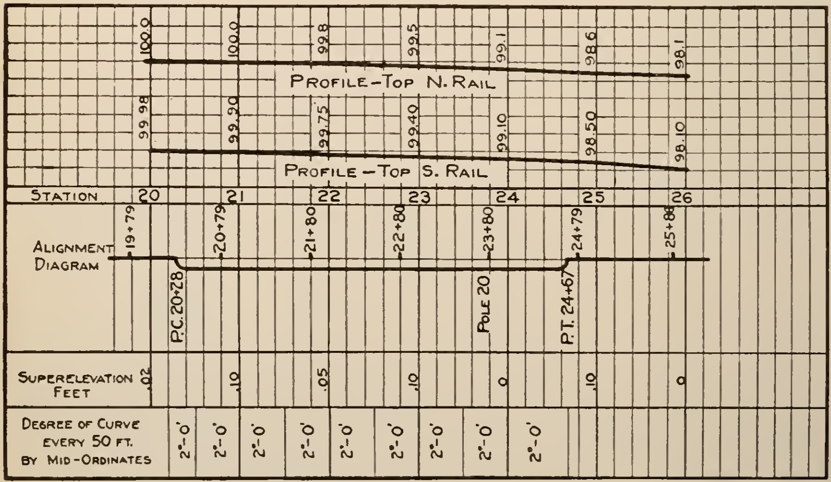

FIG. 19. PROFILE AND ALIGNMENT DIAGRAM OF THE TRACK ON THE 2°–0′ CURVE

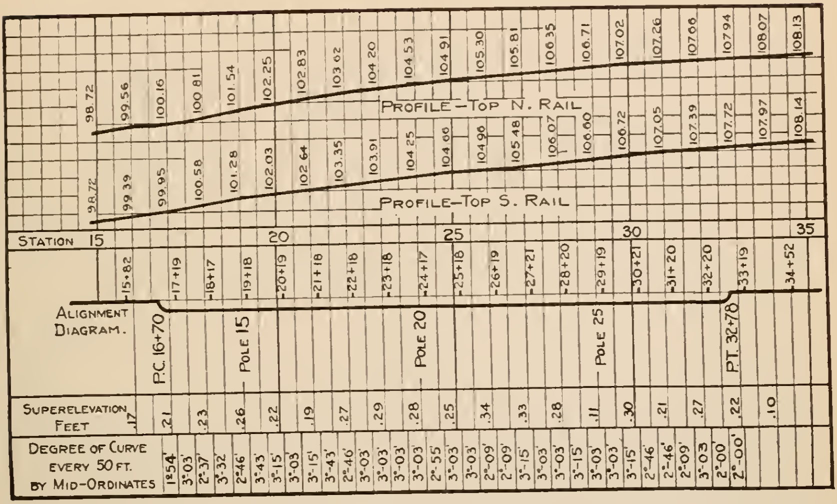

FIG. 20. PROFILE AND ALIGNMENT DIAGRAM OF THE TRACK ON THE 2°–50′ CURVE

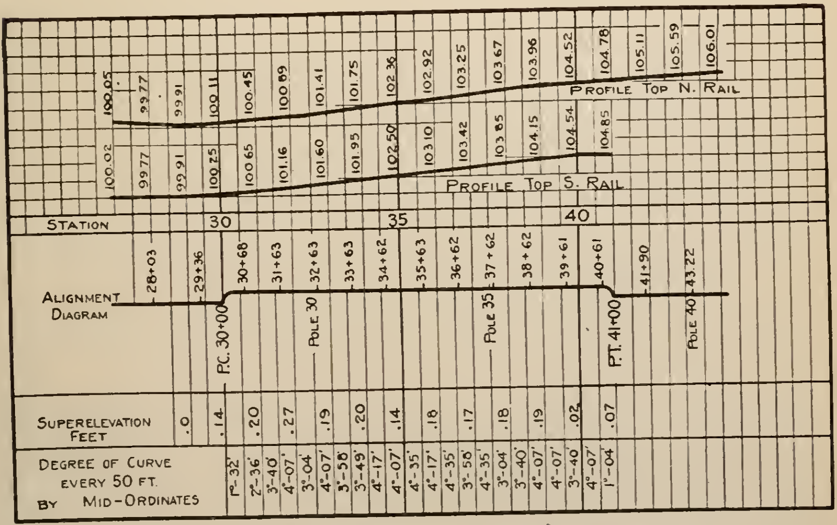

FIG. 21. PROFILE AND ALIGNMENT DIAGRAM OF THE TRACK ON THE 3°–40′ CURVE

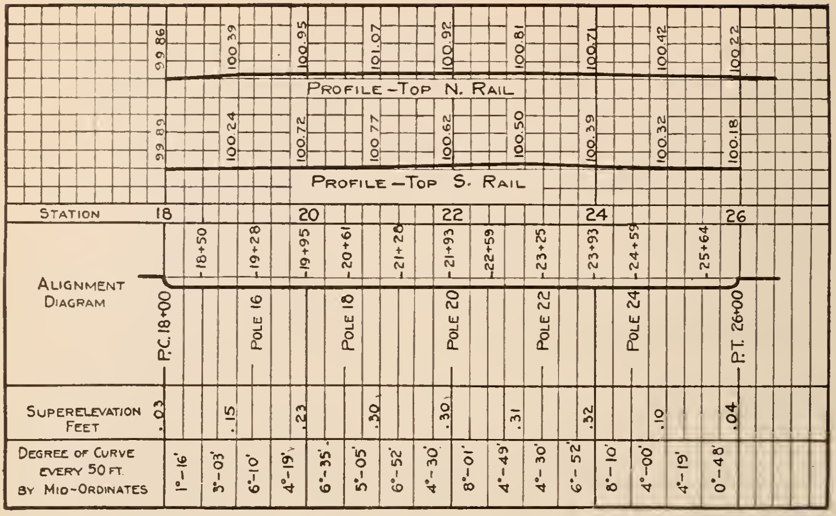

FIG. 22. PROFILE AND ALIGNMENT DIAGRAM OF THE TRACK ON THE 5°–0′ CURVE

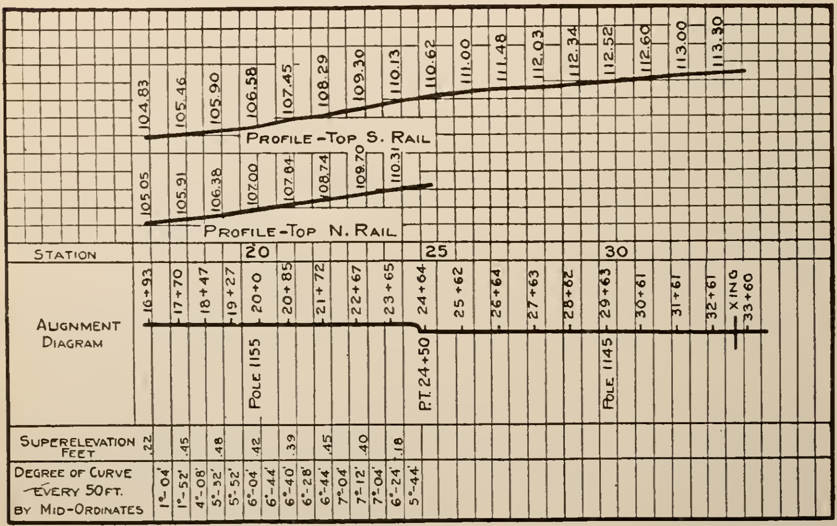

FIG. 23. PROFILE AND ALIGNMENT DIAGRAM OF THE TRACK ON THE 6°–30′ CURVE

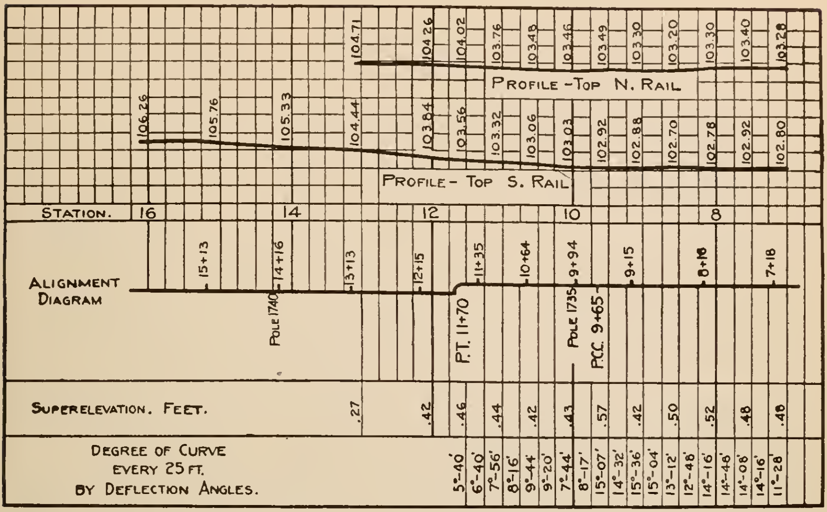

FIG. 24. PROFILE AND ALIGNMENT DIAGRAM OF THE TRACK ON THE 8°–0′ CURVE

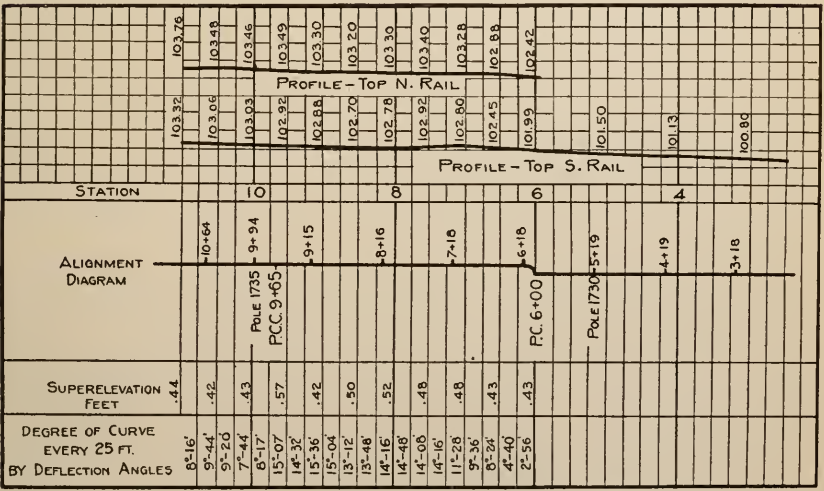

FIG. 25. PROFILE AND ALIGNMENT DIAGRAM OF THE TRACK ON THE 14°–30′ CURVE

APPENDIX II

TEST METHODS AND METHODS OF CALCULATION

This appendix is intended to supplement what has been stated in Chapter IV concerning the test methods and to present also certain details concerning the methods used in calculating the values of resistance and speed which constitute the immediate results of the tests.

Methods Used in Producing the Curves of Figs. 3 to 13.—As has been previously explained, the points in Figs. 3 to 13 occur in pairs, one point in each pair relating to a run in which the wind helped the car, the other to a run in which the wind opposed the car. The curves drawn in the figures define the mean of the values represented by these points. If wind resistance varied directly with speed, an arithmetical mean or average of the values represented by the points would define a point on this curve. Since wind resistance varies, however, about as the square of the speed, a mere arithmetical mean of these values does not serve exactly to define the mean curve; but this mean must be established by another and somewhat more elaborate process. Such a process has been set forth on page 34 of Bulletin 74 of the University of Illinois Engineering Experiment Station. When the wind velocities are low, the difference between the curves defined by taking an arithmetical mean and by this more exact process is negligible. During these tests the wind velocities were generally low, and the points representing runs with and against the wind are generally closely interwoven. For this reason the mean curves in Figs. 3 to 13 were all produced by taking the arithmetical mean of the resistance values represented by the points in the figures. Preliminary to drawing these curves, the average coördinates of various groups of points in each figure were determined, and these average coördinates were then plotted as auxiliary points through which the curve was passed as nearly as possible.

Methods of Calculation.—The data recorded during these tests are enumerated in Chapter III. All the data which pertained directly to car operation were graphically recorded upon the test car charts. A portion of the chart obtained during Test 121 is reproduced as Fig. 26, upon which the section limits and certain explanatory lettering have been added.

When the track sections were surveyed, markers were placed beside the track at the ends of the curves and tangents, and during a test the positions of these numbered markers were recorded upon the chart. In this way the chart made during a run over the test track was divided into two or more sections as illustrated by Fig. 26. That

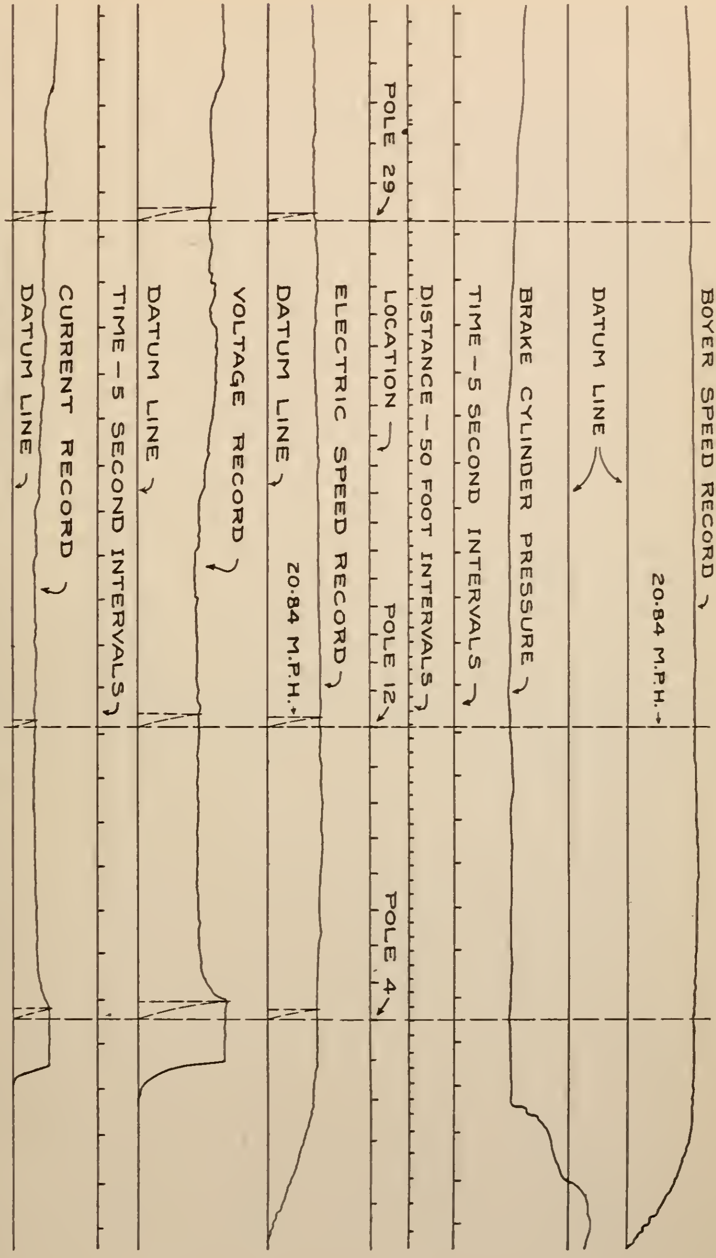

Fig. 26. Reproduced from a Portion of One of the Test Car Charts

part of the chart lying between the poles numbered 12 and 29 was produced during a run over a section of curved track, while that portion lying between the poles numbered 4 and 12 was made during a run over a section of tangent track. In making the calculations only those chart sections were considered over which there were no brake applications, no large variation of the current or voltage from the average value, and no large difference in the speed of the car. It has been found desirable to choose both track and chart sections so that the energy consumed by grade and acceleration will be as small as possible, and the errors in their calculation will have slight effect on the accuracy of the values of average net car resistance. When the variation in speed is large the energy required to produce acceleration is alone frequently greater than that required to overcome all other resistances combined. In all the tests reported in this bulletin the variations in speed in passing the track sections have exceeded 2 miles per hour in only 11 per cent of the total number of resistance determinations, and in only 3 cases out of 392 has this speed variation exceded 5 miles per hour. The maximum variation over any section was less than 10 miles per hour.

The result desired from each chart section is a value of average net car resistance. These values are given in Column 18 of Tables 5 to 11 in Appendix III. Full explanations of the significance of the various items in these tables and of the methods by which they were derived are presented in Appendix II of Bulletin 74 of the Engineering Experiment Station.

APPENDIX III

THE IMMEDIATE RESULTS OF THE TESTS

This appendix presents in tabular form the fundamental data, some of the intermediate results, and the final values of car resistance and speed for each of the curves. The corresponding data for tangents D, W, S, and R, are given in Tables 9, 10, 11, and 12, respectively, of Bulletin 74 of the Engineering Experiment Station. The items in these tables have all been derived by the processes explained in Appendix II. The points in Fig. 3 to 13 were plotted by using as coördinates the values of resistance and speed which appear in Columns 18 and 19 of these tables.

TABLE 5
THE IMMEDIATE RESULTS OF THE TESTS ON THE 2°–0′ CURVE, GIVING THE RESULTS OF RESISTANCE AND SPEED USED IN PRODUCING FIG. 3

1	2	3	4	5	6	7	8	9
							Motor Data	
Item No.	Test No.	Wind Opposing or Helping	Section Limits Trolley Line Pole Numbers	Grade Rise or Fall Over Section +Up —Down	Length of Track Section	Time to Run Over Section	Number in Use and Connection *	Efficiency of Motors and Gears
		O or H		Feet	Feet	Sec.		Per cent
1	124	H	16–21	—1.47	500	9.8	4M	73.1
2	123	O	21–16	+ "	"	14.8	4S	73.0
3	123	O	21–16	+ "	"	27.6	"	70.0
4	124	H	16–21	— "	"	10.5	4M	72.2
5	123	O	21–16	+ "	"	9.5	"	81.4
6	124	H	16–21	— "	"	11.3	"	77.6
7	124	H	16–21	— "	"	10.6	"	69.8
8	123	O	21–16	+ "	"	14.2	4S	71.5
9	124	H	16–21	— "	"	14.6	"	67.1
10	117	H	21–16	+ "	"	12.7	"	77.9
11	124	H	16–21	— "	"	27.6	"	64.7
12	123	O	21–16	+ "	"	28.6	"	66.5
13	124	H	16–21	— "	"	16.1	"	72.7
14	123	O	21–16	+ "	"	23.3	"	68.4
15	124	H	16–21	— "	"	27.6	"	63.8
16	118	O	16–21	— "	"	10.5	4M	83.2
17	118	O	16–21	— "	"	19.1	4S	76.7
18	117	H	21–16	+ "	"	9.9	4M	85.2
19	118	O	16–21	— "	"	9.9	"	85.7
20	117	H	21–16	+ "	"	8.7	"	85.0
21	118	O	16–21	— "	"	9.2	"	85.6
22	117	H	21–16	+ "	"	8.2	"	84.3
23	118	O	16–21	— "	"	8.7	"	84.4
24	117	H	21–16	+ "	"	8.6	"	84.5

*S = Series-Multiple. M = Multiple.

TABLE 5 (Continued)

THE IMMEDIATE RESULTS OF THE TESTS ON THE 2°–0′ CURVE, GIVING THE RESULTS OF RESISTANCE AND SPEED USED IN PRODUCING FIG. 3

10	11	12	13	14	15	16	17	18	19
	Speed				Energy Imparted to the Car				
Item No.	At Entrance to the Section	At Exit from the Section	Average Voltage	Average Current	By the Current	By the Change in Kinetic Energy	By the Grade	Net Car Resistance	Average Speed Over the Section
	M.P.H.	M.P.H.	Volts	Amp.	Ft. Lb.	Ft. Lb.	Ft. Lb.	Lb. per Ton	M.P.H.
1	33.84	33.84	358	103.0	194820	0	+83420	19.61	34.79
2	23.40	22.50	270	57.6	247840	+84690	— "	17.56	23.03
3	10.80	11.34	142	55.7	225410	—24510	— "	8.28	12.35
4	32.94	32.76	354	100.0	197920	+24240	+ "	21.54	32.47
5	33.66	33.66	445	134.8	342130	0	— "	18.23	35.88
6	29.70	30.96	378	119.6	292370	—156690	+ "	15.44	30.17
7	32.94	32.40	342	94.3	175980	+72330	+ "	23.38	32.16
8	23.58	22.86	268	53.9	216320	+68550	— "	14.20	24.01
9	23.04	23.04	229	46.8	154830	0	+ "	16.79	23.35
10	27.00	26.10	592	53.0	228960	+97010	—82610	17.32	26.84
11	11.16	12.78	121	46.6	148550	—79510	+83420	10.75	12.35
12	11.34	11.34	126	49.4	174630	0	— "	6.43	11.92
13	19.98	20.70	245	57.6	243600	—60040	+ "	18.82	21.17
14	14.40	14.22	157	51.1	188650	+10560	— "	8.16	14.64
15	10.80	12.42	122	45.4	143890	—77110	+ "	10.59	12.35
16	32.04	33.48	470	148.4	449390	—191530	+82610	24.23	32.47
17	18.18	19.08	405	55.1	241150	—68070	+ "	18.20	17.85
18	35.82	36.18	541	168.4	566700	—52620	— "	30.71	34.43
19	34.02	36.00	552	182.8	631350	—281440	+ "	30.78	34.43
20	40.14	40.68	592	157.4	508190	—88590	— "	23.99	39.18
21	36.36	38.34	573	168.5	560760	—300250	+ "	24.42	37.05
22	42.48	42.12	586	143.6	429000	+61830	— "	29.05	41.57
23	39.60	40.32	562	150.8	458950	—116810	+ "	30.23	39.18
24	39.96	40.14	565	151.4	458440	—29270	— "	24.67	39.64

TABLE 6

THE IMMEDIATE RESULTS OF THE TESTS ON THE 2°–50′ CURVE, GIVING THE RESULTS OF RESISTANCE AND SPEED USED IN PRODUCING FIG. 5

1	2	3	4	5	6	7	8	9
Item No.	Test No.	Wind Opposing or Helping	Section Limits Trolley Line Pole Numbers	Grade Rise or Fall Over Section +Up —Down	Length of Track Section	Time to Run Over Section	Motor Data	
							Number in Use and Connection *	Efficiency of Motors and Gears
		O or H		Feet	Feet	Sec.		Per cent
1	119	H	29–12	—8.52	1737	39.2	4M	77.4
2	120	O	12–29	+ ”	”	38.1	4M	83.7
3	120	O	12–29	+ ”	”	41.0	4M	78.4
4	120	O	12–29	+ ”	”	46.0	4M	78.9
5	119	H	29–12	— ”	”	35.4	4M	67.8
6	120	O	12–29	+ ”	”	42.6	4M	82.5
7	119	H	29–12	— ”	”	38.3	2M	77.6
8	120	O	12–29	+ ”	”	49.4	2M	85.3
9	119	H	29–12	— ”	”	49.2	2M	78.2
10	120	O	12–29	+ ”	”	54.5	2M	83.1
11	119	H	29–12	— ”	”	48.8	4 S	67.5
12	120	O	12–29	+ ”	”	61.9	4 S	82.1
13	119	H	29–12	— ”	”	50.1	4 S	69.7
14	120	O	12–29	+ ”	”	57.7	4 S	82.0
15	119	H	29–12	— ”	”	47.7	4 S	72.4
16	120	O	12–29	+ ”	”	41.3	4M	80.3
17	120	O	12–29	+ ”	”	46.0	2M	85.7
18	120	O	12–29	+ ”	”	45.9	2M	85.5
19	119	H	29–12	— ”	”	46.0	4 S	71.2
20	120	O	12–29	+ ”	”	59.2	4 S	80.7
21	121	O	29–12	— ”	”	51.3	4 S	62.8
22	122	H	12–29	+ ”	”	57.2	4 S	77.9
23	121	O	29–12	— ”	”	52.5	4 S	61.6
24	122	H	12–29	+ ”	”	64.3	4 S	76.8
25	121	O	29–12	— ”	”	35.3	4M	65.1
26	121	O	29–12	— ”	”	36.6	4M	72.0
27	122	H	12–29	+ ”	”	61.2	4Sw	76.0
28	121	O	29–12	— ”	”	59.0	4Sw	55.6
29	121	O	29–12	— ”	”	71.3	2M	73.4
30	121	O	29–12	— ”	”	56.0	4 S	64.9
31	122	H	12–29	+ ”	”	77.2	4 S	75.1
32	121	O	29–12	— ”	”	56.9	4 S	64.1
33	121	O	29–12	— ”	”	32.7	4M	77.7
34	122	H	12–29	+ ”	”	34.4	4M	83.8
35	121	O	29–12	— ”	”	30.1	4M	73.8
36	122	H	12–29	+ ”	”	78.2	4 S	75.4
37	121	O	29–12	— ”	”	83.9	C	—
38	121	O	29–12	— ”	”	106.5	C	—

*S = Series-Multiple. M = Multiple. Sw = Switching. C = Coasting.

TABLE 6 (Continued)

THE IMMEDIATE RESULTS OF THE TESTS ON THE 2°–50′ CURVE, GIVING THE RESULTS OF RESISTANCE AND SPEED USED IN PRODUCING FIG. 5

10	11	12	13	14	15	16	17	18	19
Item No.	Speed		Average Voltage	Average Current	Energy Imparted to the Car			Net Car Resistance	Average Speed Over the Section
	At Entrance to the Section	At Exit from the Section			By the Current	By the Change in Kinetic Energy	By the Grade		
	M.P.H.	M.P.H.	Volts	Amp.	Ft. Lb.	Ft. Lb.	Ft. Lb.	Lb. per Ton	M.P.H.
1	26.96	32.18	358	123.2	987020	—632860	+483510	17.00	30.21
2	30.38	31.46	436	159.9	1639720	—136910	— ”	20.68	31.08
3	31.10	29.12	365	127.8	1105890	+244430	— ”	17.59	28.89
4	26.42	26.06	340	139.2	1266870	+ 38730	— ”	16.68	25.75
5	31.82	33.08	330	90.1	526250	—167640	+ ”	17.09	33.46
6	26.78	27.86	397	152.3	1567090	—120970	— ”	19.53	27.80
7	31.10	31.10	380	59.5	495590	0	+ ”	19.87	30.92
8	23.90	24.62	417	119.2	1544830	— 71620	— ”	20.08	23.97
9	22.46	25.88	325	69.3	639000	—338910	+ ”	15.90	24.07
10	22.46	21.02	355	101.5	1203440	+128350	— ”	17.21	21.73
11	23.54	25.52	425	41.7	430480	—199130	+ ”	14.50	24.27
12	19.58	19.76	488	67.8	1240200	— 14520	— ”	15.06	19.13
13	22.10	24.98	438	43.5	490600	—277960	+ ”	14.13	23.64
14	21.92	21.38	516	65.9	1186400	+ 47930	— ”	15.24	20.53
15	23.72	26.06	472	46.1	554210	—238790	+ ”	16.22	24.83
16	30.02	29.12	378	138.9	1284060	+109110	— ”	18.46	28.68
17	24.98	26.24	458	121.1	1612430	—132300	— ”	20.22	25.75
18	25.16	25.52	443	114.3	1465370	— 37400	— ”	19.16	25.80
19	23.54	26.60	482	44.5	518120	—314530	+ ”	13.94	25.75
20	21.38	19.76	488	62.7	1078150	+136630	— ”	14.84	20.01
21	22.10	22.10	196	42.4	394880	0	+ ”	17.82	23.09
22	20.84	20.48	278	75.3	1375940	+ 30490	— ”	18.72	20.70
23	21.56	21.74	189	41.4	373260	— 15980	+ ”	17.06	22.56
24	18.86	18.50	239	74.3	1293620	+ 27570	— ”	17.00	18.42
25	33.26	33.08	288	86.5	422170	+ 24480	+ ”	18.87	33.55
26	31.64	32.54	327	104.3	662910	+118410	+ ”	20.86	32.36
27	18.50	17.78	223	71.8	1098370	+ 53550	— ”	13.56	19.35
28	19.76	19.94	160	37.2	287900	+ 14650	+ ”	15.36	20.07
29	14.90	17.06	203	61.3	480140	—141520	+ ”	16.68	16.61
30	19.58	20.84	200	44.6	478180	—104400	+ ”	17.39	21.15
31	15.08	15.62	200	69.5	1188680	— 33980	— ”	13.62	15.34
32	19.58	20.48	194	43.8	457010	— 73910	+ ”	17.58	20.81
33	35.24	36.50	420	113.8	895690	—185300	+ ”	24.22	36.22
34	34.34	32.72	479	155.7	1585580	+222710	— ”	26.88	34.43
35	38.30	38.66	407	98.9	659400	— 56800	+ ”	22.04	39.35
36	12.74	14.18	203	71.1	1255410	— 79470	— ”	14.05	15.14
37	16.34	13.46	..	..	0	+175940	+ ”	13.38	14.12
38	13.10	11.30	..	..	0	+ 90040	+ ”	11.64	11.12

TABLE 7

THE IMMEDIATE RESULTS OF THE TESTS ON THE 3°–40′ CURVE, GIVING THE RESULTS OF RESISTANCE AND SPEED USED IN PRODUCING FIG. 7

1	2	3	4	5	6	7	8	9
							Motor Data	
Item No.	Test No.	Wind Opposing or Helping	Section Limits Trolley Line Pole Numbers	Grade Rise or Fall Over Section +Up —Down	Length of Track Section	Time to Run Over Section	Number in Use and Connection *	Efficiency of Motors and Gears
		O or H		Feet	Feet	Sec.		Per cent
1	126	H	27–38	+4.81	1125	25.3	4M	78.0
2	125	O	38–27	— ”	”	24.1	”	66.6
3	125	O	38–27	— ”	”	25.2	”	71.0
4	126	H	27–38	+ ”	”	28.2	”	79.8
5	126	H	27–38	+ ”	”	24.7	”	83.5
6	126	H	27–38	+ ”	”	23.8	”	84.2
7	126	H	27–38	+ ”	”	38.0	4S	75.5
8	125	O	38–27	— ”	”	29.7	”	71.5
9	126	H	27–38	+ ”	”	34.6	”	75.7
10	128	..	27–38	+ ”	”	36.2	4M	78.1
11	127	..	38–27	— ”	”	75.6	”	23.5
12	128	..	27–38	+ ”	”	48.7	4S	71.7
13	129	H	38–27	— ”	”	29.9	”	48.7
14	130	O	27–38	+ ”	”	38.1	”	75.3
15	129	H	38–27	— ”	”	40.8	”	35.0
16	130	O	27–38	+ ”	”	43.1	”	74.2
17	129	H	38–27	— ”	”	20.3	4M	63.3
18	130	O	27–38	+ ”	”	23.9	”	82.3
19	129	H	38–27	— ”	”	21.1	”	26.7
20	130	O	27–38	+ ”	”	24.7	”	78.1
21	130	O	27–38	+ ”	”	25.4	”	78.2
22	129	H	38–27	— ”	”	25.7	”	30.5
23	130	O	27–38	+ ”	”	23.4	”	83.7
24	129	H	38–27	— ”	”	26.9	”	64.2
25	130	O	27–38	+ ”	”	23.7	”	80.6
26	129	H	38–27	— ”	”	55.6	4S	53.3

*S=Series-Multiple. M=Multiple.

TABLE 7 (Continued)

THE IMMEDIATE RESULTS OF THE TESTS ON THE 3°–40′ CURVE, GIVING THE RESULTS OF RESISTANCE AND SPEED USED IN PRODUCING FIG. 7

10	11	12	13	14	15	16	17	18	19
Item No.	Speed		Average Voltage	Average Current	Energy Imparted to the Car			Net Car Resistance	Average Speed Over the Section
	At Entrance to the Section	At Exit from the Section			By the Current	By the Change in Kinetic Energy	By the Grade		
	M.P.H.	M.P.H.	Volts	Amp.	Ft. Lb.	Ft. Lb.	Ft. Lb.	Lb. per Ton	M.P.H.
1	32.56	29.68	344	131.1	656380	+371050	—275850	23.30	30.32
2	33.64	32.20	275	90.7	295220	+196260	+ "	23.79	31.83
3	30.58	30.40	294	104.8	406550	+22720	+ "	21.86	30.44
4	28.42	26.62	340	147.0	829490	+205080	— "	23.52	27.20
5	31.84	32.02	407	162.0	1002830	—23790	— "	21.80	31.05
6	32.56	32.56	449	166.9	1107550	0	— "	25.78	32.23
7	21.04	20.32	239	67.5	682800	+61640	— "	14.52	20.19
8	25.18	26.44	262	54.0	443210	—134640	+ "	18.12	25.83
9	23.02	20.86	264	66.2	675120	+196200	— "	18.46	22.17
10	20.28	20.46	280	152.4	889700	—15220	—276580	18.49	21.19
11	9.66	8.94	91.8	40.4	48610	+27790	+ "	10.91	10.15
12	17.94	16.68	173	58.4	520450	+90510	— "	10.34	15.75
13	26.28	25.02	155	33.1	110180	+133800	+275850	16.12	25.65
14	20.52	20.52	236	66.9	668180	0	— "	12.16	20.13
15	18.54	18.18	93.6	26.5	52240	+27360	+ "	11.02	18.80
16	18.90	18.18	207	64.4	628790	+55260	— "	12.65	17.80
17	39.24	37.62	304	81.8	235690	+257740	+ "	23.85	37.79
18	32.04	30.78	395	151.0	865310	+163850	— "	23.35	32.09
19	38.52	36.00	151	43.3	27170	+388730	+ "	21.44	36.35
20	33.30	31.50	344	132.0	646040	+241440	— "	18.96	31.05
21	30.24	28.98	270	156.3	618180	+154460	— "	15.40	30.20
22	30.24	28.44	140	47.1	38100	+218640	+ "	16.51	29.85
23	32.58	32.58	436	160.4	1010100	0	— "	22.76	32.78
24	28.80	29.16	222	86.8	245430	—43190	+ "	14.82	28.51
25	34.74	32.40	383	139.9	754830	+325210	— "	24.93	32.36
26	11.16	14.76	103	36.5	164350	—193160	+ "	7.66	13.80

Table 8

The Immediate Results of the Tests on the 5°–0′ Curve, Giving the Results of Resistance and Speed Used in Producing Fig. 9

1	2	3	4	5	6	7	8	9
				Grade			Motor Data	
Item No.	Test No.	Wind Opposing or Helping	Section Limits Trolley Line Pole Numbers	Rise or Fall Over Section +Up —Down	Length of Track Section	Time to Run Over Section	Number in Use and Connection *	Efficiency of Motors and Gears
		O or H		Feet	Feet	Sec.		Per cent
1	125	O	24–14	—0.15	714	14.0	4M	82.9
2	126	H	14–24	+ ’’	’’	15.2	4M	80.5
3	126	H	14–24	+ ’’	’’	17.4	4M	83.1
4	125	O	24–14	— ’’	’’	15.6	4M	77.3
5	126	H	14–24	+ ’’	’’	15.6	4M	85.3
6	126	H	14–24	+ ’’	’’	14.7	4M	83.4
7	126	H	14–24	+ ’’	’’	13.6	4M	85.8
8	126	H	14–24	+ ’’	’’	24.8	4S	76.5
9	125	O	24–14	— ’’	’’	19.0	4S	70.3
10	126	H	14–24	+ ’’	’’	21.4	4S	76.3
11	127	..	24–14	— ’’	’’	25.0	4M	74.9
12	128	..	14–24	+ ’’	’’	23.9	4M	75.8
13	127	..	24–14	— ’’	’’	47.3	4S	70.8
14	128	..	14–24	+ ’’	’’	30.4	4S	73.4
15	129	H	24–14	— ’’	’’	19.9	4S	68.7
16	130	O	14–24	+ ’’	’’	23.7	4S	74.6
17	129	H	24–14	— ’’	’’	27.9	4S	69.9
18	130	O	14–24	+ ’’	’’	31.0	4S	75.0
19	130	O	14–24	+ ’’	’’	15.4	4M	85.5
20	130	O	14–24	+ ’’	’’	14.9	4M	79.0
21	129	H	24–14	— ’’	’’	12.4	4M	83.4
22	130	O	14–24	+ ’’	’’	16.0	4M	85.1
23	130	O	14–24	+ ’’	’’	14.6	4M	82.8

*S=Series-Multiple. M=Multiple.

TABLE 8 (Continued)

THE IMMEDIATE RESULTS OF THE TESTS ON THE 5°–0′ CURVE, GIVING THE RESULTS OF RESISTANCE AND SPEED USED IN PRODUCING FIG. 9

10	11	12	13	14	15	16	17	18	19
	Speed				Energy Imparted to the Car				
Item No.	At Entrance to the Section	At Exit from the Section	Average Voltage	Average Current	By the Current	By the Change in Kinetic Energy	By the Grade	Net Car Resistance	Average Speed Over the Section
	M.P.H.	M.P.H.	Volts	Amp.	Ft. Lb.	Ft. Lb.	Ft. Lb.	Lb. per Ton	M.P.H.
1	34.72	35.26	462	146.7	580160	—78220	+8600	24.93	34.77
2	33.10	32.02	383	138.6	479000	+145580	— ’’	30.09	32.03
3	28.42	28.24	387	164.6	679280	+21110	— ’’	33.79	27.98
4	32.74	31.84	354	123.2	387840	+120310	+ ’’	25.24	31.21
5	30.22	32.20	460	196.4	886580	—255830	— ’’	30.39	31.21
6	32.74	32.56	436	155.8	614200	+24330	— ’’	30.77	33.12
7	33.82	35.08	524	195.9	883380	—179700	— ’’	33.95	35.80
8	19.60	20.32	243	71.9	488880	—59500	— ’’	20.55	19.63
9	26.26	24.82	245	51.8	250010	+152260	+ ’’	20.07	25.62
10	22.30	22.30	268	68.1	439530	0	— ’’	21.05	22.75
11	18.84	19.56	221	131.9	402550	—57370	+8630	17.23	19.47
12	20.10	19.56	245	136.5	446780	+44440	— ’’	23.51	20.37
13	10.74	10.92	121	59.8	357380	—8090	+ ’’	17.44	10.29
14	16.32	16.68	189	62.7	390010	—24650	— ’’	17.38	16.01
15	24.30	23.76	220	49.3	218690	+53720	+ ’’	13.73	24.46
16	19.98	19.80	230	64.3	385700	+14820	—8600	19.14	20.54
17	17.64	18.54	166	54.0	257880	—67400	+ ’’	9.72	17.45
18	16.02	16.74	194	69.9	465020	—48830	— ’’	19.91	15.70
19	30.06	32.58	480	199.6	930380	—326760	— ’’	29.06	31.61
20	33.48	31.86	380	128.4	423550	+219110	— ’’	30.97	32.67
21	39.06	38.88	482	150.6	553640	+29040	+ ’’	28.88	39.26
22	27.54	30.24	441	194.1	859580	—322930	— ’’	25.79	30.43
23	34.02	33.66	431	150.0	576390	+50440	— ’’	30.20	33.34

TABLE 9

THE IMMEDIATE RESULTS OF THE TESTS ON THE 6°–30′ CURVE, GIVING THE RESULTS OF RESISTANCE AND SPEED USED IN PRODUCING FIG. 10

1	2	3	4	5	6	7	8	9
Item No.	Test No.	Wind Opposing or Helping	Section Limits — Trolley Line Pole Numbers	Grade — Rise or Fall Over Section — +Up —Down	Length of Track Section	Time to Run Over Section	Motor Data	
							Number in Use and Connection *	Efficiency of Motors and Gears
		O or H		Feet	Feet	Sec.		Per cent
1	133	H	1155–1151	+3.22	365	18.3	4S	73.1
2	134	O	1151–1155	— "	"	13.4	"	38.0
3	133	H	1155–1151	+ "	"	25.1	"	74.5
4	134	O	1151–1155	— "	"	17.7	"	49.8
5	133	H	1155–1151	+ "	"	16.5	"	74.5
6	134	O	1151–1155	— "	"	12.7	"	45.1
7	133	H	1155–1151	+ "	"	17.0	"	75.9
8	134	O	1151–1155	— "	"	12.1	"	49.3
9	133	H	1155–1151	+ "	"	20.2	"	75.6
10	133	H	1155–1151	+ "	"	15.0	"	78.3
11	134	O	1151–1155	— "	"	9.1	4M	57.0
12	133	H	1155–1151	+ "	"	9.9	"	83.5
13	134	O	1151–1155	— "	"	7.9	"	54.1
14	134	O	1151–1155	— "	"	7.9	"	76.3
15	134	O	1151–1155	— "	"	13.0	4S	53.7
16	133	H	1155–1151	+ "	"	20.9	"	75.6
17	134	O	1151–1155	— "	"	14.4	"	62.2
18	133	H	1155–1151	+ "	"	12.9	"	78.4
19	134	O	1151–1155	— "	"	11.4	"	50.9
20	133	H	1155–1151	+ "	"	25.7	"	75.2
21	133	H	1155–1151	+ "	"	13.6	"	77.4

*S=Series-Multiple. M=Multiple.

TABLE 9 (Continued)

THE IMMEDIATE RESULTS OF THE TESTS ON THE 6°–30′ CURVE, GIVING THE RESULTS OF RESISTANCE AND SPEED USED IN PRODUCING FIG. 10

10	11	12	13	14	15	16	17	18	19
	Speed				Energy Imparted to the Car				
Item No.	At Entrance to the Section	At Exit from the Section	Average Voltage	Average Current	By the Current	By the Change in Kinetic Energy	By the Grade	Net Car Resistance	Average Speed Over the Section
	M.P.H.	M.P.H.	Volts	Amp.	Ft. Lb.	Ft. Lb.	Ft. Lb.	Lb. per Ton	M.P.H.
1	14.59	12.38	173	63.2	215730	+123080	—184510	14.75	13.60
2	18.84	18.84	106	13.2	10510	0	+ ”	18.65	18.57
3	9.49	10.00	146	78.4	315710	—20530	— ”	10.58	9.91
4	13.23	14.08	97.2	34.6	43730	—47940	+ ”	17.24	14.06
5	15.61	14.25	202	66.3	242830	+83860	— ”	13.60	15.08
6	18.84	19.52	149	31.4	39530	—53870	+ ”	16.27	19.60
7	14.93	14.25	202	74.4	286030	+40980	— ”	13.63	14.64
8	19.69	20.03	140	33.7	41510	—27890	+ ”	18.95	20.57
9	12.04	11.53	182	77.5	317720	+24820	— ”	15.11	12.32
10	16.29	16.46	248	84.5	363020	—11500	— ”	15.97	16.59
11	27.17	27.17	225	73.8	63500	0	+ ”	23.71	27.35
12	24.45	24.11	380	180.0	417000	+34090	— ”	25.49	25.14
13	31.76	31.42	230	69.8	50600	+44360	+ ”	26.72	31.50
14	31.59	31.93	365	115.8	187890	—44600	+ ”	31.35	31.50
15	18.84	19.18	158	35.9	58400	—26690	+ ”	20.68	19.14
16	11.19	11.19	184	77.2	331060	0	— ”	14.01	11.91
17	16.12	17.48	169	42.4	94670	—94360	+ ”	17.68	17.28
18	19.69	19.18	282	77.9	327710	+40940	— ”	17.61	19.29
19	20.71	21.05	169	34.1	49320	—29320	+ ”	19.56	21.83
20	9.49	11.53	157	83.4	373260	—88550	— ”	9.58	9.68
21	18.67	17.82	256	75.6	300490	+64050	— ”	17.21	18.30

TABLE 10

THE IMMEDIATE RESULTS OF THE TESTS ON THE 8°–0′ CURVE, GIVING THE RESULTS OF RESISTANCE AND SPEED USED IN PRODUCING FIG. 12

1	2	3	4	5	6	7	8	9
							Motor Data	
Item No.	Test No.	Wind Opposing or Helping	Section Limits — Trolley Line Pole Numbers	Grade — Rise or Fall Over Section — +Up —Down	Length of Track Section	Time to Run Over Section	Number in Use and Connection *	Efficiency of Motors and Gears
		O or H		Feet	Feet	Sec.		Per cent
1	142	O	1735 –1737.5	+0.65	176	6.7	4S	80.3
2	142	O	1735 –1737.5	+ "	"	6.4	"	66.8
3	141	H	1737.5–1735	— "	"	6.2	"	41.2
4	142	O	1735 –1737.5	+ "	"	13.9	"	73.3
5	142	O	1735 –1737.5	+ "	"	8.1	"	64.8
6	142	O	1735 –1737.5	+ "	"	6.7	"	78.0
7	141	H	1737.5–1735	— "	"	4.9	4M	21.5
8	142	O	1735 –1737.5	+ "	"	4.9	"	65.4
9	142	O	1735 –1737.5	+ "	"	4.5	"	72.3
10	142	O	1735 –1737.5	+ "	"	5.3	"	73.9
11	141	H	1737.5–1735	— "	"	8.1	4S	33.0
12	142	O	1735 –1737.5	+ "	"	8.8	4M	74.0
13	141	H	1737.5–1735	— "	"	7.7	4S	28.2
14	141	H	1737.5–1735	— "	"	10.3	"	21.2
15	142	O	1735 –1737.5	+ "	"	7.7	4M	75.9
16	141	H	1737.5–1735	— "	"	7.95	4S	49.6
17	142	O	1735 –1737.5	+ "	"	15.3	"	70.1
18	141	H	1737.5–1735	— "	"	6.1	"	52.8
19	142	O	1735 –1737.5	+ "	"	4.5	4M	85.2
20	141	H	1737.5–1735	— "	"	9.4	4S	56.4
21	141	H	1737.5–1735	— "	"	6.5	"	59.1
22	141	H	1737.5–1735	— "	"	6.1	"	62.5
23	142	O	1735 –1737.5	+ "	"	5.6	4M	69.7
24	142	O	1735 –1737.5	+ "	"	3.5	"	86.1
25	142	O	1735 –1737.5	+ "	"	5.6	"	74.6
26	141	H	1737.5–1735	— "	"	8.3	4S	59.7
27	141	H	1737.5–1735	— "	"	10.7	C	
28	142	O	1737.5–1735	+ "	"	5.4	4M	69.6
29	142	O	1735 –1737.5	+ "	"	9.4	"	73.4
30	153	..	1737.5–1735	— "	"	12.2	4S	28.5
31	153	..	1737.5–1735	— "	"	9.0	"	43.8
32	154	..	1735 –1737.5	+ "	"	6.8	"	74.0
33	154	..	1735 –1737.5	+ "	"	11.7	"	74.7
34	153	..	1737.5–1735	— "	"	7.8	"	66.8
35	153	..	1737.5–1735	— "	"	7.4	"	40.4
36	153	..	1737.5–1735	— "	"	6.2	"	51.5
37	154	..	1735 –1737.5	+ "	"	7.5	"	76.1
38	153	..	1737.5–1735	— "	"	7.46	"	37.3
39	153	..	1737.5–1735	— "	"	6.2	"	54.8
40	153	..	1737.5–1735	— "	"	12.2	"	46.7

*S=Series-Multiple. M=Multiple. C=Coasting.

TABLE 10 (Continued)

THE IMMEDIATE RESULTS OF THE TESTS ON THE 8°–0′ CURVE, GIVING THE RESULTS OF RESISTANCE AND SPEED USED IN PRODUCING FIG. 12

10	11	12	13	14	15	16	17	18	19
	Speed				Energy Imparted to the Car				
Item No.	At Entrance to the Section	At Exit from the Section	Average Voltage	Average Current	By the Current	By the Change in Kinetic Energy	By the Grade	Net Car Resistance	Average Speed Over the Section
	M.P.H.	M.P.H.	Volts	Amp.	Ft. Lb.	Ft. Lb.	Ft. Lb.	Lb. per Ton	M.P.H.
1	17.52	18.96	290	94.9	218390	—109530	—37570	14.02	17.91
2	18.78	17.88	203	46.7	59780	+ 68790	— "	17.89	18.75
3	19.32	19.32	401	24.1	36420	0	+ "	14.55	19.35
4	7.62	9.06	128	71.4	137360	— 50080	— "	9.77	8.63
5	16.26	15.18	162	45.5	57060	+ 70800	— "	17.75	14.81
6	17.88	18.42	260	79.3	158950	— 40870	— "	15.83	17.91
7	25.26	24.90	117	43.1	3920	+ 37650	+ "	15.56	24.49
8	26.34	25.26	236	89.2	49750	+116190	— "	25.24	24.49
9	27.06	26.52	301	109.7	79230	+ 60330	— "	20.05	26.67
10	21.12	20.58	258	120.3	89660	+ 46950	— "	19.47	22.64
11	14.82	14.28	86	27.7	9440	+ 32760	+ "	15.68	14.81
12	13.92	13.56	171	136.4	112000	+ 20630	— "	18.69	13.64
13	16.08	15.72	59	26.2	4980	+ 23870	+ "	13.06	15.58
14	11.76	11.40	56	22.6	4060	+ 17380	+ "	11.60	11.65
15	15.54	15.54	212	145.3	132750	0	— "	18.71	15.58
16	15.90	15.90	110	34.3	21960	0	+ "	11.70	15.09
17	7.26	8.16	108	58.8	100460	— 28940	— "	6.67	7.84
18	20.40	20.04	166	35.3	27840	+ 30350	+ "	18.83	19.67
19	25.26	26.34	432	204.6	249930	—116190	— "	18.91	26.67
20	13.02	13.02	97	38.9	29560	0	+ "	13.20	12.77
21	19.68	19.68	169	39.8	38110	0	+ "	14.88	18.46
22	20.04	20.04	169	42.7	40570	0	+ "	15.36	19.67
23	22.74	22.20	236	101.3	68830	+ 50600	— "	16.09	21.43
24	34.08	34.44	539	208.3	249520	— 51430	— "	31.56	34.29
25	21.48	20.94	243	126.7	94860	+ 47760	— "	20.65	21.43
26	14.28	14.28	112	41.2	33730	0	+ "	14.02	14.46
27	11.94	11.40	0	0	0	+ 26280	+ "	12.55	11.21
28	23.10	22.56	262	100.0	72620	+ 51410	— "	17.00	22.22
29	12.66	12.66	171	130.0	113120	0	— "	14.85	12.77
30	10.04	9.68	57	26.3	7690	+ 14800	+37640	11.80	9.84
31	13.64	13.64	107	31.6	19650	0	+ "	11.24	13.33
32	17.42	17.42	220	62.5	102060	0	— "	12.64	17.65
33	10.22	11.66	153	78.1	154050	— 65690	— "	9.96	10.26
34	15.08	15.62	165	48.1	60980	— 34570	+ "	12.57	15.38
35	16.52	15.80	130	29.9	17130	+ 48520	+ "	20.27	16.22
36	20.66	20.12	179	34.3	28920	+ 45910	+ "	22.07	19.35
37	15.62	15.62	207	75.4	131410	0	— "	18.40	16.00
38	16.16	15.98	116	28.9	13750	+ 12060	+ "	12.45	16.09
39	20.12	19 76	165	36.6	30270	+ 29930	+ "	19.20	19.35
40	9.86	9.50	68	33.5	19160	+ 14530	+ "	14.00	9.84

TABLE 11

THE IMMEDIATE RESULTS OF THE TESTS ON THE 14°–30′ CURVE, GIVING THE RESULTS OF RESISTANCE AND SPEED USED IN PRODUCING FIG. 13

1	2	3	4	5	6	7	8	9
Item No.	Test No.	Wind Opposing or Helping	Section Limits Trolley Line Pole Numbers	Grade Rise or Fall Over Section +Up —Down	Length of Track Section	Time to Run Over Section	Motor Data	
							Number in Use and Connection *	Efficiency of Motors and Gears
		O or H		Feet	Feet	Sec.		Per cent
1	141	O	1735–1732	— 0.17	276	10.1	4 S	74.2
2	142	H	1732–1735	+ ''	''	10.7	''	72.9
3	141	O	1735–1732	— ''	''	9.0	''	60.7
4	142	H	1732–1735	+ ''	''	9.9	''	78.4
5	141	O	1735–1732	— ''	''	9.9	''	61.7
6	142	H	1732–1735	+ ''	''	22.7	''	72.4
7	141	O	1735–1732	— ''	''	8.7	''	74.4
8	141	O	1735–1732	— ''	''	9.1	''	60.0
9	142	H	1732–1735	+ ''	''	10.7	''	75.0
10	142	H	1732–1735	+ ''	''	11.6	''	77.2
11	141	O	1735–1732	— ''	''	7.1	4M	84.2
12	142	H	1732–1735	+ ''	''	7.2	''	83.0
13	141	O	1735–1732	— ''	''	10.2	4 S	72.0
14	142	H	1732–1735	+ ''	''	8.8	4M	80.0
15	141	O	1735–1732	— ''	''	9.2	4 S	76.8
16	142	H	1732–1735	+ ''	''	27.5	''	72.2
17	141	O	1735–1732	— ''	''	13.2	''	70.3
18	141	O	1735–1732	— ''	''	12.7	''	60.6
19	141	O	1735–1732	— ''	''	12.0	''	69.3
20	142	H	1732–1735	+ ''	''	25.0	''	72.9
21	141	O	1735–1732	— ''	''	9.2	''	75.7
22	141	O	1735–1732	— ''	''	11.5	''	76.8
23	142	H	1732–1735	+ ''	''	7.6	4M	78.2
24	141	O	1735–1732	— ''	''	8.1	4 S	73.8
25	141	O	1735–1732	— ''	''	14.1	''	69.2
26	141	O	1735–1732	— ''	''	9.6	''	73.9
27	141	O	1735–1732	— ''	''	9.6	''	75.0
28	142	H	1732–1735	+ ''	''	38.1	''	71.7
29	141	O	1735–1732	— ''	''	9.8	''	72.9
30	141	O	1735–1732	— ''	''	9.5	''	77.7
31	141	O	1735–1732	— ''	''	14.5	''	66.5
32	142	H	1732–1735	+ ''	''	5.6	4M	86.1
33	141	O	1735–1732	— ''	''	12.6	4 S	68.7
34	142	H	1732–1735	+ ''	''	21.1	''	75.2
35	141	O	1735–1732	— ''	''	9.3	''	74.7
36	142	H	1732–1735	+ ''	''	8.4	4M	83.8
37	142	H	1732–1735	+ ''	''	7.6	''	79.9
38	153	. .	1735–1732	— ''	''	8.6	4 S	74.5
39	154	. .	1732–1735	+ ''	''	9.0	''	77.4
40	153	. .	1735–1732	— ''	''	11.7	''	63.2
41	153	. .	1735–1732	— ''	''	12.6	''	75.1
42	153	. .	1735–1732	— ''	''	29.0	''	67.7
43	153	. .	1735–1732	— ''	''	9.7	''	71.6
44	153	.	1735–1732	— ''	''	10.3	''	70.8
45	153	. .	1735–1732	— ''	''	8.5	''	67.7
46	153		1735–1732	— ''	''	8.1	''	77.0
47	153	.	1735–1732	— ''	''	22.3	''	68.2
48	153	. .	1735–1732	— ''	''	9.0	''	75.9
49	153	. .	1735–1732	— ''	''	28.9	''	48.9
50	153	. .	1735–1732	— ''	''	15.4	''	64.7

*S = Series-Multiple. M = Multiple.

TABLE 11 (Continued)

THE IMMEDIATE RESULTS OF THE TESTS ON THE 14°–30′ CURVE, GIVING THE RESULTS OF RESISTANCE AND SPEED USED IN PRODUCING FIG. 13

10	11	12	13	14	15	16	17	18	19
	Speed				Energy Imparted to the Car				
Item No.	At Entrance to the Section	At Exit from the Section	Average Voltage	Average Current	By the Current	By the Change in Kinetic Energy	By the Grade	Net Car Resistance	Average Speed Over the Section
	M.P.H.	M.P.H.	Volts	Amp.	Ft. Lb.	Ft. Lb.	Ft. Lb.	Lb. per Ton	M.P.H.
1	18.96	18.96	224	63.2	156470	0	+9830	20.85	18.63
2	18.24	17.52	194	60.1	134150	+ 53680	— ''	22.32	17.59
3	21.48	20.22	167	41.1	55320	+109550	+ ''	21.90	20.91
4	18.60	18.78	260	82.4	245280	— 14030	— ''	27.76	19.01
5	19.32	18.06	420	35.9	135870	+ 98200	+ ''	30.58	19.01
6	7.62	7.62	112	67.9	184360	0	— ''	21.88	8.29
7	21.66	21.48	268	61.2	156580	+ 16190	+ ''	22.89	21.63
8	21.12	19.50	178	40.3	57780	+137200	+ ''	25.68	20.68
9	18.60	17.88	225	66.4	176840	+ 54760	— ''	27.80	17.59
10	14.28	15.18	227	80.2	240470	— 55280	— ''	21.98	16.22
11	26.16	26.52	383	202.7	342260	— 39540	+ ''	39.18	26.50
12	26.34	26.34	383	164.5	277660	0	— ''	33.58	26.14
13	19.32	18.78	193	57.5	120240	+ 42900	+ ''	21.69	18.45
14	21.30	21.12	312	164.3	266140	+ 15920	— ''	34.13	21.38
15	19.86	20.04	260	71.6	194000	— 14970	+ ''	23.68	20.45
16	5.82	5.82	91.8	70.9	190650	0	— ''	22.67	6.84
17	14.28	13.38	142	56.4	109640	+ 51900	+ ''	21.48	14.26
18	15.72	13.56	113	42.0	53860	+131870	+ ''	24.52	14.82
19	15.90	15.00	164	52.7	106040	+ 57980	+ ''	21.80	15.68
20	6.00	7.26	106	75.4	214790	— 34840	— ''	21.33	7.53
21	20.04	19.86	247	67.8	172010	+ 14970	+ ''	24.67	20.45
22	16.98	17.52	220	78.3	224390	— 38840	+ ''	24.49	16.36
23	26.52	25.26	290	150.0	190670	+136030	— ''	39.72	24.76
24	22.74	22.38	264	59.8	139180	+ 33870	+ ''	22.93	23.23
25	13.02	12.66	140	54.0	108800	+ 19280	+ ''	17.29	13.35
26	19.68	19.32	230	61.4	147760	+ 29270	+ ''	23.43	19.60
27	19.68	19.32	243	65.2	168270	+ 29270	+ ''	26.00	19.60
28	3.48	4.02	93.6	67.2	253450	— 8440	— ''	29.48	4.94
29	19.14	19.14	218	59.0	135510	0	+ ''	18.22	19.20
30	20.04	20.04	264	76.7	220480	0	+ ''	28.87	19.81
31	13.38	12.30	128	49.2	89600	+ 57830	+ ''	19.72	12.98
32	33.90	34.08	573	194.7	396700	— 25510	— ''	45.30	33.60
33	14.28	14.28	164	51.4	107630	0	+ ''	14.73	14.93
34	6.18	9.96	151	88.6	313150	—127200	— ''	22.08	8.92
35	20.22	20.40	243	63.8	158880	— 15240	+ ''	19.24	20.23
36	21.48	23.10	370	199.1	382450	—150580	— ''	27.84	22.40
37	24.00	23.64	318	149.1	212320	+ 35760	— ''	29.87	24.76
38	21.56	21.56	266	61.6	154840	0	+9840	20.61	21.88
39	21.56	21.74	279	72.3	207240	— 16250	— ''	22.67	20.91
40	17.06	15.44	144	44.1	69260	+109780	+ ''	23.64	16.08
41	13.82	15.26	189	71.8	189400	— 87310	+ ''	14.01	14.93
42	8.06	5.18	75	54.7	118880	+ 79500	+ ''	26.06	6.49
43	20.12	19.76	226	55.0	127340	+ 29930	+ ''	20.92	19.40
44	18.86	18.32	223	53.5	128320	+ 41860	+ ''	22.53	18.27
45	22.46	21.20	217	47.7	87850	+114700	+ ''	26.58	22.14
46	22.10	21.74	300	68.0	187670	+ 32910	+ ''	28.83	23.23
47	8.96	8.60	96	54.2	116760	+ 13180	+ ''	17.49	8.44
48	20.66	20.30	272	66.5	182270	+ 30740	+ ''	27.89	20.91
49	9.68	0.00	49.5	34.8	35960	+195370	+ ''	30.18	6.51
50	13.64	11.66	120	46. 5	82010	+104450	+ ''	24.57	12.22

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A PRELIMINARY STUDY OF THE ALLOYS OF CHROMIUM, COPPER, AND NICKEL

BY
D. F. McFARLAND
AND
O. E. HARDER

CHARTERED 1867

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

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A PRELIMINARY STUDY OF THE ALLOYS OF CHROMIUM, COPPER, AND NICKEL

BY

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ASSISTANT PROFESSOR OF APPLIED CHEMISTRY,

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A PRELIMINARY STUDY OF THE ALLOYS OF CHROMIUM, COPPER, AND NICKEL*

I. Introduction

1. *Purpose of the Investigation.*—The growing interest in special acid-resisting alloys and the many uses found for them has stimulated both the search for efficient materials of this nature and the study of the causes underlying their inertness. The alloys developed by Professor S. W. Parr for use in calorimeter construction have shown this quality of high resistance to corrosion to a marked degree. The almost perfect insolubility of these alloys in nitric and other acids seems to be conditioned upon a proper mixture of chromium, copper, and nickel, together with smaller quantities of such added metals as tungsten or molybdenum.†

These additions have so marked an effect in improving both the acid resisting properties and the casting qualities of the alloys that it has seemed desirable to study their effects more systematically in order that they may be used to the best advantage. The complexity of the mixtures used, however, has made the problem a very difficult one and has shown the necessity of first obtaining a more complete knowledge of the ternary alloys of chromium, copper, and nickel, and of the binary alloys of copper and nickel, copper and chromium, and chromium and nickel. With this information in hand it should be possible to understand better the effects produced by additions of a fourth metal.

In the work presented in this bulletin a preliminary survey has been made of the binary and ternary alloys and a somewhat systematic study made of their properties.

2. *Present State of Information.*—Previous investigators have studied in some detail the equilibrium conditions in the three binary systems. The freezing point curves for copper-nickel and chromium-nickel alloys have been fairly well established, but that for the chromium-copper series has been determined only in part, owing to the unusual difficulties attending the work. According to available information, these alloys belong to three different classes; copper-nickel

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*The manuscript for this bulletin was originally prepared by Mr. Oscar E. Harder and submitted by him in partial fulfillment of the requirements for the degree of Doctor of Philosophy, University of Illinois. Since its presentation as a thesis, it has been reviewed and in certain particulars revised by Dr. D. F. McFarland.

†S. W. Parr, Orig. Com. 8th Inter. Congr. Appl. Chem. Vol. 2, p. 209, 1912. Also Jour. Am. Chem. Soc. Vol. 37, pp. 2515-2522, 1915.

forming a continuous series of solid solutions (or mixed crystals), chromium-nickel consisting of solid solutions with a minimum point which is said to be a eutectic point, and chromium-copper having two eutectic points with a wide range in which the metals do not alloy.

The alloys of copper and nickel are commercially important in the form of Monel metal (Cu 30 per cent, Ni 70 per cent), Constantan (Cu 60 per cent, Ni 40 per cent) and other resistance alloys, and nickel coin (Cu 75 per cent, Ni 25 per cent). Chromium-nickel alloys such as ''Nichrome'' and similar resistance alloys have important uses. Chromium-copper alloys of a limited range of composition are also obtainable on the market. The solubility of chromium in copper has been placed at from 0.5 to 10.0 per cent by various investigators, while the solubility of copper in chromium has been placed at less than 5.0 per cent.

3. *Plan of Work.*—It was originally intended that this investigation should include the following:

a. The preparation of samples of both the binary and ternary alloys which would represent all of the possible combinations with variations of 10 per cent of the different constituents.

b. Physical and mechanical examinations which would include color, appearance, specific gravity, hardness, tensile strength, reduction of area, elongation, and modulus of elasticity.

c. Corrosion tests in solutions of nitric acid, hydrochloric acid, sulfuric acid, sodium hydroxide, ammonium hydroxide, and sodium chloride, and in fatty acids.

d. Measurements of the relative electromotive forces of the alloys in contact with salt solutions.

e. Thermal analysis and heat treatment.

f. Microscopic examination.

This program has been carried out with the exception of the thermal analysis and heat treatment. Considerable time and effort have been expended on the attempt to work out the former, but the experimental difficulties have been so great that it has been necessary to reserve this analysis for further work and future report. It was thought best also to postpone the study of heat treatment until the equilibrium diagram could be obtained.

4. *Summary of Results and Conclusions.*—The results obtained together with the conclusions drawn from them may be summarized as follows:

a. Methods have been developed for making castings of alloys of chromium, copper, and nickel; and twenty-one binary and thirty ternary alloys have been prepared. From this part of the work the following conditions have been drawn.

(1) Castings of chromium and copper containing as much as 13 per cent of chromium can be prepared by melting and pouring the metals at about 1600 degrees C.

(2) Chromium-copper alloys containing 6.08 per cent or more of chromium show a separation of chromium or of a chromium-rich constituent, if they are cooled slowly.

(3) If equal weights of chromium and copper are heated together to a temperature well above the melting point of chromium and slowly cooled, the alloy is not homogeneous, but consists of two layers; the lower rich in copper, and the upper rich in chromium.

(4) The addition of nickel to alloys of chromium and copper tends to prevent the separation of the chromium or chromium-rich constituent; when the amount of nickel is more than three times the amount of copper present, the alloys become practically homogeneous.

b. Physical and mechanical tests have been made with the following results:

(1) The specific gravity at 25 degrees C. of the alloys tested varies from 8.92 to 7.89 and decreases with an increase of chromium.

(2) The Brinell hardness number varies from that of pure copper to that of tool steel and increases with an increase of chromium.

(3) The modulus of elasticity of the sixteen alloys tested varies from less than 15,000,000 to more than 40,000,000 pounds per square inch. Generally it increases with an increase of chromium.

(4) The ultimate tensile strength of the eighteen alloys tested varies from less than 10,000 to more than 50,000 pounds per square inch.

(5) The reductions and elongations are small in all cases.

(6) The stress-deformation curves are similar to those of cast iron.

c. More than three hundred corrosion tests have been made. Results show that:

(1) The amount of corrosion is not proportional to the strength of the acid or base.

(2) A triangular system of plotting shows certain fairly well-defined areas which are highly resistant to corrosion.

(3) Not only the alloys in the region approximating the composition of the alloy developed by Professor Parr are highly noncorrodible, but others have shown equally good resistance to corrosion. In general the ternary alloys are less corroded than the binary, though there are some exceptions.

d. An attempt has been made to find some relation between the relative electromotive forces obtained by placing the alloys in contact with 4 Normal sodium chloride solution and their relative resistance to corrosion, but no such relation has been found from the experimental data obtained.

e. A microscopic study of the alloys has been made and the following agreements with earlier investigators have been found:

(1) The results agree with Voss' conclusions that chromium and nickel form a series of solid solutions (mixed crystals) over the range of 100 to 50 per cent of nickel and that they form a eutectic, or, as Guertler called it, a pseudoeutectic, containing about 42 per cent of nickel.

(2) The results on the copper nickel series agree with those of Guertler and Tammann in showing a continuous series of solid solutions.

(3) All nickel-rich alloys, both binary and ternary, show well-defined polyhedral crystals.

f. In general the results indicate that the alloys of chromium-copper-nickel which show possibilities of becoming of commercial importance are limited to certain rather well-defined ranges of composition. In Fig. 1 is shown diagrammatically what seems to be the most promising field for future investigation. In the nickel-rich corner of the diagram the alloys have very large polyhedral crystals and a coarse texture, and have usually developed blow-holes in casting. There is a possibility that the texture can be improved by the addition of a fourth metal as Professor Parr has done by the use of tungsten or molybdenum. The alloys containing large percentages of chromium have such high melting points and are so hard to prepare that, unless they find special and important applications, there is little chance of their being used commercially. Alloys containing large percentages of copper with chromium show such a marked segregation that they do not machine well and their mechanical properties are poor. The region which promises best from the mechanical and physical standpoint is also, as a general rule, highly resistant to corrosion in the various solutions tested.

II. Preparation of the Alloys.

5. *Materials.*—All the materials used in the preparation of the different alloys were of good quality. The chromium was secured from the Goldschmidt Thermit Company and was labeled 98.99 per cent

chromium. An analysis of one sample showed 98.2 per cent chromium with the remainder consisting largely of silica, slag, etc. The nickel was C. A. F. Kahlbaum's "Nickel in Wurfeln" and an analysis showed that it contained 99.6 per cent of nickel, a small amount of iron, and only a trace of cobalt. Two different lots of copper were used, both of which were electrolytic; one purchased from the J. T. Baker Chemical Company and the other from Kahlbaum.

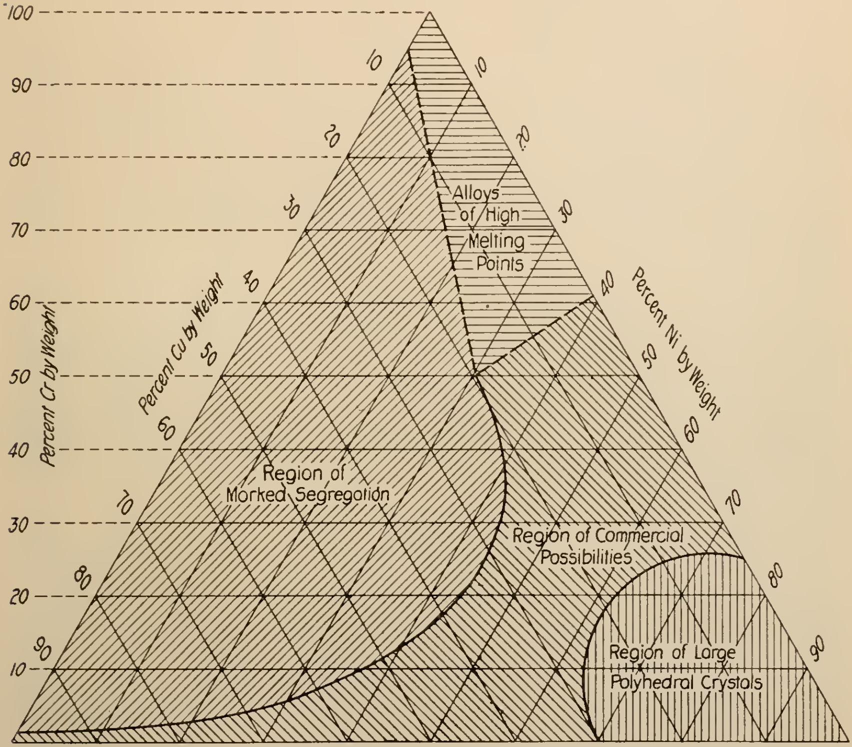
100
90
80
70
60
50
40
30
20
10
Percent Cr by Weight
10
20
30
40
50
60
70
80
90
Percent Cu by Weight
10
20
30
40
50
60
70
80
90
Percent Ni by Weight
Alloys of High Melting Points
Region of Marked Segregation
Region of Commercial Possibilities
Region of Large Polyhedral Crystals

FIG. 1. DIAGRAM SHOWING ALLOYS HAVING POSSIBLE COMMERCIAL VALUES.

6. *Melting the Metals.*—The samples, with the exception of No. 1 to No. 6, inclusive, were melted in Crescent Safety crucibles in a Hoskins electric furnace of the carbon plate resistor type. These crucibles are sand crucibles covered with graphite. They withstood temperatures as high as 1600 degrees C., and in only one or two cases did they seem to be softened by that heat. It was not possible to use a crucible for more than one melt because of the corrosion of the sand lining. Samples No. 1 to No. 6, inclusive, were melted in a gas-fired

furnace in fire-clay crucibles. The metals were protected by a cover of powdered cryolite ($NaAlF_4$) which melted easily below the melting point of copper and effectively prevented oxidation of the chromium. It was not volatilized at the temperatures used and was the most satisfactory cover, although a number of other covers and fluxes were tried.

The charges of metals were of uniform size, 300 grams, in all cases. The furnace used from 25 to 30 kw. per hour and the time required for a melt varied from two to three hours.

7. *Casting the Samples.*—The molten metals were poured into asbestos-lined iron moulds ¾ inches in diameter and about 8 inches in length, which had been heated to a bright red temperature and packed in amorphous silica. By taking a reasonable amount of care it was possible to prevent contact of the molten alloy with the iron mould and to insure its easy removal from the mould when cold. The silica, being a good non-conductor of heat, allowed the casting to cool very slowly.

8. *Composition of Alloys.*—The intended composition of the different alloys is given in Table 1. The percentages are expressed in both atomic per cent and weight per cent. The weight per cent was calculated from the atomic per cent by means of the following formula in which *A*, *B*, and *C* represent respectively the three metals chromium, copper, and nickel.

$$\text{Weight per cent } A = \frac{\text{At. \% } A \times \text{at. wt. } A \times 100}{\text{At. \% } A \times \text{at. wt. } A \div \text{at. \% } B \times \text{at. wt. } B \div \text{at. \% } C \times \text{at. wt. } C.}$$

In these calculations the values Cr = 52.00, Cu = 63.57, and Ni = 58.68 were used as the atomic weights. Table 1 shows also the weight per cent composition of the castings found by analysis.

9. *Analysis.*—Three methods of sampling were used. For some of the softer alloys the sample was obtained by making drillings; for the specimens which were turned to test pieces the turnings were used for analysis; in all other cases a piece was taken from one end of the specimen. The copper and nickel were determined electrolytically.* Chromium was determined by precipitating it as the hydroxide, and by igniting and weighing it as the oxide. In a few cases one of the constituents was determined by difference as indicated in Table 1.

10. *Changes in Composition.*—A comparison of the composition of the charge with the composition of the casting, as found by analysis,

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*The preparation of the samples for analysis and the electrolytic determination of copper and nickel in all of the alloys were done by Sydney M. Hull.

TABLE 1

COMPOSITION OF ALLOYS

Number	Atomic Per Cent Composition			Weight Per Cent Composition			Weight Per Cent Composition by Analysis		
	Cu	Cr	Ni	Cu	Cr	Ni	Cu	Cr	Ni
1	100	. . .	. . .	100.	. . .	. . .	. . .	. . .	. . .
2	90	. . .	10	90.71	. . .	9.29	90.84	. . .	9.06
3	80	. . .	20	81.27	. . .	18.73	81.07	. . .	18.76
4	70	. . .	30	71.65	. . .	28.35	71.16	. . .	28.46
5	60	. . .	40	61.90	. . .	38.10	61.63	. . .	38.25
6	50	*	50	52.00	. . .	48.00	48.96	. . .	49.90
7	40	. . .	60	41.94	. . .	58.06	69.13	. . .	30.59
8	30	. . .	70	31.70	. . .	68.30	41 14	. . .	58.27
9	20	. . .	80	21.31	. . .	78.69	20.65	. . .	79.35†
10	10	. . .	90	10.74	. . .	89.26	10.57	. . .	88.90
11	. . .	. . .	100	. . .	. . .	100.00	00.00	. . .	99.66
12	90	10	. . .	91.68	9.32	. . .	94.20	6.08	00.00
13	80	10	10	82.12	8.40	9.48	84.36	7.79	9.28
14	70	10	20	72.43	8.45	19.12	74.63	8.25	16.05
15	60	10	30	62.58	8.53	28.89	66.32	10.63	22.94
16	50	10	40	52.57	8.60	38.83	54.71	15.94	29.35
17	40	10	50	42.40	8.67	48.93	42.56	10.13	48.00
18	30	10	60	32.05	8.75	59.20	32.69	13.97	54.16
19	20	10	70	21.55	8.81	69.64	20.85	11.80	66.25
20	10	10	80	10.87	8.89	80.24	11.83	11.90	76.27†
21	. . .	10	90	. . .	8.96	91.04	00.00	19.37	78.99
22	80	20	. . .	83.02	16.98	. . .	87.93	13.15	00.00
23	70	20	10	73.22	17.12	9.66	80.58	10.56	9.38
24	60	20	20	63.28	17.25	19.47	56.30	14.56	29.24
25	50	20	30	53.16	17.40	29.44	66.92	13.62	19.20
26	40	20	40	42.88	17.54	39.58	44.08	19.30	36.34
27	30	20	50	32.41	17.69	49.90	36.70	15.99	47.31†
28	20	20	60	21.80	17.83	60.37	22.20	19.86	57.36
29	10	20	70	10.99	17.98	71.03	10.88	19.64	68.62
30	. . .	20	80	. . .	18.14	81.86	00.00	21.52	76.95
31	70	30	. . .	74.05	25.95	. . .	89.82	9.89	00.00
32	60	30	10	63.99	26.17	9.84	73.63	17.66	8.55
33	50	30	20	53.77	26.38	19.85	59.62	22.00	19.48
34	40	30	30	43.37	26.61	30.02	45.70	25.10	29.52
35	30	30	40	32.79	26.84	40.37	33.76	29.46	36.78†
36	20	30	50	22.05	27.06	50.89	22.58	28.10	48.42
37	10	30	60	11.12	27.29	61.59	10.90	29.70	58.12
38	. . .	30	70	. . .	27.52	72.48	00.00	28.44	71.56†
39	60	40	. . .	64.71	35.29	. . .	Not analyzed		
40	50	40	10	54.38	35.58	10.04	70.57	19.93	8.99
41	40	40	20	43.86	35.89	20.25	54.16	31.63†	14.21
42	30	40	30	33.17	36.20	30.63	33.60	38.16	26.78
43	20	40	40	22.31	36.50	41.19	22.68	41.32	34.60
44	10	40	50	11.25	36.82	51.93	11.02	43.30	46.46
45	. . .	40	60	. . .	37.14	62.86	00.00	44.93	56.55
46	50	50	. . .	55.20	44.80	. . .	Not prepared		
47	40	50	10	44.38	45.37	10.25	Not prepared		
48	30	50	20	33.57	45.76	20.67	28.42	54.92	17.12
49	20	50	30	22.58	46.16	31.26	24.12	47.54	26.28
50	10	50	40	11.39	46.56	42.05	. . .	. . .	. . .
51	. . .	50	50	. . .	46.96	53.04	00.00	57.40	41.66
52	40	60	. . .	44.89	55.11	. . .	. . .	. . .	
53	30	60	10	33.96	55.59	10.45	. . .	. . .	. . .
54	20	60	20	22.85	56.06	21.09	. . .	. . .	. . .
55	10	60	30	11.53	56.56	31.91	Not analyzed		
56	. . .	60	40	. . .	57.07	42.93	Not analyzed		
57	30	70	. . .	34.37	65.63	. .	. . .	. . .	. . .
58	20	70	10	23.13	66.21	11.56	. . .	. . .	. . .
59	10	70	20	11.66	66.80	21.54	. . .	. . .	. . .
60	. . .	70	30	. . .	67.40	32.60	Not analyzed		
61	20	80	. . .	23.41	76.59	. . .	. . .	. . .	. . .
62	10	80	10	11.81	77.29	10.90	. . .	. . .	. . .
63	. . .	80	20	. . .	78.00	22.00	Not analyzed		
64	10	90	. . .	11.96	88.04	. . .	. . .	. . .	. . .
65	. . .	90	10	. . .	88.86	11.14	Not analyzed		
66	. . .	100	. . .	. . .	100.00	. . .	. . .	98.21	. .

*No. 6 was made up to be 50 per cent copper and 50 per cent nickel by weight.
†Determined by difference.

shows that there was a remarkable loss in chromium in the chromium-copper alloys. The highest chromium content found was in No. 22 which showed 13.15 per cent, whereas it should have shown about 17 per cent. In No. 31 the chromium should have been about 26 per cent, but analysis showed only 9.89 per cent. All attempts to prepare alloys of chromium and copper containing higher percentages of chromium failed. Even in the case of Alloy No. 12, which was slowly cooled and which contained only 6.08 per cent of chromium, there is a separation of pure chromium or of a chromium-rich constituent. (See Fig. 25.)

It has been stated by Hindrichs that at higher temperatures chromium and copper form an emulsion in which the chromium may be in a finely divided condition, and that on slow cooling the chromium collects in larger particles. It seems more probable, however, that chromium should be more soluble in copper at higher temperatures than at the melting point. In either case rapid cooling would tend to prevent such a separation and to produce a more homogeneous structure. Evidence to this effect is shown by the fact that the alloy containing 10 per cent of chromium and 90 per cent of copper which is offered for sale by the Goldschmidt Thermit Company is almost homogeneous. This alloy is said to be made by an aluminothermic method. The aluminothermic reaction produces a very high temperature, and if, as appears to be the case, the alloy is cooled suddenly, there would not be sufficient time for the separation of the chromium or chromium-rich constituent.

In the case of the other alloys there were losses of different constituents depending upon a number of conditions. In the ternary alloys rich in copper there was generally a loss in chromium, probably due to the insolubility of the chromium in the other metals present. In the ternary alloys rich in nickel and in the binary alloys of chromium and nickel, there was generally a loss of nickel, but that may be accounted for by the fact that in making up the charges precaution was taken to protect the chromium by putting it in the bottom of the crucible, thus the nickel was left more exposed to oxidation. The changes in composition may be studied in Table 1. It is certain that in making a series of alloys by such a method as has been outlined, it is not safe to assume that the composition of the alloy obtained will be the same as that of the charge melted.

11. *Alloy of* 50 *Per Cent Cr and* 50 *Per Cent Cu.*—In order to determine whether or not a homogeneous alloy containing equal parts

of chromium and copper could be prepared, the following experiments were conducted.

A charge of 30 grams of chromium and 30 grams of copper was heated in an electric furnace. A little copper was put in the bottom of the crucible, the chromium was added, and then the remainder of the copper. The charge was covered with cryolite. It was heated well above the melting point of chromium and kept at that temperature for at least fifteen minutes. The current was then shut off and the furnace allowed to cool slowly. An examination showed that the chromium had been melted. The copper which had been put on top of the charge was found at the bottom of the melt, and a fairly well defined line of separation between the copper-rich and the chromium-rich parts of the melt could be seen. (See Figs. 2 and 3.) The above experiment was later repeated with similar results. By this method it has been possible to get a division of the melt into two fairly well defined layers. From a microscopic examination (Figs. 2 and 3) it was evident that the lower layer contained some chromium, and it appeared that the upper layer contained some copper. Hindrichs stated that he was not able to get the two metals to separate into two sharply defined layers.

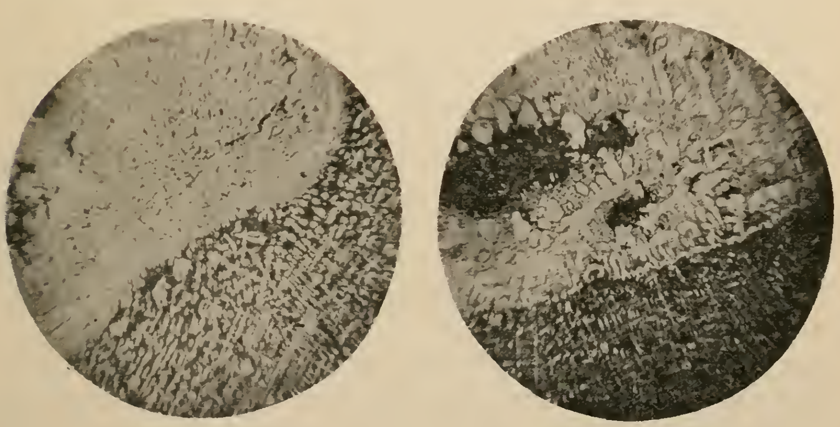

Fig. 2

Fig. 3

Microphotographs Showing Separation of Copper from Chromium in a 50 Per cent Cu, 50 Per cent Cr. Alloy. Etched in 1 per cent $FeCl_3$ in 1: 1 HCl and then Stained in I in Alcohol. The Dark Portion is the Copper-rich Constituent. Magnified 30 Diameters.

III. Physical and Mechanical Properties

12. *Color and Appearance.*—The colors of the different alloys depend upon the amount of copper as compared with the sum of the amounts of chromium and nickel present. The chromium and nickel colors are more persistent than the copper color, and an alloy containing 50 per cent of copper and 50 per cent of nickel, or of chromium and nickel, has the color of nickel and does not show any color of copper. The alloys near the pure copper corner of the diagram (see Fig. 4) show the greatest tendency to tarnish when exposed to the air in the laboratory. On the other hand, the alloys near the nickel corner of the diagram are more porous and more likely to contain blowholes.

13. *Method of Plotting Diagrams.*—The following method has been adopted for plotting the relation between the compositions and the various properties of the alloys as shown in Fig. 4.

The three metals, chromium, copper, and nickel, are represented by the corners of the equilateral triangle. The compositions of the different alloys, as found by analysis, have been represented by the centers of the circles so that they can be read directly from the diagram. The number of the alloy has been placed inside of the circle. All of the binary alloys fall on the sides of the triangle and all of the ternary alloys fall within the triangle. One example will serve to show the method of reading the composition of the different alloys from their positions on the diagram. No. 28 is approximately on the line marked 20 per cent chromium which runs parallel to the copper-nickel side of the triangle and between the lines marked 60 and 50 per cent of nickel which run parallel to the chromium-copper side of triangle. Therefore it can be estimated that the alloy contains approximately 57 per cent of nickel. About 23 per cent is left for copper. The same result can be read from the distance of the center of the circle to the line marked 20 per cent copper which runs parallel to the chromium-nickel side of the triangle. By this method it is possible to read directly the composition of any alloy which has been plotted on the diagram. The alloys which show the copper color are Nos. 2, 3, 4, 5, 12, 13, 14, 22, 23, 31, 32, 40, and 48. Their compositions can be read from Fig. 4 or can be found in Table 1. Table 2 shows in a brief way the kinds of castings which were obtained for the different alloys.

14. *Specific Gravity.*—The specific gravities of the different alloys, as cast, are given in Table 2, and the same results are shown in Fig. 4. In the diagram the specific gravity of the alloy has been written above the circle representing the composition of the alloy.

The alloys were weighed in air and in distilled water at 25 degrees C., and the specific gravities were determined from the formula:

$$\text{Sp. gr.} = \frac{\text{Wt. in air at } t^{0}\text{C. x sp. gr. } H_2O \text{ at } t^{0}\text{C.}}{\text{Loss of wt. in } H_2O \text{ at } t^{0}\text{C.}}$$

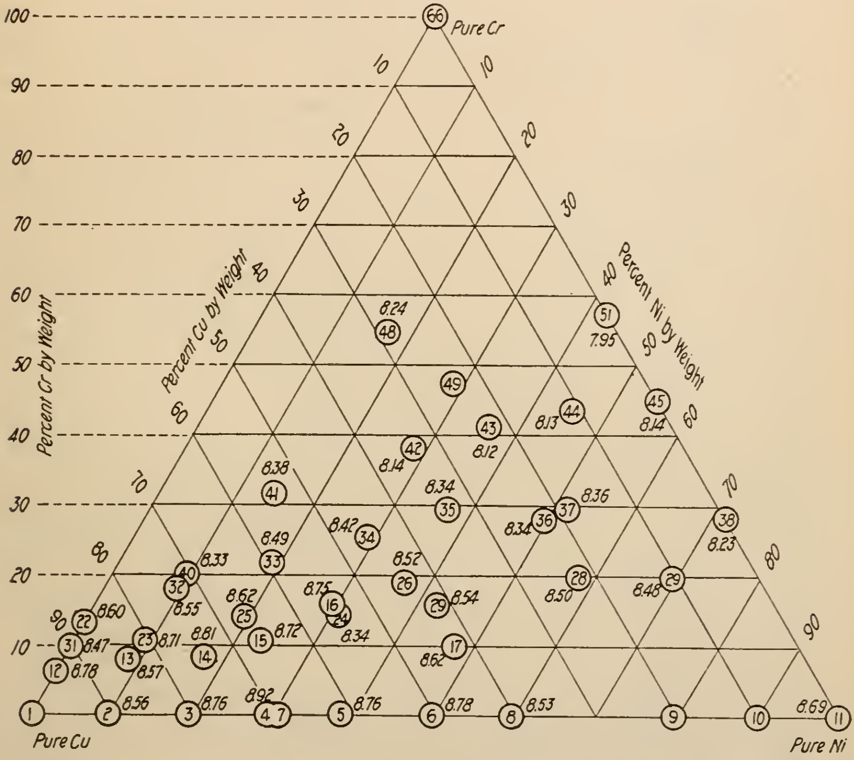

Fig. 4. Diagram Showing Composition of Alloys and Specific Gravities at 25 Degrees C.

If the alloys containing a constant amount of chromium with varying amounts of copper and nickel are considered, it will be seen that there is little variation in the specific gravity. On the other hand, if the alloys containing a constant amount of copper with varying amounts of chromium and nickel are considered, it will be seen that the specific gravity decreases as the percentage of chromium increases. Similar results may be obtained if the alloys containing constant amounts of nickel and varying amounts of chromium and copper are examined. If the specific gravities are plotted as ordinates and the percentages of chromium as abscissae, fairly regular curves will be obtained. When

TABLE 2

HARDNESS, SPECIFIC GRAVITY, AND KIND OF CASTING

Number	Diameter of Impression	Brinell Hardness Number	Specific Gravity at 25° C.	Kind of Casting
1	...	...	...	Good.
2	Broke in test.		8.56	Good.
3	Cracked in test.		8.76	Medium.
4	Cracked in test.		8.92	Blowholes in places.
5	Specimen too small.		8.76	Blowholes in places.
6	Specimen too small.		8.78	Good.
7	7.20	Below 68	8.53	Good.
8	...	...	...	Casting had blowholes.
9	...	...	...	Blowholes all through.
10	...	...	...	Blowholes in places.
11	Broke in test. (5.4 mm. thick)		8.69	Large crystals and small blowholes.
12	7.20	Below 68	8.78	Good.
13	6.30	86	8.57	Sound.
14	5.80	103	8.81	Sound.
15	5.00	143	8.72	Sound.
16	4.73	161	8.75	Small, but sound.
17	4.70	163	8.62	Fair.
18	...	...	...	Small blowholes.
19	...	...	...	Fair, except for blowholes.
20	...	...	...	Fair, except for blowholes.
21	...	...	...	Large blowholes.
22	6.90	69	8.60	Sound throughout.
23	6.40	82	8.71	Good.
24	6.20	89	*8.34	Good.
25	5.90	99	8.62	Good.
26	4.80	156	8.52	Very good.
27	4.50	175	8.54	Sound.
28	4.50	179	8.50	Sound.
29	5.30 (Specimen small)	126	8.48	Sound.
30	...	...	...	Large blowholes.
31	7.30	68	8.47	Medium.
32	5.46	117	8.55	Small, but sound.
33	5.50	116	8.49	Excellent. There was some segregation of Cr at top of casting.
34	5.30	126	8.42	Good.
35	4.92	148	8.34	Excellent.
36	4.60	170	8.34	Excellent.
37	4.70	163	8.36	Excellent.
38	4.48	181	8.23	Good.
39	Not prepared.			
40	5.40	121	8.33	Small, but sound.
41	5.50	116	8.38	Good.
42	5.20	131	8.14	Excellent.
43	4.80	156	8.12	Excellent.
44	4.70	163	8.13	Excellent.
45	4.93 (Cracked in test.)	147(?)	8.14	Good.
46	Not prepared.			
47	Not prepared.			
48	5.70	107	8.24	Excellent.
49	...	...	...	Sound.
50	4.25	202	7.89	Good.
51	4.50	179	7.95	Good.
55	4.00	228	...	Sound.
56	4.50 (Specimen too small)	179(?)	...	Sound.
60	...	...	...	Sound.
63	...	...	...	Sound.
65	...	...	...	Sound.

*This value is in error because of a concealed blowhole.

the exact composition of the alloy is known, it is possible to get some idea of the relative porosity of the specimen by a study of the specific gravity.

15. *Brinell Hardness Number.*—The hardness measurements were made in the Materials Testing Laboratory of the Department of Theoretical and Applied Mechanics of the University of Illinois by using a Brinell instrument with a 3,000 kilogram load and a ball of 10 mm. diameter, the pressure being applied for 15 seconds. The diameters of the impressions were carefully measured, and the hardness numbers corresponding to those diameters were found in a table supplied by the makers of the instrument. The pieces tested had been ground smooth and were from 4 to 8 mm. thick and about 20 mm. in diameter. Only one test was made on each piece, and if it was noticed that the piece had bulged or cracked from the pressure, that fact was recorded or the results were rejected.

Specimen No. 11 requires special mention. It was a sample of pure nickel which had been melted in a magnesia crucible and slowly cooled. It had very large crystals (see microphotograph of Alloy No. 11, Fig. 24), and when a piece 5.4 mm. thick and 22 mm. in diameter was tested, it broke at a load of about 2,000 kilograms.

A comparison with the values which are given in Table 3* shows that some of the specimens were especially hard.

TABLE 3

HARDNESS TESTS BY CANADA DEPARTMENT OF MINES

Material	Date	Load	Brinell Hardness
Copper, rolled sheet, unannealed	Jan. 1913	1,000 lbs.	65.6
" " " "	" 1914	1,000 "	67.4
" " " "	" 1914	3,500 "	75.0
" " " "	" 1914	3,500 "	81.9
Swedish iron	Jan. 1913	3,500 "	90.7
" "	" 1914	1,000 "	68.6
" "	" 1914	3,500 "	75.2
Wrought iron	Jan. 1913	3,500 "	92.0
" "	" 1914	1,000 "	83.1
" "	" 1914	3,500 "	100.2
Cast iron	Jan. 1913	3,500 "	97.8
" "	" 1914	1,000 "	84.4
" "	" 1914	3,500 "	104.5
Mild steel	Jan. 1913	3,500 "	109.9
" " cold rolled shafting	" 1914	3,500 "	126.2
Tool steel	Jan. 1913	3,500 "	153.8
" " "Crescent"	" 1914	3,500 "	130.2
Spring steel	Jan. 1913	3,500 "	160.3
" "	" 1914	3,500 "	178.0
Tool steel self-hardening	Jan. 1913	3,500 "	180.0
" " " " "Rex" (before hardening)	" 1914	3,500 "	162.1
Tool steel, self-hardening "Rex" (after hardening)	" 1914	3,500 "	240.0
Tool steel, self-hardening, from work shop (School of Mines)	Jan. 1914	3,500 "	259.0
Cobalt		3,500 "	124.0

*Canada Department of Mines, Report No. 309, Part II, p. 9, by Kalmus and Harper.

It is evident that some of these alloys are as hard as tool steel, and No. 55 approaches the hardness of self-hardening tool steel after hardening. It is likely that some of the alloys in the series have still greater hardness than the one referred to since some of the others contain more chromium.

These specimens were extremely difficult to work. A saw blade was ruined on some of them without cutting more than 1 mm. deep. It has been found that by keeping the saw blades moist with turpentine they will last somewhat longer.

The results obtained by the hardness tests are shown in Table 2.

16. *Tensile Strength and Stress-Deformation Tests.*—Eighteen specimens of the alloys of various compositions were turned down to test pieces and their ultimate tensile strengths determined. In all except two cases their elongations under stresses were measured.

Test Pieces.—The test pieces which were from 3 to 4 inches in total length were turned to a diameter of approximately 0.300 inches for a length of 2.0 inches, and were threaded at the ends so that they could be screwed into half-inch grips. It was necessary to make the measurements on these short test pieces instead of the usual 8-inch specimens, because the original castings were only 4 to 5 inches long and pieces had been cut off for microscopic examination, for corrosion tests, etc. It is likely that some of the irregularities observed in the values obtained for the modulus of elasticity and for the ultimate tensile strength should be attributed to the use of the short test pieces.

Testing Machine and Method of Loading.—The testing machine on which the tests were made was an Olsen Universal Screw-Power Testing Machine of 10,000 pounds capacity. The loading was by hand and was slow except in the case of specimens No. 17 and No. 28 when the loading was done with the motor. The loading was continuous and not repeated.

Extensometer.—A Ewing extensometer having a gage of 1.25 inches was used in measuring the elongations. The instrument was sensitive and the elongations could be read accurately to 0.00008 inches and estimated to 0.000008 inches. The initial or zero extensometer reading was taken with a small load on the machine. After a satisfactory number of readings had been made the extensometer was removed and the load increased until the specimen broke.

Calculation of Modulus of Elasticity.—Curves were plotted with stresses as ordinates and elongations as abscissae. Tangents to the curves were drawn and extended to a point which corresponded to an elongation of 0.001 inch on a length of 1.0 inch, and the stress corresponding to that elongation was read from the cross-ruled paper. This stress minus the initial stress for zero elongation gave the modulus of elasticity. The values are given in round numbers.

The results are shown in Table 4, and the specimens have been tabulated so that the percentage of chromium increases from the top to the bottom of the column.

TABLE 4

COMPOSITION, MODULUS OF ELASTICITY, AND ULTIMATE TENSILE STRENGTH

Number	Composition of the Alloy as Found by Analysis			Modulus of Elasticity in lb. per sq. in.	Ultimate Tensile Strength in lb. per sq. in.
	Cr	Cu	Ni		
14	8.25	74.63	16.05	18,000,000	19,978
17	10.13	42.56	48.00	....	23,833
25	13.62	66.92	19.20	7,000,000	14,880*
24	14.56	56.30	29.24	20,000,000	17,521
26	19.30	44.08	36.34	12,000,000	14,802
28	19.64	22.20	57.36	....	56,255
29	19.64	10 88	68.62	7,000,000	19,846†
33	22.00	59.26	19.48	17,600.000	22,650
34	25.10	45.70	28.52	20,000,000	25,768
36	28.10	22.58	48.42	23,700,000	32,654 plus
38	28.44	....	71.56	15,800,000	36,958
35	29.46	33.76	36.78	16,300,000	38,734
37	29.70	10.90	58.12	22,000,000	29,630
42	38.16	33.60	26.78	26,400,000	7,449
44	43.30	11.02	46.46	16,300,000	29,587
49	47.54	24.12	26.28	43,200,000	22,336 plus
48	54.92	28.42	17.12	57,000,000	33,842†
51	57.40	....	41 66	48,000,000	37,200

*Showed a flaw at the fracture.

†The threads were not straight and these values are considered less reliable than the others.

The following is given as a typical set of data. The complete data for each specimen is not included.

ALLOY NO. 35

DIAMETER 0.305 IN. CROSS SECTION, 0.07306 SQ. IN.

Extensometer Readings	Elongation in .0008 in.	Load in Pounds	Unit Load	Remarks
3.00	0.0000	150	2,053	
3.30	0.30	380	5,201	
3.63	0.63	625	8,554	
3.93	0.93	894	12,236	
4.23	1.23	1140	15,603	
4.55	1.55	1385	18,956	
4.83	1.83	1625	22,241	
5.18	2.18	1790	24,500	
5.60	2.60	2000	27,374	
Ultimate		2830	38,734	

Reduction and Elongation.—The per cent of reduction of area was quite small in all cases, in fact, it was so slight that measurements with a micrometer would not give appreciable decreases in diameters. The curves in Fig. 5 show that the elongations were small. These alloys do not show well-defined yield points or elastic limits but a gradual flattening of the stress deformation curves until the ultimate is reached. The curves are similar to those for cast iron.

Ultimate Strength.—The ultimate tensile strength varied from 7,449 to 56,255 pounds per square inch cross-section. The binary alloy of chromium and nickel had more ultimate tensile strength than the ternary alloy in which 10 per cent of the nickel had been replaced by copper. (See Nos. 38 and 37.) It does not seem possible, however, to draw any general conclusions from these results as to which composition will have the greatest tensile strength. The remarkably low ultimate strength for No. 42 is probably due to the fact that it contains a large amount of chromium (38.16 per cent) and at the same time a large amount of copper (33.60 per cent), which produces an unstable condition because it has not been possible to prepare binary alloys of copper and chromium containing more than about 13 per cent of chromium.

Modulus of Elasticity.—Although there are some irregularities in the values obtained, it seems that the modulus of elasticity increases with an increase in chromium content. The values obtained vary from 15,000,000 to more than 40,000,000, or by from ½ to 1½ times the value for iron and steel.

A general idea of the properties of these alloys may be obtained from the stress-deformation curves which are given in Fig. 5.

IV. Corrosion Tests

17. *Reason for Tests.*—In order to determine the power of the different alloys to resist corrosion, over three hundred tests were made. It was hoped that such a series of tests would show that some of these alloys were highly resistant to corrosion and suited for industrial purposes requiring that property.

18. *Materials.*—The corroding reagents used were normal solutions of nitric acid, hydrochloric acid, sulfuric acid, sodium hydroxide, ammonium hydroxide, sodium chloride, and molten fatty acids. The normal solutions had been carefully standardized and were known to be reasonably accurate. The fatty acids were secured from Swift & Co. of Chicago and were listed as double distilled. An analysis showed that they had a mean molecular weight of 273, an iodine value of 72, and consisted of a mixture of fatty acids. The alloy specimens used in the tests were usually about 5 mm. thick and about 15 to 20 mm. in diameter. They were ground and then polished with

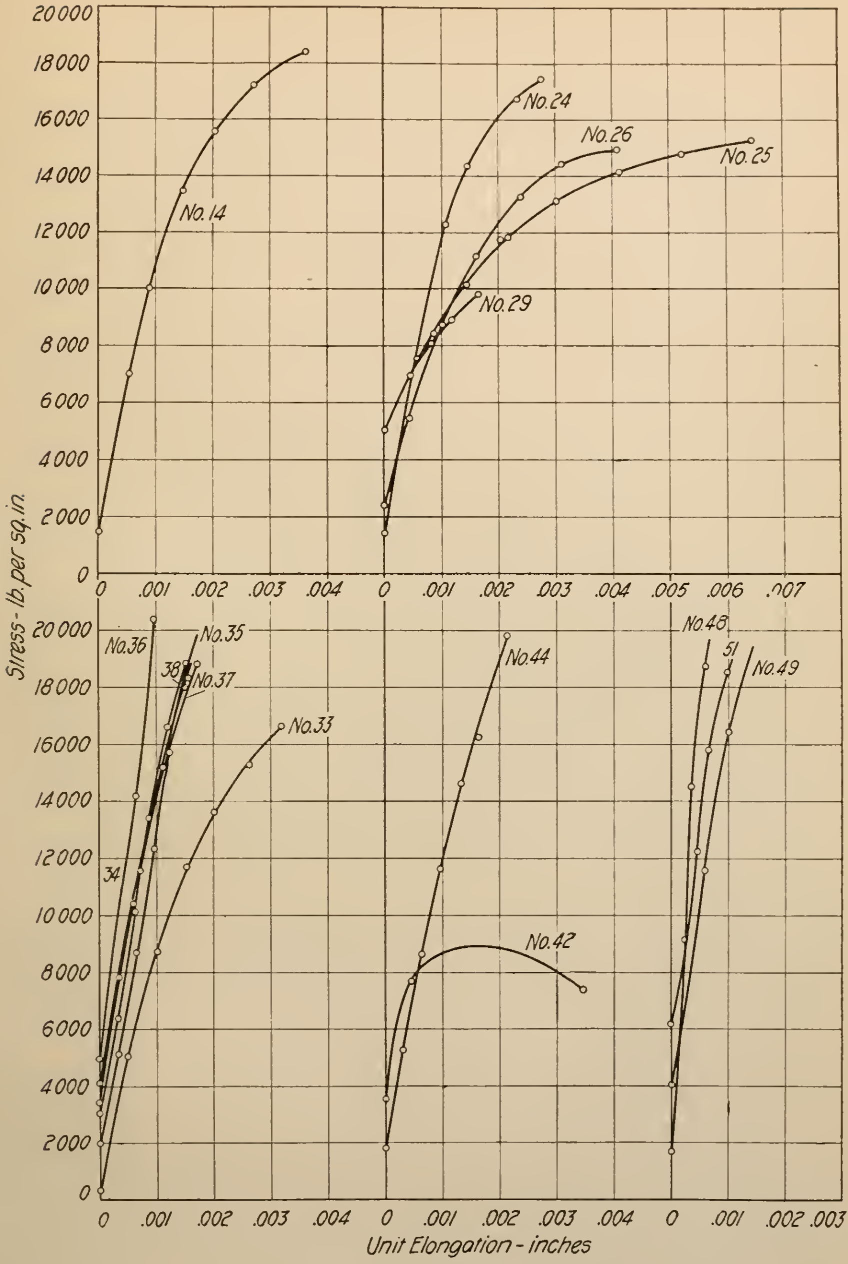

FIG. 5. STRESS DEFORMATION CURVES FOR CR-CU-NI ALLOYS.

No. 000 Hubert Emery paper. The three metals were included in the tests with the alloys. Fifty different samples were tested.

19. *Methods.*—There does not seem to be any well-established method for making corrosion tests. Different investigators have used a variety of methods. For the work in hand, the scheme used has been reasonably satisfactory and it will be described somewhat in detail.

Solutions.—In the case of the normal solutions mentioned under Section 18, 400 cc. were used for each test. No attempt was made to remove any dissolved gases, such as carbon dioxide. The solutions were contained in beakers of such size that they were filled almost to the top. Of course, the solutions changed slightly in concentration because of evaporation and of corrosion of the specimens, but that could not be avoided. The loss by evaporation was made very small by covering the beakers with watch glasses. With such a large volume of solution the change in concentration was not important in most cases. If it was observed that a specimen was dissolving very rapidly, it was removed and from the amount which had been corroded a calculation was made to find how much would have corroded in one week. One hundred grams of the fatty acids were taken for each of the tests that were made at 105 degrees C. and 200 grams for those made at 85 degrees C.

Suspending the Specimens.—Cotton thread was used to suspend the specimens in the alkaline solutions and wool thread was used in the acid solutions. The specimens were completely immersed in the liquid so that no part was in contact with the air. In some cases there was a tendency for the solutions to creep out of the beakers by way of the threads, but that was prevented by putting a little paraffin on the thread.

Temperature.—The corrosion tests were made at the temperature of the laboratory. In order to be able to approximate the mean temperature, a maximum and minimum thermometer was kept on the desk where the tests were being made and readings were taken from time to time. A calculation showed that the mean temperature for the whole period of time was 20 degrees C.

Time.—It was intended to leave the specimens in the solutions for one week, but in some cases it was necessary to remove them before the end of that period.

Calculations.—The amount of surface exposed to corrosion was determined by measuring the thickness and the diameter of the piece to the nearest 0.1 mm. and then calculating the total surface. Unless there was known to have been a big loss in corrosion, the surface was not recalculated when the piece was used for a second test. The results have been calculated to loss in weight per week per square inch of surface exposed and have been expressed in milligrams.

Accuracy of the Method.—The accuracy of the method has not been all that might be desired. The specimens were weighed before and after the tests and the loss in weight was taken as the loss due to corrosion. In a few cases a gain in weight was observed. It was hard to get all of the salts, etc. out of some of the porous pieces, but by boiling in distilled water and drying at 100 degrees C. most of that trouble was avoided. While some of the results show irregularities, as a whole they may be considered reasonably accurate and reliable.

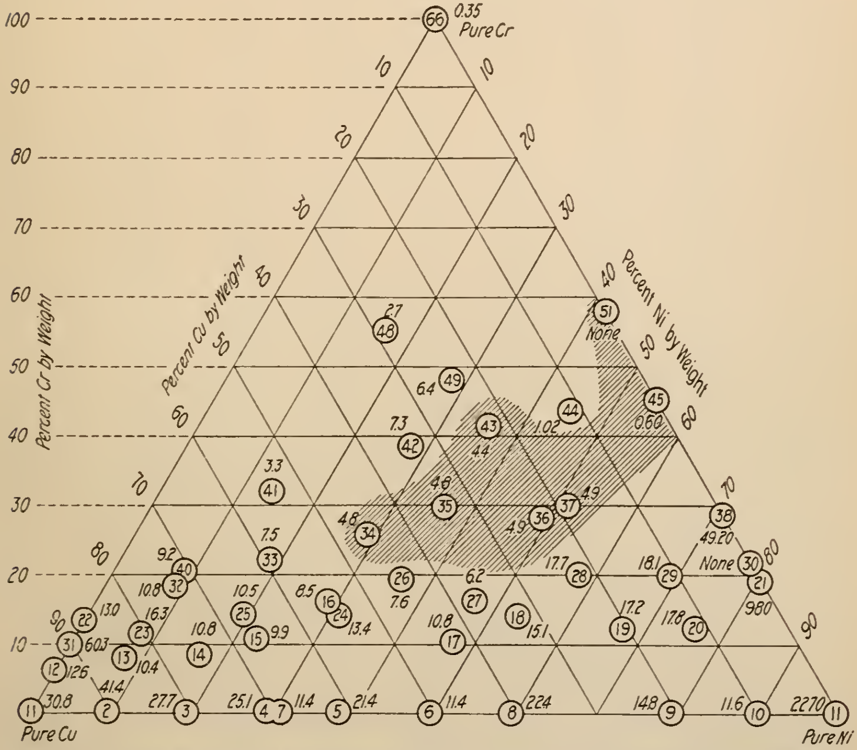

Fig. 6. Diagram Showing Composition of Alloys and Corrosion in Normal Nitric Acid Solution.

20. *Results.*—The results of the corrosion tests are shown on triangular diagrams as explained previously (p. 14). In each diagram the loss in weight in milligrams per square inch per week for each

alloy tested is shown above the circle, the center of which represents the composition of the alloy as determined by analysis.

Complete data of the tests with normal nitric acid are given in Table 5, and by comparison of this table with the corresponding diagram (Fig. 6), the method of plotting can be readily understood. Such detailed data has been omitted for the tests with other solvents, the results being shown only in the graphical form. Areas of minimum corrosion are designated on the diagrams by shading.

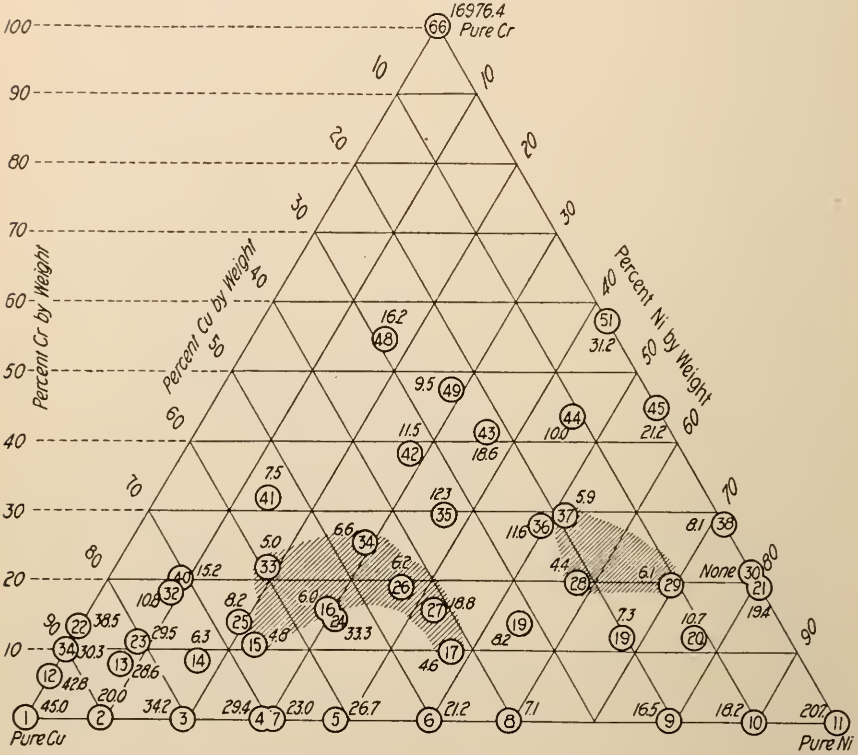

FIG. 7. DIAGRAM SHOWING COMPOSITION OF ALLOYS AND CORROSION IN NORMAL HYDROCHLORIC ACID SOLUTION.

Nitric Acid.—Table 5 and Fig. 6 show the results obtained with a normal solution of nitric acid. Nickel, especially, is attacked, and alloys with high percentages of this metal show great corrosion. In the chromium-nickel alloys there is a gradual decrease in the loss by corrosion as the per cent of chromium increases, with an abrupt drop at Alloy No. 45. There is also a marked change in the micro-structure

TABLE 5

CORROSION IN NORMAL NITRIC ACID

Number	Weight Before	Weight After	Loss in weight mg.	Surface		Time in solution hrs.	Loss in wt. in mg. per wk.	
				sq. in.	sq. cm.		sq. in.	sq. cm.
1	5.5125	5.4945	18.0	0.71	4.58	138	30.80	4.78
2	8.6232	8.5882	35.0	1.03	6.67	138	41.40	6.42
3	5.6478	5.6323	15.5	0.68	4.41	138	27.70	4.30
4	4.5331	4.5205	12.6	0.61	3.95	138	25.10	3.38
5	4.5007	4.4898	10.9	0.64	4.12	138	21.40	3.33
6	4.5918	4.5859	5.9	0.63	4.04	138	11.40	1.77
7	16.9032	16.8890	14.2	1.52	9.83	138	11.40	1.77
8	11.3342	11.3092	25.0	1.36	8.77	138	22.40	3.47
9	9.8259	9.8086	14.5	1.28	8.27	138	14.80	2.13
10	10.0842	10.0724	11.8	1.24	7.98	138	11.60	1.80
11	9.8700	9.4410	429.0	1.32	8.49	24	2270.00	352.00
12	11.7920	11.7783	13.7	1.33	8.58	137	12.60	1.98
13	13.1763	13.1647	11.6	1.35	8.69	137	10.40	1.62
14	10.0412	10.0310	10.2	1.16	7.47	137	10.80	1.67
15	8.4206	8.4123	8.3	1.03	6.66	137	9.90	1.53
16	7.3881	7.3807	7.4	1.07	6.94	137	8.50	1.32
17	12.8800	12.8684	11.6	1.32	8.51	137	10.80	1.67
18	10.5626	10.5474	15.2	1.24	7.98	137	15.10	2.33
19	14.3148	14.2931	21.7	1.54	9.92	137	17.20	2.67
20	10.6661	10.6490	17.1	1.18	7.61	137	17.80	2.76
21	13.2433	12.8326	410.7	1.47	9.43	48	980.00	152.00
22	9.0392	9.0390	10.2	1.07	6.91	122	13.10	2.03
23	9.1265	9.1151	11.4	1.03	6.67	122	16.30	2.22
24	10.5944	10.5821	12.3	1.26	8.10	122	13.40	2.08
25	11.1470	11.1376	9.4	1.23	7.92	122	10.50	1.63
26	13.1489	13.1412	7.7	1.39	8.95	122	7.60	1.18
27	9.7288	9.7222	6.6	1.08	7.00	168	6.20	0.95
28	9.8098	9.7948	15.0	1.17	7.56	122	17.70	2.74
29	8.0788	8.0655	13.3	1.03	6.65	122	18.10	2.80
30	Too many holes.							
31	11.1653	11.0955	69.8	1.16	7.46	168	60.30	9.35
32	12.1926	12.1777	14.9	1.38	8.92	168	10.80	1.67
33	9.7367	9.7270	9.7	1.29	8.33	168	7.50	1.16
34	11.6059	11.5994	6.5	1.37	8.82	168	4.80	0.74
35	8.8904	8.8847	5.7	1.18	7.62	168	4.80	0.74
36	9.6290	9.6241	5.9	1.20	7.73	168	4.90	0.76
37	10.8796	10.8738	5.8	1.31	8.43	168	4.90	0.76
38	9.2554	9.0129	242.5	1.18	7.62	70	492.00	76.00
39	Not prepared.							
40	13.7427	13.7295	13.2	1.45	9.33	168	9.20	1.43
41	11.5077	11.4963	11.4	1.37	8.83	168	8.30	1.29
42	10.1818	10.1725	9.3	1.28	8.23	168	7.30	1.13
43	11.5401	11.5341	6.0	1.35	8.70	168	4.40	0.69
44	11.8160	11.8146	1.4	1.37	8.81	168	1.02	0.16
45	10.5332	10.5325	0.7	1.16	7.48	168	0.60	0.09
46	Not prepared.							
47	Not prepared.							
48	10.7279	10.7154	12.5	1.29	8.32	168	9.70	1.50
49	9.1910	9.1854	5.6	1.21	7.81	122	6.40	0.99
50	10.4293	10.4198	9.5	1.28	8.23	168	7.40	1.15
51	12.4458	12.4458	0.0	1.42	9.19	168	0.00	0.00
55	9.7665	9.7559	10.6	1.38	8.89	168	7.70	1.19
56	8.4517	8.4513	0.4	1.14	7.33	168	0.35	0.05
66	2.0776	2.0774	0.2	0.57	3.66	168	0.35	0.05

of the alloys in the same region as may be seen by comparing microphotographs of Alloys No. 11, 21, 30, 38, and 45. (Figs. No. 24, 34, 43, 51, and 58.)

Hydrochloric Acid.—The results from these tests are shown in Fig. 7. It will be seen that all of the metals showed considerable corrosion in hydrochloric acid and that chromium was especially soluble in this acid. However, there is a fairly well-defined area in the triangle over which the corrosion was small.

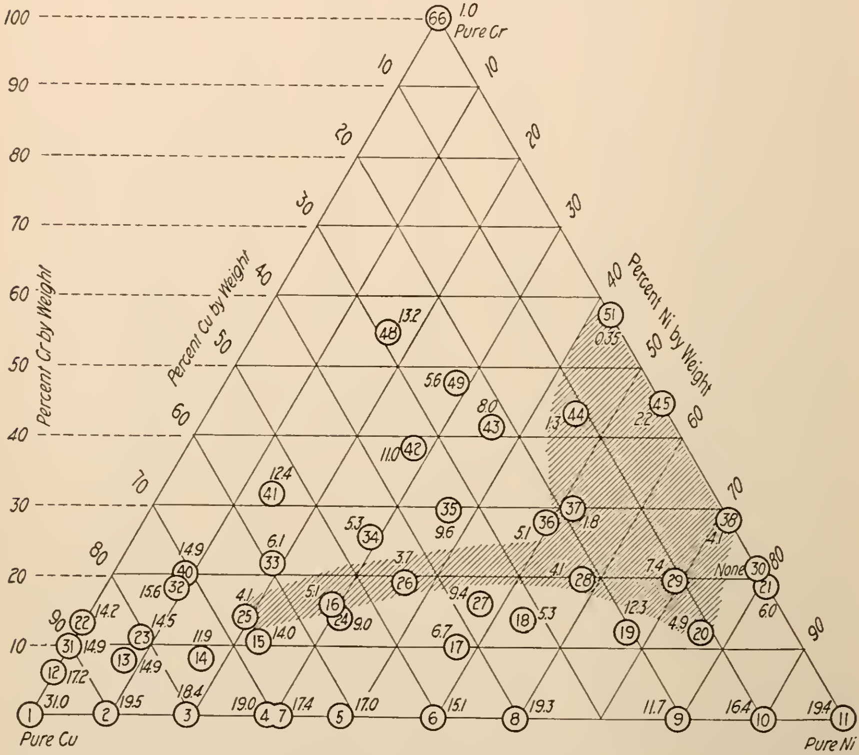

FIG. 8. DIAGRAM SHOWING COMPOSITION OF ALLOYS AND CORROSION IN NORMAL SULFURIC ACID SOLUTION.

Sulfuric Acid.—The data obtained from the corrosion tests in normal sulfuric acid are shown in Fig. 8. The results do not require any special explanation.

Sodium Hydroxide.—Fig. 9 shows the results obtained by the corrosion tests in normal sodium hydroxide. Copper and nickel show appreciable losses, while chromium is little attacked. In general the losses for the alloys are small.

Ammonium Hydroxide.—The alloys which best resist corrosion by ammonia are near the nickel corner of the triangle and on the chromium-nickel side. The results are shown in Fig. 10.

The copper-chromium and the copper-rich alloys show a selective corrosion in ammonium hydroxide; that is, the copper is removed and some of the chromium is left. This fact suggests that good results might be obtained by using ammonia as an etching reagent in the preparation of specimens for microscopic examination.

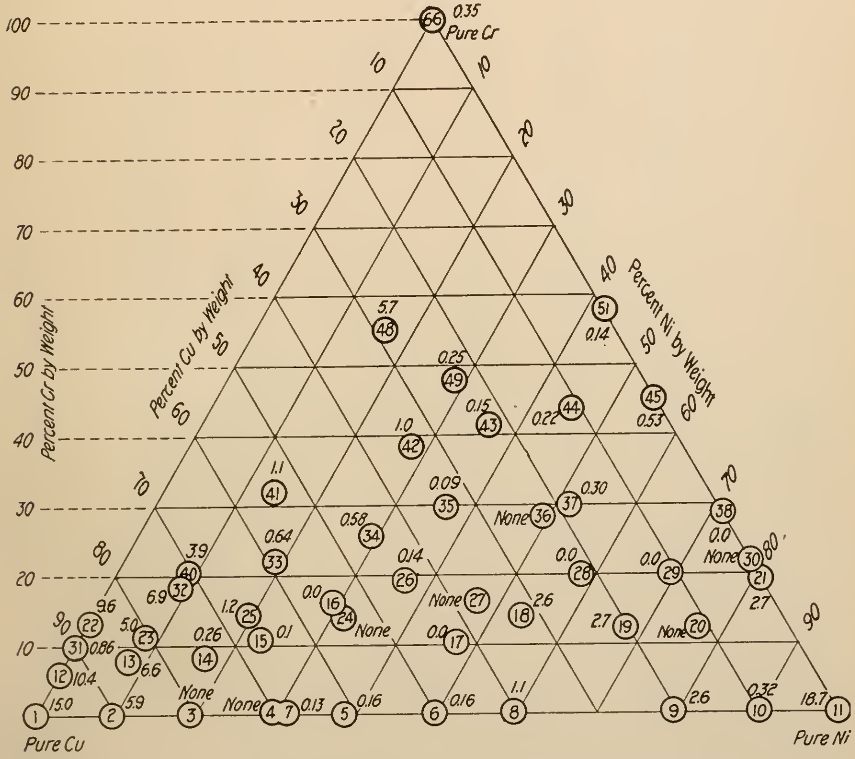

FIG. 9. DIAGRAM SHOWING COMPOSITION OF ALLOYS AND CORROSION IN NORMAL SODIUM HYDROXIDE SOLUTION.

Sodium Chloride.—Losses in normal sodium chloride solution are, in general, small. These losses are largest for alloys near the copper corner of the triangle. (See Fig. 11.)

Fatty Acids.—Only twenty-four specimens were corroded in the fatty acids. Twelve of the tests were made at 105 degrees C. by heating in an electric oven. The other tests were made at approximately

85 degrees C. by heating on a steam bath. The results are shown in Fig. 12. The greatest losses are near the copper corner of the triangle. This suggests that instead of using a copper container for fatty acids it might be advisable to use an alloy of copper and nickel, or possibly one containing copper and nickel with a little chromium.

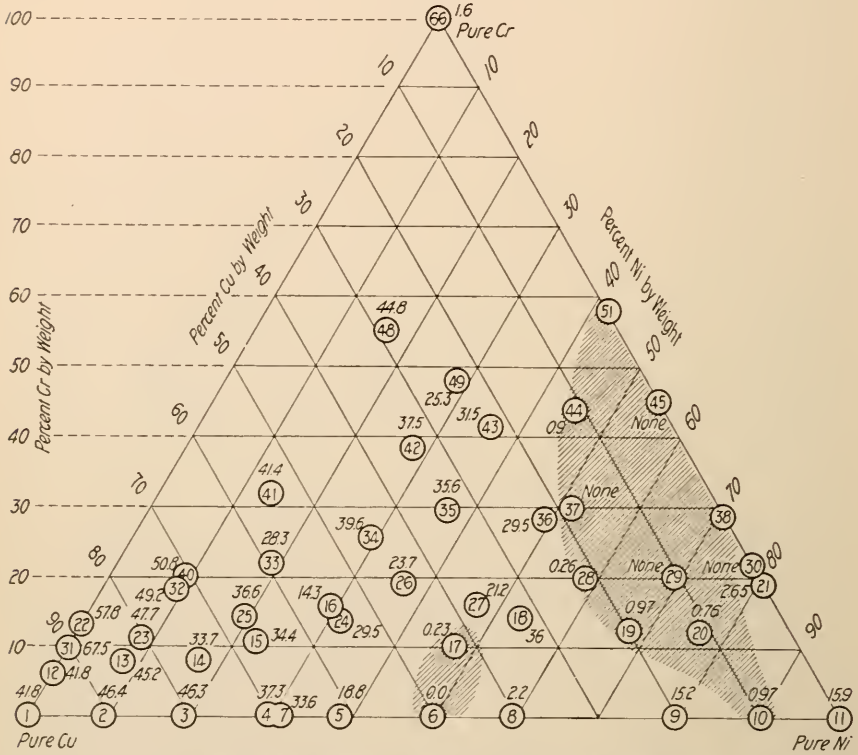

FIG. 10. DIAGRAM SHOWING COMPOSITION OF ALLOYS AND CORROSION IN NORMAL AMMONIUM HYDROXIDE SOLUTION.

Comparison of Corrosions.—Table 6, page 31, shows the relative corrosions in the different reagents which have been used. The values are losses in weight in milligrams per square inch per week.

21. *Conclusions.*—The corrosion tests show that the amount of loss in the different reagents is not proportional to the strengths of the different acids or bases.

So far, it has not been possible to show any definite relation between the relative electromotive forces and corrosion losses.

TABLE 6

COMPARISON OF THE CORROSIONS IN THE DIFFERENT SOLUTIONS

THE VALUES ARE LOSS IN WEIGHT IN MILLIGRAMS PER WEEK PER SQUARE INCH

Number	NaCl	HCl	H_2SO_4	HNO_3	NaOH	NH_4OH	Fatty acids
1	5.08	45.0	31.00	30.80	15.00	41.80	8.90
2	4.18	20.0	19.50	41.40	5.90	46.40	...
3	1.60	34.2	18.40	27.70	None	46.30	5.80
4	1.65	29.4	19.00	25.10	None	37.30	...
5	2.35	26.7	17.00	21.40	0.16	18.80	2.90
6	2.37	21.2	15.10	11.40	0.16	0.00	...
7	0.78	7.1	19.30	22.40	1.10	2.20	...
8	1.71	23.0	17.40	11.40	0.13	35.60	4.00
9	1.71	16.5	11.70	14.80	2.60	15.20	...
10	1.13	18.2	16.40	11.60	0.32	0.97	...
11	None	207.0	19.40	2270.00	18.70	15.90	...
12	4.33	42.8	17.20	12.60	10.40	41.80	10.50
13	2.55	28.6	14.90	10.40	6.60	45.20	...
14	1.89	6.3	11.90	10.80	0.26	33.70	7.70
15	1.53	4.8	14.00	9.90	0.10	34.40	...
16	0.66	6.0	5.10	8.50	0.00	14.30	...
17	0.53	4.6	6.70	10.80	0.00	0.23	None
18	0.71	8.2	5.30	15.10	2.60	3.60	...
19	0.63	7.3	12.30	17.20	2.70	0.97	1.14
20	0.60	10.7	4.90	17.80	None	0.76	...
21	None	19.4	6.00	980.00	2.70	2.65	...
22	4.00	38.3	14.20	13.10	9.60	57.80	...
23	3.65	29.5	14.50	16.30	5.00	47.70	6.40
24	0.76	33.3	9.00	13.40	None	29.50	...
25	2.13	8.2	4.10	10.50	1.20	36.60	6.60
26	0.94	6.2	3.70	7.60	0.14	23.70	3.80
27	0.85	18.8	9.40	6.20	None	21.20	0.87
28	0.37	4.4	4.10	17.70	0.00	0.26	...
29	0.51	6.1	7.40	18.10	0.00	0.00	0.25
30	Too many blowholes.						
31	4.60	30.3	16.40	60.30	0.86	67.50	10.70
32	3.30	10.8	15.60	10.80	6.90	49.20	...
33	2.62	5.0	6.10	7.50	0.64	28.30	7.50
34	1.37	6.6	5.30	4.80	0.58	39.60	...
35	1.05	12.3	9.60	4.80	0.09	35.60	3.80
36	0.37	11.6	5.10	4.90	None	29.50	...
37	...	5.9	1.80	4.90	0.30	None	...
38	0.37	8.1	4.10	492.00	0.00	None	8.30
39	Not prepared.						
40	2.95	15.2	14.90	9.20	3.90	50.80	11.30
41	1.84	7.5	12.40	8.30	1.10	41.40	...
42	1.67	11.5	11.00	7.30	1.00	37.50	2.90
43	1.25	18.6	8.00	4.40	0.15	31.50	...
44	0.45	10.0	1.30	1.02	0.22	0.90	0.34
45	0.40	21.2	2.20	0.60	0.53	0.00	...
46	Not prepared.						
47	Not prepared.						
48	2.45	16.2	13.20	9.70	5.70	44.80	3.25
49	0.87	9.5	5.60	6.40	0.25	25.30	...
50	0.37	19.5	6.40	7.40	1.00	0.90	2.40
51	0.27	31.2	0.35	0.00	0.14	0.00	...
55	0.33	11.9	3.70	7.70	1.00	0.20	...
56	0.27	32.2	1.30	0.35	0.44	0.50	...
66	2.00	16976.4	1.00	0.35	0.35	1.60	1.23

In all cases there are certain fairly well-defined ranges of composition as shown in the diagrams in which the alloys are highly resistant to corrosion.

Generally the ternary alloys are less corroded than the binary, though there are some exceptions.

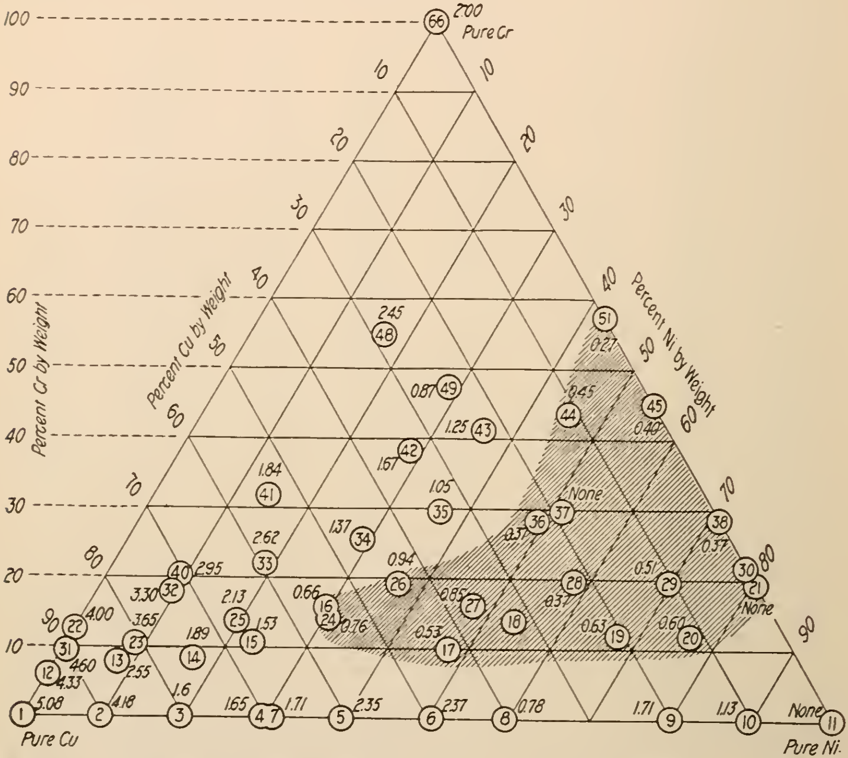

FIG. 11. DIAGRAM SHOWING COMPOSITION OF ALLOYS AND CORROSION IN NORMAL SODIUM CHLORIDE SOLUTION.

V. RELATION BETWEEN CORROSION AND RELATIVE ELECTROMOTIVE FORCES

22. *Purpose of Measurements.*—A series of measurements was made of the relative electromotive forces developed by the different alloys when placed in contact with an electrolyte, to ascertain whether or not any simple relation could be found between the values obtained and the amount of corrosion sustained by the same alloys in the same electrolyte. The measurements were made in a sodium chloride solution and the data obtained is of considerable interest, but it is not sufficient to afford a basis for any general conclusions.

23. *Method and Results of Measurements.*—After some experimentation with other methods the Poggendorf Compensation Method* of measurement was chosen. Briefly described, it consisted in balancing the current from two No. 6 French Auto Special Dry Batteries against the current from a calomel electrode and the alloy in contact with 4 Normal salt solutions. The resistance in the first circuit was 110 ohms. This resistance was varied in the second circuit until the galvanometer showed a zero deflection. A voltmeter, reading 0 to 1.5 volts, was so connected that it showed directly the potential in the calomel electrode-alloy combination. In the different series of measurements all conditions remained the same, except that different alloys were placed in salt solution. Readings were taken at one minute intervals for five minutes. The results are shown in Table 7.

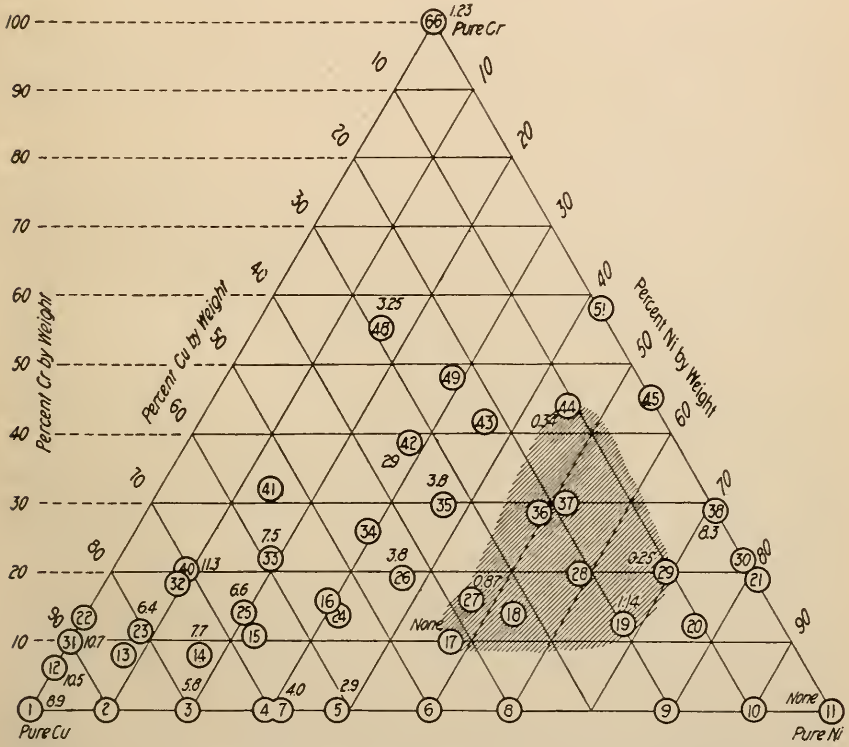

Fig. 12. Diagram Showing Composition of Alloys and Corrosion in Fatty Acids

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*Stahler, A. Handb. d. Arbeitsmethoden in der anorg. Chem., 1914.

These values are not absolute, but relative, for they were made under the same conditions. The following factors seemed to influence the results: (a) The condition of the specimen as regards polishing. (b) The depth to which the piece was immersed in the solution. The latter factor was active until the piece was at least one-half covered; after that the value did not seem to change even upon complete immersion. (c) The time that the solution and specimen were left in contact, especially if any current was allowed to flow through the system. It is believed that this effect was caused by the deposition of gases upon the specimen or possibly by gases already dissolved in them. (d) Any moving of the specimen or solution.

It was noticed that the alloys which contained high percentages of copper gave values which increased with time, whereas those containing high percentages of nickel gave values which decreased with time. These results together with the composition of the alloys are shown diagrammatically in Fig. 13. Those alloys which showed an increase have been marked with a plus sign and those which showed a decrease have been marked with a minus sign. Alloys Nos. 17, 18, and 19 seem irregular, but they contained blowholes; and if there was any concentration of the low melting constituent (copper) at the blowholes, it might be expected that they would behave like the copper-rich alloys. No. 24 is the only one which did not show a change in the five minute interval, although the changes shown by some of the others were very small. From an examination of the corrosion tables it may be seen that the alloys which showed little or no change in relative electromotive force were not immune to corrosion in the different solutions. Therefore, it does not seem that resistance to corrosion can be predicted from the fact that the relative electromotive force of the alloy in contact with an electrolyte either remained constant or showed little variation.

In the case of the corrosions in normal salt solutions it was noticed that, almost without exception, the alloys which had shown an increase in the relative electromotive force were the ones which showed a turbidity of the corroding solution, while those which had given decreasing values remained clear. Similarly, in the corrosions in ammonium hydroxide the solutions were colored a deep blue in the case of the alloys which had shown increases in the relative electromotive forces, while the others remained practically colorless. However, when the corrosion specimens were weighed, it was found that there had been losses in the salt solutions, which had remained clear, and in the ammonium hydroxide solutions, which had remained colorless.

From the results obtained it is not felt that any safe conclusions can be drawn as to the possibility of predicting the corrodibility of an alloy from such a series of measurements of the relative electromotive forces or from the changes in these values with time.

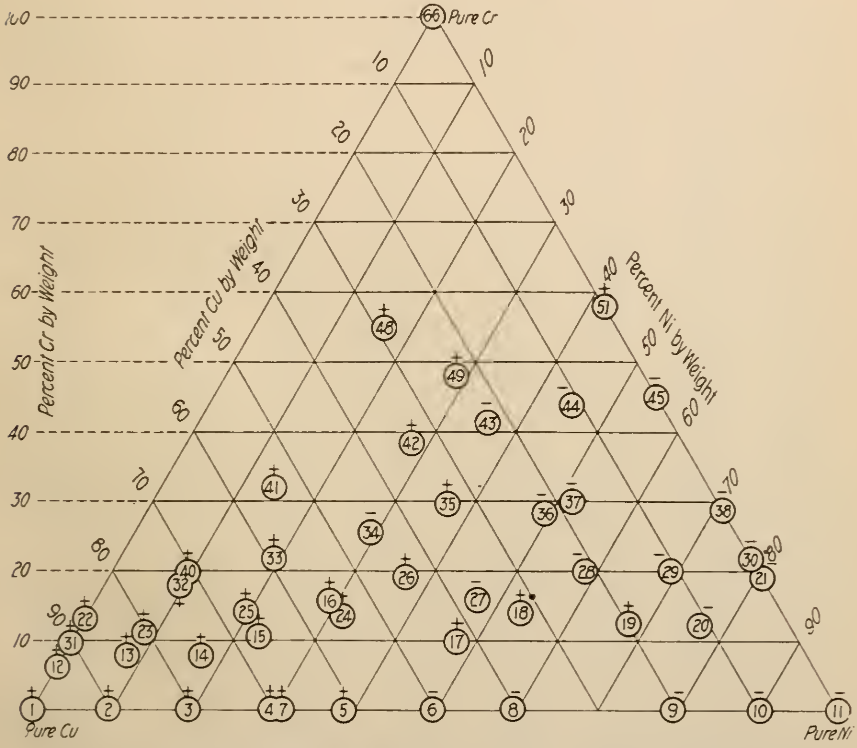

FIG. 13. DIAGRAM SHOWING COMPOSITION OF ALLOYS AND THEIR CHANGES IN ELECTROMOTIVE FORCE IN A 4 NORMAL SODIUM CHLORIDE SOLUTION.

VI. THERMAL ANALYSIS AND MICROSCOPIC EXAMINATION.

24. *Difficulties of Thermal Analysis.*—The very high melting points of the metals and alloys of this system, their great susceptibility to oxidation and absorption of impurities at high temperatures, and the extreme difficulty of finding suitable materials for pyrometers, tubes, and crucibles, all combined to make the study of freezing point curves extremely difficult, and, so far as present developments are concerned, almost impossible.

TABLE 7

RELATIVE ELECTROMOTIVE FORCES OF THE DIFFERENT ALLOYS IN CONTACT WITH 4 NORMAL SALT SOLUTIONS

Number	Relative Electromotive Force After						
	1 min.	2 min.	3 min.	4 min.	5 min.	Change	Remarks
1	...	...	...	...	0.364	...	Increasing
2	0.328	0.328	0.334	0.331	0.331	+ .003	
3	0.328	0.330	0.331	0.333	0.334	+ .006	
4	0.311	0.316	0.317	0.318	0.319	+ .009	
5	0.314	0.318	0.315	0.318	0.315	+ .001	
6	0.309	0.302	0.296	0.295	0.293	— .016	
7	0.313	0.315	0.315	0.318	0.318	+ .005	
8	0.305	0.300	0.300	0.297	0.296	— .009	
9	0.295	0.295	0.290	0.280	0.280	— .015	
10	0.318	0.309	0.300	0.294	0.289	— .029	
11	0.294	0.294	0.290	0.287	0.287	— .007	Pure nickel
12	0.338	0.341	0.345	0.346	0.348	+ .010	
13	0.345	0.343	0.348	0.348	0.350	+ .005	
14	0.340	0.341	0.341	0.339	0.339	— .001	
15	0.348	0.345	0.344	0.342	0.341	— .007	
16	0.342	0.336	0.333	0.332	0.330	— .012	
17	0.310	0.309	0.306	0.306	0.304	— .006	
18	0.307	0.304	0.303	0.303	0.303	— .004	
19	0.306	0.312	0.315	0.316	0.318	+ .012	
20	0.331	0.325	0.315	0.309	0.300	— .031	
21	0.294	0.282	0.279	0.276	0.275	— .019	
22	0.357	0.360	0.363	0.363	0.363	+ .006	
23	0.345	0.347	0.348	0.349	0.349	+ .004	
24	0.307	0.306	0.307	0.307	0.307	+ .000	
25	0.315	0.319	0.322	0.324	0.325	+ .010	
26	0.336	0.339	0.339	0.339	0.339	+ .003	
27	0.330	0.326	0.320	0.320	0.320	— .010	
28	0.355	0.344	0.339	0.332	0.330	— .025	
29	0.360	0.349	0.341	0.338	0.332	— .028	
30	0.325	0.320	0.329	0.331	0.324	— .001	
31	0.339	0.340	0.345	0.346	0.349	+ .010	
32	0.330	0.337	0.339	0.340	0.341	+ .011	
33	0.341	0.347	0 346	0.346	0.346	+ .005	
34	0.340	0.339	0.339	0.338	0.338	— .002	
35	0.333	0.333	0.334	0.334	0.334	+ .001	
36	0.334	0.330	0.329	0.320	0.320	— .014	
37	0.332	0.330	0.320	0.319	0.315	— .017	
38	0.316	0.310	0.305	0.300	0.296	— .020	
39	...	...	...	...	...	...	Not prepared
40	0.342	0.346	0.352	0.355	0.355	+ .013	
41	0.350	0.351	0.352	0.355	0.355	+ .005	
42	0.331	0.335	0.339	0.339	0.339	+ .008	
43	0.366	0.365	...	0.350	0.346	— .020	
44	0.314	0.300	0.290	0.286	0.286	— .028	
45	0.322	0.315	0.309	0.302	0.300	— .022	
46	...	...	...	...	...	...	Not prepared
47	...	...	...	...	...	...	Not prepared
48	0.338	0.345	0.349	0.351	0.350	+ .012	
49	0.345	0.347	0.349	0.349	0.349	+ .004	
50	0.298	0.299	0.298	0.295	0.295	— .003	
51	0.468	0.473	0.476	0.480	0.480	+ .012	

Note: The other specimens have either not been prepared or have not been tested.

The handicaps were overcome in part by the construction of a specially designed granular carbon resistance furnace which gave sufficiently high temperatures for the melting of the alloys. Fairly satisfactory crucibles were made of fused magnesia, which material served also as a satisfactory refractory for parts of the furnace. By means of these crucibles, melts of about 200 grams could be made.

The greatest difficulty, however, was encountered in attempting to find a protecting tube for the thermo couple which would withstand the severe conditions to which it would be exposed. Quartz tubes are good for low temperatures, but they cannot be used at the melting point of chromium. Porcelain tubes, such as were used by Hindrichs, stand slightly higher temperatures than quartz without softening, but they break easily and are attacked by chromium. An attempt was made to use an alundum tube, but it broke in the first melt. Since it was known that the fused magnesia crucibles were little attacked by chromium, it was thought that an insulating tube of the same material might be satisfactory. Some tubes were moulded, dried, and heated to about 1500 degrees C. They became hard and dense, but bent during the heating. It apears that in heating the magnesia to 1500 degrees C. it passes through a semi-fused state at which time sintering takes place, but at that same time the tubes bend so badly that they can not be used. Some magnesia tubes were moulded in which a solution of magnesium chloride was used as the binding material. These tubes, after drying at 105 degrees C., were hard and looked promising, but when heated they became brittle and crumbled to pieces at 800 to 900 degrees C. So far, all attempts to prepare satisfactory magnesia insulating tubes have failed. Because of so many difficulties and a limited amount of time, it was thought best to omit this part of the work for the time being, reserving it for later investigation.

25. *Heat Treatment.*—As was explained in the introduction it has not been considered advisable to attempt much in the way of heat treatment of the alloys until the equilibrium diagrams have been more thoroughly established.

Some annealing tests have been carried out and are mentioned at the close of this chapter.

26. *Microscopic Examination.*

a. *General Discussion.*—The metallurgical microscope used in the examination of these alloys was a Leitz "Micrometallograph." For the most part apochromatic objectives were used with projection eyepieces. Many difficulties were met with in the preparation of the different alloys for microscopic study. No two alloys had exactly the same composition. It is almost equally true that no two of the alloys would give the best results by the same methods of polishing, etching, staining, and photographing. Thus each specimen became a research problem in itself.

b. *Etching Reagents.*—A study of the different etching reagents, stains, etc., described in the literature, was made in an endeavor to find means of identifying the different constituents in the alloys. Descriptions of the two reagents which have been found most effective follow:

Ferric Chloride in Hydrochloric Acid.—This solution consisted of 1 per cent $FeCl_3$ dissolved in 1:1 HCl. It has been found very useful. In some cases it was found necessary to dilute it with one or more volumes of water because the etching was too rapid. An attempt was made to secure colorations by using a mixture of 25 cc. of this solution with 25 cc. of glycerine, 2 grams of resorcin, and 50 cc. of water, but it did not give any better results.

Iodine.—Iodine has been used as the tincture and in a solution of potassium iodide. These solutions can be used to etch the specimen in some cases, but they have more value in staining the specimen after it has been etched with some other reagent. Iodine stains copper or the copper rich constituent, giving it a dark appearance; because it has little effect on either chromium or nickel, it has proved the best method for identifying the different constituents in these alloys.

Other reagents tried, but with little success, were alkaline potassium permanganate, a mixture of picric and nitric acids in amyl and ethyl alcohols, picric acid in alcohol, sodium picrate in alcohol, and tartaric acid in water.

Additional information regarding the etching and staining used may be obtained by a study of the microphotographs. (Figs. No. 14 to 65.)

c. *Results.*—Some of the results obtained from the microscopic examination have been referred to in the previous discussions. In general the data given with the microphotographs are sufficient to explain them. The composition given is in weight per cent as found by analysis, unless otherwise stated.

Chromium-Nickel Alloys.—The microscopic examination of the alloys of chromium and nickel seems to confirm Voss' conclusions:

First, that chromium and nickel form a series of solid solutions (mixed crystals) in the alloys containing from 100 to 50 per cent of nickel. (See Figs. 24, 34, 43, 51, and 58. Alloys No. 11, 21, 30, 38, and 45.)

Second, that chromium and nickel form a eutectic which contains about 42 per cent of nickel. Guertler does not think that the metals form a true eutectic, but calls it a pseudoeutectic. Fig. 63, Alloy No. 51, is of an alloy having approximately 42 per cent of nickel and which appears to have a eutectic-like structure. Specimen No. 56, Fig. 65, which should contain more chromium, shows what appears to be the eutectic structure and an excess of chromium. Although alloys containing higher percentages of chromium have been prepared, they have not been analyzed and microphotographs of them have not been made. The reason for this has been the difficulty of cutting proper samples from the extremely hard alloys.

Copper-Nickel Alloys.—The microphotographs of the alloys of copper and nickel agree very well with those obtained by Guertler and Tammann, especially for those obtained under similar conditions; namely, with slow cooling. The alloys containing more than 80 per cent of nickel show large polyhedral crystals. Microphotographs of the pure metals and their alloys are shown in Figs. 14 to 24.

Solubility of Chromium in Copper.—The tendency for chromium or a chromium-rich constituent to separate in alloys of chromium and copper has already been discussed. This effect may be seen in Alloys Nos. 12, 22, and 31; Figs. 25, 35, and 44, and in Figs. 2 and 3. The chromium or chromium-rich constituent shows either in relief or as the light part of the photograph, if the specimen was stained with iodine.

Effect of Nickel on the Solubility of Chromium.—The effect of the addition of nickel to alloys of chromium, copper, and nickel may be studied in the microphotographs of Alloys Nos. 22 to 30, inclusive, Figs. 35 to 43, in which series there is an increase in nickel and a decrease in copper. The separation of the chromium or of the chromium-rich constituent is apparent in the Alloys Nos. 22 to 26. Those containing larger percentages of nickel are more homogeneous. In No. 29, Fig. 42, well-defined polyhedral crystals characteristic of a solid solution are shown. Similar effects will be seen if other series, such as Nos. 12 to 21, Figs. 25 to 34, and Nos. 31 to 38, Figs. 44 to 51 are studied. The alloys become practically homogeneous when the amount of nickel is more than three times the amount of copper present.

Crystals in Nickel-Rich Alloys.—It must be remembered that the alloys studied were intended to have variations of 10 per cent in their different constituents and it is not possible to say at exactly what nickel content the binary alloys begin to show well-defined polyhedral crystals. From the specimens examined, it is evident that in the case of the binary alloys those which contain as much as 80 per cent of nickel show such crystals. On the other hand, the ternary alloys show them if they contain as much as 70 per cent of nickel. Of course, the structures which have been obtained in these alloys represent what may be expected if the castings are slowly cooled, but they do not necessarily show what structure would be produced by quenching.

Annealing Tests.—Small pieces of Alloys Nos. 22 to 38, inclusive, were packed in amorphous silica in an iron pipe, 1¾ by 5 inches in dimensions and closed at both ends by caps. The pipe and contents were placed in an electric furnace and heated at a temperature of approximately 900 degrees C. for at least twenty-four hours. The specimens were repolished and examined microscopically. From a brief examination it seems that there was not any very noticeable change in the structure.

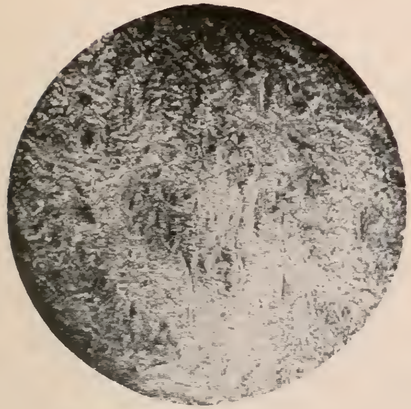

FIG. 14. ALLOY NO. 1. x 40.
CU. 100%
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

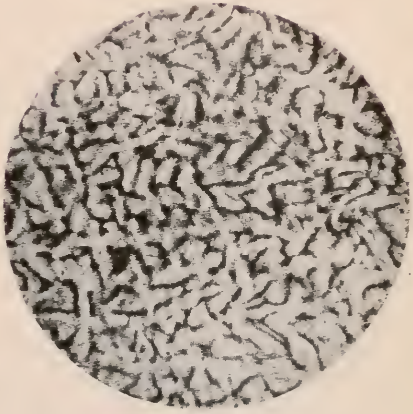

FIG. 15. ALLOY NO. 2. x 40.
90.84% CU, 9.06% NI BY WEIGHT.
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

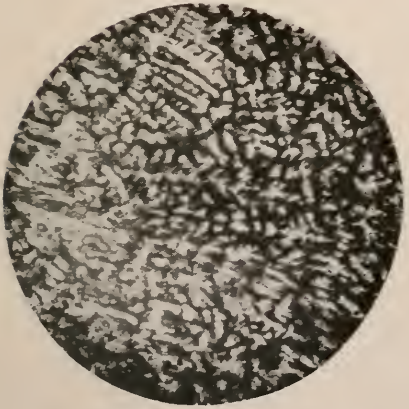

FIG. 16. ALLOY NO. 3. x 40.
81.07% CU, 18.76% NI, BY WEIGHT.
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

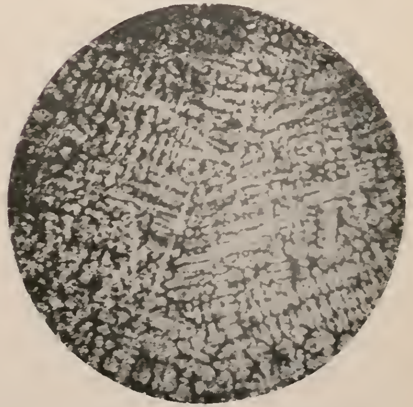

FIG. 17. ALLOY NO. 4. x 40.
71.16% CU, 28.46% NI BY WEIGHT.
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

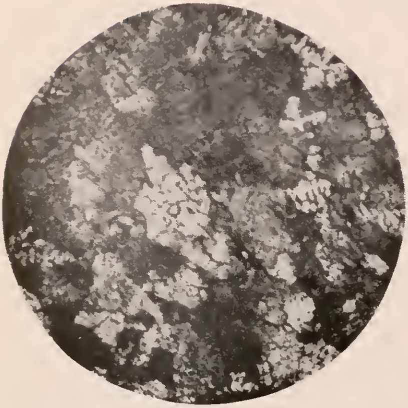

FIG. 18. ALLOY NO. 5. x 40.
61.63% CU, 38.25% NI BY WEIGHT.
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

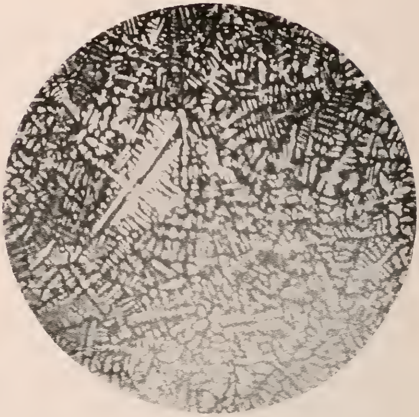

FIG. 19. ALLOY NO. 6. x 40.
48.96% CU, 49.90% NI BY WEIGHT.
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

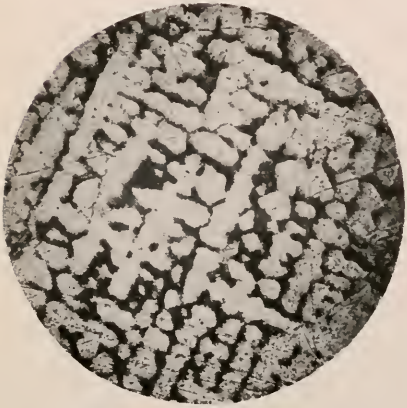

FIG. 20. ALLOY NO. 7. x 40.
69.13% CU, 30.59% NI.
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

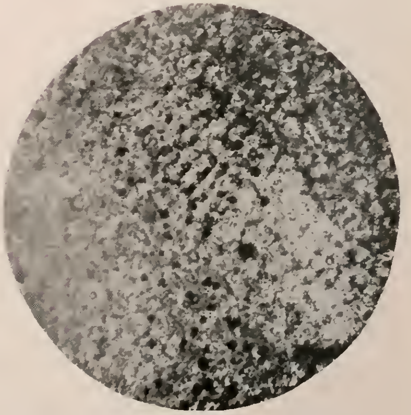

FIG. 21. ALLOY NO. 8. x 40.
41.14% CU, 58.27% NI.
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

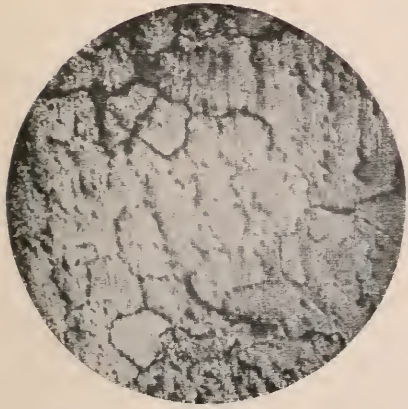

FIG. 22. ALLOY NO. 9. x 40.
20.65% CU, 79.35% NI.
ETCHED IN 1% $FeCl_3$ IN 1:1 HCl.

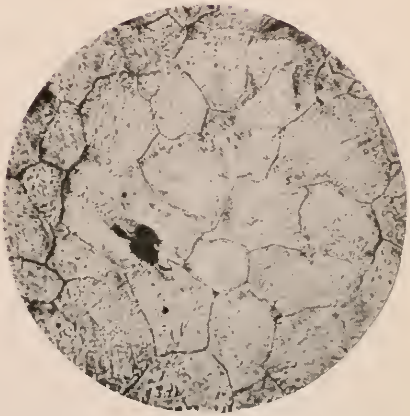

FIG. 23. ALLOY NO. 10. x 40.
10.57% CU, 88.90% NI.
ETCHED IN 1% $FeCl_3$ IN 1:1 HCl.

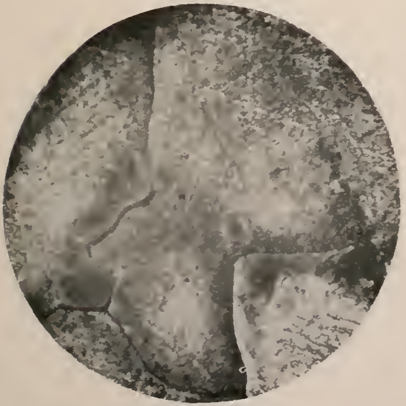

FIG. 24. ALLOY NO. 11. x 40.
99.66% NI.
ETCHED IN 1% $FeCl_3$ IN 1:1 HCl.

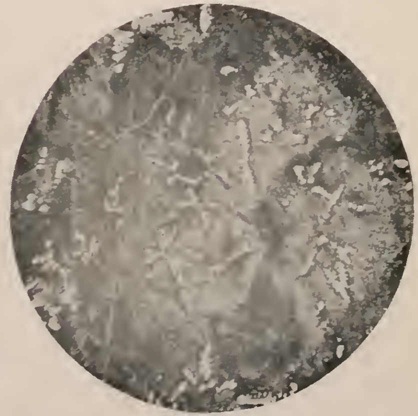

FIG. 25. ALLOY NO. 12. x 40.
6.08% CR, 94.20% CU.
ETCHED IN 1% $FeCl_3$ IN 1:1 HCl.

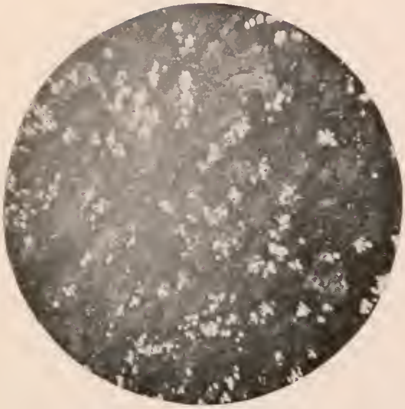

FIG. 26. ALLOY NO. 13. x 40.
7.8% CR, 84.4% CU, 9.3% NI
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

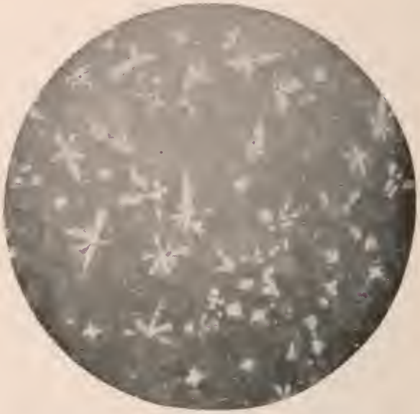

FIG. 27. ALLOY NO. 14. x 40.
8.25% CR, 74.63% CU, 16.05% NI.
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

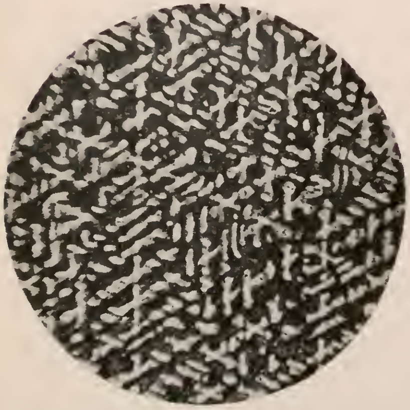

FIG. 28. ALLOY NO. 15. x 40.
10.6% CR, 66.3% CU, 22.9% NI.
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

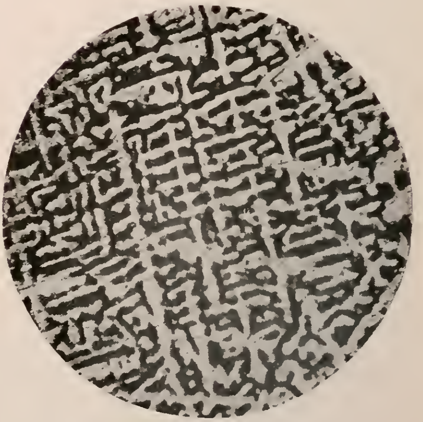

FIG. 29. ALLOY NO. 16. x 40.
15.9% CR, 54.7% CU, 29.4% NI.
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

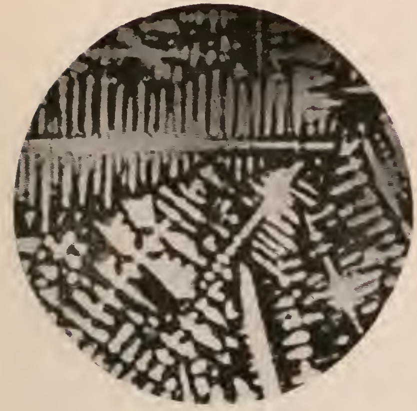

FIG. 30. ALLOY NO. 17. x 40.
10.1% CR, 42.6% CU, 48.0% NI.
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

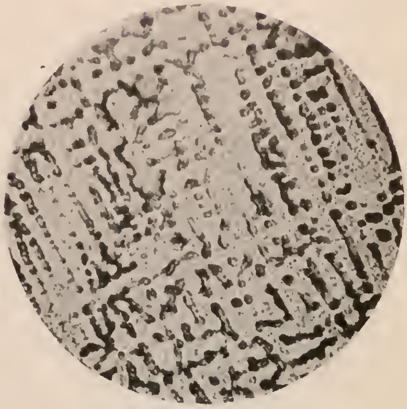

FIG. 31. ALLOY NO. 18. x 40.
13.97% CR, 32.69% CU, 54.16% NI.
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

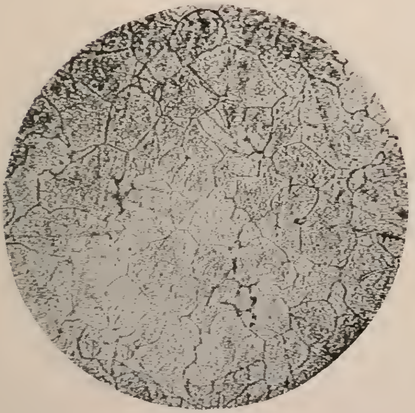

FIG. 32. ALLOY NO. 19. x 40.
11.80% CR, 20.85% CU, 66.25% NI.
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

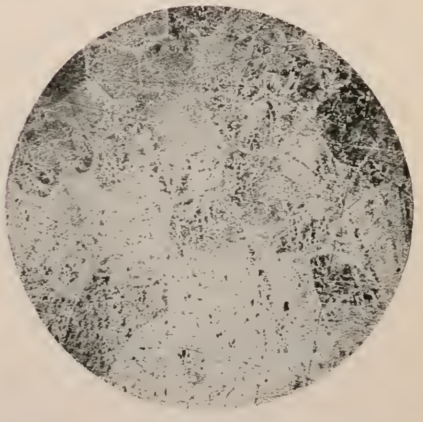

FIG. 33. ALLOY NO. 20. x 40.
11.90% CR, 11.83% CU, 76.27% NI,
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

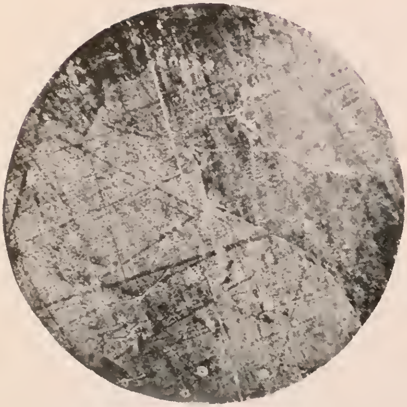

Fig. 34. Alloy No. 21. x 40.
19.37% Cr, 78.99% Ni.
Etched in 1% $FeCl_3$ in 1: 1 HCl.

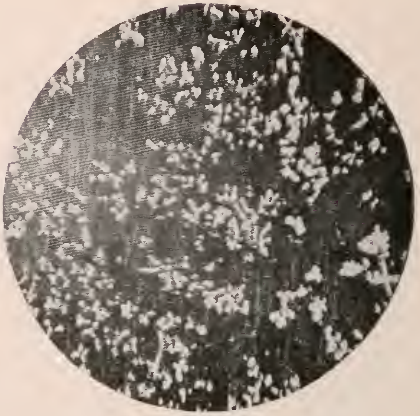

Fig. 35. Alloy No. 22. x 40.
13.15% Cr, 87.93% Cu.
Not Etched. Stained in I in KI.

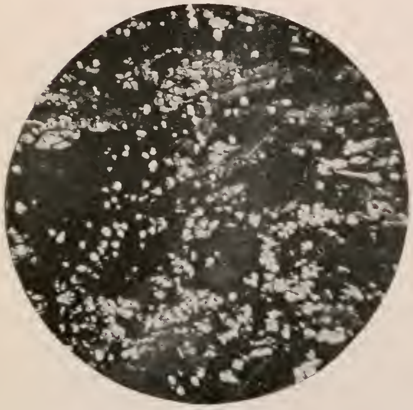

Fig. 36. Alloy No. 23. x 40.
10.56% Cr, 80.58% Cu, 9.38% Ni.
Etched Lightly in 1% $FeCl_3$ in 1: 1 HCl.

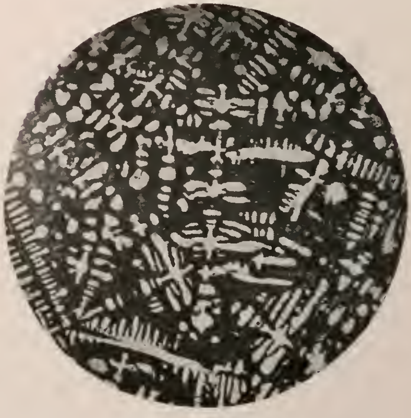

Fig. 37. Alloy No. 24. x 40.
14.56% Cr, 56.30% Cu, 29.24% Ni.
Etched and then Stained in I in KI.

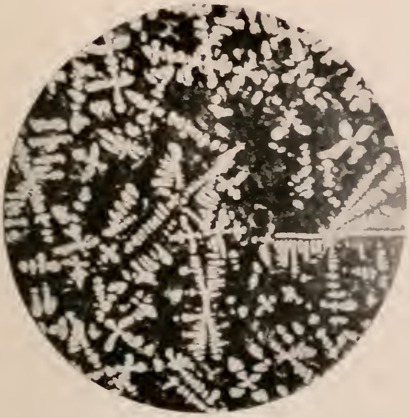

FIG. 38. ALLOY NO. 25. x 40.
13.62% CR, 66.92% CU, 19.20% NI.
ETCHED AND THEN STAINED IN I IN KI.

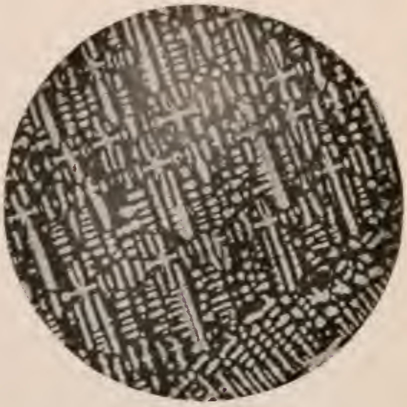

FIG. 39. ALLOYED NO. 26. x 40.
19.30% CR, 44.08% CU, 36.34% NI.
NOT ETCHED, BUT STAINED IN I IN KI.

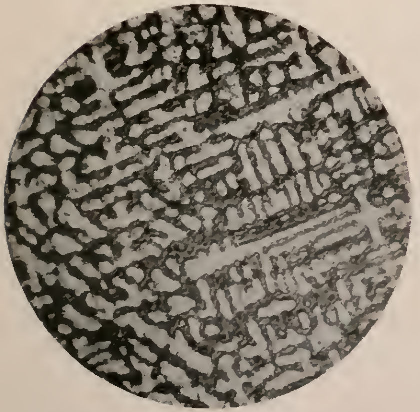

FIG. 40. ALLOY NO. 27. x 40.
15.99% CR, 36.70% CU, 47.31% NI.
STAINED IN I IN ALCOHOL.

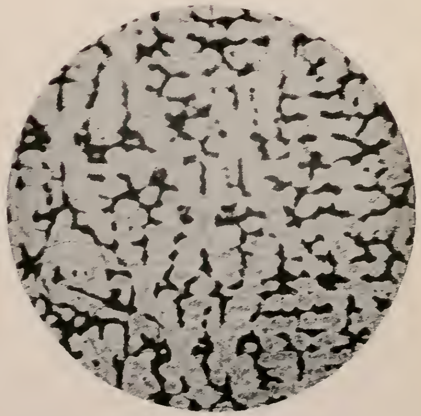

FIG. 41. ALLOY NO. 28. x 40.
19.86% CR, 22.20% CU, 57.36% NI,
STAINED IN I IN ALCOHOL.

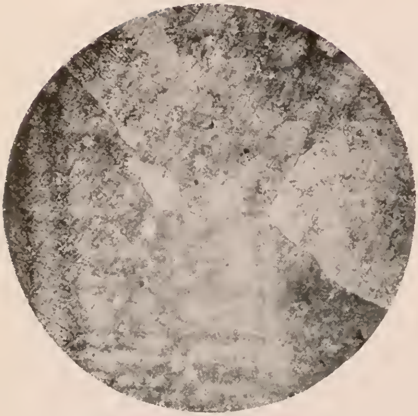

Fig. 42. Alloy No. 29. x 40.
19.64% Cr, 10.88% Cu, 68.62% Ni.
Etched in 1% $FeCl_3$ in 1: 1 HCl.

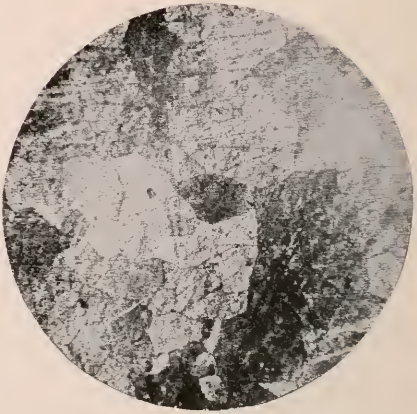

Fig. 43. Alloy No. 30. x 40.
21.52% Cr, 76.95% Ni.
Etched in 1% $FeCl_3$ in 1: 1 HCl.

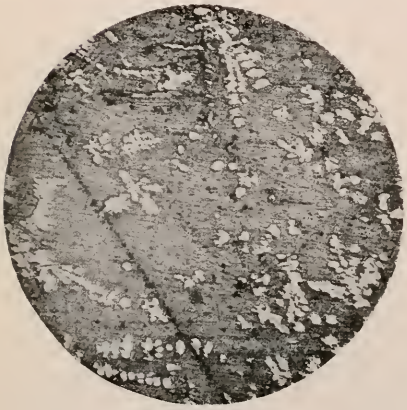

Fig. 44. Alloy No. 31. x 40.
9.89% Cr, 89.82% Cu.
Not Etched. Relief Polishing.

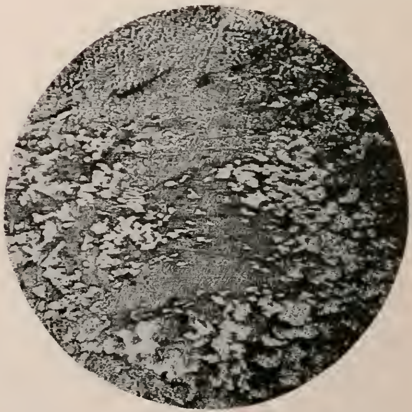

Fig. 45. Alloy No. 32. x 40.
17.66% Cr, 73.63% Cu, 8.55% Ni.
Not Etched. Relief Polishing.

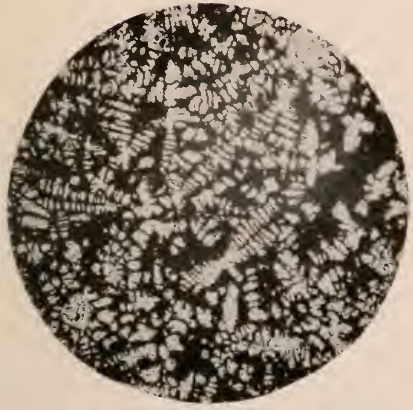

FIG. 46. ALLOY NO. 33. x 40.
22.00% CR, 59.62% CU, 19.48% NI.
ETCHED IN I IN KI.

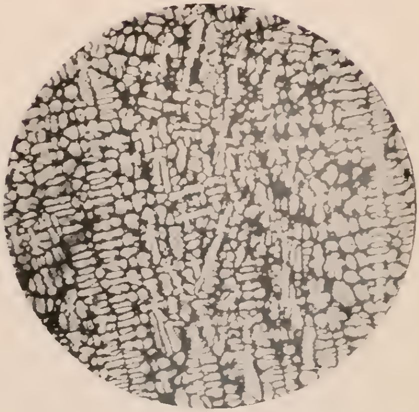

FIG. 47. ALLOY NO. 34. x 40.
25.10% CR, 45.70% CU, 29.52% NI.
ETCHED, THEN STAINED IN I IN KI.

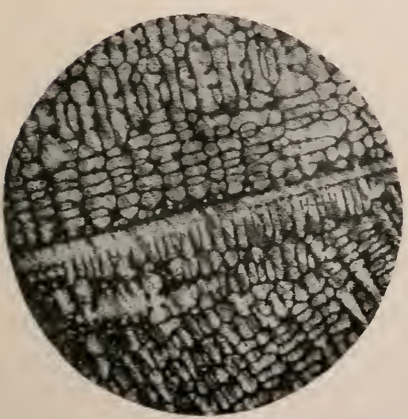

FIG. 48. ALLOY NO. 35. x 40.
29.46% CR, 33.76% CU, 36.78% NI.
ETCHED, THEN STAINED IN I IN KI.

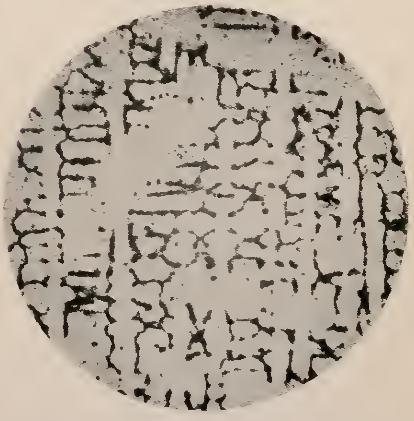

FIG. 49. ALLOY NO. 36. x 40.
28.10% CR, 22.58% CU, 48.42% NI.
ETCHED, THEN STAINED IN I IN KI.

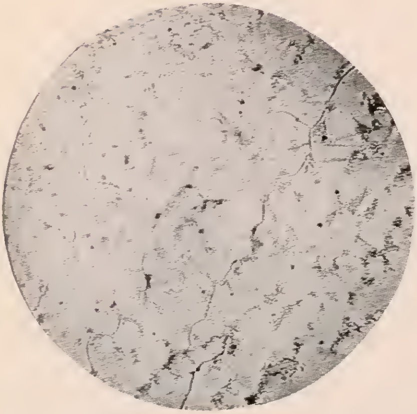

FIG. 50. ALLOY NO. 37. x 40.
29.70% CR, 10.90% CU, 58.12% NI.
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

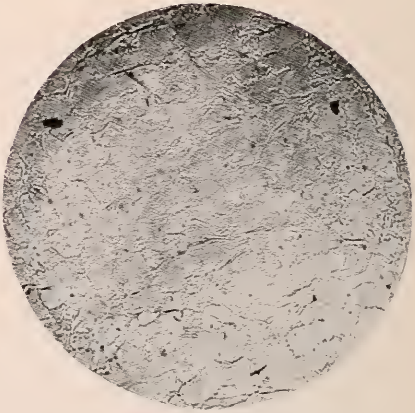

FIG. 51. ALLOY NO. 38. x 40.
28.44% CR, 71.56% NI.
ETCHED IN 1% $FeCl_3$ IN 1: 1 HCl.

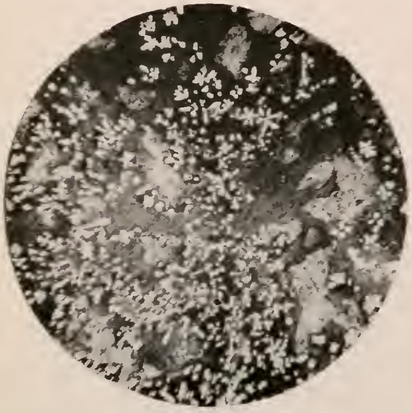

FIG. 52. ALLOY NO. 39. x 40.
APPROX. 35.29% CR, 64.71% CU.

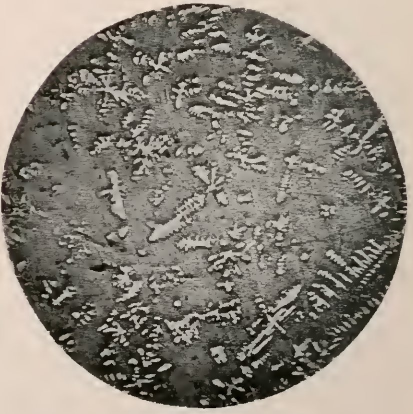

FIG. 53. ALLOY NO. 40. x 40.
19.93%CR, 70.57% CU, 8.99% NI.
NOT ETCHED. RELIEF POLISHING.

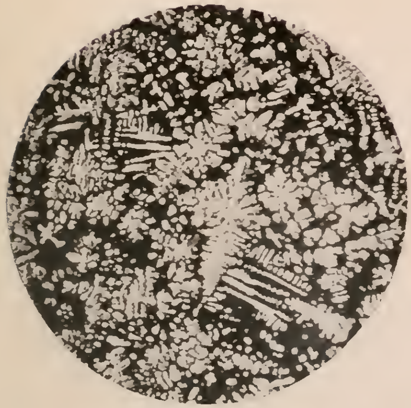

FIG. 54. ALLOY NO. 41. x 40.
31.63% CR, 54.16% CU, 14.21% NI.
STAINED IN I IN ALCOHOL.

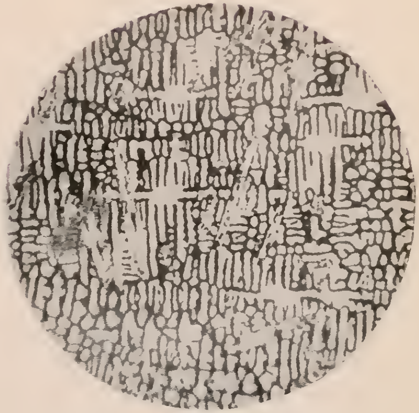

FIG. 55. ALLOY NO. 42. x 40.
38.16% CR, 33.60% CU, 26.78% NI.
ETCHED, THEN STAINED IN I IN ALCOHOL.

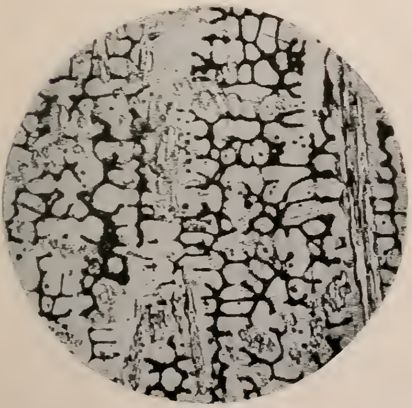

FIG. 56. ALLOY NO. 43. x 40.
41.32% CR, 22.68% CU, 34.60% NI.
ETCHED, THEN STAINED IN I IN ALCOHOL.

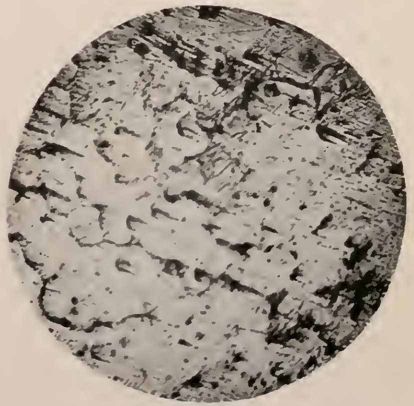

FIG. 57. ALLOY NO. 44. x 40.
43.30% CR, 11.02% CU, 46.46% NI.
ETCHED, THEN STAINED IN I IN ALCOHOL.

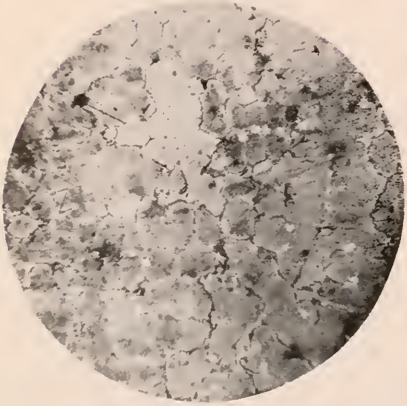

Fig. 58. Alloy No. 45. x 40.
44.93% Cr, 56.55% Ni.
Etched in Aqua Regia.

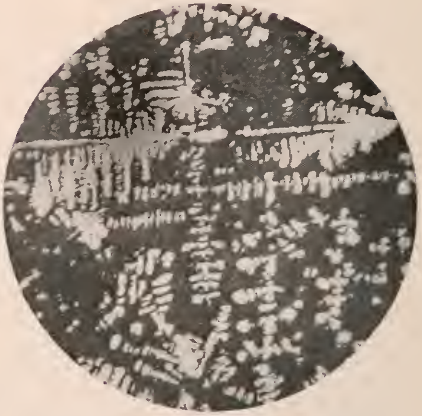

Fig. 59. Alloy No. 48. x 60.
54.92% Cr, 28.42% Cu, 17.12% Ni.
Stained in I in Alcohol.

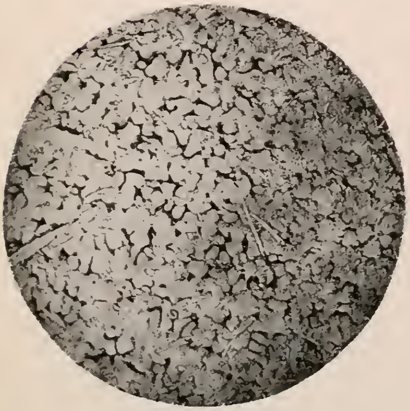

Fig. 60. Alloy No. 49. x 40.
47.54% Cr, 24.12% Cu, 26.28% Ni.
Polished. Not Etched.

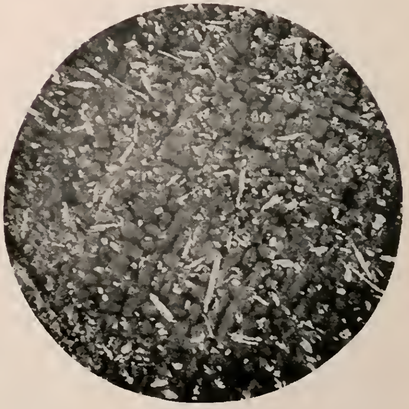

Fig. 61. Alloy No. 49. x 40.
47.54% Cr, 24.12% Cu, 26.28% Ni.
Stained in I in KI.

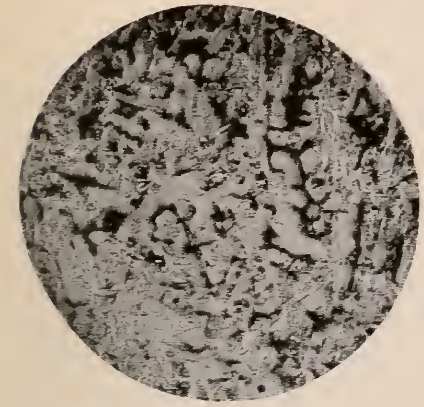

FIG. 62. ALLOY NO. 50. x 40.
APPROX. 46.6% CR, 11.4% CU, 42.0% NI.
STAINED IN I IN ALCOHOL.

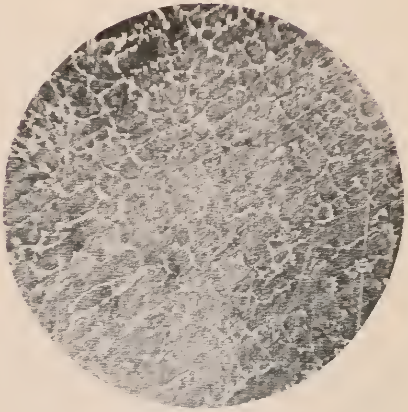

FIG. 63. ALLOY NO. 51. x 40.
57.40% CR, 41.66% NI.
ETCHED IN AQUA REGIA.

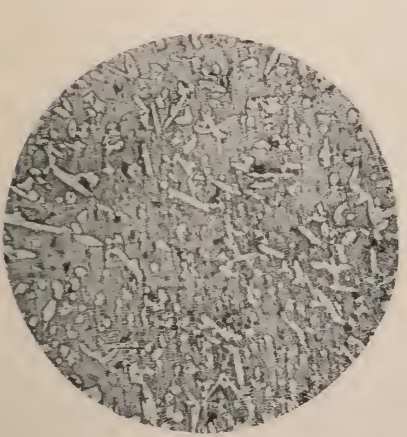

FIG. 64. ALLOY NO. 55. x 40.
APPROX. 56.6% CR, 11.5% CU, 31.9% NI.
ETCHED IN AQUA REGIA.

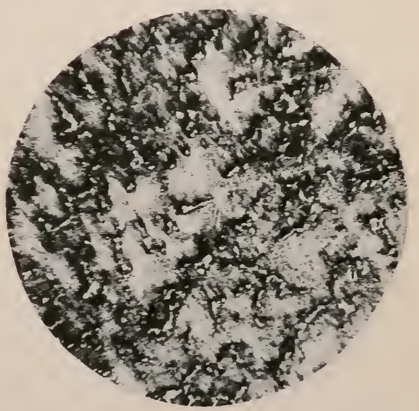

FIG. 65. ALLOY NO. 56. x 40.
APPROX. 57% CR, 43% NI.
ETCHED IN 1% $FeCl_3$ IN 1:1 HCl.
THEN STAINED IN I IN ALCOHOL.

APPENDIX

Historical Review

In this review it is intended to give only the more important results of previous investigations and references to the original publications.

1. *Copper-Nickel Alloys.*—Christofle and Bouilhet* prepared alloys containing 50 per cent of copper and 50 per cent of nickel, and 85 per cent of copper and 15 per cent of nickel, and observed some of their properties.

In 1896 H. Gautier† made a more extensive study of these alloys. He determined their freezing points and concluded that they formed a definite chemical compound having the formula CuNi which melted at 1340 degrees C. The original paper was presented by H. Moissan.

Heycock and Neville‡ in their work on the "Complete Freezing-point Curves of Binary Alloys containing Silver or Copper together with Another Metal" tried the effect of the addition of small amounts of nickel upon the freezing point of copper and found that the freezing point was raised from 1080 to 1110 degrees C. by the addition of 4.5 per cent of nickel.

Kurnakoff and Schemtschny¶ prepared alloys from electrolytic copper and nickel, determined their freezing points, plotted the freezing-point curve, studied the structure of the different specimens, and pointed out certain similarities to the alloys of iron-copper, cobalt-copper, and copper-nickel. They took 1484 degrees C. as the melting point of nickel which is now known to be too high.

Guertler and Tammann§ in their investigation of the alloys of copper and nickel showed that there was no break in either the liquidus or solidus curves. This showed that these alloys do not form a definite chemical compound as had been claimed by H. Gautier. They made also both magnetic and microscopic examinations and showed the effect of heat on the magnetic properties as well as the effect of the rate of cooling on the grain and crystal size.

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*Christofle and Bouilhet, Bul. Soc. Chem., Vol. 26, p. 419, 1876.
Compt. rend., Vol. 83, p. 29, 1876.

†H. Gautier, Compt. rend., Vol. 123, p. 172, 1896.

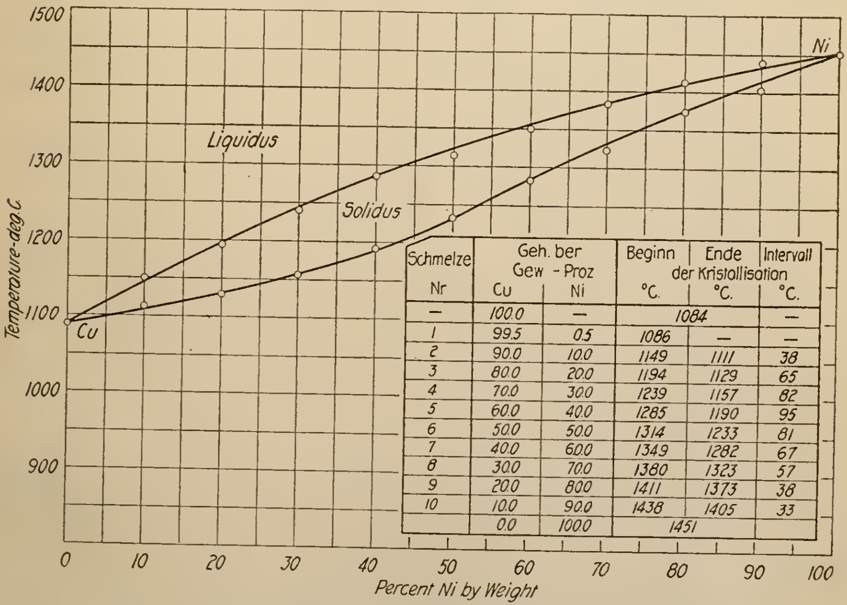
‡Heycock and Neville, Philos. Trans., 189A, p. 25, 1897.

¶Kurnakoff and Schemtschny, Z. anorg. Chem., Vol. 54, p. 149, 1907

§Guertler and Tammann, Z. anorg. Chem., Vol. 52, p. 25, 1907.

The following year (1908) Victor E. Tafel* published the results of his studies of the constitution of the binary system copper-nickel. His work seems to be the best that has been published. For convenience his results are shown diagrammatically in Fig. 66. He obtained higher values for both the liquidus and the solidus curves than did Guertler and Tammann, but that may be attributed to the fact that they used nickel which contained a considerable amount of impurities (0.47 per cent Fe, 1.86 per cent Co.) which would lower the freezing points.

E. Vigouroux†, using pure metals especially free from cobalt, prepared a series of copper-nickel alloys, but he was unable to detect any indication of definite chemical compounds by chemical investigation or by a study of the electromotive forces in the cells $\mathrm{Ni}-\frac{N}{1}$, $\mathrm{NiSO_4}$—CuNi alloy and $\mathrm{Cu}-\frac{N}{1}$, $\mathrm{NiSO_4}$—CuNi alloy. His results by this method agree with those obtained by the cooling-curve method.



Schmelze Nr	Geh. ber Gew - Proz Cu	 Ni	Beginn der Kristallisation °C.	Ende der Kristallisation °C.	Intervall °C.
—	100.0	—	1084		—
1	99.5	0.5	1086	—	—
2	90.0	10.0	1149	1111	38
3	80.0	20.0	1194	1129	65
4	70.0	30.0	1239	1157	82
5	60.0	40.0	1285	1190	95
6	50.0	50.0	1314	1233	81
7	40.0	60.0	1349	1282	67
8	30.0	70.0	1380	1323	57
9	20.0	800	1411	1373	38
10	10.0	90.0	1438	1405	33
	0.0	100.0	1451		

FIG. 66. COPPER-NICKEL EQUILIBRIUM DIAGRAM AFTER TAFEL.

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*Victor E. Tafel, Metallurgie, Vol. 5, p. 348, 1908.

†E. Vigouroux, Compt. rend., Vol. 159, p. 1378, 1909.

David H. Browne* secured U. S. patent 934,278 (Sept. 14, 1910) for the manufacture of nickel and copper-nickel alloys by electrically fusing compounds of the metals as sulfide matte with lime (CaO), forming calcium sulfide (CaS), sulfur dioxide (SO2), and an alloy of the metals.

2. *Chromium-Copper Alloys.*—H. Moissan† prepared an alloy of chromium and copper which contained about 0.5 per cent of chromium. It was more resistant to humid air than was copper and took a beautiful polish.

H. Goldschmidt‡ has described an alloy of chromium and copper containing 10 per cent of chromium and having the color of copper, but being harder. The Goldschmidt Thermit Company¶, 90 West Street, New York, now offers for sale an alloy of chromium-copper containing 10 per cent of chromium. The alloy is made by the aluminothermic method. This alloy is discussed more fully in Chapter II.

Hamilton and Smith§ heated chromium oxide and metallic copper in a carbon crucible and in the presence of carbon, by which process they obtained an alloy of gray-red color and of a hardness which placed it next to the alloys containing tungsten and molybdenum. The alloy gave the analysis: 88.18 per cent Cu, 3.22 per cent Cr, 1.35 per cent Fe, 2.38 per cent C, and 4.13 per cent gangue. The specific gravity was 8.3.

Binet de Jassonnix** stated that chromium dissolved in copper to the extent of about 1.6 per cent, but that on cooling, the chromium separated in a very finely divided condition.

G. Hindrichs†† made a more extensive study of the alloys of chromium and copper. He decided that the freezing point of copper was lowered about eight degrees by the addition of 0.5 per cent of chromium and that the maximum solubility of chromium in copper was 0.5 per cent. He, likewise, found that the freezing point of chromium was lowered from 1550 to about 1470 degrees C. by the addition of 5 per cent of copper and considered that the maximum solubility of copper in chromium was not over 5 per cent. From his researches it seems that there are two eutectic points in the chromium-copper freezing point curve. His chromium-copper diagram has been reproduced in Fig. 67. He pointed out some of the difficulties in working with chromium or chromium-copper alloys. Chromium remains viscous after melting, attacks the crucibles, insulating tubes, etc., and has a strong tendency to oxidize. He was not able to get chromium and copper to separate into two well-defined layers.

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*David H. Browne, C. A., Vol. 4, p. 41, 1910.

†H. Moissan, Compt. rend., Vol. 119, p. 185, 1894.
H. Moissan, Compt. rend., Vol. 122, p. 1302, 1896.

‡H. Goldschmidt, Liebigs Ann., Vol. 301, p. 25, 1898.

¶Thermit Carbon-Free Metals, Pamphlet No. 20, 2nd ed., p. 23.

§Hamilton and Smith, Jour. Am. Chem. Soc., Vol. 23, p. 151, 1901.

**Binet de Jassonix, Compt. rend., Vol. 144, p. 915, 1907.

††G. Hindrichs, Z. anorg. Chem., Vol. 59, p. 414, 1908.

From the foregoing paragraphs it may be seen that the different investigators have placed the solubility of chromium in copper at 0.5, 1.6, 3.22, and 10 per cents.

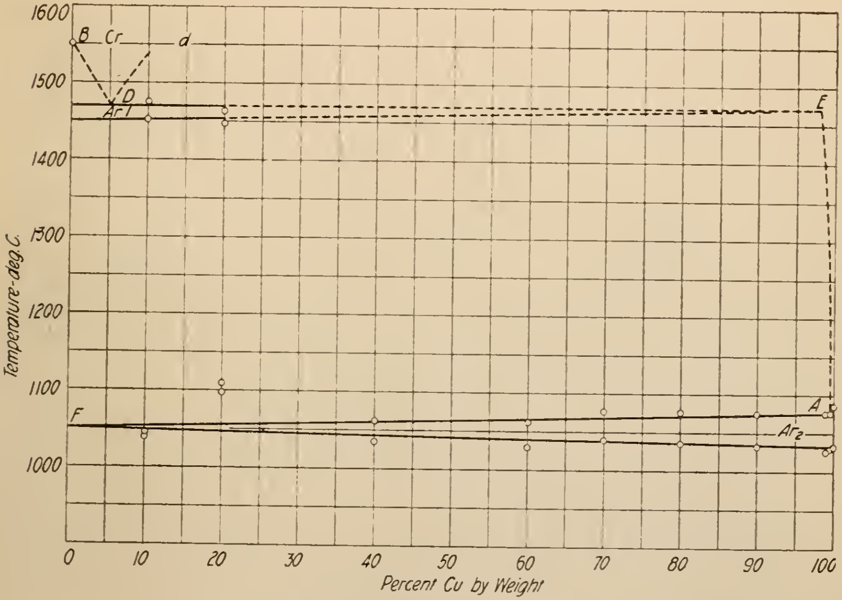

FIG. 67. COPPER-CHROMIUM EQUILIBRIUM DIAGRAM AFTER HINDRICHS.

3. *Chromium-Nickel Alloys.*—G. Voss[*] has investigated the alloys of chromium and nickel. His chromium-nickel diagram has been reproduced in Fig. 68. At the time he published his paper he stated that he was unable to find any published literature on the subject and at the present time he seems to be the only one who has published his researches. However, there have been extensive researches in the development of such alloys as "Nichrome", but these have been conducted in commercial laboratories and the re-

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[*] G. Voss, Z. anorg. Chem., Vol. 57, p. 34, 1908.

sults have not been published. Voss showed that the system consists of two series of solid solutions with a minimum freezing point of about 42 per cent of nickel. From a microscopic examination he concluded that the point represented a true eutectic, although he was not able to demonstrate the presence of a eutectic structure on either side of this point. He assumed the presence of a solution gap (Mischungslücke). On the other hand Guertler* concluded that the structure represented a condition of unstable equilibrium, which he termed a psuedoeutectic produced by the extreme viscosity of the chromium.

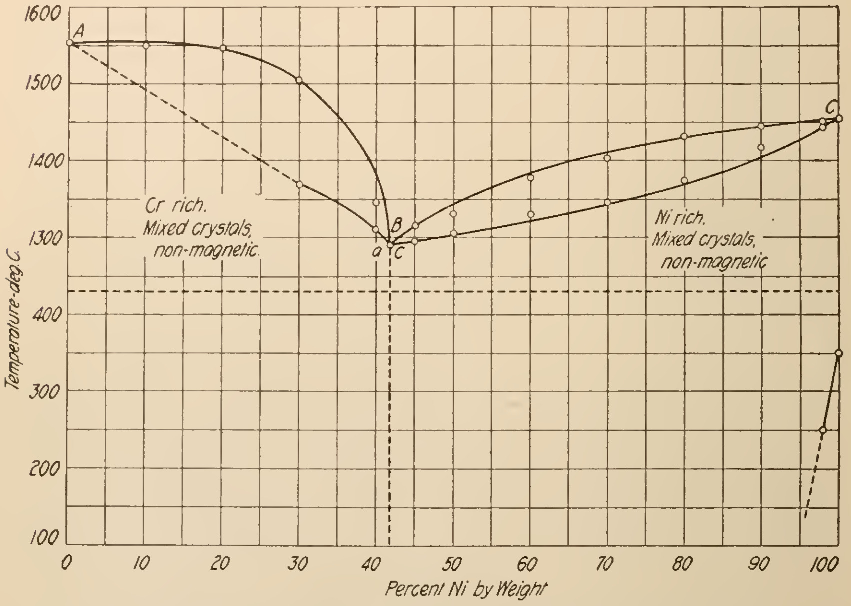

Fig. 68. Nickel-Chromium Equilibrium Diagram After Voss.

To summarize, the binary alloys belong to three different classes: copper-nickel representing those which have continuous freezing-point curves, chromium-nickel representing those which have two series of solid solutions and a minimum freezing point, and chromium-copper those which have two eutectic points. The freezing-point curves for copper-nickel and for chromium-nickel seem to be pretty well established, but the exact location of a large part of the freezing-point curve for chromium-copper is unknown.

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*Guertler, Metallographie, Vol. 1. Part I, p. 209, 1912.

4. *Ternary Alloys.*—The ternary alloys of chromium-copper-nickel have not been described, so far as is known to the writers, but four ternary systems which are somewhat closely related have been reported: copper-nickel-zinc by Victor E. Tafel,[*] copper-iron-nickel by R. Vogel,[†] copper-manganese-nickel by N. Parravano,[‡] and cobalt-copper-nickel by Wahlert.[¶]

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[*] Victor E. Tafel, Metallurgie, Vol. 5, p. 413, 1908.

[†] R. Vogel, Z. anorg. Chem., Vol. 67, p. 1, 1910.

[‡] N. Parravano, Inter. Z. Metallographie, Vol. 4, p. 171, 1913.

[¶] Wahlert. Oester. Z. Berg. Hüttenw., Vol. 62, pp. 341-6, 357-61, 374-8, 392-5, and 406-10, 1914. C.A., Vol. 8, p. 3549, 1914.

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